Search Less for a Better Price

Eren Bilen[*] Chun-Hui Miao[†]

Abstract

In many markets, sellers must spend resources to learn the costs of providing goods/services. This paper considers consumer searches in such markets. It is found that (1) even with ex ante identical consumers and sellers, there is price dispersion in the equilibrium; (2) despite price dispersion and minimal search costs, it could be optimal to search just two sellers; (3) the optimal number of searches can increase with sellers' information costs. (JEL D40, L00)

Keywords: Price dispersion, Precontract costs, Search costs, Sealed-bid Auction.

[*]Department of Data Analytics, Dickinson College, Carlisle, PA 17013. Email: bilene@dickinson.edu

[†]Department of Economics, University of South Carolina, Columbia, SC 29208. Email: miao@moore.sc.edu

1 Introduction

To make informed purchase decisions, consumers search. To earn their business, sellers provide relevant information such as prices. The standard economic models of consumer search assume that price search is costly, but price-setting is costless.[1] In many markets, however, even a simple price quote may involve nontrivial costs for the seller. For example, a mortgage lender must evaluate a borrower's creditworthiness before offering a rate quote;[2] an insurance agent must assess an applicant's risk characteristics before issuing the premium; a repair shop must diagnose the problem before giving a cost estimate. In these markets, production costs depend on consumers' individual needs. Sellers set prices after consumer search takes place. A consumer can canvass multiple sellers, but cannot contract on any seller's effort in preparing the price quote.

This paper incorporates the above features into a model of consumer search to study the market impact of precontract costs, including consumer search cost and price-setting cost. The latter cost is due to uncertainty in the production cost. As such, we call it information cost.[3] Ultimately, in a competitive market, the price quotes will reflect information costs. While this observation suggests an equivalence between consumer search and sellers' information acquisition — both are paid by the consumer — our analysis reveals an important difference: the only way to save on total search costs is to search less, but paradoxically a consumer can save on total information costs by searching more. The latter is true if sellers' willingness to incur information costs drops sharply when they face more competitors. Consequently, information costs and search costs can have different, even opposite, impacts on consumer search behavior. For example, the optimal number of searches is two when information costs are zero, but can be infinity when search costs are zero. More generally, while the optimal number of searches always *decreases* with search costs, it can sometimes *increase* with information costs.

Interestingly, despite price dispersion and zero search costs, it could be optimal to search just two sellers. This is because the consumer indirectly pays for sellers' information costs and has an

[1] It is a long tradition that began with Stigler's seminal paper (Stigler 1961). More recently, a class of search models with an "information clearinghouse" assume nearly the opposite, that is, zero (marginal) search cost but positive (fixed) advertising cost (Baye and Morgan 2001).

[2] Woodward (2008) estimates that the "dry-hole" - applications that are processed but fail to become loans - cost of a mortgage loan is somewhere in the range of \$120 to \$410.

[3] It should be distinguished from the so-called menu costs, originally introduced by Sheshinski and Weiss (1977). While menu costs are unavoidable for every price change, sellers in our model can avoid the information costs should they choose not to acquire information.

incentive to limit her number of searches. The model thus provides a possible explanation for why consumers appear to undertake surprisingly little search in relevant markets.[4]

This paper contributes to the understanding of transaction costs. Dahlman (1979) classifies transaction costs into three categories based on the stages of a contract: search and information costs (precontract), bargaining and decision costs (contract), policing and enforcement costs (post-contract). While the impact of consumer search costs on market outcome has been extensively studied,[5] its interaction with sellers' information costs has so far received scant attention. A notable exception is French and McCormick (1984), whose informal analysis of the service market anticipates many of the themes explored in this paper. After showing that the winning bidder's expected profit equals the sum of his competitors' sunk costs of bid preparation under a free-entry condition, they argue that consumers indirectly pay for service providers' information costs. The focus of their paper, however, is on firms' marketing strategies, such as how likely firms charge for their estimates or advertise, whereas the focus of this paper is on the problem faced by the consumer side.[6]

Pesendorfer and Wolinsky (2003) and Wolinsky (2005) have also considered consumer search in the presence of information costs. By assuming that service outcomes are not contractible (but price searches are costless), Pesendorfer and Wolinsky (2003) examine market inefficiencies when a consumer must rely on second opinions to pick the right contractor. Under the assumption that sellers can provide better matching via costly investments, Wolinsky (2005) shows that consumers' inability to internalize sellers' costs leads to excessive search. Despite the similarity, these two papers have a different focus than ours: they are concerned with prior information on product characteristics (so there is no price dispersion), whereas our paper is concerned with prior information on prices. Because of this difference, our results on prices do not exist in their models.[7]

[4]Lee and Hogarth (2000) find that a majority of mortgage borrowers consult less than three lenders or brokers. Woodward and Hall (2012) argue that the pecuniary search costs for mortgage loans implied by the number of searches are implausibly high. See also Honka (2014) for evidence from the US auto insurance market, Allen, Clark, and Houde (2014) and Alexandrov and Koulayev (2017) from the US and Canadian mortgage markets, respectively, and Stango and Zinman (2015) from the credit card market.

[5]See Anderson and Renault (2017) for a recent survey of consumer search theories.

[6]Due to the lack of formal game-theoretic analysis, the connection between assumptions and results is somewhat opaque in their paper. For example, it is not clear whether the predicted pattern is the result of collective behavior among sellers or the noncooperative outcome.

[7]For tractability, these two models make two assumptions that are somewhat unrealistic: (1) prices are set before diagnoses; (2) search costs are not paid until a contract offer is accepted.

The current paper assumes that the consumer can commit to any chosen number of searches. A commitment is possible if the number of price quotes is observed by sellers.[8] This assumption is satisfied in the aforementioned markets. For example, in the mortgage market, lenders can infer a consumer's search intensity from the number of credit inquiries recorded in the consumer's credit report.[9] The same is true in the market for auto, home, and life insurances.[10] Admittedly, in other markets, the number of consumer searches may not be observable. We view our current analysis of the commitment case as providing a useful benchmark. In a companion paper, Miao (2020) studies consumer search behavior in a similar setting, but the number of consumer searches is not publicly observable. It is found that, in the absence of the ability to commit, consumers may be worse off when search costs decline. The two papers are complementary and apply to different markets.[11]

Our paper is also related to Burdett and Judd (1983) ("BJ-83"). In both papers, consumers engage in fixed-sample size ("FSS") searches of ex ante identical sellers. However, they assume costless price-setting. Because of this, consumer welfare is maximized when everyone searches exactly twice, with prices being set at marginal costs. Moreover, price dispersion disappears when search costs approach zero. These results are different from those of this paper.[12]

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents some preliminary results. Section 4 analyzes equilibrium properties and shows our main results. Section 5 concludes. Any formal proofs omitted from the main text are contained in the appendix.

