

The Queen's Gambit: Explaining the Superstar Effect Using Evidence from Chess *

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Abstract. “Superstars” exist in many places – in classrooms, or in workplaces, there is a small number of people who show extraordinary talent and ability. The impact they have on their peers, however, is an ongoing research topic. In competition, they might intimidate others; causing their peers to show less effort. On the other hand, superstars might encourage higher effort, as their existence in a competition encourages them to “step up” their game. In this study, we analyze the impact of a superstar on their peers using evidence from chess. The existence of a contemporary superstar (and the current World Chess Champion) Magnus Carlsen, as well as, past world champions such as Garry Kasparov, Anatoly Karpov, or Bobby Fischer enables us to identify whether the existence of a superstar in a chess tournament has a positive or an adverse effect on other chess players’ performance. We identify errors committed by players using computer evaluation that we run on our sample with 35,000 games and 2.8 million moves with observations from 1962 to 2019, and estimate position complexity via an Artificial Neural Network (ANN) algorithm that learns from an independent sample with 25,000 games and 2 million moves. The results indicate that the effect depends on the intensity of the superstar. If the skill gap between the superstar and the rest of the competitors is large enough, an adverse effect exists. However, when the skill gap is small, there may be slight positive peer effects. In terms of head-to-head competition, the evidence shows that players commit more mistakes against superstars in similarly-complex games. Understanding the effect of superstars on peer performance is crucial for firms and managers considering to introduce a superstar associate for their team.

JEL classification: M52, J3, J44, D3

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1. Introduction

"When you play against Bobby [Fischer], it is not a question of whether you win or lose. It is a question of whether you survive."

–Boris Spassky, World Chess
Champion, 1969 - 1972.

Maximizing their employees' efforts is one of the chief goals of the firm. To this extent, firms typically encourage competition among their employees and allocate bonuses according to their performance and effort. At the same time, firms want to hire the best workers – preferably, the ones who are “superstars” in their fields. For this reason, it is not unusual to see million-dollar hiring contracts among the Forbes top 500 firms.

However, hiring a superstar employee might potentially cause unintentional side effects. [Brown \(2011\)](#) took a creative approach to analyze these potential side effects by considering a famous golf superstar: Tiger Woods. Her goal was to uncover whether Tiger Woods had a positive or negative effect on his competitors' performance. She compared performances in tournaments with and without Tiger Woods and unveiled that there was a sharp decline in performance in tournaments where Tiger Woods competed. This evidence points out that Tiger Woods, as a superstar, creates a psychological pressure on his competitors which has a discouraging effect, causing them to perform worse than their typical performance.

In this paper, we analyze the superstar effect using chess data.¹ Chess provides a clean setting to analyze the superstar effect for the following reasons: First, non-player related factors are minimal to non-existent in chess since every chess board is the same for all players.² Second, the move-level performance indicators can be obtained with the use of computer algorithms that can evaluate the quality of each move and estimate the complexity of each unique position.

¹There is growing literature studying a broad range of questions using data from chess competitions. For example, [Levitt, List and Sadoff \(2011\)](#) test whether chess masters are better at making backward induction decisions. [Gerdes and Gränsmark \(2010\)](#) test for gender differences in risk-taking using evidence from chess games played between male and female players, where they find that women choose more risk-averse strategies playing against men. On the one hand, [Backus et al. \(2016\)](#) and [Smerdon et al. \(2020\)](#) find that female players make more mistakes playing against male opponents with similar strength. On the other hand, [Stafford \(2018\)](#) has an opposite finding that women perform better against men with similar ELO ratings. [Dreber, Gerdes and Gränsmark \(2013\)](#) test the relationship between attractiveness and risk-taking using chess games.

²There is no compelling reason to expect a systematic difference in the environmental factors to directly affect a tournament performance. However, [Künn, Palacios and Pestel \(2019\)](#) and [Klingen and van Ommeren \(2020\)](#) find that indoor air quality has effects on performance and risk-taking behavior of chess players.

Third, multiple chess superstars exist who lived in different time periods and come from different backgrounds, enhancing the external validity of the study.³

To begin with, we present a two-player contest model with a "superstar." Our theory suggests that the skill gap between the superstar and the other player is crucial to determine the superstar effect on the competition. If this gap is small, then the superstar effect is positive: both players exert high effort. However, when the skill gap is large, the superstar effect is negative: both players lower their efforts. Our theory provides explanations for different superstar effects in the literature. The negative superstar effect in golf is found not only in [Brown \(2011\)](#), but also in [Tanaka and Ishino \(2012\)](#)⁴, while the positive superstar effect in track and field events is found in [Hill \(2014\)](#). He compares the performance of athletes in runs where Usain Bolt is competing and where Usain Bolt is not present, finding that athletes perform much better when Usain Bolt is competing. This can be attributed to non-superstar athletes being motivated by having Usain Bolt running just "within their reach", enabling them to push one step further and show extra effort.

Then, we test our theory on six different male and female chess superstars who come from different backgrounds and time periods: Magnus Carlsen, Garry Kasparov, Anatoly Karpov, Bobby Fischer, Hou Yifan, and Igors Rausis.⁵ We are looking for direct (individual competition with a superstar) and indirect (performance in a tournament with a superstar) superstar effects in chess tournaments. To find these effects, we analyze 2.8 million move-level observations from elite chess tournaments that took place between 1962 to 2019 with the world's top chess players. Our main performance indicator is unique to chess: the "Average Centipawn Loss" (ACPL), which shows the amount of error a player commits in a game.⁶ In chess, a player's goal is to find the optimal move(s). Failing to do so would result in mistake(s), which the ACPL metric captures. Having multiple mistakes committed in a game almost certainly means losing at the top level chess tournaments. We then test the following hypotheses:

1. **Direct effect:** Do players commit more mistakes (than they are expected to) playing head-to-head against a superstar?
2. **Indirect effect:** Do players commit more mistakes (than they are expected to) in games played against each other *if* a superstar is present in the tournament as a competitor?

Holding everything else constant, a player should be able to show the same

³In the media, "The Queen's Gambit" gives a realistic portrayal of a chess superstar. The protagonist, Beth Harmon, is a superstar who dominates her peers in tournaments. In this paper, we analyze the real-life chess superstar effect on their peers in actual tournaments.

⁴Their superstar is Masashi Ozaki who competed in the Japan Golf Tour and dominated the tournaments he participated in throughout the 1990s.

⁵We discuss why these players are superstars in Section 3.

⁶The Average Centipawn Loss is also referred to as "mean-error". We provide details on how we use this metric in Section 3.4.

performance in finding the best moves in two "similarly complex" chess positions. The difficulty of finding the optimal moves – assuming players show full effort – is related to two main factors: (1) External factors impacting a player. For instance, being under pressure can lead the player to choke, resulting in more mistakes. (2) The complexity of the position that the player faces. If both players are willing to take risks, they can opt to keep more complex positions on the board, which raises the likelihood that a player commits a mistake. To isolate the "choking effect", we construct a complexity metric using a state-of-the-art Artificial Neural Network (ANN) algorithm that is trained with an independent sample with more than 2 million moves.⁷ By controlling board complexity, we compare games with identical difficulty levels. If a player commits more mistakes against the superstar (or in the presence of a superstar) in similarly complex games, it must be that either (i) the player chokes under pressure (that is, even if the player shows full effort, the mental pressure of competing against a superstar results in under-performance) or (ii) the player gives up and does not show full effort, considering his or her ex-ante low winning chances (this results in lower performance with more mistakes committed), or both (i) and (ii).

We find a strong **direct superstar effect**: in similarly complex games, players commit more mistakes and perform below their expected level when they compete head-to-head against the superstar. This result can be explained by both the choking and the giving up effects. Consequently, players are less likely to win and more likely to lose in these games compared to their games against other opponents.

We show that the **indirect superstar effect** depends on the skill gap between the superstar and the competition. As our theory predicts, we find that if this gap is small, the indirect superstar effect is positive: it seems that the players believe they indeed have a chance to win the tournament and exert higher effort. The data shows that the top 25 percent of the tournament participants improve their performances and commit fewer mistakes. However, if the skill gap is large, then the indirect superstar effect is negative: it seems that players believe that their chances to win the tournament are slim, and/or that competing at the same tournament with a superstar creates psychological pressure. As a result, the top players show an under-performance with more mistakes and more losses. Interestingly, there is a tendency for the top players to play more complex games in tournaments with a superstar. This suggests that the choking effect is more dominant than the giving up effect.

Our results provide clear takeaways for organizations: hiring a superstar can have potential spillover effects, which could be positive for the whole organization if the superstar is slightly better than the rest of the group. However, the organization can experience negative spillover effects if the skill gap between the superstar and the rest of the group is substantial. Thus, managers should compare the marginal benefit of hiring an "extreme" superstar to the potential

⁷The details of the algorithm are provided in Section 3.5.

spillover costs on the whole organization.⁸ Moreover, hiring a marginally-better superstar can act as a performance inducer for the rest of the team.

The superstar literature started from [Rosen \(1981\)](#), who makes the first contribution in the understanding of "superstars" by pointing out how skills in certain markets become excessively valuable. One of the most recent theoretical contributions in the "superstar" literature is [Xiao \(2020\)](#), who demonstrates the possibility of having positive or negative incentive effects when a superstar participates in a tournament. These effects depend on the prize structure and the participants' abilities.

[Lazear and Rosen \(1981\)](#), [Green and Stokey \(1983\)](#), [Nalebuff and Stiglitz \(1983\)](#), and [Moldovanu and Sela \(2001\)](#) describe how to design optimal contracts in rank-order tournaments. [Prendergast \(1999\)](#) provides a review on incentives in workplaces.

The empirical sports superstar literature started from [Brown \(2011\)](#)⁹ and is ranging from professional track and field competitions to swimming. [Yamane and Hayashi \(2015\)](#) compare the performance of swimmers who compete in adjacent lanes and find that the performance of a swimmer is positively affected by the performance of the swimmer in the adjacent lane. In addition, this effect is amplified by the observability of the competitor's performance. Specifically, in backstroke competitions where observability of the adjacent lane is minimal, there appears to be no effect, whereas the effect exists in freestyle competitions with higher observability. [Jane \(2015\)](#) uses swimming competitions data in Taiwan and finds that having faster swimmers in a competition increases the overall performance of all the competitors participating in the competition.

Topcoder and Kaggle are the two largest crowdsourcing platforms where contest organizers can run online contests offering prizes to contestants who score the best in finding a solution to a difficult technical problem stated at the beginning of the contest. [Archak \(2010\)](#) finds that players avoid competing against superstars in Topcoder competitions. Studying the effect of increased competition on responses from the competitors, [Boudreau, Lakhani and Menietti \(2016\)](#) discover that lower-ability competitors respond negatively to competition, while higher-ability players respond positively. [Zhang, Singh and Ghose \(2019\)](#) suggest that there may potentially be future benefits from competitions with superstars: the competitors will learn from the superstar. This finding is similar to the pos-

⁸Mitigating the negative effects by avoiding within-organization pay-for-performance compensation schemes is a possibility. However, it is challenging to eliminate all competition in an organization.

⁹[Connolly and Rendleman \(2014\)](#) and [Babington, Goerg and Kitchens \(2020\)](#) point out that an adverse superstar effect may not be as strong as suggested by [Brown \(2011\)](#). They claim that this result is not robust to alternative specifications and suggest that the effect could work in the opposite direction – that the top competitors can perhaps bring forth the best in other players' performance. In addition, [Babington, Goerg and Kitchens \(2020\)](#) provide further evidence using observations from men's and women's FIS World Cup Alpine Skiing competitions and find little to no peer effects when skiing superstars Hermann Maier and Lindsey Vonn participate in a tournament. Our theory can suggest an explanation for these findings.

itive peer effects in the workplace and in the classroom, see [Mas and Moretti \(2009\)](#), [Duflo, Dupas and Kremer \(2011\)](#), [Cornelissen, Dustmann and Schönberg \(2017\)](#).

The rest of the paper is organized as follows: Section 2 presents a two-player tournament model with a superstar. Section 3 gives background information on chess and describes how chess data is collected and analyzed. Section 4 provides the empirical design. Section 5 presents the results, and section 6 concludes.

2. Theory

In this section, we consider a two-player contest in which player 1 competes against a superstar, player 2.¹⁰ Player 1 maximizes his expected payoff, consisting of expected benefits minus costly effort:¹¹

$$\max_{e_1} \frac{e_1}{(e_1 + \theta e_2)} V_1 - e_1,$$

where e_i is the effort of player $i = 1, 2$, V_1 is a (monetary or rating/ranking) prize which player 1 can win, and θ is the ability of player 2. We normalize the ability of player 1 at one. Player 2, a superstar, has high ability $\theta \geq 1$ and maximizes her expected payoff:

$$\max_{e_2} \frac{\theta e_2}{(e_1 + \theta e_2)} V_2 - e_2,$$

where V_2 is the prize that player 2 can win. Note that θ is not only the ability of player 2, but also the ratio of that player's abilities.

The first order conditions for players 1 and 2 are

$$\frac{\theta e_2}{(e_1 + \theta e_2)^2} V_1 - 1 = 0,$$

and

$$\frac{\theta e_1}{(e_1 + \theta e_2)^2} V_2 - 1 = 0.$$

Therefore, in an equilibrium

$$\frac{e_2}{e_1} = \frac{V_2}{V_1}.$$

We can state our theoretical results now.

Proposition 1 *Suppose that $V_1 > V_2$. Then, there exists a unique equilibrium in the two-player superstar contest model, where player $i = 1, 2$ exerts effort*

$$e_i^* = \frac{\theta V_1 V_2}{(V_1 + \theta V_2)^2} V_i.$$

¹⁰[Tullock \(1980\)](#) discussed a similar model, but did not provide a formal analysis.

¹¹We assume that costs are linear functions.

In the equilibrium, player $i = 1, 2$ wins the contest with the probability p_i^* , where

$$(p_1^*, p_2^*) = \left(\frac{V_1}{V_1 + \theta V_2}, \frac{\theta V_2}{V_1 + \theta V_2} \right).$$

We assume that the prize for the underdog is greater than the prize for the superstar in the two-player contest: everyone expects the superstar to win the competition and her victory is neither surprising, nor too rewarding. However, the underdog's victory makes him special, which is also evident from rating point calculations in chess: a lower rated player gains more rating points if he wins against a higher ranked player.¹² It follows from proposition 1 that the underdog, player 1, always exerts higher effort than the superstar, player 2, in the equilibrium, since $V_1 > V_2$. In addition, underdog's winning chances decrease in the superstar abilities. We have the following comparative statics results.

Proposition 2 *Suppose that $V_1 > V_2$. Then, individual equilibrium efforts increase in the superstar ability if $\theta^* < \frac{V_1}{V_2}$ and decrease if $\theta^* > \frac{V_1}{V_2}$. Individual equilibrium efforts are maximized if the superstar ability is $\theta^* = \frac{V_1}{V_2}$.*

Proposition 2 gives a unique value of the superstar ability which maximizes individual and total equilibrium efforts. This observation suggests the best ability of a superstar for the contest designer.

Figure 1 illustrates this proposition and shows how equilibrium efforts and winning probabilities change for different levels of superstar abilities if $V_1 = 10$ and $V_2 = 4$. When the ability ratio is small, effort levels for both players increase. As the ability ratio increases, both players decrease their efforts. In other words, if the gap between the superstar and the underdog abilities is small, the superstar effect is positive as both players exert higher efforts. However, if the superstar is much better than the underdog, then both players shirk in their efforts and the superstar effect is negative.

¹²The statement of Proposition 1 holds without the assumption about prizes. We will need this assumption for the comparative statics results.

