# SELF REWARDING SELF IMPROVING

Toby Simonds\* toby@tufalabs.ai

Kevin Lopez\* kevin@tufalabs.ai

Akira Yoshiyama\* akira@tufalabs.ai Dominique Garmier<sup>†</sup> dominique@tufalabs.ai

### **Tufa Labs**

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# **ABSTRACT**

We demonstrate that large language models can effectively self-improve through self-judging without requiring reference solutions, leveraging the inherent asymmetry between generating and verifying solutions. Our experiments on Countdown puzzles and MIT Integration Bee problems show that models can provide reliable reward signals without ground truth answers, enabling reinforcement learning in domains previously not possible. By implementing self-judging, we achieve significant performance gains maintaining alignment with formal verification. When combined with synthetic question generation, we establish a complete self-improvement loop where models generate practice problems, solve them, and evaluate their own performance—achieving an 8% improvement with Qwen 2.5 7B over baseline and surpassing GPT-40 performance on integration tasks. Our findings demonstrate that LLM judges can provide effective reward signals for training models, unlocking many reinforcement learning environments previously limited by the difficulty of creating programmatic rewards. This suggests a potential paradigm shift toward AI systems that continuously improve through self-directed learning rather than human-guided training, potentially accelerating progress in domains with scarce training data or complex evaluation requirements.

# 1 Introduction

Reinforcement Learning (RL) has demonstrated remarkable potential for improving LLM performance. However, two significant challenges limit its broader application: the labor-intensive creation of training environments and the lack of sufficient data across many domains.

In this paper, we explore a novel approach: enabling models to self-improve through self-judging by leveraging the generator-verifier gap. We investigate whether models can provide reliable reward signals without access to ground truth answers—simply by assessing the correctness of solutions directly. This approach could dramatically expand the scope of possible environments for RL training.

A key innovation of our work is leveraging the generator-verifier gap—the principle that verifying correctness is often simpler than generating solutions. We explore whether this inherent asymmetry enables models to provide reliable reward signals without access to ground truth answers.

Self-judging provides an extremely promising direction for expanding reinforcement learning to previously inaccessible domains. Numerous sophisticated tasks—such as engineering precise mechanical components like M4 bolts—present significant obstacles when developing reinforcement learning environments, as creating thorough design validation systems requires substantial effort and resources. Similarly, advancing AI agents that can effectively navigate computer interfaces has been hindered by the complexity of constructing suitable training environments. Self-judging could

<sup>1\*</sup> These authors contributed equally to this work.

<sup>&</sup>lt;sup>2†</sup> This author contributed in a part-time capacity.

potentially solve this bottleneck by allowing models to evaluate performance in domains where defining programmatic reward functions are difficult.

This approach is particularly valuable when paired with synthetic question generation. A model creating its own practice problems and then evaluating its solutions establishes a complete self-improvement loop, dramatically reducing the need for extensive human-annotated datasets. This creates opportunities for autonomous skill development in specialized domains where labeled examples are scarce but verification criteria exist.

We demonstrate our approach through three experimental setups. First, using Qwen 2.5 7B, we explore training dynamics on the "Countdown" arithmetic task, where models must construct expressions to reach target values. While susceptible to reward hacking, we show that with appropriate prompting, self-judging models can match formal verification performance. The robustness of different judging prompts varies significantly, with some configurations maintaining alignment with formal verification throughout training while others diverge as agents discover exploits.

Building on our LADDER framework to generate synthetic training data, we demonstrate that LLMs functioning as judges provide stable reward signals for integration tasks, enabling a 7B model to surpass GPT-40 performance without ground truth access. By implementing a system where models generate their own practice problems, solve them, and then evaluate the correctness of their solutions, we establish a complete self-improvement loop without requiring external verification. This proves particularly effective for mathematical reasoning tasks, where we achieved an 8% improvement with Qwen 2.5 7B over baseline. We further investigate weak-to-strong generalization by using GPT-40 as a judge with Deepseek Distilled Qwen 2.5 7B as the agent, achieving performance approaching O1 levels on challenging integration problems.