2 The Model

A consumer is willing to pay v for a good or service (henceforth, the product), which can be provided by any one of the N sellers. Sellers are assumed to be ex ante identical. This assumption

[8]See Bagwell (1995) for a classic discussion on the relationship between commitment and observability.

[9]A bank's cost of pulling consumer credit is small relative to other costs during the loan application process, so is assumed to be negligible in this paper.

[10]During insurance application interviews, insurance companies often ask whether an applicant has other applications pending (`https://www.nerdwallet.com/article/insurance/life-insurance-application`). The answer to this question provides another indicator of the applicant's search intensity.

[11]Another difference is that this paper explicitly models sellers' decisions on whether to acquire information, whereas Miao (2020) follows Lang and Rosenthal (1991) by adopting a reduced-form assumption of bidding costs. Because of this difference, the current paper obtains a richer set of results on the optimal number of searches, which would not have been obtained in the reduced-form specification.

[12]See Section 4 for more details.

ensures that equilibrium price dispersion cannot be attributed to different cost realizations across sellers. It is also consistent with empirical research in related markets.[13]

To find the best deal, a consumer visits sellers to collect price quotes. The cost of effort for each visit is s ("search cost"). All sellers face the same exogenously given production cost of c, but it can take two values: with a probability of q, $c = c_l$, and with a probability of $1 - q$, $c = c_h$. Without loss of generality, $c_l < c_h$. Moreover, we assume that $c_h < v$ so the social value is always positive. A seller can incur t ("information cost") to acquire information about the actual production cost before offering the price quote, but the consumer cannot. This assumption is made on the ground that sellers have more expertise than consumers in the relevant markets. It also ensures that sellers do not face an adverse selection problem.[14] Both the consumer and sellers are risk-neutral.

The game is played in the following order:

1. The consumer requests price quotes from n sellers;

2. Each seller, upon request, chooses whether to incur t to acquire information about the production cost: if a seller acquires information, it quotes a price from the distribution of either $F_l(p)$ or $F_h(p)$, depending on whether the cost is c_l or c_h; if a seller does not acquire information, it quotes a price from the distribution of $F_b(p)$.

3. After receiving all price quotes, the consumer buys from the seller that offers the lowest price; if multiple sellers offer the same lowest price, the consumer randomizes her purchase among these sellers.

We look for a subgame-perfect Nash equilibrium ("SPNE"), in which the consumer minimizes her expected total costs given her (correct) beliefs about sellers' bidding strategies and sellers maximize their expected profits given the number of consumer searches and their (correct) beliefs about other sellers' bidding strategies. As is standard in the literature, we restrict our attention to symmetric equilibria, in which all sellers choose the same bidding strategy. Therefore, the

[13] For example, in her study of the auto insurance market, Honka (2014) reports that over 93% of consumers kept their coverage choice the same during the last shopping occasion and were searching only for the lowest premium. Similarly, in their study of the Canadian mortgage market, Allen et al. (2014) find that contracts are homogeneous, and for a given consumer costs are mostly common across lenders due to loan securitization and a government insurance program.

[14] In a model with a similar setup, Lauermann and Wolinsky (2017) assume that an auctioneer has private information, which affects the number of bidders she solicits. Their paper, however, does not consider *bidders'* precontract costs.

equilibrium will be characterized by the consumer's choice of n and sellers' strategies in a quadruplet $\{\alpha, F_l(p), F_h(p), F_b(p)\}$, where α is the probability of a seller choosing to acquire information about the production cost, and $F_l(p), F_h(p), F_b(p)$ are the cumulative distribution functions of price quotes as defined above. Sellers use pure strategies in pricing if and only if price distributions are degenerate.

By assumption, the consumer is engaged in a fixed-sample size ("FSS") search, as opposed to a sequential search where the consumer visits sellers one-by-one and stops search once her reservation price is met. We adopt this approach for two reasons: first, existing empirical evidence suggests that FSS search provides a more accurate description of observed consumer search behavior (De Los Santos, Hortaçsu, and Wildenbeest 2012, Honka and Chintagunta 2017); second but particularly relevant to this model, costly information acquisition can cause delay and a delay is a more significant problem for sequential search than for FSS search.[15] For example, in the US mortgage market a consumer typically receives a Loan Estimate three business days after the initial request,[16] but a lender is only required to honor the terms of a Loan Estimate for ten business days.[17] Therefore, it may actually be optimal for a consumer to request price quotes from several lenders at once, rather than one after another. For the same reason, we assume that the consumer cannot engage in multiple rounds of searches.[18]

For ease of exposition, we impose the following restrictions on parameter values. First, we assume that $s << v$ so that it is never optimal for the consumer to search just one seller, in which case she minimizes the total search costs but has to pay a monopoly price for the product. Second, we assume that the pool of sellers is so large that it never constrains the consumer's number of searches unless $n \to \infty$. These restrictions cut down the number of cases we have to consider and allow us to focus on only the nontrivial cases, but they do not change any of the qualitative results.

[15]In the same vein, Morgan and Manning (1985) and Janssen and Moraga-González (2004) argue that fixed-sample size search is more appealing when a consumer needs to gather price information quickly.

[16]"*Loan Estimate and Closing Disclosure: Your guides in choosing the right home loan*", Nicole Shea, Consumer Financial Protection Bureau (CFPB), Aug 19, 2019.

[17]"*Real Estate Settlement Procedures Act*", CFPB Consumer Laws and Regulations, March 2015.

[18]While it appears that the consumer could choose to search again after extracting information from early rounds of offers, doing so would lower sellers' incentive to acquire information and render their bids uninformative, defeating the very purpose of searching multiple rounds.