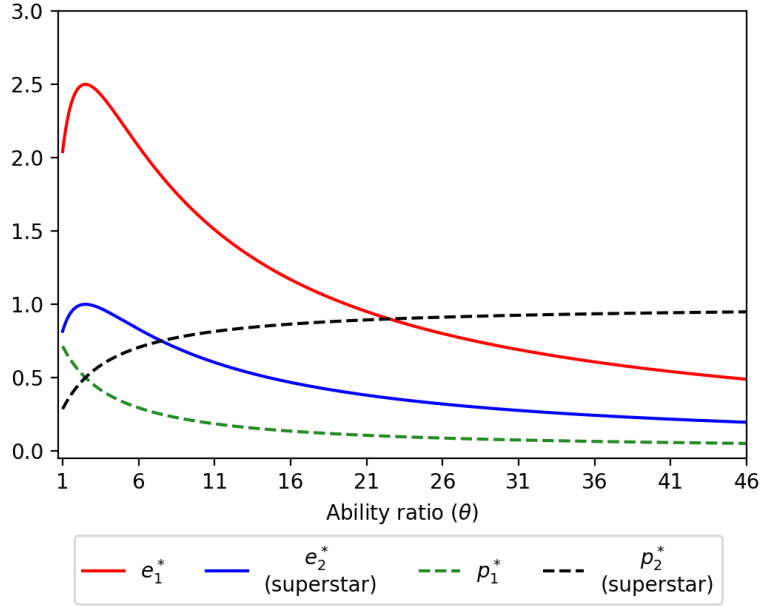


Figure 1: Equilibrium effort and winning probabilities with prizes $V_1 = 10$ and $V_2 = 4$.

3. Data

3.1 Chess: Background

"It is an entire world of just 64 squares."

—Beth Harmon, The Queen's Gambit, Netflix Mini-Series (2020)

Chess is a two-player game with origins dating back to 6th century AD. Chess is played over a 8x8 board with 16 pieces for each side (8 pawns, 2 knights, 2 bishops, 2 rooks, 1 queen, and 1 king). [Figure 2](#) shows a chess board. Players make moves in turns, and the player with the white pieces moves first. The ultimate goal of the game is to capture the enemy king. A player can get close to this goal by threatening the king through a "check": if the king has no escape, the game ends with a "checkmate". A game can end in three ways: white player wins, black player wins, or the game ends in a draw.

The possible combinations of moves in a chess game is estimated to be more than the number of atoms in the universe.¹³ However, some moves are better than others. With years of vigorous training, professional chess players learn how to find the best moves by employing backward-induction and calculating

¹³A lower bound on the number of possible moves is 10^{120} moves, per ? while the number of atoms in the observable universe is estimated to be roughly 10^{80} .

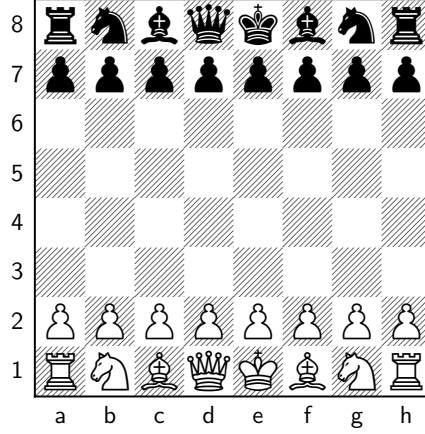


Figure 2: A chess board

consequences of moves to a certain complexity level. Failing to find the best move(s) in a position would result in a "blunder" or a "mistake" which typically leads to the player losing their game at the top level if a player commits multiple blunders or mistakes. The player who performs better overall is the player who manages to find the correct moves more often.

The standard measure of player strength in chess is the ELO rating system first adopted by FIDE in 1970. This system was created by the Hungarian physicist Arpad Elo (Elo 1978). Elo considers the performance of a player in a given game as a random variable normally distributed around her unobservable true ability. Each player gets a starting ELO rating which is updated according to the outcome of each game via

$$ELO_{R,t+1} = ELO_{R,t} + K [S_i - E_t(S_i | R_i, R_j)], \quad (1)$$

where S_i is the outcome of a game such that $S_i = 1$ if player i wins the game, $S_i = 0$ if player i losses the game, and $S_i = 0.5$ if the game ended in a draw. $E_t(S_i | R_i, R_j)$ is the expected probability of player i winning the game given the ELO ratings of the two players R_i and R_j which equals $E(S_R | R_R, R_B) = \Phi\left(\frac{R_R - R_B}{400}\right)$ where $\Phi(\cdot)$ is the c.d.f. of the normal distribution.¹⁴ K is a parameter for rate of adjustment.

This rating system allows comparisons of players' strengths. For instance, every month, FIDE publishes ELO ratings of all chess players. The Top 10 players are considered the most elite players in the world who earn significant amounts of prizes and sponsorships. Moreover, chess titles have specific ELO

¹⁴The probability that player i wins a game against player j is a function of their true abilities $P(p_i > p_j) = P(p_i - p_j > 0) = \int_0^\infty \frac{1}{\sqrt{2\sigma^2}} \Phi\left(\frac{x - [\mu_i - \mu_j]}{\sqrt{2\sigma^2}}\right) = 1 - \Phi\left(\frac{0 - [\mu_i - \mu_j]}{\sqrt{2\sigma^2}}\right) = \Phi\left(\frac{[\mu_R - \mu_B]}{\sqrt{2\sigma^2}}\right)$, where p_i is the probability that player i with true ability μ_i wins the game.

rating requirements. For instance, the highest title in chess, Grandmaster, requires the player to have an ELO rating 2500 or higher.¹⁵

Over the past decades, computer scientists have developed algorithms, or "chess engines" that exploit the game-tree structure of chess. These engines analyze each possible tree branch to come up with the best moves. The early chess engines were inferior to humans. After a few decades, however, one chess engine developed by IBM in the 1990s, Deep Blue, famously defeated the world chess champion at the time, Garry Kasparov, in 1997. This was the first time a world chess champion lost to a chess engine under tournament conditions. Since then, chess engines have passed well beyond the human skills. As of 2021, Stockfish 11 is the strongest chess engine with an ELO rating of 3497.¹⁶ In comparison, the current world chess champion, Magnus Carlsen, has an ELO rating of 2862.¹⁷

In addition to finding the best moves in a given position, a chess engine can be used to analyze the games played between human players.¹⁸ The quality of a move can be measured numerically by evaluating the move chosen by a player and comparing it to the list of moves suggested by the chess engine. If the move played by a player is considered a bad move by the engine, then that move is assigned a negative value with its magnitude depending on the engine's evaluation.¹⁹

3.2 Chess Superstars

The first official world chess champion is Wilhelm Steinitz who won the title in 1886. Since Steinitz, there have been sixteen world chess champions in total. Among these sixteen players, four have shown an extraordinary dominance over their peers: Magnus Carlsen, Garry Kasparov, Anatoly Karpov, and Bobby Fischer.²⁰ We present evidence why these players were so dominant and considered "superstars" in their eras. Specifically, we define a superstar as a player who satisfies the following conditions: (i) be the world chess champion; (ii) win at

¹⁵Our sample consists of the very elite chess players, often called "Super GMs", with ELO ratings higher than 2700 in most cases.

¹⁶Modern chess engines, such as Stockfish, have much higher ELO ratings compared to humans. Most modern computers are strong enough to run Stockfish for analyzing chess positions and finding the best moves, which is the engine we use in our analyses.

¹⁷The highest ELO rating ever achieved by a human was 2882 in May 2014 by Magnus Carlsen.

¹⁸Every chess game played at the top level is recorded, including all the moves played by the players.

¹⁹Engine evaluation scores in chess have no impact on the game outcomes. Engines are used in post-game analysis for learning and research purposes. They are also used during live broadcasting, such that the audience can see which player maintains an advantage. The use of a computer engine by a player during a game is against fair play rules and is equivalent to using Performance Enhancing Drugs (PEDs) for other sports.

²⁰In his classic series, "My Great Predecessors", [Kasparov \(2003\)](#) gives in-depth explanations about his predecessors, outlining qualities of each world champion before him. In this paper, we consider the "greatest of the greatest" world champions as "superstars" in their eras.

least 50% of all tournaments participated in,²¹ (iii) maintain an ELO rating at least 50 points above the average ELO rating of the world's top 10 players (this condition must hold for the post-1970 era when ELO rating was introduced); (iv) have such a high ELO rating that just winning an elite tournament is not sufficient to gain ELO rating points. We define an elite tournament, a tournament which has (1) at least two players from the world's Top 10 and (2) the average ELO rating in the tournament is within 50 points of the average ELO rating in tournaments with a superstar.

Magnus Carlsen is the current world chess champion, who first became champion in 2013 at age 22. He reached the highest ELO rating ever achieved in history. Garry Kasparov was the world champion from 1985-2000 and was the number one ranked chess player for 255 months, setting a record for maintaining the number one position for the longest duration of time. For comparison, Tiger Woods was the number one ranked player in the world for a total of 683 weeks, the longest ever in golf history. Anatoly Karpov was the world champion before Kasparov in the years 1975-1985. He won over 160 tournaments, which is a record for the highest number of tournaments won by a chess player.²² Bobby Fischer was the world champion before Karpov between 1972 - 1975, winning all U.S. championships he played in from 1957 (at age 14) to 1966. Fischer won the 1963 U.S. chess championship with a perfect 11 out of 11 score, a feat no other player has ever achieved.²³

In addition to the four male superstars, we consider a female chess superstar: Hou Yifan, a four time women's world chess champion between the years 2010-2017. She played three women's world chess championship matches in this period and did not lose a single game against her opponents, dominating the tournaments from 2014 until she decided to stop playing in 2017.²⁴

Figures 5–9 show how the four world chess champions: Carlsen, Kasparov,

²¹For comparison, Tiger Woods won 24.2 percent of his PGA Tour events.

²²Kasparov (2003) shares an observation on Karpov's effect on other players during a game in Moscow in 1974: *"Tal, who arrived in the auditorium at this moment, gives an interesting account: 'The first thing that struck me (I had not yet seen the position) was this: with measured steps Karpov was calmly walking from one end of the stage to the other. His opponent was sitting with his head in his hands, and simply physically it was felt that he was in trouble. 'Everything would appear to be clear,' I thought to myself, 'things are difficult for Polugayevsky.' But the demonstration board showed just the opposite! White was a clear exchange to the good – about such positions it is customary to say that the rest is a matter of technique. Who knows, perhaps Karpov's confidence, his habit of retaining composure in the most desperate situations, was transmitted to his opponent and made Polugayevsky excessively nervous."* p. 239 "My Great Predecessors" Vol 5.

²³Kasparov (2003) on Fischer's performance in 1963 U.S. championship: *"Bobby crushed everyone in turn, Reshevsky, Steinmeyer, Addison, Weinstein, Donald Byrne... Could no one really withstand him?! In an interview Evans merely spread his hands: 'Fantastic, unbelievable...' Fischer created around himself such an energy field, such an atmosphere of tension, a colossal psychological intensity, that this affected everyone."* See p. 310 "My Great Predecessors" Vol 4.

²⁴Not losing a single game in world championship matches is a very rare phenomenon since the world champion and the contestant are expected to be at similar levels.

Karpov and Hou Yifan performed compared to their peers across years.²⁵ The ELO rating difference between each superstar and the average of world's top 10 players in each corresponding era is about 100 points. This rating gap is very significant, especially at top-level competitive chess. For instance, the expected win probabilities between two players with a gap of 100 ELO rating points are approximately 64%-36%.

Figures 10–14 show individual tournament performances across years for each superstar with the vertical axis showing whether the superstar gained or lost rating points at the end of a tournament. For instance in 2001, Kasparov played in four tournaments and won all of them. In one of these tournaments, he even lost rating points despite winning. For the world's strongest player, winning a tournament is not sufficient to maintain or gain rating points because he also has to win decisively.

We also consider a chess grandmaster, Igors Rausis, who competed against non-masters in the years between 2012-2019. He was in the top 100 chess players in the world and played against players about 500 ELO rating points below his rating. The ELO rating difference between him and his average opponent in the tournaments is similar to the rating difference between the current world champion Magnus Carlsen, with ELO rating of 2882, and the chess computer program Stockfish 11, with ELO rating 3497. Figure 15 illustrates an example of the ELO rating distribution of Rausis' opponents in a tournament. His participation in such tournaments created a unique setting where a strong chess grandmaster plays against much lower rated non-master opponents. Obviously, Rausis was a superstar in these tournaments.²⁶

Table 1 presents statistics of the superstars' dominance. Panels A-E include the World's Top 10 chess players for the corresponding era and a summary of their tournament performances. For example, Magnus Carlsen participated in 35 tournaments with classical time controls between 2013 and 2019, winning 21 of them. This 60% tournament win rate is two times higher than World's #2 chess player, Fabiano Caruana, who has a tournament win rate of 30%. A more extreme case is Anatoly Karpov, who won 26 out of 32 tournaments, which converts to an 81% tournament win rate while the runner up Jan Timman had a tournament win rate of 22%.²⁷ The final panel, panel F, shows the tournament performances for Rausis and his top-performing opponents. Rausis showed an outstanding performance by winning all eight tournaments in the sample without facing a single loss.

²⁵ ELO rating information is not available for Fischer's era. FIDE adopted the ELO rating system in 1970.

²⁶ Igors Rausis was banned by FIDE, the International Chess Federation, in July 2019 due to cheating by using a chess engine on his phone during tournaments.

²⁷ Restricting the runner-ups' tournament wins to tournaments in which a superstar participated lowers their tournament win rate significantly. Tables are available upon request.

Table 1: World's Top 10 chess players and their tournament performances

years: 2013-2019											
PANEL A											
Name	# of tournament wins	# of tournaments played	% tournament wins	\overline{ELO}	proportion of games won	proportion of draws	proportion of games lost	\overline{ACPL}	complexity	# of moves	# of games
Carlsen, Magnus	21	35	60%	2855	0.352	0.576	0.072	14.413	26.802	16,104	303
Caruana, Fabiano	15	49	30%	2802	0.283	0.592	0.126	16.758	28.280	22,205	447
So, Wesley	6	27	22%	2777	0.226	0.666	0.108	14.928	25.717	11,622	263
Aronian, Levon	5	33	15%	2788	0.196	0.662	0.142	16.059	25.654	13,455	294
Giri, Anish	3	31	9%	2770	0.149	0.719	0.131	14.873	26.202	14,224	304
Karjakin, Sergey	3	28	10%	2768	0.168	0.689	0.143	15.947	26.938	12,764	281
Mamedyarov, Shakhriyar	3	22	13%	2777	0.172	0.674	0.154	15.050	26.339	9,405	216
Nakamura, Hikaru	3	35	8%	2779	0.218	0.622	0.160	15.823	27.398	15,349	327
Vachier Lagrave, Maxime	3	26	11%	2777	0.163	0.703	0.134	14.539	26.842	10,227	232
Grischuk, Alexander	0	15	0%	2777	0.183	0.633	0.184	18.081	27.539	6,852	146
years: 1995-2001											
PANEL B											
Name	# of tournament wins	# of tournaments played	% tournament wins	\overline{ELO}	proportion of games won	proportion of draws	proportion of games lost	\overline{ACPL}	complexity	# of moves	# of games
Kasparov, Garry	17	22	77%	2816	0.439	0.510	0.051	17.595	28.509	18,082	488
Kramnik, Vladimir	12	30	40%	2760	0.322	0.621	0.058	16.743	26.071	22,782	610
Anand, Viswanathan	9	25	36%	2762	0.305	0.595	0.099	19.337	28.328	18,879	517
Topalov, Veselin	6	26	23%	2708	0.279	0.514	0.207	21.193	22,081	28,801	515
Ivanchuk, Vassily	4	17	23%	2727	0.255	0.582	0.164	19.626	27.045	13,673	362
Adams, Michael	3	22	13%	2693	0.255	0.575	0.169	19.096	27.382	17,472	421
Short, Nigel D	3	18	16%	2673	0.272	0.475	0.253	22.717	29.080	13,538	348
Svidler, Peter	3	13	23%	2684	0.234	0.599	0.167	19.340	27.573	9,458	260
Karpov, Anatoly	2	12	16%	2742	0.214	0.679	0.107	18.292	26.457	8,966	211
Shirov, Alexei	2	26	7%	2706	0.288	0.460	0.253	21.865	29.014	22,277	529