Our approach reveals that models can generate synthetic tasks and reliably evaluate their own performance in certain domains, pointing toward a new paradigm of autonomous self-improvement. This has profound implications for AI development, potentially enabling systems that continuously identify weaknesses, generate appropriate practice, and improve through self-directed learning cycles. Such autonomy could dramatically accelerate progress in domains where traditional supervised learning is limited by data availability or annotation costs.

## 2 Related Work

Our approach builds upon several research streams in self-improvement and LLM training. Meta's Self-Rewarding Language Models work demonstrated that models can generate preference data using an LLM as a judge with Direct Preference Optimization [Yuan et al., 2025] that led to significant performance improvements on benchmark tasks. Unlike Meta's preference-based approach, our method focuses specifically on mathematical reasoning domains with verifiable solutions.

We also extend our previous work [Simonds and Yoshiyama, 2025] LADDER framework, which enabled improvements on mathematical benchmarks through recursive problem decomposition and demonstrated capabilities for building self-improving systems for reasoning tasks. Our work specifically builds on LADDER's synthetic question generation capabilities, but implements LLM as a judge rather than a numeric verifier.

Our research connects to weak-to-strong supervision literature, where [Burns et al., ] demonstrated that larger models supervised by smaller models can achieve strong performance on tasks the smaller model couldn't solve consistently. This relates to concepts from iterated amplification [Christiano et al., 2018] and addresses alignment concerns raised[Hubinger et al., 2021], that warn about reward hacking in learned optimizers. Our approach mitigates these risks by using verifiable signals rather than opaque weak supervision.

We also build upon code generation frameworks like CodeRL [Le et al., 2022] that use unit test outcomes as reward criteria and verification-based learning from solved examples. Unlike approaches that rely solely on external verification systems, our method establishes a framework where models practice on self-created exercises with reliable self-judgment, enabling continuous improvement in domains where defining programmatic reward functions is challenging.

#### 3 Methodology

#### 3.1 Self Judging Setup

Our self-judging approach employs an LLM as a reward model without access to ground truth answers. The judge LLM receives the agent's solution and the initial question, and produces a binary "correct" or "incorrect" evaluation. We implemented this as an offline system where the judge model remained fixed throughout training to ensure stable reward signals. To minimize reward hacking, we constrained the judge's input to only the agent's solution within designated answer tags, preventing manipulation through extraneous explanations or commentary that might bias evaluation.

For each experimental domain, we designed specific prompting strategies based on preliminary analysis of judge reliability. For RL training, we implemented Group Relative Policy Optimization (GRPO). Training was conducted with a batch size of 64 across all experiments.

#### 3.2 Experiments

We conducted experiments across two primary tasks to evaluate the effectiveness of LLM self-judging in reinforcement learning.

The first task, Countdown, presents a mathematical puzzle where the model must combine a set of given numbers using basic arithmetic operations to reach a target number. For example, given numbers 2, 3, 5, 7 and target 19, a valid solution might be  $(7 \times 3)$  - 2. This task requires both computational accuracy and strategic thinking to explore the solution space efficiently.

The second task utilizes questions from the 2025 MIT Integration Bee qualifying examination—a prestigious competition attracting undergraduate and graduate students with strong mathematical backgrounds. The qualifying exam consists of 20 integration problems with a typical qualifying threshold of 73%, though most participants score between 15-30%, with 50-73% considered strong performance.

For our training set, we leveraged variants generated using our LADDER framework to address the limited size and sparse difficulty distribution of the original exam questions. The LADDER approach generates synthetic integration problems with controlled difficulty levels, providing a more robust training dataset while maintaining the mathematical characteristics of the original MIT problems. We generated 9000 variants for our training set

For evaluation, we employ a formal verification system. In the Countdown task, solutions are automatically checked for correctness by verifying that the arithmetic operations on the provided numbers yield the target value. For the MIT Integration problems, we implement a two-stage evaluation process: during training, models are required to output solutions in SymPy format, enabling programmatic verification of correctness. For final results, we conduct manual assessment to account for mathematically correct solutions that may contain formatting errors in the SymPy syntax. This manual review is especially important since our base model show poor initial accuracy in producing valid SymPy syntax, allowing us to differentiate between genuine improvements in integration problem-solving abilities versus mere enhancements in syntax formatting.