3 Preliminary Results

3.1 A Benchmark Result

A useful point of departure is to consider what happens if $t = 0$, i.e., sellers can costlessly learn the production cost. This is not only the assumption of a frictionless market, but also the working assumption of almost all consumer search models. In this case, since sellers have the same production cost, based on the standard Bertrand style argument, we can see that the prices will be set equal to the (realized) production cost as long as there are at least two sellers. A consumer cannot do better by visiting more sellers because she cannot get a better offer, nor can she do better by visiting just one seller, who will charge a monopoly price. Therefore, it is sufficient for the consumer to visit just two sellers to obtain competitive price quotes while economizing on the search costs. The consumer earns a surplus of $v - c_E - 2s$, where $c_E = qc_l + (1-q)\, c_h$ is the expected production cost. This serves as a natural benchmark for the current analysis.

Zero information cost, however, is not a necessary condition for the above benchmark outcome. The same outcome can be obtained even if the information cost is positive. It is not difficult to see why: acquiring information about the production cost allows sellers to earn information rents, but it is wasteful from the consumer's point of view.[19] If she and sellers could contract on the latter's information acquisition efforts, then the consumer would prevent sellers from earning information rents by simply requiring sellers not to acquire product cost information. The sellers would again compete *a la* Bertrand, with each of them quoting a price of c_E and earning zero profits. The consumer would earn the same amount of expected surplus as in the benchmark. Albeit straightforward, this result demonstrates that the information cost, in itself, does not necessarily cause welfare loss for the consumer. Rather, any loss of efficiency is due to the inability to contract on sellers' information acquisition efforts. Of course, if the cost of effort is so high that it exceeds the private value of information, which equals $q\,(1-q)\,(c_h - c_l)$, then neither seller will make an effort even if they are not contractually precluded from doing so.

[19] It should be noted, however, this is true because we assume $c_h < v$. Otherwise, information acquisition will not be a pure waste. We are grateful to an anonymous reviewer for making this point. To analyze the case where $c_h > v$ will require us to significantly expand the game and is thus beyond the scope of this paper.

In all the above cases, consumer surplus is maximized when the consumer obtains two competing price quotes. There is no further gain from requesting additional price quotes.[20] Accordingly, we summarize the results in the following proposition:

Proposition 1 *Consumer surplus is maximized in the unique equilibrium if either (i) $t = 0$; or (ii) $t \geq q(1-q)(c_h - c_l)$, or (iii) the consumer and sellers can contract on the latter's information acquisition effort. In all three cases, the optimal number of searches is two.*

3.2 The Subgame Equilibrium at the "Bidding" Stage

For the remainder of the paper, we focus on the more interesting case, in which the cost of information acquisition is small but positive, i.e., $t \in (0, q(1-q)(c_h - c_l))$. In the (stage 2) subgame, the $n \geq 2$ sellers that have received requests for price quotes decide simultaneously whether or not to acquire production cost information and what prices to quote. Recall that we look for symmetric equilibria where all sellers use the same strategy $\{\alpha, F_l(p), F_h(p), F_b(p)\}$.

Lemma 1 *If $t \in (0, q(1-q)(c_h - c_l))$, there is no symmetric SPNE in which sellers learn the production cost with probability $\alpha \in \{0, 1\}$.*

Lemma 1 implies that a symmetric equilibrium in the subgame at the bidding stage must be in mixed strategies. It involves two randomizations for sellers: first, sellers randomize between submitting an informed quote and submitting a blind quote; second, sellers randomize their price quotes. The solution is given by Lemma 2.

Lemma 2 *If $t \in (0, q(1-q)(c_h - c_l))$, each seller chooses to acquire information about the production cost with a probability of $\alpha = 1 - \left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)^{1/(n-1)}$. If a seller learns that the production cost is c_h, he quotes a price of c_h, i.e., $F_h(p) = 0$ for $p < c_h$ and $F_h(p) = 1$ for $p \geq c_h$; otherwise, he quotes a price according to the distribution of $F_l(p) = \frac{1-\left(\frac{t}{q(p-c_l)}\right)^{1/(n-1)}}{\alpha}$ on the support of $[\underline{p}_l, \bar{p}_l]$, with $\underline{p}_l = c_l + t/q$ and $\bar{p}_l = c_h - t/(1-q)$. If a seller does not acquire information, he quotes a price according to the price distribution of $F_b(p) = 1 - \frac{\alpha}{1-\alpha}\left(\left(\frac{q(p-c_l)}{(1-q)(c_h-p)}\right)^{\frac{1}{n-1}} - 1\right)^{-1}$ on the support of $[\underline{p}_b, \bar{p}_b]$, with $\underline{p}_b = \bar{p}_l$ and $\bar{p}_b = c_h$. The expected price paid is $c_E + n\alpha t$.*

[20] It goes without saying, the result will be different if the sellers do not have the same production cost. In MacMinn (1980) and Spulber (1995), sellers' price setting is equivalent to bidding in a private value auction. Price dispersion arises from cost heterogeneity of sellers. Alternatively, even if sellers have the same cost, consumer heterogeneity in captivity leads to an asymmetric mixed strategy Nash equilibrium (Gilgenbach 2015).

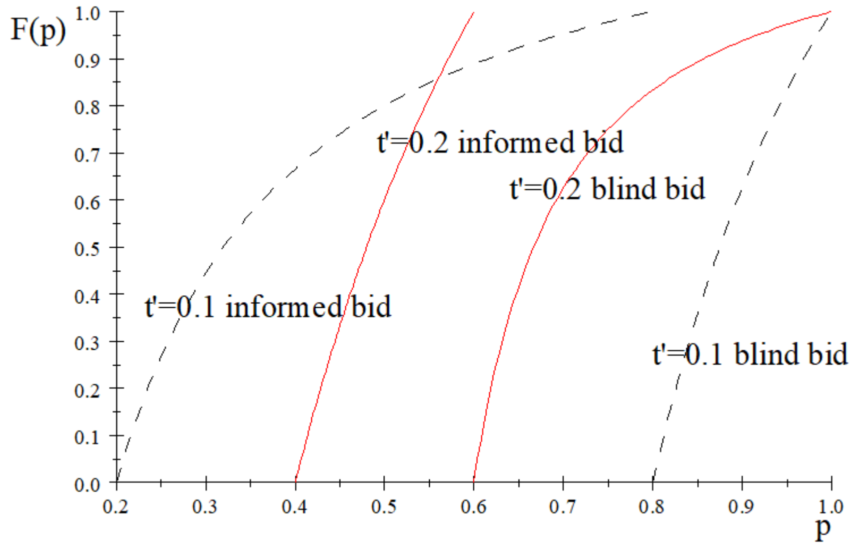

Figure 1: ($q = 1/2$) The red solid curve depicts the price distribution when $t' = t/(c_h - c_l) = 0.2$ and the black dashed curve when $t' = 0.1$.