Table 1 (cont): World's Top 10 chess players and their tournament performances

PANEL C											
years: 1976-1983											
Name	# of tournament wins	# of tournaments played	% tournament wins	\overline{ELO}	proportion of games won	proportion of draws	proportion of games lost	\overline{ACPL}	complexity	# of moves	# of games
Karpov, Anatoly	26	32	81%	2707	0.432	0.524	0.044	17.429	25.244	14,581	391
Timman, Jan H	7	32	21%	2607	0.333	0.525	0.142	20.777	26.074	17,690	305
Larsen, Bent	5	27	18%	2607	0.383	0.342	0.274	23.726	27.251	17,495	193
Kasparov, Garry	3	5	60%	2652	0.429	0.487	0.084	20.459	25.634	2,207	53
Portisch, Lajos	3	20	15%	2634	0.324	0.516	0.160	20.471	25.729	10,744	204
Tal, Mihail	3	11	27%	2632	0.271	0.652	0.077	19.898	24.915	5,505	131
Petrosian, Tigran V	2	12	16%	2608	0.244	0.652	0.103	20.810	23.089	5,257	100
Spassky, Boris Vasilievich	2	16	12%	2624	0.196	0.697	0.107	19.789	24.771	6,115	157
Belavsky, Alexander G	1	5	20%	2596	0.320	0.457	0.223	23.923	28.210	2,758	58
Kortschnoj, Viktor Lvovich	1	2	50%	2672	0.558	0.292	0.150	22.860	28.550	1,106	23
PANEL D											
years: 1962-1970											
Name	# of tournament wins	# of tournaments played	% tournament wins	\overline{ELO}	proportion of games won	proportion of draws	proportion of games lost	\overline{ACPL}	complexity	# of moves	# of games
Fischer, Robert James	12	16	75%	.	0.641	0.286	0.073	18.405	28.622	10,706	252
Kortschnoj, Viktor Lvovich	7	12	58%	.	0.469	0.459	0.072	19.964	26.378	7,728	197
Keres, Paul	4	8	50%	.	0.420	0.547	0.032	19.080	25.084	4,830	139
Spassky, Boris Vasilievich	4	9	44%	.	0.410	0.570	0.020	18.240	24.341	4,365	138
Botvinnik, Mikhail	3	5	60%	.	0.529	0.414	0.056	18.769	26.065	2,251	63
Geller, Efim P	2	12	16%	.	0.425	0.506	0.069	18.519	25.024	8,432	220
Tal, Mihail	2	8	25%	.	0.466	0.402	0.133	21.894	26.928	4,615	123
Petrosian, Tigran V	1	13	7%	.	0.334	0.621	0.045	20.311	24.675	7,622	227
Reshevsky, Samuel H	1	11	9%	.	0.258	0.505	0.237	24.871	25.505	5,253	140
Bronstein, David Ionovich	0	6	0%	.	0.283	0.628	0.089	21.604	25.199	3,043	94

Table 1 (cont): World's Top 10 chess players and their tournament performances

PANEL E											
years: 2014-2019											
Name	# of tournament wins	# of tournaments played	% tournament wins	\overline{ELO}	proportion of games won	proportion of draws	proportion of games lost	\overline{ACPL}	complexity	# of moves	# of games
Hou, Yifan	4	4	100%	2644	0.614	0.364	0.023	16.222	26.820	4,028	88
Ju, Wenjun	3	6	50%	2563	0.400	0.523	0.077	16.565	26.079	5,826	130
Koneru, Humpy	2	6	33%	2584	0.379	0.424	0.197	20.033	27.516	5,832	132
Dzagnidze, Nana	0	6	0%	2540	0.359	0.347	0.295	24.728	28.335	6,080	128
Goryachkina, Aleksandra	0	1	0%	2564	0.364	0.636	0.000	14.322	27.547	1,116	22
Kosteniuk, Alexandra	0	7	0%	2532	0.297	0.508	0.195	22.841	29.092	6,878	150
Lagno, Kateryna	0	2	0%	2544	0.227	0.682	0.091	16.283	26.304	1,666	44
Muzchuk, Anna	0	5	0%	2554	0.218	0.582	0.200	18.416	26.901	4,814	110
Muzchuk, Mariya	0	3	0%	2544	0.242	0.576	0.182	18.302	27.178	3,056	66
Zhao, Xue	0	6	0%	2519	0.288	0.424	0.288	24.058	27.521	5,990	132
PANEL F*											
years: 2012-2019											
Name	# of tournament wins	# of tournaments played	% tournament wins	\overline{ELO}	proportion of games won	proportion of draws	proportion of games lost	\overline{ACPL}	complexity	# of moves	# of games
Rausis, Igors	8	8	100%	2578	0.783	0.217	0.000	18.319	25.436	3,864	105
Naumkin, Igor	1	4	25%	2444	0.667	0.228	0.106	24.891	26.162	1,746	48
Patuzzo, Fabrizio	1	3	33%	2318	0.600	0.133	0.267	28.208	26.425	1,322	25
Reinhardt, Bernd	1	4	25%	2207	0.423	0.215	0.362	32.870	27.644	1,299	27
Bardone, Lorenzo	0	3	0%	2095	0.433	0.267	0.300	27.199	26.779	890	24
Gascon del Nogal, Jose R	0	2	0%	2469	0.625	0.250	0.125	23.560	28.387	1,162	32
Lubbe, Melanie	0	4	0%	2316	0.401	0.263	0.336	25.243	25.287	1,730	40
Lubbe, Nikolas	0	4	0%	2467	0.527	0.442	0.031	24.480	25.623	1,676	48
Luciani, Valerio	0	4	0%	2158	0.485	0.083	0.432	35.437	27.656	1,352	35
Montilli, Vincenzo	0	3	0%	2117	0.311	0.422	0.267	32.871	26.972	1,278	26

Notes: Panels A-E show the tournament performance for the World's Top 10 chess players for the corresponding time period. ELO rating was first adopted by the International Chess Federation (FIDE) in 1970, hence this information is absent in Panel D for Fischer's sample. Variables and their definitions are presented in table [Table A.1](#).

*: Panel F shows the tournament performance for Rausis and his top performing opponents.

3.3 ChessBase Mega Database

Our data comes from the 2020 ChessBase Mega Database containing over 8 million chess games dating back to the 1400s. Every chess game is contained in a PGN file which includes information about player names, player sides (White or Black), ELO ratings, date and location of the game, tournament name, round, and the moves played. An example of a PGN file and a tournament table is provided in the appendix. See [Table A.2](#) and [Figure A.1](#).

[Table A.1](#) in the appendix provides a summary of variables used and their definitions. [Table 2](#) presents the summary statistics for each era with tournaments grouped according to the superstar presence. In total, our study analyzes over 2 million moves from approximately 35,000 games played in over 300 tournaments between 1962 and 2019.²⁸

3.4 Measuring Performance

3.4.1 Average Centipawn Loss

Our first metric comes from computer evaluations where we identify mistakes committed by each player in a given game.²⁹ A chess game g consists of moves $m \in \{1, \dots, M\}$ where player i makes an individual move m_{ig} . A chess engine can evaluate a given position by calculating layers with depth n at each decision node and make suggestions about the best moves to play. Given a best move is played, the engine provides the relative (dis)advantage in a given position $C_{igm}^{computer}$. This evaluation is then compared to the actual evaluation score C_{igm}^{player} once a player makes his or her move. The difference in scores reached via the engine's top suggested move(s) and the actual move a player makes can be captured by

$$error_{igm} = \left| C_{igm}^{computer} - C_{igm}^{player} \right|. \quad (2)$$

If the player makes a top suggested move, the player has committed zero error, i.e., $C_{igm}^{computer} = C_{igm}^{player}$. We can think of chess as a game of attrition where the player who makes less mistakes eventually wins the game. While staying constant if top moves are played, the evaluation shows an advantage for the opponent if a player commits a mistake by playing a bad move.

We then take the average of all the mistakes committed by player i in game g via

$$\overline{error}_{ig} = \frac{\sum_{m=1}^M \left| C_{igm}^{computer} - C_{igm}^{player} \right|}{M}, \quad (3)$$

which is a widely accepted metric named Average Centipawn Loss (ACPL). ACPL is the average of all the penalties a player is assigned by the chess engine for the

²⁸A list of the tournaments is provided in the appendix.

²⁹[Guid and Bratko \(2011\)](#) and [Regan, Biswas and Zhou \(2014\)](#) are two early examples of implementations of computer evaluations in chess.

mistakes they committed in a game. If the player plays the best moves in a game, his ACPL score will be small where a smaller number implies the player performed better. On the other hand, if the player makes moves that are considered bad by the engine, the player's ACPL score would be higher.

We used Stockfish 11 in our analysis with depth $n = 19$ moves. For each move, the engine was given half a second to analyze the position and assess $|C_{igm}^{computer} - C_{igm}^{player}|$. Figure 16 shows an example of how a game was analyzed. For instance, at move 30, the computer evaluation is +3.2, which means that the white player has the advantage by a score of 3.2: roughly the equivalent of being one piece (knight or bishop) up compared to his opponent. If the white player comes up with the best moves throughout the rest of the game, the evaluation can also stay 3.2 (if the black player also makes perfect moves) or only go up leading to a possible win toward the end of the game. In the actual game, the player with the white pieces lost his advantage by making bad moves and eventually lost the game. The engine analyzes all 162 moves played in the game and evaluates the quality of each move. Dividing the sum of mistakes committed by player i to the total number of moves played by player i gives the player-specific ACPL score.

3.4.2 Board Complexity

Our second measure that reinforces our ACPL metric is "board complexity" which we obtain via an Artificial Neural Network (ANN) approach. The recent developments with AlphaGo and AlphaZero demonstrated the strength of using heuristic-based algorithms that perform at least as good as the traditional approaches, if not better.³⁰ Instead of learning from self-play, our neural-network algorithm "learns" from human players.³¹ To train the network, we use an independent sample published as part of a Kaggle contest consisting of 25,000 games and more than 2 million moves, with Stockfish evaluation included for each move.³² The average player in this sample has an ELO rating of 2280, which corresponds to the "National Master" level according to the United States Chess Federation (USCF).³³

The goal of the network is to predict the probability of a player making a mistake with its magnitude. This task would be trivial to solve for positions that were previously played. However, each chess game reaches a unique position after the opening stage which requires accurate extrapolation of human play in

³⁰<https://en.chessbase.com/post/leela-chess-zero-alphazero-for-the-pc>

³¹Sabatelli et al. (2018) and McIlroy-Young et al. (2020) are two recent implementations of such architecture.

³²<https://www.kaggle.com/c/finding-elo/data>

³³<http://www.uschess.org/index.php/Learn-About-Chess/FAQ-Starting-Out.html>

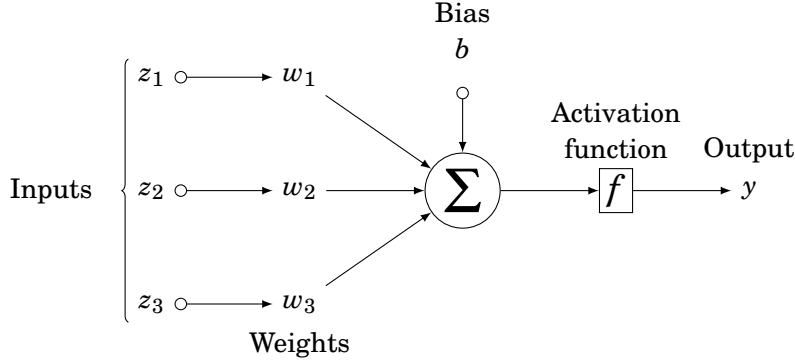


Figure 3: Example of a simple perceptron, with 3 input units (each with its unique weight) and 1 output unit.

order to predict the errors.³⁴ We represent a chess position through the use of its 12 binary features, corresponding to the 12 unique pieces on the board (6 for White, 6 for Black). A chess board has $8 \times 8 = 64$ squares. We split the board into 12 separate 8×8 boards (one for each piece) where a square gets "1" if the piece is present on that particular square and gets "0" otherwise. In total, we represent a given position using $12 \times 8 \times 8 = 768$ inputs. We add one additional feature to represent the players' turn (white to move, or black to move) and thus have $768 + 1 = 769$ inputs in total per position. Figure 4 illustrates.³⁵

The neural network "learns" from 25,000 games by observing each of the approximately two million positions and estimates the optimal weights by minimizing the error rate that results from each possible set of weights with the Gradient Descent algorithm. The set of 1,356,612 optimal weights uniquely characterizes our network. We use two networks to make a prediction on two statistics for a given position: (i) probability that a player commits an error and (ii) the amount of error measured in centipawns. For a full game, the two statistics multiplied (and averaged out across moves) gives us an estimate for the ACPL that each player is expected to get as the result of the complexity of the game

$$E(\overline{error}_{ig}) = \frac{\sum_{m=1}^M P\left(\left|C_{igm}^{computer} - C_{igm}^{network}\right| > 0\right) \left|C_{igm}^{computer} - C_{igm}^{network}\right|}{M}, \quad (4)$$

³⁴This approach is vastly different than traditional analysis with an engine such as Stockfish. Engines are very strong and can find the best moves. However, they cannot give any information about how a human would play in a given situation because they are designed to find the best moves without any human characteristics. Our neural-network algorithm is specifically designed to learn how and when humans make mistakes in given positions from analyzing mistakes committed by humans from a sample of 2 million moves.

³⁵We use a network architecture with three layers. Figure 3 illustrates. The layers have 1048, 500, and 50 neurons, each with its unique weight. In order to prevent overfitting, a 20% dropout regularization on each layer is used. Each hidden layer is connected with the Rectified Linear Unit (ReLU) activation function. The Adam optimizer was used with a learning rate of 0.001.

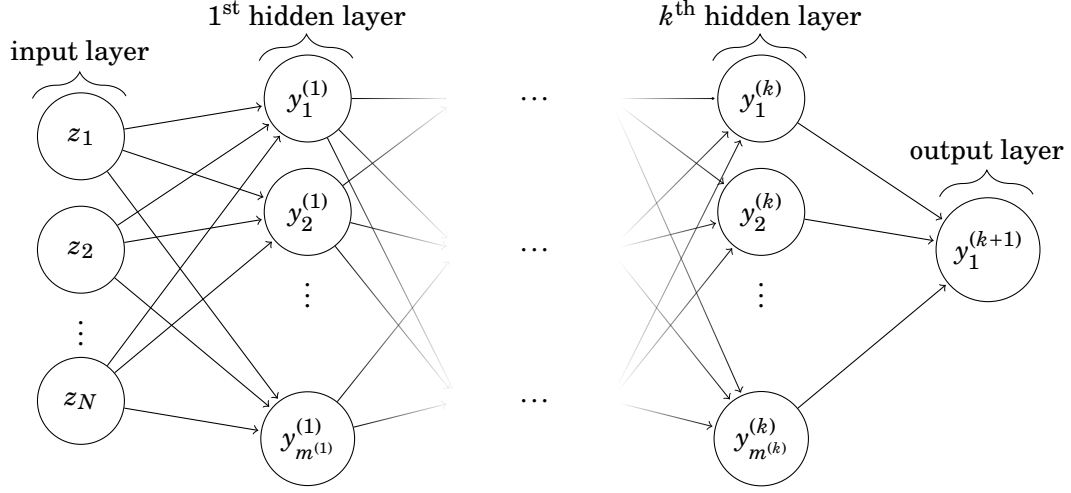


Figure 4: Network graph of a multilayer neural network with $(k + 1)$ layers, N input units, and 1 output unit. Each neuron collects unique weights from each previous unit. The k^{th} hidden layer contains $m^{(k)}$ neurons.

where $|C_{igm}^{computer} - C_{igm}^{network}|$ is the expected centipawn loss in a given position predicted by the neural network. We test our network's performance on our main "superstars" sample.³⁶ The mean ACPL for the whole sample with 35,000 games is 25.87, and our board complexity measure, which is the expected ACPL that we obtained through our network, is 26.56.³⁷ Figure 19 shows a scatterplot of ACPL and the expected ACPL. The slope coefficient is 1.14, which implies that a point increase in our complexity measure results in a 1.14 point increase in the actual ACPL score.³⁸ Figure 20 shows the distributions of ACPL and the expected ACPL.

The board complexity measure addresses the main drawback of using only ACPL scores. The ACPL score of a player is a function of his or her opponent's strength and their strategic choices. For instance, if both players find it optimal to not take any risks, they can have a simple game where players make little to no mistakes, resulting in low ACPL scores. Yet, this would not imply that players showed a great performance compared to their other –potentially more complex– games. Being able to control for complexity of a game enables us to

³⁶Figures 17–18 show our network's prediction on a game played by Magnus Carlsen.