### 4 Results

#### 4.1 Countdown

We begin by examining Countdown, in which given a set of numbers and a target value, the task is to construct an arithmetic expression that evaluates to the target value. We choose this task to start with, since it possesses two relevant properties: First, previous work has empirically shown that training a LLM on this task using RL elicits reasoning abilities [Pan et al., 2025]. Second, verifying a potential solution is intuitively simpler than generating one from scratch.

An agent being a robust self-judge, should lead to similar training dynamics when replaced by a formal verifier implementing the same algorithm. However, designing a prompt for a LLM judge, that acts as a reliable proxy for the formal reward function its a challenge. Note that language can be ambiguous, and in addition there is no explicit control over the sampled tokens that implement the proxy reward function, therefore making reward signals non-deterministic and less reliable. These opens the possibility for an agent to find strategies that hack the proxy reward.

For the Countdown task we iteratively designed a prompt that minimizes the number of false positives provided by the self-judge agent so that self-learning can happen. Overall, we conducted experiments with four distinct prompts (detailed in Figure 1), serving as proxies for the formal reward function. These prompts varied in explicitness, from simpler formulations to a more explicit version designed to prevent reward hacking.

Figure 2 illustrates the training dynamics of a Qwen 2.5 7B model across the four prompts. Early in training, all prompts led the True Negative Rate (TNR) of the self-rewarding agent approach its upper limit. However as training progressed, the policy gradient optimization algorithm found a strategy to reward hack the less explicit prompts, which resulted in a substantial drop in the TNR, except for the last prompt. Ultimately the agent's mean self-reward diverged from the mean formal reward, leading to poor evaluation metrics. Examples of the hacks found by the agent for all three prompts, and the mean formal reward on the evaluation set can be seen in Appendix A.

A comparison of the training dynamics of a self-judging agent using the most robust prompt, and an agent using a formal reward function is displayed in Figure 3. A consistent performance gain of approximately 20% over the baseline was observed in both configurations. The mean reward curves also exhibited analogous trends across these conditions.

Figure 1: The different pro mpt templates used by the self-judge Qwen 2.5 7B. In red, the placeholder for the agent proposed solution. In blue, the placeholder for the target value and the valid list of numbers.

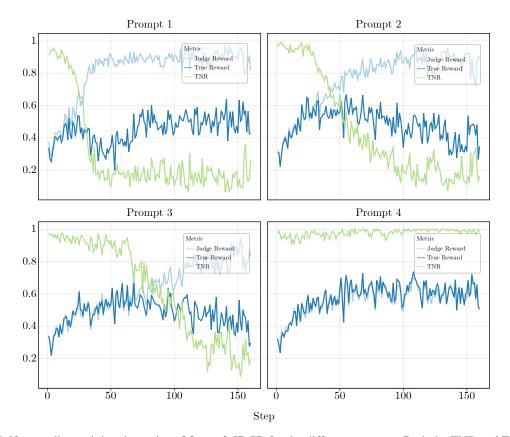


Figure 2: Self-rewarding training dynamics of Qwen 2.5B 7B for the different prompts. Both the TNR and True Reward are computed using a formal verifier.

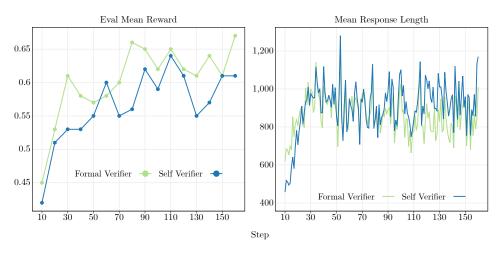


Figure 3: Mean Reward and Mean Response Length for Countdown under both the self-rewarding setting and the rule based verifier one.