Figure 1 illustrates how the information cost, t, affects price distributions. The red solid curve depicts the price distribution when $t = 0.2(c_h - c_l)$ and the black dashed curve when $t = 0.1(c_h - c_l)$. For each level of information cost, there are two segments of price distributions, corresponding to informed bids and blind bids.

From the graph, we can see that sellers are more likely to set low blind bids when t is large. Intuitively, a larger t makes sellers less likely to acquire costly information and this means uninformed sellers are less likely to suffer from the winner's curse. As a result, uninformed sellers bid more aggressively. The effects of a larger t on the bidding behavior of a seller informed of a low cost, however, are more complicated: on the one hand, having fewer competing bids raises the lower bound of informed bids; on the other hand, more aggressive bidding by uninformed sellers lowers the upper bound of informed bids. Therefore, a larger t decreases the degree of dispersion in informed bids.[21] These observations are summarized in Lemma 3:

Lemma 3 (i) *The range of informed bids decreases with t;* (ii) *The range of blind bids increases with t; moreover, blind bids for t first-order stochastically dominates blind bids for t' if $t < t'$.*

[21] A standard measure of dispersion is the variance in prices. Unfortunately, we do not have an analytical solution for the variance. Other commonly used metrics to measure price dispersion include the range of prices (Baye, Morgan, and Scholten 2006), which is the one used here.

4 The Optimal Number of Searches

Lemma 2 also shows that, when sellers acquire information (i.e., $t \in (0, q(1-q)(c_h - c_l))$), the consumer pays for information costs indirectly in the form of a higher expected price. This gives us the basic intuition of why prices can increase with the number of searches and why the consumer may want to limit her number of searches (on top of the incentive to save on search costs). However, this does not mean that the consumer should always search as little as possible, since a smaller number of searches will give each seller a greater incentive to earn information rents, as α decreases with n. In other words, neither total information costs nor the expected price is monotonic in n. Basically, as n increases, there are two competing effects on prices: a competitive effect that pushes down prices and a compensation effect that drives up prices. The relative importance of these two effects determines the optimal number of searches.

The consumer surplus can be written as $v - c_E - \min_{n \geq 2}(\alpha t + s)n$. Relative to the benchmark case, it is lower by $\min_{n \geq 2}(\alpha t + s)n - 2s$. The term $(\alpha t + s)n$ captures the overall impact of precontract costs, including consumer search cost and seller information cost, on consumer welfare. It does not contribute to sellers' profit margin and is merely a waste caused by market frictions, but for the lack of better names we shall call it the expected markup and denote it by $\psi(n, s, t)$. For ease of exposition, following Wolthoff (2017), we treat n as a continuous variable.[22]

We start by examining how the expected markup (essentially the negative of consumer surplus) varies with the number of searches for different combinations of parameter values. Due to its technical nature, we relegate the details of the analysis to the appendix (Lemma 4) but summarize the results here. According to Lemma 4, there are three possible patterns of how the expected markup varies with the number of searches. Figure 2 illustrates these possibilities. When the information cost t is small (the blue dashed curve at the bottom), the expected markup monotonically increases with n; when t is large but s is zero (the blue dotted curve in the middle), the expected markup first increases with n, then decreases; when t is large and s is positive (the red solid curve at the top), the expected markup first increases, then decreases between n_1 and n_2, and then increases again for $n > n_2$, where n_1 is a local maximum and n_2 is a local minimum as defined in Lemma 4.

[22] Treating n as a continuous variable simplifies the analysis in two aspects. First, when the minimizing solution is not an integer, e.g., 2.5, we don't have to compare $\psi(2)$ and $\psi(3)$ to find the optimal n. Second, when doing comparative statics, it allows us to take differentiation and use the envelope theorem.

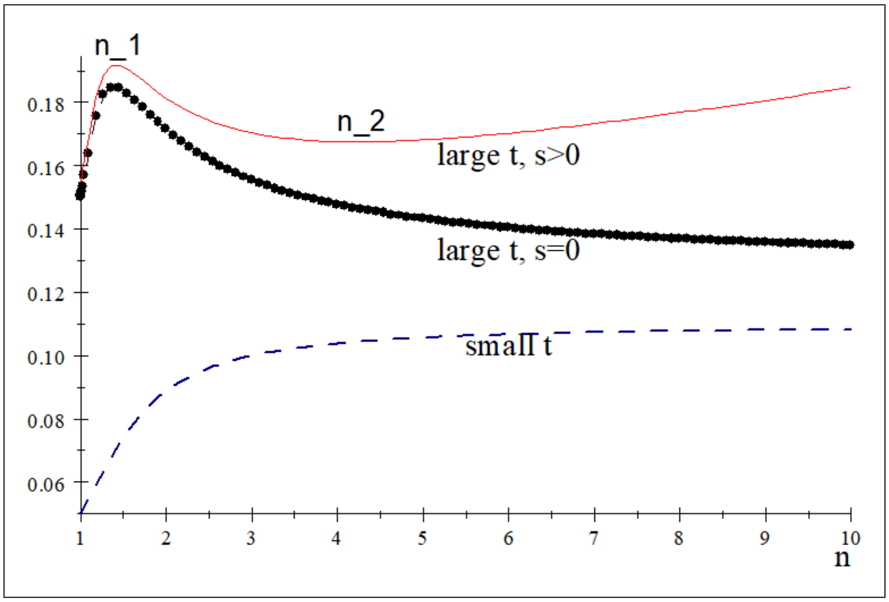

Figure 2: $(q = 1/2)$ The expected markup as a function of n. Blue dashed $(t = 0.05\,(c_h - c_l)\,, s = 0)\,,$ Black dotted $(t = 0.15\,(c_h - c_l)\,, s = 0)\,,$ Red solid $(t = 0.15\,(c_h - c_l)\,, s = 0.005\,(c_h - c_l))\,.$

Hence, there are at most three candidates for the optimal number of price quotes: 2, n_2, (the local minimum) or infinity, with the last candidate, infinity, being optimal only if $s = 0$. Therefore, the optimal number of price quotes can simply be determined by comparing three numbers, $\psi(2)$, $\psi(n_2)$ and $\psi(\infty)$. The remaining difficulty is to determine $\psi(n_2)$, as n_2 does not have an analytical solution, but we rely on studying the monotonicity and variation of the expected markup as a function of n to accomplish the task.