³⁷The reason why our network –which was trained with games played at on average 2280 ELO level– makes a close estimate for the ACPL in the main sample is that the estimates come from not a single player with ELO rating 2280, but rather from a "committee" of players with ELO rating 2280 on average. Hence the network is slightly "stronger" compared to an actual 2280 player.

³⁸The highest ACPL prediction of the network is 50.2 while about 8% of the sample has an actual ACPL > 50.2 . These extreme ACPL cases are under-predicted by the network due to the network's behavior as a "committee" rather than a single player, where the idiosyncratic shocks are averaged out.

compare mistakes committed in similarly-complex games.

3.4.2 Game outcomes

The third measure we use is game-level outcomes. Every chess game ends in a win, a loss, or a draw. The player who wins a tournament is the one who accumulates more wins and fewer losses, as the winner of a game receives a full point toward his or her tournament score.³⁹ In other words, a player who has more wins in a tournament shows a higher performance. In terms of losses, the opposite is true. If a player has many losses in a tournament, their chances to win the tournament are slim. Of course, a draw is considered better than a loss and worse than a win.

4. Empirical Design

Our baseline model compares a player’s performance in a tournament where a superstar is present with their performance in a tournament without a superstar. This can be captured by the following equation:

$$\begin{aligned} Performance_{i,j} = & \beta_0 + \beta_1 Superstar_j \times HighELO_{i,j} + \beta_2 Superstar_j \times MidELO_{i,j} \\ & + \beta_3 Superstar_j \times LowELO_{i,j} + \beta_4 HighELO_{i,j} + \beta_5 MidELO_{i,j} \\ & + \Theta X_{i,j} + \eta_i + \epsilon_{i,j}, \quad (5) \end{aligned}$$

where $Performance_{i,j}$ is the performance of player i in tournament j , measured by methods discussed in section 3.4. $Superstar_j$ is an indicator for a superstar being present in tournament j . $\epsilon_{i,j}$ is an idiosyncratic shock. Having negative signs for coefficients β_1 , β_2 , and β_3 means that the superstar presence creates an adverse effect: players are discouraged to demonstrate their best efforts resulting in worse performance outcomes. $HighELO_{i,j}$ equals one if the player has an ELO rating within the top quartile in the ELO rating distribution of the tournament participants. $MidELO_{i,j}$ captures the second and third quartiles, and $LowELO_{i,j}$ captures the bottom quartile. $\Theta X_{i,j}$ contains the game and tournament level controls. In addition to tournament level specifications, chess allows for a game level analysis which can be specified as the following:

$$Performance_{i,j,k} = \alpha_0 + \alpha_1 AgainstSuperstar_{i,j,k} + \Phi X_{i,j} + \eta_i + \epsilon_{i,j,k}, \quad (6)$$

where $AgainstSuperstar_{i,j,k}$ equals one if player i in tournament j plays against a superstar in round k . In this specification, α_1 captures the effect of head-to-head competition against a superstar.

³⁹A draw brings half a point, while a loss brings no points in a tournament.

Note that chess superstars as a rule play in the strongest tournaments, which guarantees more money and higher prestige. However, it is not possible to play in all top-level tournaments.⁴⁰ Typically, if a player competes in a world championship match in a given year, (s)he tends to play in fewer tournaments in that particular year to be able to prepare for the world championship match.⁴¹ In years without a world championship match, the world champion typically picks a certain number of tournaments to participate in. (S)he may play in fewer tournaments if (s)he believes that the schedule does not allow for adequate preparation for each tournament. We control for the average ELO rating in a tournament to account for any selection issues.⁴²

5. Results

Table 3 shows the performance of non-superstar players playing against a superstar for each sample. There is a distinct pattern that is true for all superstars: playing against them is associated with a higher ACPL score, more blunders, more mistakes, lower chances to win, and higher chances to lose. What is more, games played against superstars are more complex. This higher complexity could be due to the superstar’s willingness to reach more complex positions in order to make the ability-gap more salient. It could also be linked to a non-superstar player taking more risk.⁴³ Taken as a whole, players commit more blunders and mistakes, holding board complexity constant. For instance, a player who plays against Fischer shows an ACPL that is 4.3 points higher compared to his games against other players with a similar complexity level. His likelihood is 10 percentage points less for a win, 18 percentage points less for a draw, and 29 percentage points higher for a loss compared to his typical games. This implies that in terms of direct competition, these superstars have a strong dominance over their peers. Rausis, unsurprisingly, and Hou Yifan, surprisingly, show the strongest domination, with Fischer closely following in third position

⁴⁰In our sample with elite tournaments, a tournament with a superstar has, on average, an average ELO score that is 50 points higher compared to the tournaments without a superstar. This shows that chess superstars indeed play in the strongest tournaments.

⁴¹We indeed document a negative correlation between the number of tournaments a superstar plays and world championship years. These results are available upon request.

⁴²Linnemer and Visser (2016) document self-selection in chess tournaments with stronger players being more likely to play in tournaments with higher prizes. A central difference between their sample and ours is the level of tournaments, with their data coming from the World Open tournament, which is an open tournament with non-master participants with Elo ratings between 1400-2200. Meanwhile, our sample consists of players from a much more restricted sample with only the most elite Grandmasters having Elo ratings often above 2700. Moreover, these high-level tournaments are invitation based; i.e., tournament organizers offer invitations to a select group of strong players, with these restrictions working against any possible selection issues.

⁴³It is not a trivial task to identify which player initiates complexity. Typically, complex games are reached with mutual agreement by players, avoiding exchanges and keeping the tension on the board.

behind these two players. The magnitudes for ACPL, win, and loss probabilities are stronger for these players compared to the rest of the samples.⁴⁴

Tables 4–12 show the effect of a superstar’s presence on the performance of other competitors. Rausis, the most dominant superstar according to Table 3, has a negative effect on the top players’ performances.⁴⁵ However, focusing on the last three rounds – the rounds where the winner is determined by the Swiss pairing format – a large adverse effect exists for the top players. These players have an ACPL score that is 2.4 points higher, about 65% higher chances of committing a blunder, and 23% less chances of winning.⁴⁶ A similar adverse effect on the top players is true for the second most dominant superstar, Hou Yifan. Her presence is associated with an ACPL score that is 4.5 points higher, 11% less chances of winning, and 17% higher chances of losing for the top players in a tournament. For Fischer, the coefficients for ACPL, blunders, and mistakes are positive but imprecise. Fischer’s opponents indeed had more draws and less complex-games, which agrees with the findings in Moul and Nye (2009) on Soviet collusion.

Another situation with intense competition is when two superstars, Kasparov and Karpov, both participate in a tournament. This means that for a given player, he or she will have to face both Kasparov and Karpov and perform better than both of them in order to win the tournament. This tough competition appears to lead to more decisive games and less draws, with players committing fewer blunders and mistakes. The top quartile of players, who usually try to compete with Kasparov and Karpov, experience a lot of pressure and as a result, increased losses. However, these players also win more games.⁴⁷

Players perform better if they face only Kasparov or Karpov in the tournament compared to facing both superstars in the tournament. With one superstar, either Kasparov or Karpov, in the tournament, players have higher chances of winning the tournament and as a result, play more accurately and manage to get more wins, with substantial gains in the ACPL score and less mistakes committed. This improvement is the strongest for the top quartile of players.

Surprisingly, Carlsen is the least dominant superstar in our sample because his presence appears to create a slight positive effect on performance. Players play more accurately and make fewer mistakes under higher challenges with more complex positions. The positive effect appears to apply to all the tourna-

⁴⁴Taken as a whole, these findings verify that the superstars considered in our study indeed show greater performance compared to their peers.

⁴⁵Unsurprisingly, Rausis’ effect on *all* participants is minimal. The average number of participants in an open Swiss-format tournament is typically more than fifty with very heterogeneous participants.

⁴⁶The top players perform better at the beginning of a tournament with Rausis, and their motivation appears to vanish toward the end of the tournament when critical games for the tournament victory are decided. We focus on the latter results due to higher stakes in the final rounds.

⁴⁷In fact, the bottom quartile of players shows better performance with Kasparov and Karpov’s presence. These players have more wins and fewer losses as a result of a worse performance by the upper quartile.

ment participants.⁴⁸

Table 11 shows the impact of superstar presence for all samples aggregated. Table 12 and Figure 21 show the aggregate superstar effect broken down to each sub-sample for the top quartile players. Moving from Carlsen to Rausis, we observe increases in the committed mistakes, in the overall complexity, in blunders and mistakes, and in the loss rates. At the same time, the win and draw rates decrease, confirming our theory that an increase in the intensity of a superstar is associated with stronger adverse responses from the top players. When the gap between the superstar and the rest of the group is not too wide, all players perform better, but as the gap widens, performance drops are anticipated.

6. Conclusion

The empirical superstar literature finds both positive and negative superstar effect evidence. For example, in golf, the effort level decreases with the superstar presence. However, in the 100-meter running or swimming contests, the effort level appears to increase. In this paper, we show theoretically and empirically that the superstar effect depends on the intensity of the superstar: if the skill gap between the superstar and the rest of the field is small, then the superstar effect is positive; otherwise, if the skill gap is large, the superstar effect is negative.

Using the chess data, we empirically show that the direct superstar (head-to-head) effect is always negative and the indirect superstar effect depends on the skill gap. We find a strong choking effect in head-to-head games with a superstar. Players commit more mistakes than they are expected to when they face a superstar in a head-to-head game.

The takeaway for firms seeking to hire a superstar employee is that hiring a superstar employee may create a positive or an adverse effect on the cohort's performance depending on the skill level gap. If the gap is too large, there may be negative side effects of hiring a superstar employee. In these settings, a highly skilled team member would destroy competition and create an adverse effect on the rest of the team members.

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⁴⁸Draw rates are higher in tournaments with Carlsen. Many chess fans criticize modern chess for a lot of draws, which we document in our analysis.

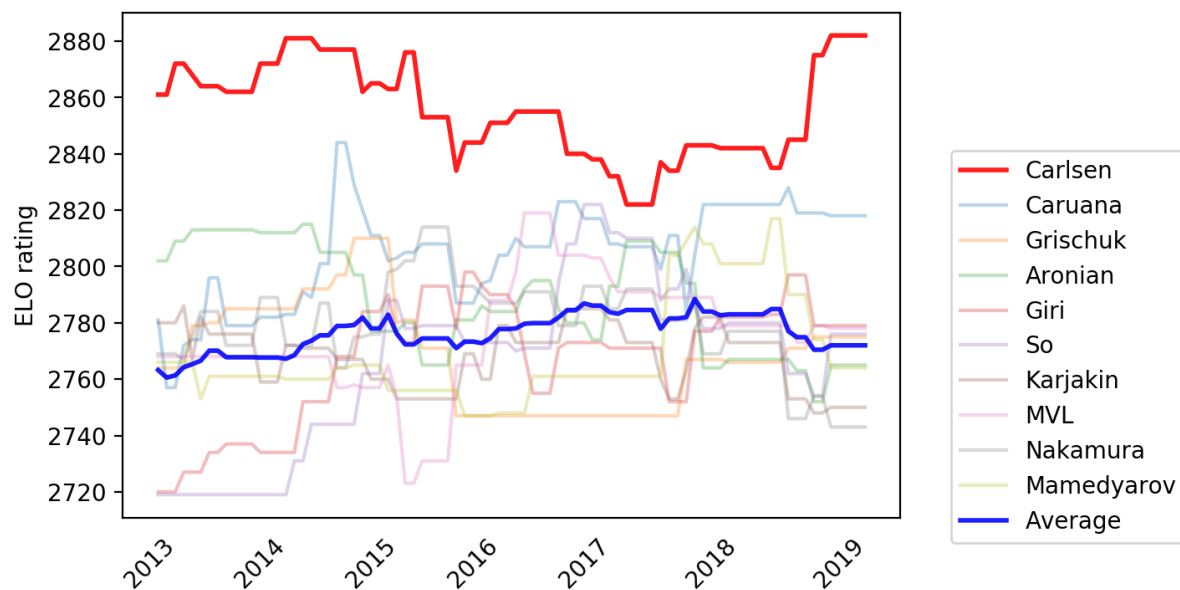
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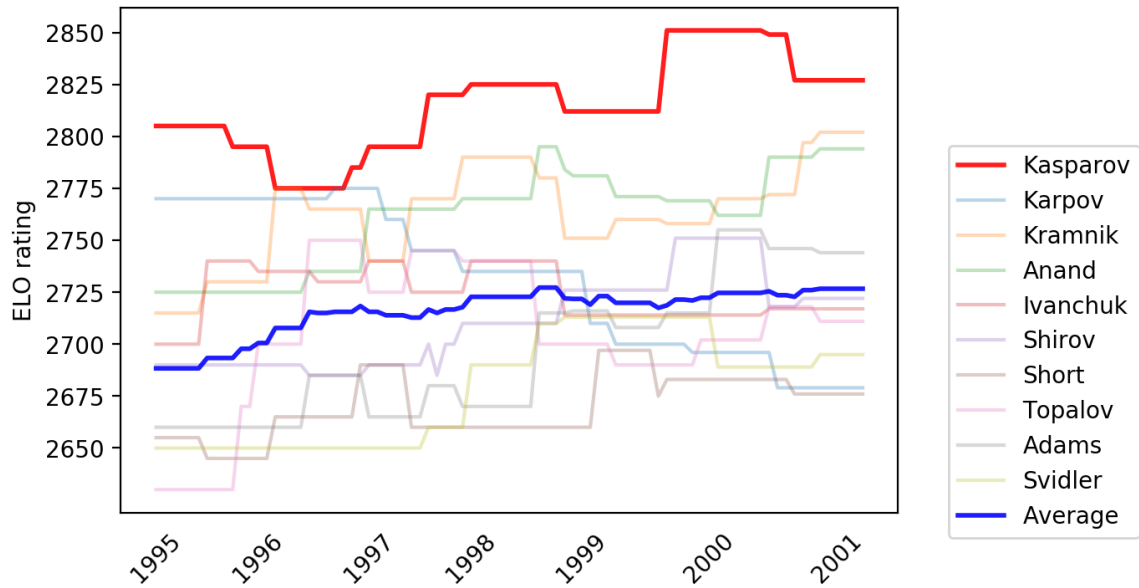
Tables & Figures

Figure 5: ELO ratings of top chess players between 2013-2019.



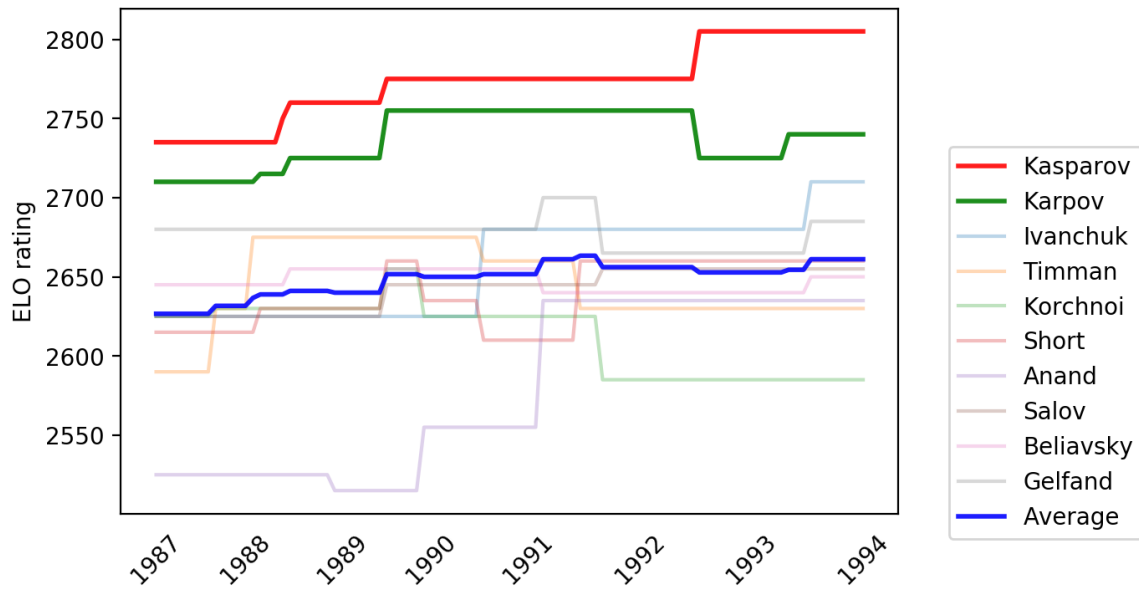
Note: The blue line shows the average ELO rating of top chess players other than Carlsen (World ranking 2–10). ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 6: ELO ratings of top chess players between 1995-2001.