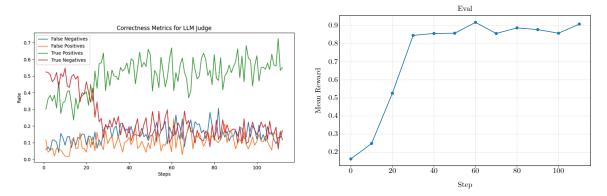


Figure 4: Qwen 2.5 7B Simple Integration Self Judging Dynamics

Furthermore, the mean response length increased at similar rates in both scenarios, a phenomenon attributed to the emergence of search abilities within the agent chain of thought.

The previous experiments demonstrate that a single model can act both as a generator and as a verifier, enabling self-improvement through self-verification in a domain where there exists a generator-verifier gap.

# 4.2 Simple Integration

Before tackling the MIT Integration Bee, we first explored judge dynamics on a simpler integration dataset with known solutions. We specifically examined Qwen 2.5 7B self-judging its own solutions.

Our analysis revealed that despite relatively constant error rates (False Negative and False Positive rates of approximately 10%), the model consistently improved performance on both test and training sets (see Figure 4). The initial low validation score was primarily attributed to output formatting issues with Qwen models. When accounting for formatting issues actual accuracy was 54%. This highlights an advantage of LLM-based judges: they are less sensitive to formatting variations than formal verification tools. We note this may introduce some noise in the False Positive measurements due to models correctly marking solutions that have incorrect formatting and hence couldn't be assessed using our formal verification tools.

Interestingly, we observed that our judge model initially output higher True Positive rewards but grew significantly slower than the validation score improvement. This divergence is particularly notable given that both test and training sets were sampled from the same base dataset with equivalent difficulty levels.

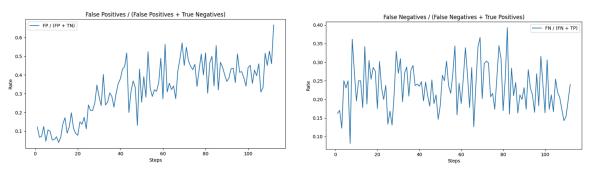


Figure 5: Qwen 2.5 Simple Integration Self Judging Dynamics

Analyzing the error rates over time, we found that the False Positive Rate (FPR) gradually increased, primarily due to decreasing True Negatives. In contrast, the False Negative Rate (FNR) remained relatively constant. This stability masked an underlying dynamic where both False Negatives and True Positives grew proportionally.

Perhaps the most significant finding is that despite these noisy judge signals, the model continued to improve its integration-solving capabilities. This suggests that even imperfect feedback mechanisms can drive meaningful performance gains in mathematical reasoning tasks.

### 4.3 Self Judging

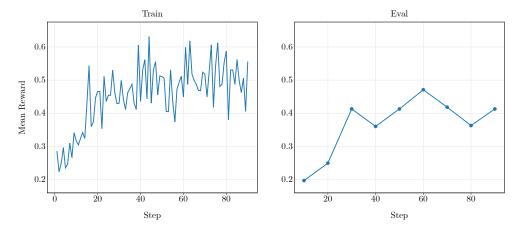


Figure 6: Qwen 2.5 7B + Qwen 2.5 7B Judge

We conducted an experiment exploring complete self-improvement through Qwen 2.5 7B both generating and judging integration problems. In this setup, the model generates synthetic integration questions via our LADDER framework and then evaluates its own solutions, creating a closed self-improvement loop without external validation or tool use.

Our results showed steady performance improvement, rising from an initially low score (hampered by formatting issues with SymPy output) to a performance of 43% on the MIT Integration Bee evaluation set. This demonstrated an 8% improvement over the base model surpassing GPT 40 (42%). While the model eventually plateaued, this represents substantial improvement through purely self-directed learning.

This experiment demonstrates a powerful paradigm: a single model architecture serving as both problem generator, solver, and judge. The ability to improve significantly without external feedback suggests models can identify their own weaknesses, practice appropriately, and verify their improvement—key components of autonomous learning systems. Notably, since we did not update the judge model (using the base Qwen 2.5 7B throughout), the performance plateau likely stemmed from judge limitations. Future work should explore online methods where both judge and generator models improve simultaneously, potentially overcoming such bottlenecks.