Suppose that n^o is the optimal number of price quotes, then we must have $\psi(n^o, s, t) \leq \psi(n, s, t)$ for all $n \geq 2$. Proposition 2 summarizes the choices of n^o for different parameter values of s and t.

Proposition 2 *If $s = 0$, then $n^o > 2$ if and only if $\frac{t}{c_h - c_l} > \frac{q(1-q)}{4.92(1-q)+q}$; otherwise, $n^o = 2$;*

if $s > 0$, then $n^o > 2$ if and only if (i) $\frac{t}{c_h - c_l} > \frac{q(1-q)}{(1-q)e+q}$ and $s/t < e^{-\gamma}(2\gamma + 1) - 1$, or (ii) $\frac{t}{c_h - c_l} < \frac{q(1-q)}{(1-q)e+q}$ and $s/t < e^{\gamma - 2}(4/\gamma - 1) - 1$ and $2(1 + s/t - e^{-\gamma}) > \frac{(z^* + \gamma)^2}{\gamma} \exp(-z^*)$, where $\gamma = -\ln \frac{t(1-q)}{q((c_h - c_l)(1-q) - t)}$ and z^* is the smaller root for $\gamma \frac{(1+s/t)\exp z - 1}{z(z+\gamma)} = 1$; otherwise, $n^o = 2$.

Proposition 2 shows that, frequently, the optimal number of searches is two.[23] We can see this result more clearly from Figure 3, where $q = 1/2$. The optimal number of searches is two in all

[23] Honka (2014) finds that consumers get on average 2.96 quotes with the majority of consumers collecting two or three quotes when purchasing auto insurance policies.

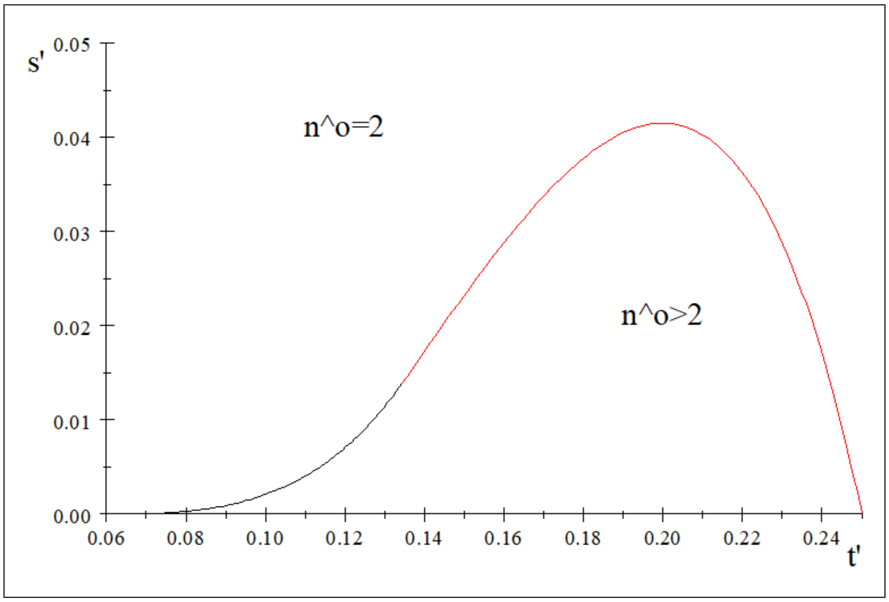

Figure 3: $(q = 1/2)$ The Optimal Number of Searches, where $t' = t/(c_h - c_l)$ and $s' = s/(c_h - c_l)$.

regions but the lower-right corner. It is true even when the search cost goes to zero, in which case the consumer searches twice as long as the information cost is not too large. (For reasons elaborated below, the consumer may search more when the information cost increases.) If the search cost is positive, then the optimal number of searches is not monotonic in t, a result formally established in Proposition 3. It does not pay to search more than two sellers unless t is in the mid-range.

At first glance, the result that a consumer only needs to search twice in a market of homogeneous sellers may not be surprising. For example, the same result holds in BJ-83 based on the standard Bertrand style argument. However, there is a crucial difference. In BJ-83, if a consumer searches twice, then there will be no price dispersion, eliminating the need for further search. In the present model, a consumer searches twice despite price dispersion, because additional searches would change sellers' bidding strategies, potentially raising prices. It follows that, if search costs are zero, the "law of one price" will hold in BJ-83, but not in the present model.

Our result is more similar to that of Che and Gale (2003), who find it optimal to include only two contestants in a research contest.[24] In their model, restricting entry to two competitors decreases the coordination problem of competing contestants and minimizes the duplication of fixed costs.

[24]A similar result is also found in auctions with entry (e.g., Harstad 1990, Levin and Smith 1994), and for research tournaments (Taylor 1995, Fullerton and McAfee 1999). However, in all these models, the number of bidders is revealed before bidding takes place.

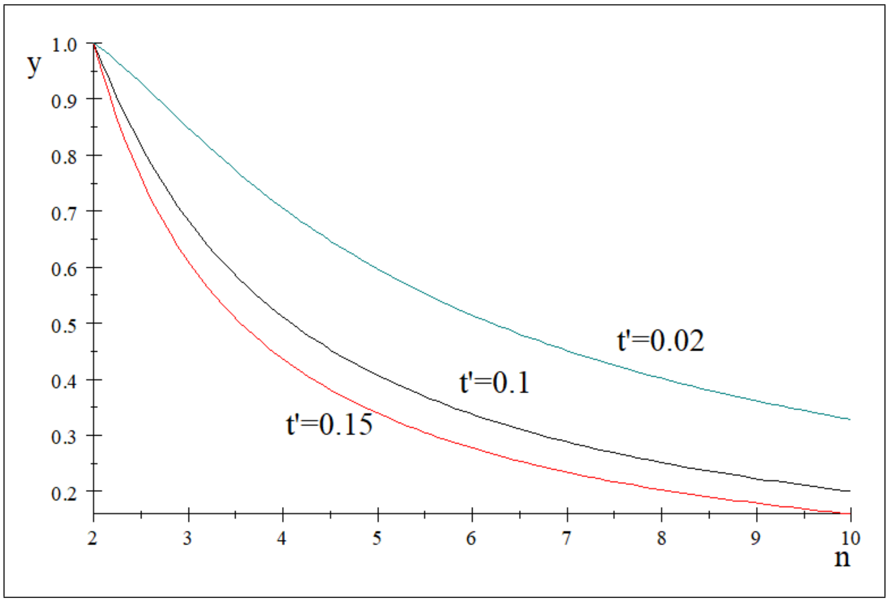

Figure 4: $(q = 1/2)$ $y = \alpha(n)/\alpha(2)$, when $t' = t/(c_h - c_l) = 0.02, 0.1, 0.15$