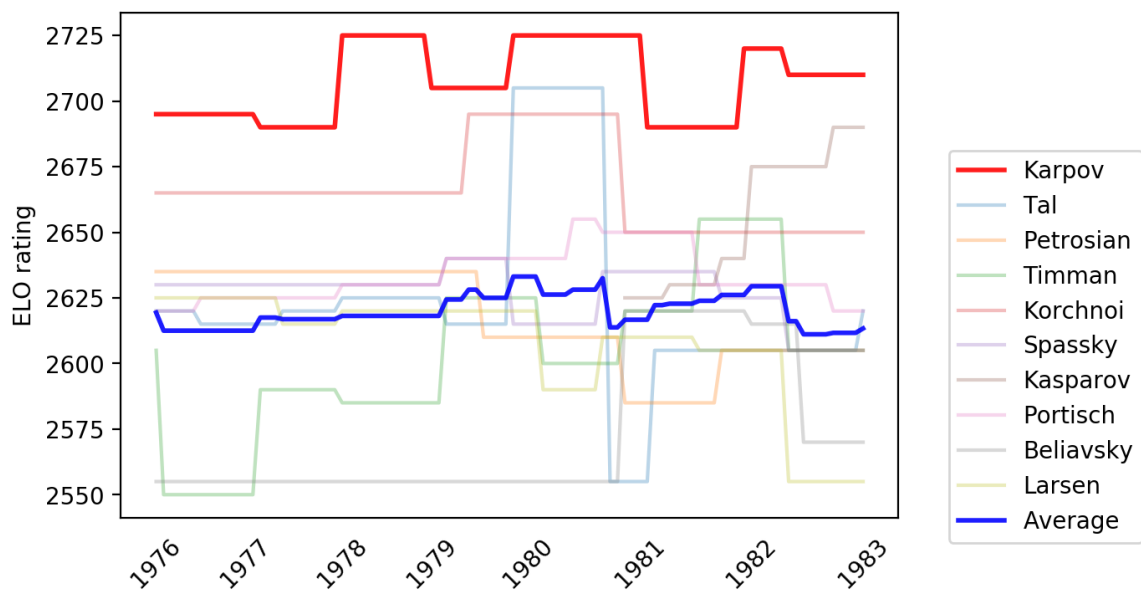
Note: ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 7: ELO ratings of top chess players between 1987-1994.



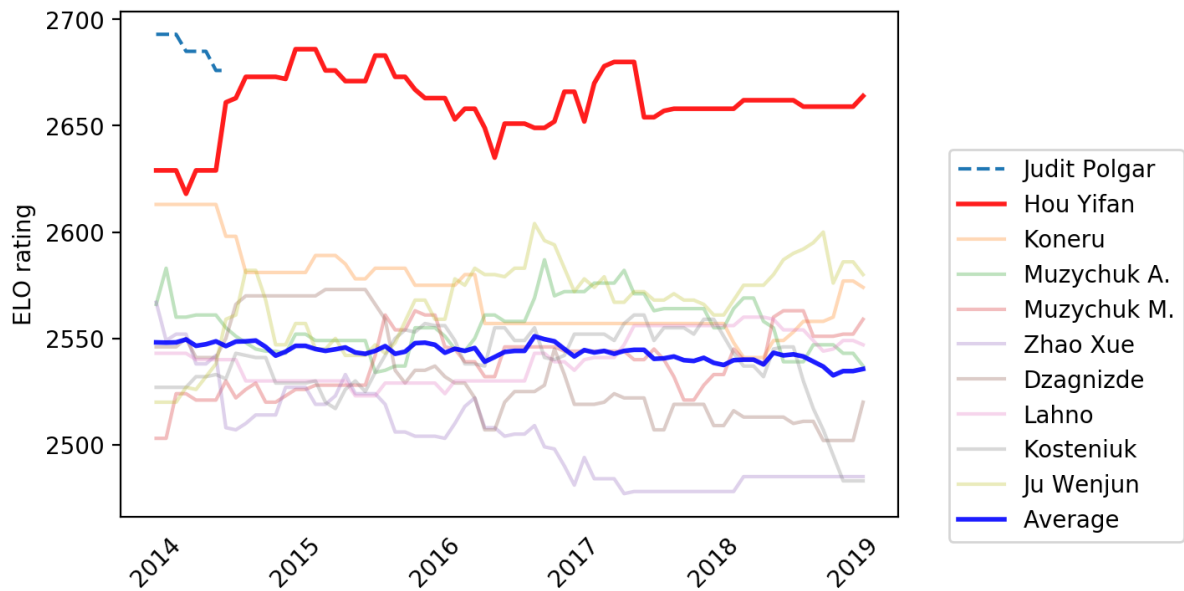
Note: ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 8: ELO ratings of top chess players between 1976-1983.



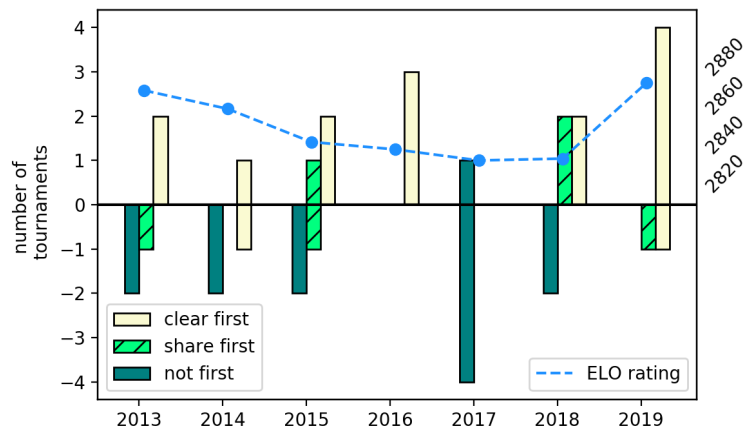
Note: ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 9: ELO ratings of top female chess players between 2010-2015.



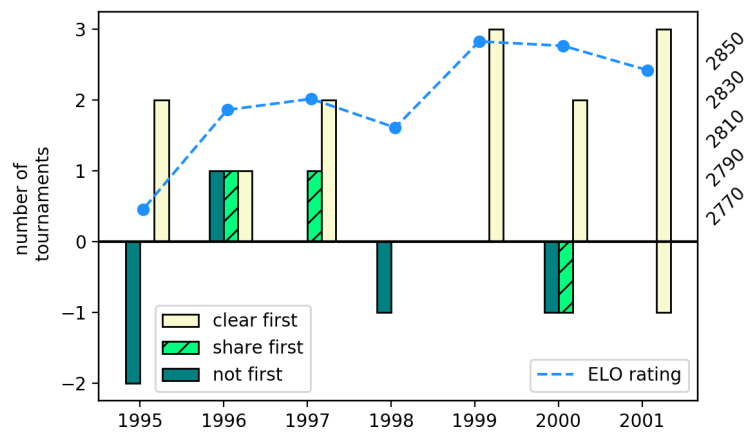
Note: Judit Polgar is considered the strongest female chess player of all time, however she stopped competing in female tournaments in 1990 when she was 14 years old. Hou Yifan stopped competing in female tournaments after 2015. ELO rating data is obtained from FIDE available online at <https://ratings.fide.com>

Figure 10: Carlsen's tournament performance (classical)



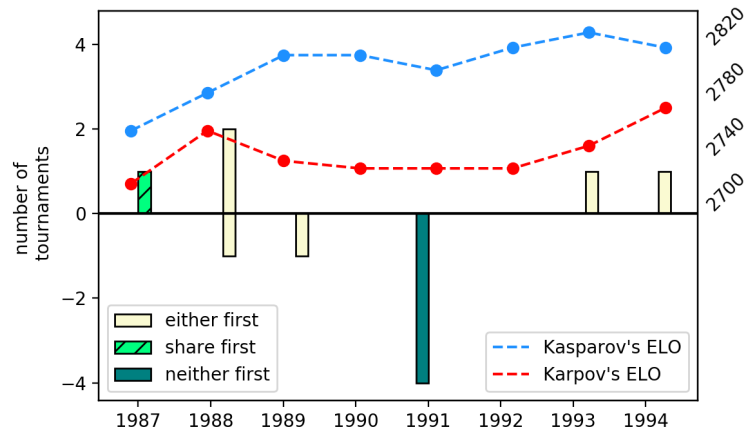
Note: Carlsen's ELO rating data is obtained from FIDE.

Figure 11: Kasparov's tournament performance (classical)



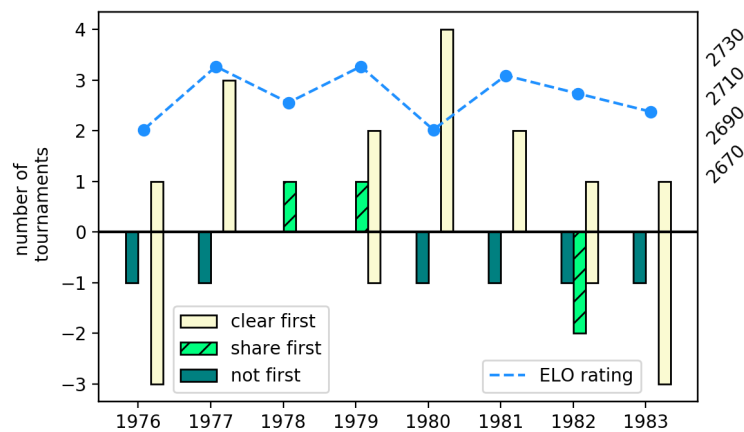
Note: Kasparov's ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 12: Kasparov and Karpov's tournament performance (classical)



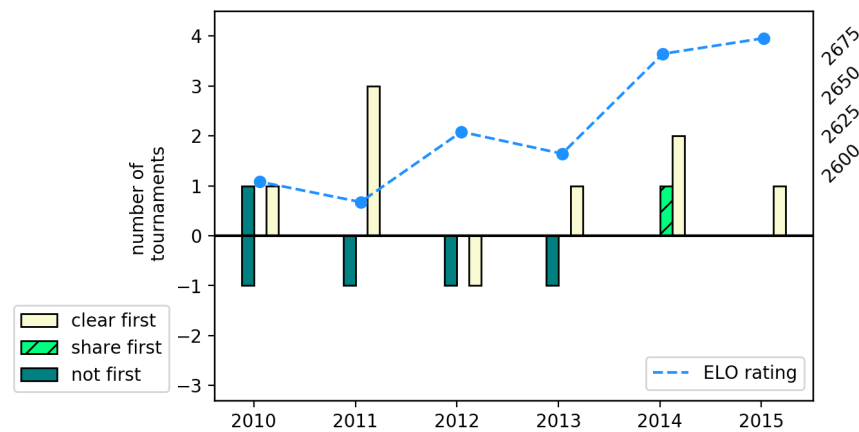
Note: Kasparov's and Karpov's ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 13: Karpov’s tournament performance (classical)



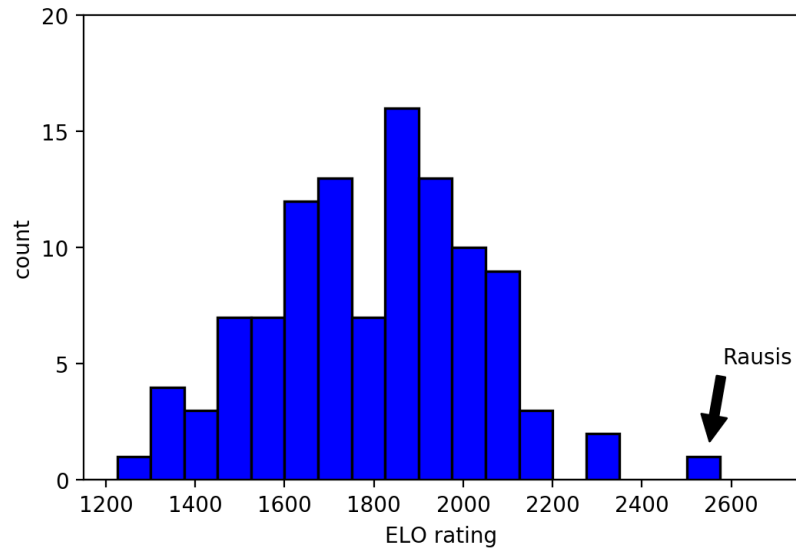
Note: Karpov’s ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 14: Hou Yifan’s tournament performance (classical)



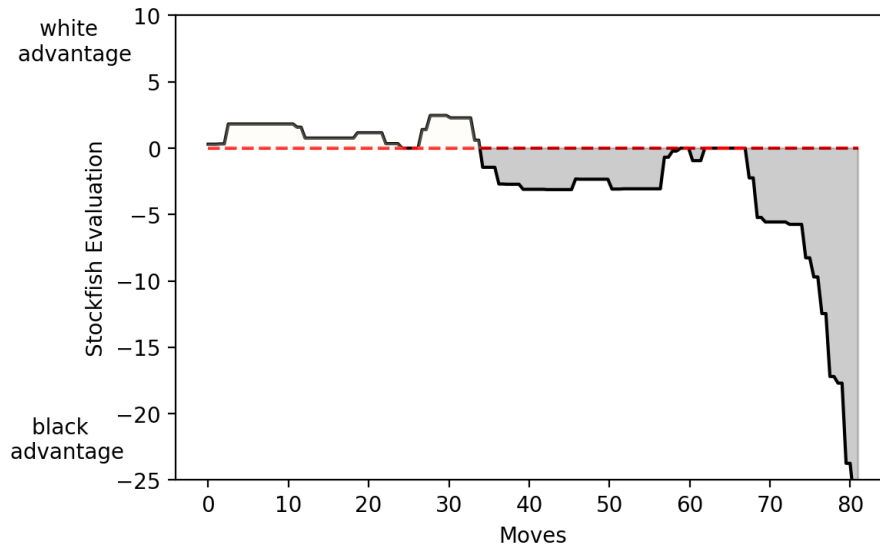
Note: Hou Yifan’s ELO rating data is obtained from Chessbase Mega Database 2020.

Figure 15: Elo rating distribution of a tournament Rausis competed in 2012.



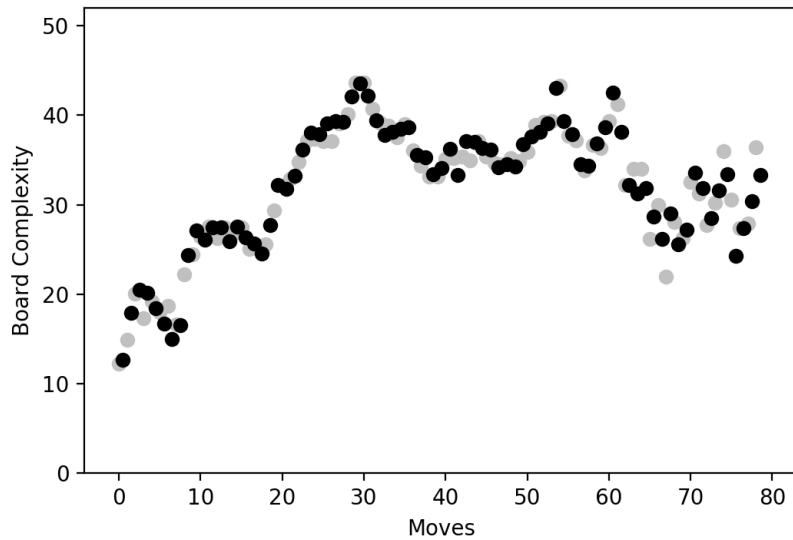
Note: The tournament is Tres Cantos Open played in June 2012 in Spain. Rausis had an ELO rating of 2514. His nine opponents had an average ELO rating of 2046. ELO rating information is obtained from Chessbase Mega Database 2020.