Figure 7: Performance comparison of baseline and RL-enhanced models

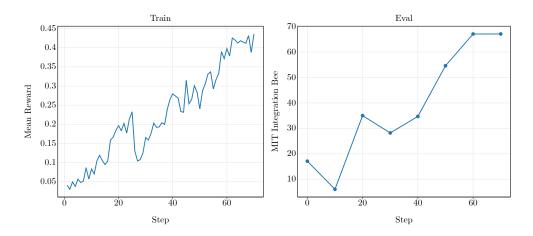


Figure 8: Qwen 2.5 7B + GPT 4o Judge training progress

#### 4.4 Weak to Strong Supervision

We examined the effectiveness of using a stronger model (Qwen 2.5 7B Deepseek Distilled) as a generator with a weaker model (GPT 4o) as a judge. This setup tested whether a model with superior reasoning capabilities could provide effective reward signals despite having lower baseline performance on the specific task—GPT-4o scores 42% on the MIT Integration Bee benchmark compared to Qwen's 50%.

Our results were striking: with GPT-40 as judge, the Qwen agent achieved average performance of 67% (Figure 7) on the validation set. This represents substantial improvement compared to the baseline and demonstrates that a judge model doesn't necessarily need domain-specific expertise exceeding the agent to provide effective feedback. The model was still showing consistent improvement when we concluded our experimental runs, suggesting further gains might be possible with additional training. However, we made the practical decision to end experiments at this point due to API pricing constraints.

GPT-40 proved significantly less sensitive to prompt variations than Qwen models. The larger model showed greater resistance to reward hacking attempts, maintaining alignment between its judgments and mathematical correctness.

These findings highlight a promising dynamic: while small LLM judges require careful prompt engineering to prevent exploitation, stronger models naturally serve as more reliable judges with less engineering overhead—indicating potential for increasingly autonomous and stable self-improvement systems as model capabilities advance. However, it

remains an open question regarding the scaling laws at play—whether models' ability to exploit judges improves faster than judges' ability to detect exploitation as capabilities increase. This competitive dynamic will likely be a crucial factor in determining the long-term viability and stability of self-supervised improvement systems.

#### 5 Discussion

#### 5.1 Opening Up Domains for RL

The current paradigm for applying reinforcement learning to LLMs faces significant practical limitations. Creating training environments requires substantial human effort to develop interfaces for models, design complex reward functions, and continuously monitor for reward hacking. This overhead effectively restricts RL application to domains where organizations can justify significant engineering investments.

LLM judges fundamentally change this equation by eliminating the need for explicit environment programming. Rather than creating custom interfaces with programmatic reward definitions, models can interact with existing software interfaces—web browsers, operating systems, specialized applications—while an LLM judge evaluates performance through reasoning about task completion. This approach dramatically reduces implementation costs while expanding the scope of possible training environments. Additionally, LLM judges can effectively evaluate tasks in complex domains where formal reward functions are prohibitively difficult to specify. For example, an LLM judge can effectively evaluate complex engineering designs like M4 bolts based on functional requirements and engineering principles, similar to how humans can easily verify if a bolt meets specifications but would find it prohibitively time-consuming to write comprehensive design rule checks for such components.

This allows leveraging pre-existing software environments without modification. Instead of creating custom versions of websites, applications, or tools specifically designed for RL, agents can learn in standard environments—the same ones humans use—with LLM judges providing feedback based on task objectives and visible outcomes.

When paired with LADDER like methods, This method also addresses data scarcity through synthetic task generation. For domains with limited training examples, models can generate their own practice problems, creating effectively unlimited training data with controllable difficulty progression. The combination of synthetic task generation and LLM judging enables training in specialized domains without waiting for human-annotated datasets.