Similarly, in the present model, limiting the number of bidders reduces duplication in wasteful information acquisition. However, unlike other papers with a similar result, the present paper also shows that the consumer can sometimes benefit from expanding her search effort, especially when the search cost is small and the information cost is relatively large. The first effect is quite obvious, but the second one is not. When the information cost rises, one might expect the consumer to search less since she has to indirectly pay for sellers' information costs, but this intuition is incomplete because it ignores the effect of an increase in n on sellers' incentive to acquire information. To see this effect clearly, we plot $\alpha(n)/\alpha(2)$ in Figure 4. The graph illustrates three results: first, the propensity to acquire information, α, decreases with n across all ranges of t; second, sellers are less likely to acquire information when t is large; these two results are obvious. Less obvious is the third result, namely, α decreases at a faster rate when t is large. This means that increasing the number of searches can potentially reduce the wasteful and duplicative information acquisition efforts for a large t.

4.1 Comparative Statics

Next, we examine the impact of search cost and information cost on the equilibrium outcome. The first result is immediate from the preceding discussion.

Proposition 3 *The optimal number of searches n^o*

(*i*) *decreases with s;*

(*ii*) *is non-monotonic in t if $s > 0$, but increases with t if $s = 0$.*

According to Proposition 3, search costs and information costs have different impacts on consumers' search behavior, even though they both contribute to the overall precontract costs. This means that it is not only the total costs, but also the composition of the costs, that matter to consumer search. A similar result exists for two-sided markets, but the underlying mechanism is much different. In two-sided markets the composition of costs matters because the costs imposed on one side cannot be fully internalized by the other side of the market, whereas in this model it matters despite consumers' full internalization of sellers' costs.

Let $\varphi(s,t) = \min_{n \geq 2} (\alpha t + s) n$ denote the minimized expected markup as a function of s and t. Proposition 4 examines the welfare impact of the two costs.

Proposition 4 *The expected markup $\varphi(s,t)$*

(*i*) *monotonically increases with s;*

(*ii*) *is unimodal in t.*

Perhaps not surprisingly, the consumer benefits from a lower search cost. More interestingly, the consumer can benefit from a further increase in the information cost when it is already large. This directly follows from our earlier observation that a high information cost can discourage sellers from engaging in wasteful and duplicative information acquisition efforts, which are indirectly paid by the consumer. Another way to understand why the expected total expense is not monotonic in the information cost is to recall Proposition 1: when $t = 0$, by definition the cost of information acquisition effort will be zero; when $t \geq q(1-q)(c_h - c_l)$, there will be no wasteful expenditure on information acquisition because the cost exceeds the value of the information. Between the two extremes, however, there will be positive amounts of (wasteful) information acquisition efforts.

4.2 Economic Significance of Information Costs

While the above model generates some interesting results, they will not matter much if the information cost only has a small impact on consumer welfare. In order to evaluate its economic

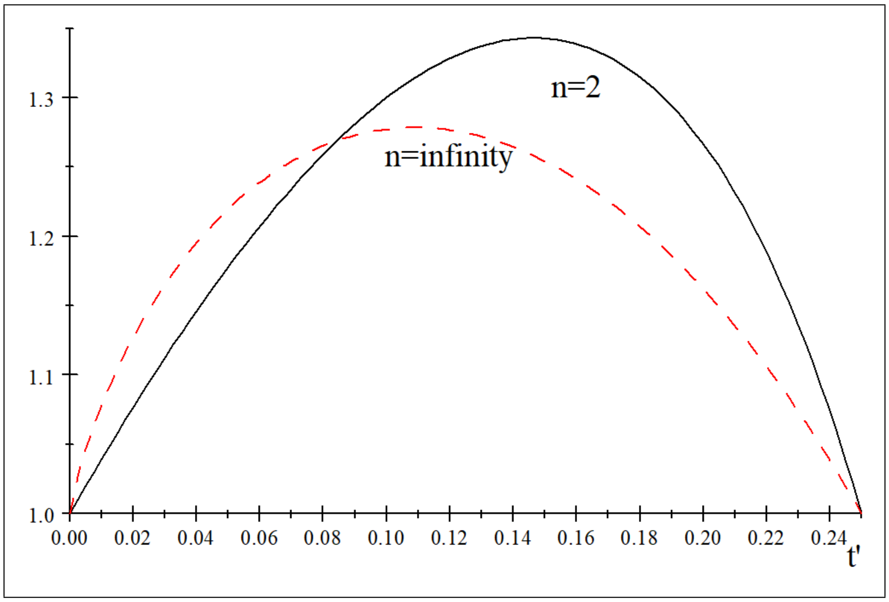

Figure 5: $(q = 1/2)\ \varphi(0,t)/c_E$ as a function of $t' = t/(c_h - c_l)$. The solid curve has $n = 2$ and the dashed $n \to \infty$.

significance, we compare the expected total expense under costly information acquisition to the benchmark case, in which the total expense is just the expected production cost plus the search costs, $c_E + 2s$. To focus on the impact of information cost, we further assume $s = 0$, in which case n^o is either 2 or ∞. Another measure that can be used is the expected price plus the information costs, but it generates the same qualitative result.[25]

Figure 5 plots $\varphi(0,t)/c_E$ when $q = 1/2$, with the solid curve corresponding to the case of $n = 2$ and the dashed $n \to \infty$, so the lower envelope is the expected cost when n is optimally chosen. As we can see from the graph, the existence of a small incontractible information cost can potentially increase the consumer's total expense by more than thirty percent. This demonstrates that it is not a negligible cost and should be taken seriously not only for its theoretical interests, but also for its practical importance. Interestingly, it offers an alternative explanation for why car buyers obtain significantly more of the surplus available under customer rebates than under dealer discounts, a finding that is counter to the simple invariance of incidence analogy (Busse, Silva-risso, and Zettelmeyer 2006). Busse et al. (2006) test several hypotheses and find evidence consistent with the asymmetric information hypothesis, that is, car buyers are disadvantaged in negotiations because they are less informed than dealers about the availability of dealer discounts. In contrast,

[25] Consumer surplus is a less appropriate measure because it depends on v, an arbitrary parameter in the model.

the parties are symmetrically informed about the availability of customer rebates, which are always publicized to potential customers, often in prime-time television advertisements. Note that their explanation is based on the assumption that the information about dealer discounts is readily accessible to dealers. However, these discounts are often in the form of conditional discounts, depending on the geographical location and/or the specific equipment package, or "trim level". This means that there may be higher information costs for dealer discounts than for customer rebates.[26] Thus, the result that dealer discounts have a smaller pass-through can also be predicted by our model.