Figure 16: Computer evaluation of a game played by Carlsen in 2019.



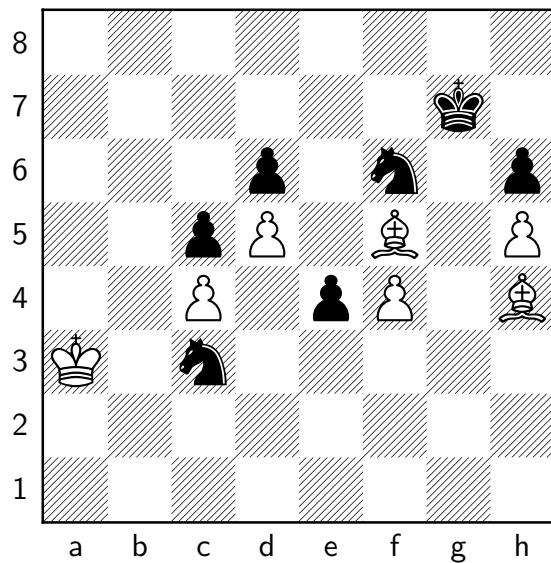
Note: The game was played between Vincent Keymer (White) and Magnus Carlsen (Black) on April 20, 2019 during the first round of Grenke Chess Classic 2019. Keymer's Average Centipawn Loss (ACPL) was 35.22 and Carlsen's 26.17 using our algorithm. A higher ACPL means the player made more mistakes according to the chess engine. The chess engine used for evaluations is Stockfish 11 with a depth of 19 moves.

Figure 17: Complexity evaluation of a game played by Carlsen in 2019 using an Artificial Neural Network (ANN) algorithm.



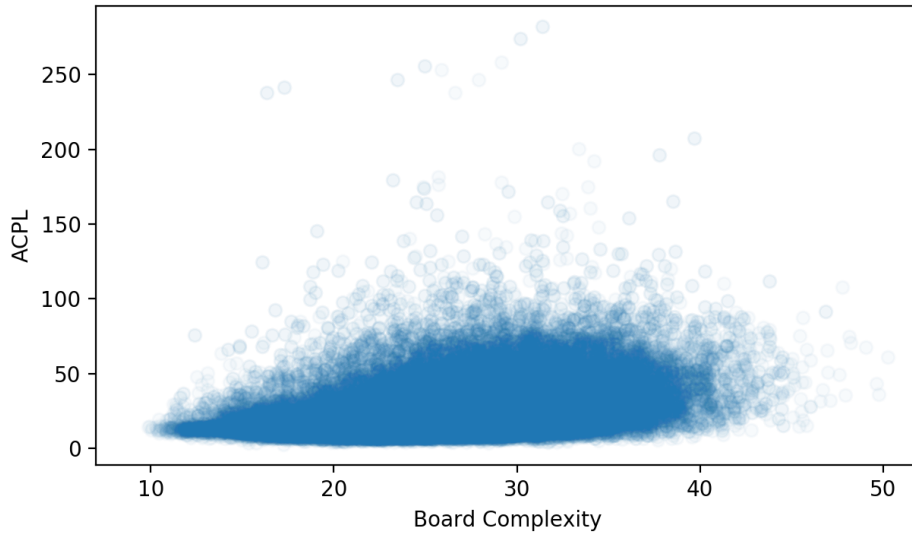
Note: The game was played between Vincent Keymer (White) and Magnus Carlsen (Black) on April 20, 2019 during the first round of Grenke Chess Classic 2019. Keymer's Average Centipawn Loss (ACPL) was 35.22 and Carlsen's 26.17 using our algorithm. Our neural-network board complexity estimate assigns an expected ACPL score of 34.87. This score is substantially higher than the sample average, 26.56. The game is within the top 10% of the sample in terms of complexity.

Figure 18: A position from Keymer vs. Carlsen (2019).



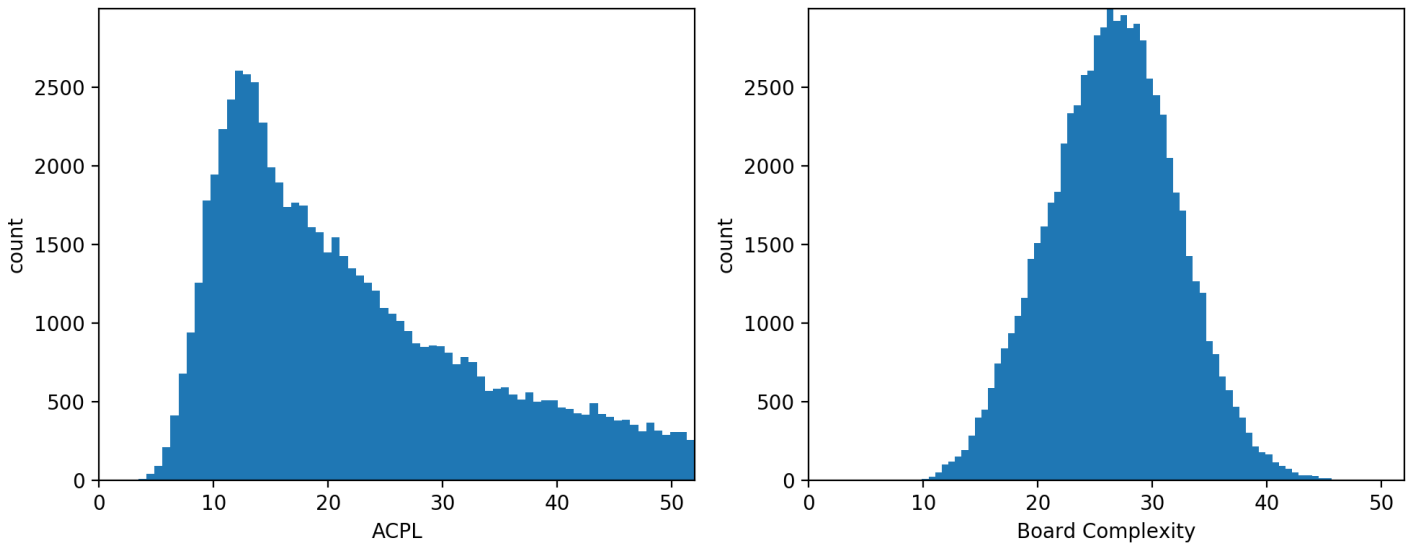
Note: This position is from Vincent Keymer (White) vs. Magnus Carlsen (Black), Grenke Chess Classic 2019 (white to play). Our neural-network algorithm calculates the probability of making an error as 0.52 (about twice as high as the sample average) in an amount of 65 centipawns. In the game, white blundered (by playing Bf2) in an amount of 180 centipawns, according to Stockfish. Before this blunder, the position was a forced draw.

Figure 19: Scatterplot of board complexity and ACPL scores.



Note: The board complexity measure is obtained via a neural-network algorithm. It is the "expected ACPL score" according to the AI, depending on the complexity of a game. The estimated slope is 1.14 for the overall sample of 36,000 games and 2.8 million moves.

Figure 20: Distribution of ACPL and board complexity scores.



Note: The board complexity measure is obtained via a neural-network algorithm. It is the "expected ACPL score" according to the AI which depends on the complexity of a game. The average ACPL score in the sample is 25.49 and the board complexity score is 26.57 for the overall sample with 36,000 games and 2.8 million moves. The neural-network was trained with an independent sample consisting of 25,000 games and 2 million moves with games played between players with "National Master" ranking on average.

Table 2: Summary statistics for all samples.

years: 2013-2019					years: 1995-2001			
variable	with Carlsen		without Carlsen		with Kasparov		without Kasparov	
	mean	sd	mean	sd	mean	sd	mean	sd
ACPL	17.810	11.766	16.681	10.836	21.316	12.992	20.759	12.782
Difficulty	27.048	5.419	26.796	5.278	27.298	5.525	27.867	5.501
TotalBlunder	.229	.576	.188	.515	.285	.677	.241	.561
TotalMistake	1.682	1.941	1.457	1.817	1.907	2.039	1.786	1.882
win	.205	.403	.166	.372	.227	.419	.208	.406
draw	.593	.491	.643	.479	.544	.498	.549	.498
loss	.203	.402	.191	.393	.229	.420	.243	.429
ELO	2714	80.37	2758	47.43	2644	65.33	2685	59.27
Moves	45.21	17.68	43.59	15.97	38.82	15.69	38.34	15.92
#of tournaments	=35		=40		=22		=44	
#of games	=1,336		=1,774		=1,727		=3,696	
#of moves	=114,898		=160,362		=133,184		=286,787	
years: 1987-1994					years: 1976-1983			
	with Kasparov & Karpov		without Kasparov & Karpov		with Karpov		without Karpov	
ACPL	21.578	13.226	20.949	12.335	25.694	14.907	21.805	13.495
Difficulty	26.838	5.820	27.040	5.501	26.010	5.755	25.403	5.928
TotalBlunder	.318	.685	.272	.610	.307	.697	.276	.642
TotalMistake	1.921	2.029	1.829	1.917	2.575	2.323	1.840	2.036
win	.233	.423	.195	.396	.254	.435	.207	.406
draw	.533	.499	.561	.496	.493	.500	.552	.497
loss	.235	.424	.243	.429	.253	.435	.241	.427
ELO	2590	56.02	2627	59.77	2529	76.13	2558	68.08
Moves	39.66	16.87	39.04	16.77	37.67	17.01	36.67	17.48
#of tournaments	=11		=37		=32		=39	
#of games	=874		=1,989		=2,036		=3,634	
#of moves	=68,012		=157,668		=149,122		=76,167	

Notes: Superstar player observations are excluded in each sample. Data comes from Chessbase Mega Database 2020.

Table 2: Summary statistics for all samples. (cont.)

<i>years: 1962-1970</i>					<i>years: 2014-2019</i>			
	<i>with Fischer</i>		<i>without Fischer</i>		<i>with Hou Yifan</i>		<i>without Hou Yifan</i>	
ACPL	25.284	15.663	24.196	15.649	34.717	22.405	23.022	14.525
Difficulty	26.078	5.750	26.024	5.600	26.750	5.217	26.863	5.198
TotalBlunder	.349	.746	.347	.764	.717	1.107	.377	.693
TotalMistake	2.311	2.254	2.206	2.134	3.400	2.687	2.482	2.475
win	.250	.433	.242	.428	.353	.478	.246	.431
draw	.500	.500	.479	.500	.290	.454	.462	.499
loss	.250	.433	.279	.449	.357	.479	.293	.455
ELO	2099	296.55	2493	68.85
Moves	36.10	15.63	38.43	16.42	39.30	16.13	45.28	18.69
<i>#of tournaments</i>	=16		=81		=4		=6	
<i>#of games</i>	=1,660		=7,832		=440		=748	
<i>#of moves</i>	=126,578		=565,611		=40,324		=69,084	
<i>years: 2012-2019</i>								
	<i>with Rausis</i>		<i>without Rausis</i>					
ACPL	34.642	22.113	39.216	23.904				
Difficulty	26.536	5.278	27.121	5.122				
TotalBlunder	.714	1.108	.818	1.162				
TotalMistake	3.395	2.688	3.773	2.750				
win	.352	.478	.386	.487				
draw	.294	.455	.209	.407				
loss	.354	.478	.405	.491				
ELO	2096	281.02	1984	276.78				
Moves	38.988	16.231	37.655	14.955				
<i>#of tournaments</i>	=8		=30					
<i>#of games</i>	=2,106		=6,357					
<i>#of moves</i>	=159,052		=495,482					

Notes: Superstar player observations are excluded in each sample. Data comes from Chessbase Mega Database 2020.

*: ELO rating system was first adopted by FIDE beginning 1970.

Table 3: Performance against a superstar.

<i>Panel A. Classical</i>								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
	ACPL	TotalBlunder	TotalMistake	win	draw	loss	Difficulty	# of games
								# of moves
Against Carlsen 2013-2019	1.868*** (0.449)	0.080*** (0.026)	0.174** (0.085)	-0.069*** (0.015)	-0.100*** (0.023)	0.169*** (0.025)	0.172 (0.252)	3,316
Against Kasparov 1995-2001	2.300*** (0.634)	0.093** (0.046)	0.226** (0.095)	-0.104*** (0.012)	-0.049 (0.035)	0.153*** (0.037)	1.485*** (0.346)	5,770
Against Kasparov/Karpov 1987-1994	2.757*** (0.656)	0.174*** (0.033)	0.184* (0.093)	-0.102*** (0.016)	-0.078*** (0.027)	0.180*** (0.030)	-0.427 (0.326)	2,768
Against Karpov 1976-1983	3.171*** (0.579)	0.167*** (0.036)	0.150* (0.083)	-0.106*** (0.012)	-0.089*** (0.024)	0.195*** (0.024)	0.522** (0.253)	5,326
Against Fischer 1962-1970	4.379*** (0.949)	0.150*** (0.040)	0.222 (0.149)	-0.106*** (0.022)	-0.186*** (0.032)	0.292*** (0.040)	2.255*** (0.361)	9,626
Against Hou Yifan 2014-2019	4.415** (1.530)	0.203*** (0.047)	0.502 (0.437)	-0.111*** (0.031)	-0.203*** (0.034)	0.314*** (0.029)	0.839** (0.331)	1,232
Against Rausis 2012-2019	9.275*** (1.741)	0.400*** (0.092)	0.635** (0.257)	-0.295*** (0.024)	-0.135** (0.055)	0.430*** (0.057)	0.457 (0.708)	7,924
Against Superstar 1962-2019	2.721*** (0.312)	0.100*** (0.016)	0.018 (0.055)	-0.115*** (0.008)	-0.059*** (0.014)	0.174*** (0.015)	0.627*** (0.155)	36,266
								2,801,958

Notes: All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament (except for pre-1970 games, as ELO rating was adopted in 1970 by FIDE), player's ELO rating (except pre-1970 games), board complexity measured by our neural-network algorithm (except in column 7), opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Performance in tournaments with and without Rausis.

<i>Panel A. Classical</i>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	ACPL	TotalBlunder	TotalMistake	win	draw	loss	Difficulty
Superstar effect for							
All players	-1.504 (1.041)	-0.099 (0.074)	0.109 (0.097)	-0.005 (0.027)	0.060* (0.034)	-0.055* (0.028)	0.289 (0.266)
Top players ⁺	-2.101** (0.783)	-0.070 (0.057)	-0.067 (0.112)	0.052 (0.038)	0.038 (0.037)	-0.090*** (0.023)	0.224 (0.350)
Last 3 Rounds	2.446** (1.128)	0.220*** (0.069)	-0.117 (0.166)	-0.230*** (0.062)	0.213*** (0.055)	0.017 (0.020)	1.309 (0.989)
<i>Number of moves</i>	654,534	654,534	654,534	654,534	654,534	654,534	654,534
<i>Number of games</i>	8,463	8,463	8,463	8,463	8,463	8,463	8,463

Notes: Rausis' games are excluded. The sample consists of open tournaments in Swiss system as opposed to the previous tables in which the samples consist of invitation-based Round robin tournaments. The third row indicates the last three rounds in the tournament where competition is more intense in a Swiss-format tournament, as winners in each round get paired with other winners. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

⁺: These are the players with the top 2 highest ELO rating in a given tournament following Rausis' ELO rating.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Performance in tournaments with and without Hou Yifan.

<i>Panel A. Classical</i>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	ACPL	TotalBlunder	TotalMistake	win	draw	loss	Difficulty
Superstar effect for							
Top 25% players	4.344** (1.846)	-0.012 (0.112)	0.807** (0.305)	-0.104* (0.054)	-0.048 (0.054)	0.152** (0.066)	1.204 (0.753)
Mid 50% players	1.143 (1.039)	0.015 (0.083)	0.076 (0.207)	-0.068 (0.042)	0.085* (0.047)	-0.017 (0.056)	1.172** (0.447)
Bottom 25% players	1.511 (1.699)	-0.017 (0.104)	0.138 (0.236)	-0.050 (0.054)	-0.032 (0.044)	0.082 (0.075)	0.390 (0.690)
<i>Number of moves</i>	109,408	109,408	109,408	109,408	109,408	109,408	109,408
<i>Number of games</i>	1,188	1,188	1,188	1,188	1,188	1,188	1,188

Notes: Hou Yifan's games are excluded. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Performance in tournaments with and without Fischer.