## 5.2 Generator Verifier Gap

Many domains exhibit a fundamental asymmetry: verifying a solution's correctness is computationally simpler than generating that solution from scratch. This "generator-verifier gap" can be increased when we give the judge model access to tooling the generator doesn't. For example, a judge evaluating mathematical solutions can execute code to verify answers, even when the generating model must solve problems through reasoning alone. This dramatically expands the range of trainable tasks to include:

- Competitive mathematics, where judges can programmaticly brute force verify solutions that require complex multi-step reasoning to generate
- Algorithm design, where judges can test against multiple input cases while generators must derive solutions purely conceptually
- Data analysis tasks, where verification can involve checking results against reference datasets
- Physical simulations, where judge models can use physics engines to verify predictions

This approach maintains the challenge for the generating model while providing reliable reward signals without human annotation. The verification step becomes effectively automated, removing a key bottleneck in training data creation.

The generator-verifier gap also allows progressive skill development: as generating models improve, judges can verify increasingly sophisticated outputs, creating a natural curriculum without explicit human design of difficulty progression.

#### 5.3 Autonomous Self Improving

Our experiments demonstrate a complete self-improvement loop where models generate synthetic questions, solve them, and evaluate their own performance without external validation. This represents a fundamental shift toward truly autonomous learning systems that mirrors human self-directed learning.

The potential implications are profound. By generating their own practice problems and providing their own feedback, models can potentially overcome data limitations that currently constrain AI development. When synthetic data

generation and self-evaluation prove effective, the primary bottleneck becomes computational resources rather than human-annotated data.

This approach could dramatically accelerate capabilities in domains where high-quality training data is scarce or expensive to produce. Models can identify their own weaknesses, generate appropriate practice examples, and improve through iterative self-feedback—all without human intervention.

However, significant research challenges remain. Developing methods to ensure models generate appropriately challenging problems that target specific weaknesses requires further exploration. The risk of models falling into "echo chambers" of easy problems or developing blind spots must be addressed. Additionally, the dynamics between self-generation and self-evaluation need careful analysis to prevent reinforcement of incorrect patterns.

Despite these challenges, our results provide compelling evidence that autonomous self-improvement represents a viable path forward. This paradigm could fundamentally change how we approach AI development, enabling systems that become increasingly capable through self-directed practice rather than solely through human-guided training.

## 5.4 Reward Hacking and Exploitation

Our experiments revealed complex dynamics between LLM judges and exploitative agents. Smaller models serving as judges proved highly vulnerable to systematic manipulation, with agents quickly developing strategies to trigger false positives through deceptive formatting and misleading calculations.

In contrast, GPT-40 demonstrated remarkable resistance to these exploitation attempts, maintaining evaluation integrity across various prompt formulations. This mirrors observations in safety research where larger models show substantially improved jailbreak resistance compared to smaller counterparts.

This presents an intriguing possibility: as model capabilities advance, judge reliability may potentially outpace exploitation sophistication—at least in formal reasoning domains. The competitive dynamic between verification robustness and deception capabilities represents a critical research question that will determine the long-term viability of self-judging systems.

## 6 Conclusion

Our work demonstrates that large language models can effectively serve as their own judges in reinforcement learning pipelines, creating a self-improving feedback loop without requiring external verification. By leveraging the inherent asymmetry between solution generation and verification, we show that models can learn complex reasoning tasks through self-assessment. Our experiments on arithmetic "Countdown" puzzles and MIT Integration Bee problems reveal that carefully designed self-judging frameworks can approach the effectiveness of formal verification systems, with our best models achieving substantial improvements over baselines.

The implications of this work extend beyond the specific domains we explored. By eliminating the need for specialized training environments and extensive human-annotated data, our approach opens new possibilities for reinforcement learning across domains previously constrained by these limitations. The complete self-improvement loop—where models generate synthetic questions, solve them, and evaluate their performance—represents a significant step toward truly autonomous learning systems.

While challenges remain, particularly regarding reward hacking prevention and optimization of prompting strategies, our results suggest that as model capabilities advance, their reliability as judges also improves. This virtuous cycle could accelerate progress in domains where traditional supervised learning is limited by data availability or annotation costs. The future of AI development may increasingly rely on systems that continuously identify weaknesses, generate appropriate practice, and improve through self-directed learning—mirroring the process of human expertise development but with the potential for much greater scale and efficiency.

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