5 Conclusion

When consumers search, they incur costs. To provide consumers with the information they search for, sellers may also incur costs. This paper departs from the extant literature by assuming that sellers must make an effort to learn the cost of providing the goods/services before they bid against other sellers. Recent empirical studies have documented surprisingly few searches conducted by consumers when shopping for financial products. The lack of consumer search has been attributed to high search costs and non-price preferences. Our result, however, suggests that the choice of a small sample size when consumers search is not necessarily due to high search costs. It is also consistent with the existence of information costs. Empirical studies that do not take into account sellers' information costs may overestimate consumer search costs or the impact of other factors.

It is worth noting that underlying our analysis a key assumption is that the number of searches is publicly observable. More realistically, the number of searches may be observed by some sellers but not all. Or, some measure of search intensity is observable, but the precise number of searches is not. In other words, the number of searches may be observed imperfectly. Although we believe our qualitative results will continue to hold, we hope to relax the assumption of perfect observability and explore related issues in future researches.

[26] According to a website specialized in automobile markets: "Even if you are the only customer in the dealership, there is still no guarantee you'll be able to get a deal offer in a flash. If you're taking out a loan, the sales manager might have to run your credit to get your credit score. He'll call the finance department to get your interest rate, and then look up specials and incentives on your car to make sure you're getting the right program offer for the right car. Sometimes it just takes a while to get all the information together." Matt Jones, "Behind the Scenes at A Car Dealership", April 29th, 2016, `https://www.edmunds.com/car-buying/behind-the-scenes-at-a-car-dealership.html`.

Another potential extension of the model is to study search intermediaries such as brokers, who play a prominent role in relevant markets. In a two-sided matching model, Shi and Siow (2014) find that brokers can help reduce the costs of market participants through inventory management, thereby improving welfare. Similarly, introducing intermediaries into our model and analyzing the welfare consequence will help us better understand their roles in search markets.

References

Alexandrov, A. and S. Koulayev (2017): "No shopping in the US mortgage market: Direct and strategic effects of providing more information," Consumer Financial Protection Bureau Office of Research Working Paper No. 2017-01.

Allen, J., R. Clark, and J.-F. Houde (2014): "The effect of mergers in search markets: Evidence from the Canadian mortgage industry," *American Economic Review*, 104, 3365–3396.

Anderson, S. and R. Renault (2017): "Firm pricing with consumer search," in *Handbook of Game Theory and Industrial Organization*, ed. by L. Corchon and M. Marini, Edward Elgar Publishing, chap. 8, 177–224.

Bagwell, K. (1995): "Commitment and observability in games," *Games & Economic Behavior*, 8, 271–280.

Baye, M. R. and J. Morgan (2001): "Information gatekeepers on the internet and the competitiveness of homogeneous product markets," *American Economic Review*, 454–474.

Baye, M. R., J. Morgan, and P. Scholten (2006): "Information, search, and price dispersion," *Handbook on Economics and Information Systems*, 1, 323–375.

Burdett, K. and K. L. Judd (1983): "Equilibrium price dispersion," *Econometrica*, 51, 955–969.

Busse, M., J. Silva-risso, and F. Zettelmeyer (2006): "$1,000 cash back: The pass-through of auto manufacturer promotions," *American Economic Review*, 96, 1253–1270.

Che, Y.-K. and I. Gale (2003): "Optimal design of research contests," *American Economic Review*, 93, 646–671.

Dahlman, C. J. (1979): "The problem of externality," *Journal of Law & Economics*, 22, 141–162.

De Los Santos, B., A. Hortaçsu, and M. R. Wildenbeest (2012): "Testing models of consumer search using data on web browsing and purchasing behavior," *American Economic Review*, 102, 2955–2980.

Estrada, R. and M. Pavlovic (2017): "L'Hôpital's monotone rule, Gromov's theorem, and operations that preserve the monotonicity of quotients," *Publications de l'Institut Mathematique*, 101, 11–24.

French, K. R. and R. E. McCormick (1984): "Sealed bids, sunk costs, and the process of competition," *The Journal of Business*, 57, 417.

Fullerton, R. L. and R. P. McAfee (1999): "Auctioning entry into tournaments," *Journal of Political Economy*, 107, 573–605.

Gilgenbach, R. G. (2015): "Can a decline in search cost increase prices?" *Canadian Journal of Economics/Revue canadienne d'économique*, 48, 1381–1402.

Harstad, R. M. (1990): "Alternative common-value auction procedures: Revenue comparisons with free entry," *Journal of Political Economy*, 98, 421–429.

Honka, E. (2014): "Quantifying search and switching costs in the US auto insurance industry," *The RAND Journal of Economics*, 45, 847–884.

Honka, E. and P. Chintagunta (2017): "Simultaneous or sequential? Search strategies in the US auto insurance industry," *Marketing Science*, 36, 21–42.

Janssen, M. C. W. and J. L. Moraga-González (2004): "Strategic pricing, consumer search and the number of firms," *Review of Economic Studies*, 71, 1089–1118.

Lang, K. and R. W. Rosenthal (1991): "The contractors' game," *The RAND Journal of Economics*, 22, 329–338.

Lauermann, S. and A. Wolinsky (2017): "Bidder solicitation, adverse selection, and the failure of competition," *American Economic Review*, 107, 1399–1429.

LEE, J. AND J. M. HOGARTH (2000): "Relationships among information search activities when shopping for a credit card," *Journal of Consumer Affairs*, 34, 330–360.

LEVIN, D. AND J. L. SMITH (1994): "Equilibrium in auctions with entry," *American Economic Review*, 84, 585–599.

MACMINN, R. D. (1980): "Search and market equilibrium," *Journal of Political Economy*, 88, 308–327.

MIAO, C.-H. (2020): "Search and bidding costs," *The B.E. Journal of Economic Analysis and Policy*, 20, Letter 1–7.