<i>Panel A. Classical</i>							
	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
<i>Superstar effect for</i>							
All players	0.435 (0.322)	0.022 (0.014)	0.016 (0.057)	-0.011 (0.009)	-0.013 (0.012)	0.024* (0.012)	-0.736*** (0.119)
Top players	0.594 (0.616)	0.016 (0.031)	0.153 (0.107)	-0.029 (0.028)	0.043* (0.026)	-0.014 (0.026)	-0.819*** (0.243)
<i>Number of moves</i>	692,072	692,072	692,072	692,072	692,072	692,072	692,072
<i>Number of games</i>	9,491	9,491	9,491	9,491	9,491	9,491	9,491

Notes: Fischer's games are excluded. Top 10 players are the top chess players in the world from 1962-1970 other than Fischer.⁺ All regressions include player and year fixed effects, round fixed effects, event site fixed effects, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

⁺: These players are Tigran Petrosian, Viktor Korchnoi, Boris Spassky, Vasily Smyslov, Mikhail Tal, Mikhail Botvinnik, Paul Keres, Efim Geller, David Bronstein, and Samuel Reshevsky.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Performance in tournaments with and without Kasparov & Karpov.

<i>Panel A. Classical</i>							
	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
<i>Superstar effect when both present</i>							
Top 25% players	-0.174 (1.048)	-0.057 (0.052)	-0.034 (0.185)	0.014 (0.031)	-0.072* (0.038)	0.058 (0.041)	0.692 (0.452)
Mid 50% players	0.088 (0.568)	-0.006 (0.050)	-0.093 (0.084)	0.021 (0.021)	-0.028 (0.026)	0.007 (0.023)	0.285 (0.347)
Bottom 25% players	-2.137** (0.935)	-0.099* (0.049)	-0.174 (0.146)	0.063** (0.025)	-0.006 (0.036)	-0.057* (0.031)	0.316 (0.418)
<i>Number of moves</i>	207,880	207,880	207,880	207,880	207,880	207,880	207,880
<i>Number of games</i>	2,624	2,624	2,624	2,624	2,624	2,624	2,624

Notes: Kasparov and Karpov's games are excluded. Top 25% is defined as having an ELO rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an ELO in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Performance in tournaments with and without Kasparov.

	<i>Panel A. Classical</i>						
	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	-1.253* (0.696)	-0.061* (0.034)	-0.111 (0.103)	0.025 (0.025)	0.045 (0.030)	-0.069*** (0.023)	0.496 (0.407)
Mid 50% players	-0.229 (0.538)	-0.002 (0.027)	-0.054 (0.072)	0.028* (0.017)	0.003 (0.023)	-0.031 (0.020)	-0.191 (0.387)
Bottom 25% players	0.559 (0.688)	-0.005 (0.041)	0.049 (0.096)	-0.027 (0.020)	0.012 (0.024)	0.014 (0.029)	-0.097 (0.490)
<i>Number of moves</i>	420,348	420,348	420,348	420,348	420,348	420,348	420,348
<i>Number of games</i>	5,427	5,427	5,427	5,427	5,427	5,427	5,427

Notes: Kasparov's games are excluded. Top 25% is defined as having an ELO rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an ELO in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by player-year) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Performance in tournaments with and without Karpov.

	<i>Panel A. Classical</i>						
	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	-1.843 (2.357)	-0.205** (0.098)	-0.148 (0.399)	-0.050 (0.055)	0.137* (0.071)	-0.088 (0.100)	2.316** (0.895)
Mid 50% players	-0.915 (2.103)	-0.249*** (0.095)	0.091 (0.289)	-0.033 (0.047)	0.047 (0.064)	-0.014 (0.084)	1.445 (1.042)
Bottom 25% players	2.375 (2.304)	-0.225* (0.135)	0.820** (0.397)	-0.245*** (0.076)	-0.066 (0.190)	0.311 (0.245)	1.445 (1.023)
<i>Number of moves</i>	381,460	381,460	381,460	381,460	381,460	381,460	381,460
<i>Number of games</i>	5,120	5,120	5,120	5,120	5,120	5,120	5,120

Notes: Karpov's games are excluded. Top 25% is defined as having an ELO rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an ELO in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by player-year) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Performance in tournaments with and without Carlsen.

	<i>Panel A. Classical</i>						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	ACPL	TotalBlunder	TotalMistake	win	draw	loss	Difficulty
Superstar effect for							
Top 25% players	-0.080 (0.698)	0.040 (0.032)	-0.128 (0.097)	0.010 (0.031)	0.022 (0.038)	-0.032 (0.032)	0.756** (0.362)
Mid 50% players	-1.221* (0.680)	0.008 (0.033)	-0.186* (0.097)	-0.002 (0.029)	0.072** (0.030)	-0.069** (0.030)	0.241 (0.314)
Bottom 25% players	-0.680 (0.993)	-0.004 (0.044)	-0.073 (0.128)	-0.013 (0.035)	0.059 (0.041)	-0.046 (0.039)	0.770* (0.423)
<i>Number of moves</i>	275,260	275,260	275,260	275,260	275,260	275,260	275,260
<i>Number of games</i>	3,110	3,110	3,110	3,110	3,110	3,110	3,110

Notes: Carlsen's games are excluded. Top 25% is defined as having an ELO rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an ELO in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by player-year) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 11: Performance in tournaments with and without a Superstar (overall effect).

	<i>Panel A. Classical</i>						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	ACPL	TotalBlunder	TotalMistake	win	draw	loss	Difficulty
Superstar effect for							
Top 25% players	-0.391 (0.310)	-0.061*** (0.021)	0.064 (0.049)	-0.004 (0.014)	0.011 (0.013)	-0.007 (0.013)	0.338** (0.162)
Mid 50% players	-0.259 (0.265)	-0.023 (0.017)	-0.017 (0.037)	-0.001 (0.008)	0.003 (0.012)	-0.002 (0.009)	0.232 (0.145)
Bottom 25% players	0.211 (0.398)	-0.049** (0.021)	0.076 (0.049)	0.012 (0.011)	-0.017 (0.013)	0.004 (0.014)	0.004 (0.166)
<i>Number of moves</i>	1,858,099	1,858,099	1,858,099	1,858,099	1,858,099	1,858,099	1,858,099
<i>Number of games</i>	24,083	24,083	24,083	24,083	24,083	24,083	24,083

Notes: Superstars' games are excluded. Top 25% is defined as having an ELO rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an ELO in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament, player's ELO rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 12: Performance in tournaments with and without a superstar for top players.

	<i>Panel A. Classical</i>							# of games	# of moves
	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty		
Carlsen present 2013-2019	-0.080 (0.698)	0.040 (0.032)	-0.128 (0.097)	0.010 (0.031)	0.022 (0.038)	-0.032 (0.032)	0.756** (0.362)	3,110	275,260
Kasparov present 1995-2001	-1.427** (0.696)	-0.086** (0.035)	-0.142 (0.102)	0.025 (0.025)	0.028 (0.033)	-0.053** (0.025)	0.694 (0.466)	5,427	420,348
Kasparov&Karpov present 1987-1994	-0.848 (1.030)	-0.062 (0.059)	-0.124 (0.168)	0.031 (0.033)	-0.082** (0.036)	0.051 (0.039)	0.377 (0.436)	2,624	207,880
Karpov present 1976-1983	-1.843 (2.357)	-0.205** (0.098)	-0.148 (0.399)	-0.050 (0.055)	0.137* (0.071)	-0.088 (0.100)	2.316** (0.895)	5,120	381,460
Fischer present ⁺ 1962-1970	0.431 (0.655)	0.007 (0.031)	0.138 (0.103)	-0.040 (0.031)	0.042* (0.025)	-0.002 (0.028)	-0.768*** (0.267)	9,491	692,072
Hou Yifan present 2014-2019	4.557** (1.999)	-0.041 (0.112)	0.970*** (0.341)	-0.118** (0.058)	-0.055 (0.057)	0.172** (0.071)	0.734 (0.752)	1,188	109,408
Rausis present ⁺⁺ 2012-2019	3.482** (1.471)	0.239** (0.088)	-0.101 (0.124)	-0.227*** (0.058)	0.175*** (0.048)	0.052* (0.027)	1.589 (1.021)	8,463	654,534
Aggregate effect ⁺⁺⁺ 1962-2019	-0.403 (0.324)	-0.009 (0.018)	-0.031 (0.052)	0.012 (0.011)	-0.010 (0.016)	-0.002 (0.013)	0.384** (0.170)	17,167	1,362,162

Notes: Superstars' games are excluded. A top player is defined as having an ELO rating in the top 25% among the competitors at the time of the tournament. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average ELO rating in the tournament (except for pre-1970 games, as ELO rating was adopted in 1970 by FIDE), player's ELO rating (except pre-1970 games), board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

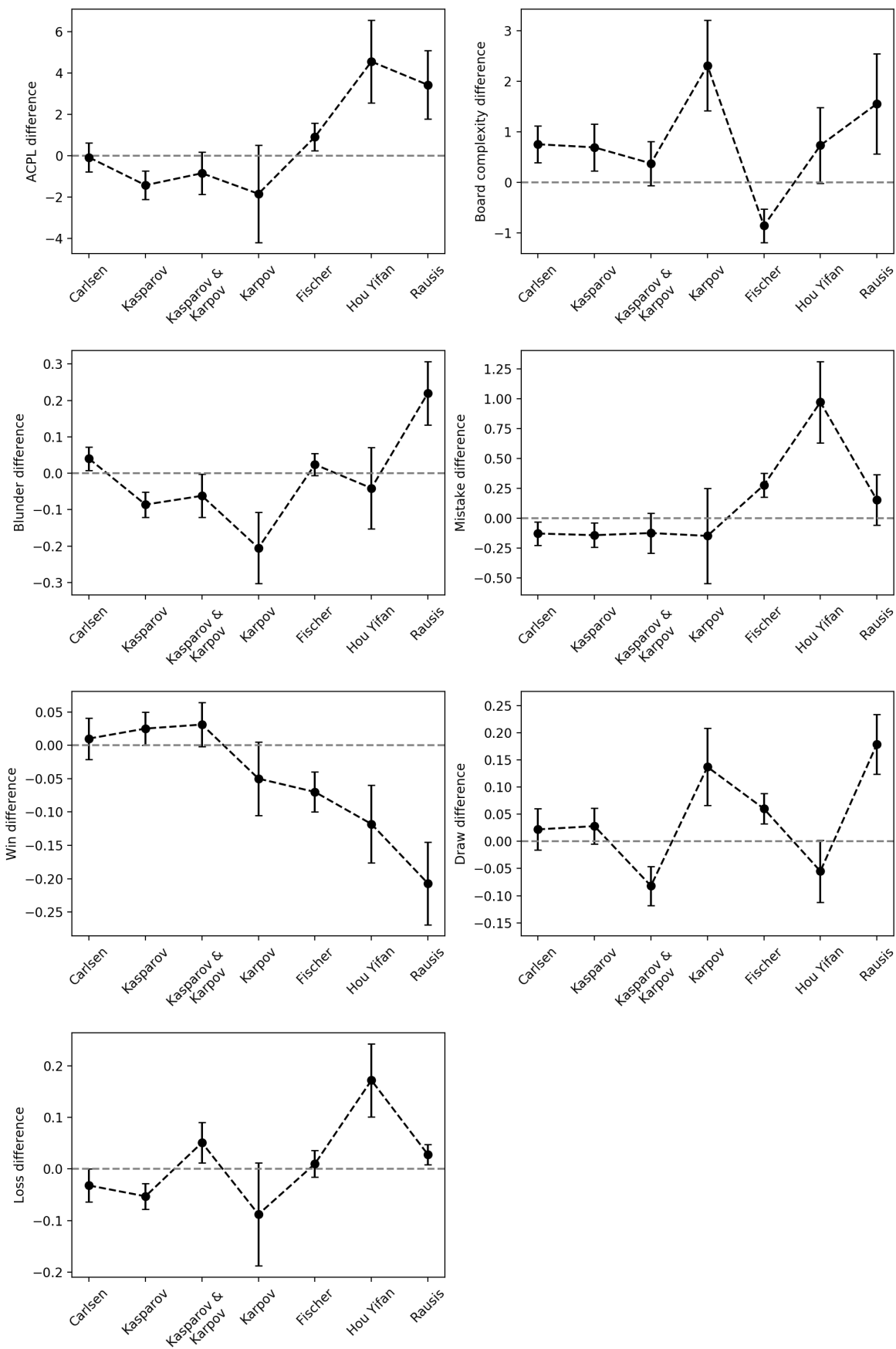
⁺: Since no ELO rating information was available in Fischer's era, we define the top players as the top chess players in the world from 1962-1970 other than Fischer. These players are Tigran Petrosian, Viktor Korchnoi, Boris Spassky, Vasily Smyslov, Mikhail Tal, Mikhail Botvinnik, Paul Keres, Efim Geller, David Bronstein, and Samuel Reshevsky. [Kasparov \(2003\)](#) provides a detailed overview on each of these players.

⁺⁺: The estimates are for the top 5% players with the highest ELO ratings in a given tournament following Rausis' ELO rating during the second half of the tournament. In a Swiss type open tournament, only the very top players have a chance to win the tournament, and the competition gets more intense as winners in each round are paired with other winners.

⁺⁺⁺: The sample restricted to Round-robin tournaments with average ELO information available.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Figure 21: Superstar presence coefficients for top players over different superstar intensity levels.



Note: The figure presents the coefficients in [Table 12](#) with confidence intervals over different superstar intensity levels.

Appendix

Table A.1: Variables list.

Variable Name	Variable Meaning
Superstar Present	=1 if a superstar is present in a tournament.
Against Superstar	=1 if a game is played against a superstar.
ELO	ELO rating of a player.
ACPL	Average Centipawn Loss of a player in a game.
TotalBlunder	Total number of blunders committed by a player in a game. A move is considered a blunder if the change in centipawn score is more than 300 centipawns.
TotalMistake	Total number of mistakes committed by a player in a game. A move is considered a mistake if the change in centipawn score is between 100-300 centipawns.
Difficulty	The board complexity metric estimated via an Artificial Neural Network algorithm.
win	=1 if a player wins his or her game.
draw	=1 if a games ends in a draw.
loss	=1 if a player loses his or her game.
white	=1 if a player's side is white.
moves	Total number of moves played by a player in a game.
Round-robin tournament	An invitation based tournament system with a limited number of participants. Each participant plays against participants once or twice, depending on the tournament length. The participant who accumulates the highest number of points wins the tournament.
Swiss tournament	A tournament system that is used typically in open tournaments with a large pool of participants. Following the results in the first round, winners are matched with other winners. Towards the end of the tournament, strongest players with the highest number of scores match. The participant who accumulates the highest number of points wins the tournament.











Table A.2: An example pgn file.

[Event "GRENKE Chess Classic"]
[Site "Karlsruhe GER"]
[Date "2019.04.20"]
[EventDate "2019.04.20"]
[Round "1"]
[Result "0-1"]
[White "Vincent Keymer"]
[Black "Magnus Carlsen"]
[ECO "A56"]
[WhiteElo "2516"]
[BlackElo "2845"]
[PlyCount "162"]

1 d4 ♘f6 2 c4 c5 3 d5 g6 4 ♘c3 d6 5 e4 ♙g7 6 ♘f3 O-O 7 ♙e2 e5 8 O-O ♘e8 9
 ♘e1 f5 10 e×f5 g×f5 11 f4 ♘d7 12 ♘d3 e4 13 ♘f2 ♙×c3 14 b×c3 ♘df6 15 ♙e3 ♘g7
 16 ♙e1 ♙d7 17 ♘d1 ♙a4 18 h3 ♙×d1 19 ♙×d1 ♙e8 20 ♖f2 ♙g6 21 ♙g1 ♖h8 22 a4
 ♙g8 23 ♙f1 ♘fh5 24 g3 ♙af8 25 ♙g2 ♙f6 26 ♙ac1 ♙d8 27 ♙h2 ♘f6 28 g4 ♘d7 29
 g5 ♙a5 30 g6 h6 31 ♙b1 ♙b8 32 ♙g3 ♙d8 33 ♖e1 ♘e8 34 ♖d2 ♘f8 35 ♙f2 ♙e7 36
 ♖e3 ♙f6 37 ♖d2 ♘×g6 38 h4 ♘e7 39 ♙h3 ♙×g1 40 ♙×g1 ♙f7 41 h5 ♘f6 42 ♙h4 b6
 43 ♙b1 ♙f8 44 ♙g1 ♙f7 45 ♙b1 ♙g7 46 ♙g1 ♙f8 47 ♖c2 ♘fg8 48 ♖d2 ♙f7 49 ♖c2
 ♙f8 50 ♖d2 ♙e8 51 ♙a1 ♙f7 52 a5 b×a5 53 ♙×a5 ♘c8 54 ♙a1 ♙f8 55 ♙b1 ♘b6 56
 ♙g1 ♙g7 57 ♙×g7 ♖×g7 58 ♙g3+ ♖h8 59 ♙g6 a5 60 ♙f1 a4 61 ♖c2 a3 62 ♖b3 ♘a4
 63 ♙h3 ♙g7 64 ♙×g7+ ♖×g7 65 ♙×f5 ♘f6 66 ♖×a3 ♘×c3 67 ♙f2 ♘e2 68 ♖a4 ♘×h5
 69 ♖a5 ♘f6 70 ♖b6 ♖f7 71 ♖c7 ♖e7 72 ♙e3 ♘d4 73 ♙g6 h5 74 ♙f2 ♘f3 75 ♙f5 ♘d2
 76 ♙h4 e3 77 ♙d3 ♘f3 78 ♙×f6+ ♖×f6 79 ♖×d6 h4 80 ♖c7 ♘d4 81 ♖c8 e2 0-1

Figure A.1: An example of a tournament table.

Grenke Chess Classic 6th 2019

			1	2	3	4	5	6	7	8	9	10		TB	Perf.	+/-
1		Carlsen, Magnus	2845	*	½	½	1	½	1	1	1	1	7.5 / 9		2990	+14
2		Caruana, Fabiano	2819	½	*	1	½	½	½	½	1	1	6.0 / 9		2833	+3
3		Naiditsch, Arkadij	2695	½	0	*	½	1	½	0	½	1	5.0 / 9	19.00	2766	+9
4		Vachier Lagrave, Maxime	2773	0	½	½	*	½	½	½	1	1	5.0 / 9	18.25	2758	-1
5		Anand, Viswanathan	2774	½	½	0	½	*	½	½	1	0	4.5 / 9	19.75	2719	-6
6		Aronian, Levon	2763	0	½	½	½	½	*	1	½	½	4.5 / 9	18.75	2720	-5
7		Svidler, Peter	2735	0	½	1	½	½	0	*	½	1	4.5 / 9	17.75	2723	-1
8		Vallejo Pons, Francisco	2693	0	½	½	½	0	½	½	*	½	4.0 / 9		2689	-1
9		Meier, Georg	2628	0	0	0	0	1	½	0	½	*	2.0 / 9	8.75	2518	-12
10		Keymer, Vincent	2516	0	0	0	0	0	½	½	0	1	2.0 / 9	6.50	2529	+1

Average Elo: 2724 <=> Cat: 19

gm = 3.24 m = 1.44

(45 Games)

Note: The tournament table is obtained from Chessbase Mega Database 2020.

Table A.3: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Carlsen Present</i>	
2019	GCT Croatia 2019, Grenke Chess Classic 2019, Gashimov Memorial 2019, Norway Chess 2019, Sinquefield 2019, Tata Steel 2019
2018	Gashimov Memorial 2018, Sinquefield 2018, Biel 2018, Norway Chess 2018, Grenke Chess Classic 2018, Tata Steel 2018
2017	London Classic 2017, Norway Chess 2017, Sinquefield 2017, Grenke Chess Classic 2017, Tata Steel 2017
2016	Norway Chess 2016, Tata Steel 2016, Bilbao Masters 2016
2015	London Classic 2015, Sinquefield 2015, Norway Chess 2015, Gashimov Memorial 2015, Grenke Chess Classic 2015, Tata Steel 2015
2014	Norway Chess 2014, Zuerich Chess Challenge 2014, Sinquefield 2014, Gashimov Memorial 2014
2013	Moscow Tal Memorial 2013, Norway Chess 2013, Candidates Tournament 2013, Tata Steel 2013, Sinquefield 2013
<i>Panel B. Carlsen Not Present</i>	
2019	U.S. Championship 2019, Dortmund 2019
2018	Candidates Tournament 2018, U.S. Championship 2018, Dortmund 2018
2017	U.S. Championship 2017, Dortmund 2017, Gashimov Memorial 2017
2016	London Classic 2016, Sinquefield 2016, Gashimov Memorial 2016, Candidates Tournament 2016, Moscow Tal Memorial 2016, U.S. Championship 2016, Dortmund 2016
2015	Dortmund 2015, Zuerich Chess Challenge 2015, Tbilisi FIDE GP 2015, Khanty-Mansiysk FIDE GP 2015, Capablanca Memorial 2015, U.S. Championship 2015
2014	Beijing Sportaccord Basque 2014, London Classic 2014, Tashkent FIDE GP 2014, Dortmund 2014, Tata Steel 2014, U.S. Championship 2014, Candidates Tournament 2014, Baku FIDE GP 2014, Capablanca Memorial 2014, Bergamo ACP Golden Classic 2014
2013	Paris FIDE GP 2013, Dortmund 2013, Thessaloniki FIDE GP 2013, Zug FIDE GP 2013, Beijing FIDE GP 2013, Zuerich Chess Challenge 2013, Grenke Chess Classic 2013, Capablanca Memorial 2013, U.S. Championship 2013

Table A.4: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Kasparov Present</i>	
2001	Astana 2001, Zuerich 2001, Linares 2001, Corus Wijk aan Zee 2001
2000	Fujitsu Siemens Giants 2000, Sarajevo Bosnia 2000, Linares 2000, Corus Wijk aan Zee 2000
1999	Sarajevo Bosnia 1999, Linares 1999, Hoogovens Wijk aan Zee 1999
1998	Linares 1998
1997	Tilburg 1997, Novgorod 1997, Linares 1997
1996	Las Palmas 1996, Dos Hermanas 1996, Amsterdam Euwe Memorial 1996
1995	Horgen 1995, Amsterdam Euwe Memorial 1995, Novgorod 1995 Riga Tal Memorial 1995
<i>Panel B. Kasparov Not Present</i>	
2001	Sigeman & Co 2001, Biel 2001, Dortmund 2001, Pamplona 2001, Dos Hermanas 2001
2000	Japfa Classic 2000, Dortmund 2000, Sigeman & Co 2000, Biel 2000
1999	Pamplona 1999, Lost Boys Amsterdam 1999, Dortmund 1999, Sigeman & Co 1999 Dos Hermanas 1999, Biel 1999
1998	Hoogovens Wijk aan Zee 1998, Tilburg 1998, Dortmund 1998, Madrid 1998, Pamplona 1998
1997	Hoogovens Merrillville 1997, Sigeman & Co 1997, Ubeda 1997, Hoogovens Wijk aan Zee 1997 Dos Hermanas 1997, Lost Boys 1997, Dortmund 1997, Madrid 1997, Belgrade Investbank 1997
1996	Koop Tjuchem 1996, Donner Memorial 1996, Hoogovens Wijk aan Zee 1996, Tilburg 1996, Dortmund 1996, Dos Hermanas 1996, Madrid 1996
1995	Belgrade Investbank 1995, Horgen 1995, Donner Memorial 1995, Biel 1995, Madrid 1995, Dos Hermanas 1995, Groningen 1995, Dortmund 1995

Table A.5: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Kasparov & Karpov Both Present</i>	
1994	Linares 1994
1993	Linares 1993
1992	
1991	Reggio Emilia 1991, Tilburg 1991, Amsterdam Euwe Memorial 1991, Linares 1991
1990	
1989	World Cup Skelleftea 1989
1988	USSR Championship 1988, World Cup Belfort 1988, Optiebeurs Amsterdam 1988
1987	Brussels 1987
<i>Panel A. Kasparov & Karpov Neither Present</i>	
1994	Donner Memorial 1994, Dortmund 1994, Hoogovens Wijk aan Zee 1994, Groningen 1994, Munich 1994
1993	Antwerp 1993, Amsterdam VSB 1993, Madrid 1993, Las Palmas 1993, Munich 1993
1992	Alekhine Memorial 1992, Amsterdam Euwe Memorial 1992, Hoogovens Wijk aan Zee 1992, Groningen 1992, Munich 1992
1991	World Cup Reykjavik 1991, Hoogovens Wijk aan Zee 1991, Groningen 1991, Munich 1991
1990	Tilburg 1990, Hoogovens Wijk aan Zee 1990, Prague 1990, Groningen 1990, Munich 1990
1989	Hoogovens Wijk aan Zee 1989, Groningen 1989, Munich 1989, Amsterdam Euwe Memorial 1989
1988	Amsterdam Euwe Memorial 1988, OHRA Amsterdam 1988, Linares 1988, Hastings 1988
1987	Belgrade Investbanka 1987, Hoogovens Wijk aan Zee 1987, Interpolis 1987, OHRA Amsterdam 1987, Reykjavik 1987

Table A.6: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Karpov Present</i>	
1983	Interpolis 1983, International DSB Mephisto GM 1983, USSR Final 1983, Bath 1983, Linares 1983
1982	Interpolis 1982, Turin 1982, Hamburg 1982, London Phillips 1982, Mar del Plata Clarin Masters 1982
1981	IBM Herinnerungs Toernooi 1981, Moscow 1981, Linares 1981
1980	Buenos Aires 1980, Interpolis 1980, IBM Kroongroep 1980, Bugojno 1980, Bad Kissingen 1980
1979	Interpolis 1979, Waddinxveen KATS 1979, Montreal International 1979, GER International 1979
1978	Bugojno 1978
1977	Interpolis 1977, October Revolution 1977, Las Palmas 1977, GER International 1977
1976	USSR Final 1976, Montilla 1976, Manila Marlboro 1976, Amsterdam 1976, Skopje Solidarnost 1976
<i>Panel B. Karpov Not Present</i>	
1983	Jakarta International 1983, Hoogovens Wijk aan Zee 1983
1982	Bugojno 1982, Moscow Interzonal 1982, Las Palmas Interzonal 1982, Toluca Interzonal 1982, Niksic International 1982, Hoogovens Wijk aan Zee 1982
1981	Las Palmas 1981, IBM Herinnerungs Toernooi 1981, Interpolis 1981, Hoogovens Wijk aan Zee 1981
1980	Buenos Aires 1980, London Phillips 1980, Hoogovens Wijk aan Zee 1980, Las Palmas 1980, Reykjavik International 1980
1979	Buenos Aires Clarin 1979, Riga Interzonal 1979, Buenos Aires Interzonal 1979, Vidmar Memorial 1979, IBM 1979, Hoogovens Wijk aan Zee 1979, Buenos Aires Konex 1979
1978	Interpolis 1978, Reykjavik International 1978, Hoogovens Wijk aan Zee 1978, Las Palmas 1978, IBM 1978, Clarin 1978
1977	Geneve 1977, Vidmar Memorial 1977, Hoogovens Wijk aan Zee 1977, IBM 1977
1976	Interzonal 1976, Las Palmas 1976, Reykjavik International 1976, Hoogovens Wijk aan Zee 1976, IBM 1976

Table A.7: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Fischer Present</i>	
1970	Interzonal 1970, Buenos Aires 1970, Rovinj Zagreb 1970
1969	
1968	Vinkovci 1968, Nathanya 1968,
1967	Skopje 1967, Monaco Grand Prix 1967, Interzonal 1967
1966	Piatigorsky Cup 1966, U.S. Championship 1966
1965	U.S. Championship 1965, Capablanca Memorial 1965
1964	
1963	U.S. Championship 1963
1962	U.S. Championship 1962, Candidates Tournament 1962, Interzonal 1962
<i>Panel B. Fischer Not Present</i>	
1970	Vinkovci 1970, IBM Amsterdam 1970, Budapest 1970, Sarajevo 1970, Caracas 1970, Hoogovens Wijk an Zee 1970, Costa del Sol 1970, Skopje 1970, Rubinstein Memorial 1970, Christmas Congress 1970
1969	Monaco Grand Prix 1969, Hoogovens Wijk an Zee 1969, Venice 1969 U.S. Championship 1969, Palma de Mallorca 1969, IBM Amsterdam 1969, Sarajevo 1969, Christmas Congress 1969, Rubinstein Memorial 1969, Capablanca Memorial 1969
1968	Rubinstein Memorial 1968, Christmas Congress 1968, Palma de Mallorca 1968, U.S. Championship 1968, Bamberg 1968, IBM Amsterdam 1968, Sarajevo 1968 Hoogovens Wijk an Zee 1968, Monaco Grand Prix 1968, Skopje 1968
1967	Winnipeg 1967, October Revolution Leningrad 1967, October Revolution Moscow 1967, Capablanca Memorial 1967, Palma de Mallorca 1967, Sarajevo 1967, Hoogovens Beverwijk 1967, Christmas Congress 1967, Rubinstein Memorial 1967, Venice 1967, IBM Amsterdam 1967
1966	IBM Amsterdam 1966, Sarajevo 1966, Palma de Mallorca 1966 Hoogovens Beverwijk 1966, Venice 1966, Rubinstein Memorial 1966, Christmas Congress 1966
1965	ZSK International 1965, Zagreb 1965, Mer del Plata 1965, IBM Amsterdam 1965, Sarajevo 1965, Hoogovens Beverwijk 1965, Christmas Congress 1965, Rubinstein Memorial 1965
1964	Buenos Aires 1964, Capablanca Memorial 1964, Rubinstein Memorial 1964, Interzonal 1964, IBM Amsterdam 1964, Sarajevo 1964, Hoogovens Beverwijk 1964, Christmas Congress 1964, ZSK International 1964
1963	Piatigorsky Cup 1963, Alekhine Memorial 1963, IBM Amsterdam 1963, Sarajevo 1963, Hoogovens Beverwijk 1963, Rubinstein Memorial 1963, Christmas Congress 1963, Capablanca Memorial 1963
1962	Mer del Plata 1962, IBM Amsterdam 1962, Sarajevo 1962, Hoogovens Beverwijk 1962, Rubinstein Memorial 1962, Christmas Congress 1962, Capablanca Memorial 1962

Table A.8: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Hou Yifan Present</i>	
2015	Monte Carlo FIDE GP 2015
2014	Lopota FIDE GP 2014, Khanty-Mansiysk FIDE GP 2014, Sharjah FIDE GP 2014
<i>Panel B. Hou Yifan Not Present</i>	
2019	Skolkovo FIDE GP 2019, Saint Louis Cairns Cup 2019
2016	Khanty-Mansiysk FIDE GP 2016, Chengdu FIDE GP 2016, Batumi FIDE GP 2016, Tehran FIDE GP 2016

Table A.9: List of tournaments (classical)

Year	Tournament Name
<i>Panel A. Rausis Present</i>	
2019	Lugano op 2019
2018	Sautron op 2018
2017	
2016	Salon de Provence op 2016
2015	
2014	Chemnitz op 2014, Biella op 2014
2013	Charleroi op 2013, Lueneburg op 2013
2012	Tres Cantos op 2012
<i>Panel B. Rausis Not Present</i>	
2019	Locarno op 2019, Ascona op 2019, Porto San Giorgio op 2019
2018	Erfurt op 2018, Pfarrkirchen Rottal op 2018, Locarno op 2018, Paderborn op 2018, Forchheim op 2018
2017	Pfarrkirchen Rottal op 2017, Porto San Giorgio op 2017 Lugano op 2017, Sautron op 2017
2016	Wasselonne op 2016, Heraklion op 2016
2015	Salon de Provence op 2015, Biella op 2015, Porto San Giorgio op 2015, Erfurt op 2015, Lugano op 2015, Ascona op 2015, Forchheim op 2015
2014	Salon de Provence op 2014, Paderborn op 2014, Erfurt op 2014 Arco op 2014, Ascona op 2014, Forchheim op 2014
2013	Biella op 2013, Forchheim op 2013