MORGAN, P. AND R. MANNING (1985): "Optimal search," *Econometrica*, 53, 923–944.

PESENDORFER, W. AND A. WOLINSKY (2003): "Second opinions and price competition: inefficiency in the market for expert advice," *Review of Economic Studies*, 70, 417–437.

SHESHINSKI, E. AND Y. WEISS (1977): "Inflation and costs of price adjustment," *The Review of Economic Studies*, 44, 287–303.

SHI, X. AND A. SIOW (2014): "Information externalities and intermediaries in frictional search markets," *Canadian Journal of Economics/Revue canadienne d'économique*, 47, 1131–1152.

SPULBER, D. F. (1995): "Bertrand competition when rivals' costs are unknown," *The Journal of Industrial Economics*, 1–11.

STANGO, V. AND J. ZINMAN (2015): "Borrowing high versus borrowing higher: price dispersion and shopping behavior in the US credit card market," *The Review of Financial Studies*, 29, 979–1006.

STIGLER, G. J. (1961): "The economics of information," *Journal of Political Economy*, 69, 213–225.

TAYLOR, C. R. (1995): "Digging for golden carrots: An analysis of research tournaments," *American Economic Review*, 872–890.

WOLINSKY, A. (2005): "Procurement via sequential search," *Journal of Political Economy*, 113, 785–810.

Wolthoff, R. (2017): "Applications and interviews: Firms' recruiting decisions in a frictional labour market," *The Review of Economic Studies*, 85, 1314–1351.

Woodward, S. E. (2008): "A study of closing costs for FHA mortgages," *US Department of Housing and Urban Development, Office of Policy Development and Research.*

Woodward, S. E. and R. E. Hall (2012): "Diagnosing consumer confusion and sub-optimal shopping effort: Theory and mortgage-market evidence," *American Economic Review*, 102, 3249–76.

Appendix

A Proof

Proof of Lemma 1. If all sellers learn the true cost, since this is a common value auction, they will all quote the same price. As a result, net of the information cost t, their profits are negative. If a seller deviates by not incurring the information cost and quoting c_h, his profit will be zero, so the deviation is profitable. Conversely, if no seller learns the true cost, then the price must be c_E and sellers' profits will be zero. If a seller learns the true cost, he can charge $c_E - \varepsilon$ if the production cost is revealed to be c_l. This happens with a probability of q, so his expected profit is $q\left(c_E - c_l\right) = q\left(1-q\right)\left(c_h - c_l\right)$. It is a profitable deviation if $t < q\left(1-q\right)\left(c_h - c_l\right)$. ■

Proof of Lemma 2. First, $F_h\left(p\right)$ must be a degenerate distribution, where $F_h\left(p\right) = 0$ for $p < c_h$ and $F_h\left(p\right) = 1$ for $p \geq c_h$. Suppose to the contrary that $F_h\left(p\right) \sim \left[\underline{p_h}, \bar{p}_h\right]$. Then p_h must be greater than c_h, otherwise profits will be negative. Also, $f_b\left(p\right)$ cannot be positive on $\left[\underline{p_h}, \bar{p}_h\right]$, otherwise an uninformed seller can earn a positive profit. Thus, if $c = c_h$, then the profit from charging $\bar{p}_h$ must be 0. In turn, due to the indifference principle, the profit from p_h must be 0, too. This means that $p_h = c_h$. Hence, $\bar{p}_h = \underline{p_h}$, otherwise a $p \in \left(\underline{p_h}, \bar{p}_h\right)$ will generate a positive profit, contradiction.

Second, neither $F_l\left(p\right)$ nor $F_b\left(p\right)$ has an atom on its support. Suppose to the contrary that $F_l\left(p\right)$ has an atom at $p_m \in \left[\underline{p_l}, \bar{p}_l\right]$. Given that all other informed sellers bid according to $F_l\left(p\right)$, consider an individual seller that deviates from $F_l\left(p\right)$ by moving the mass point from p_m to $p_m - \varepsilon$. The deviation increases the seller's winning probability by a discrete amount. Because ε is arbitrarily small, the seller's profit in the case of winning is arbitrarily close to the profit before deviation. Thus, the deviation increases the seller's expected profit by a discrete amount, contradicting the fact that $F_l\left(p\right)$ is an equilibrium bid distribution. Similarly, $F_b\left(p\right)$ does not have a mass point.

Third, the supports of the two price distributions $F_l\left(p\right)$ and $F_b\left(p\right)$ do not overlap. Suppose to the contrary that they overlap. If seller i chooses not to learn the cost, then his expected profit is

$$\pi_i = \int_{\underline{p_b}}^{\bar{p}_b} dF_b\left(p\right) \left(\begin{array}{c} q\left(p - c_l\right) \sum_{k=0}^{n-1} \binom{n-1}{k} \alpha^k \left(1 - \tilde{F}_l\left(p\right)\right)^k \left(1-\alpha\right)^{n-1-k} \left(1 - \tilde{F}_b\left(p\right)\right)^{n-1-k} \\ + \left(1-q\right)\left(p - c_h\right) \sum_{k=0}^{n-1} \binom{n-1}{k} \alpha^k \left(1-\alpha\right)^{n-1-k} \left(1 - \tilde{F}_b\left(p\right)\right)^{n-1-k} \end{array} \right);$$

if he chooses to learn the cost, then his expected profit is

$$\pi_i = q\int_{\underline{p_l}}^{\bar{p}_l} (p-c_l)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}\left(1-\tilde{F}_l(p)\right)^k\left(1-\tilde{F}_b(p)\right)^{n-1-k}dF_l(p)-t,$$

where $\tilde{F}_l(p)$ and $\tilde{F}_b(p)$ are the corresponding price distributions for the $n-1$ other sellers. Because of symmetry, $\tilde{F}_l(p)\equiv F_l(p)$ and $\tilde{F}_b(p)\equiv F_b(p)$. In a mixed strategy equilibrium, seller i must be indifferent among all choices of p on the support. Hence,
$q(p-c_l)[\alpha(1-F_l(p))+(1-\alpha)(1-F_b(p))]^{n-1}+(1-q)(p-c_h)[\alpha+(1-\alpha)(1-F_b(p))]^{n-1} = \Lambda_1$ for $p\in\left[\underline{p_b},\bar{p}_b\right]$ and $q(p-c_l)[(\alpha(1-F_l(p))+(1-\alpha)(1-F_b(p)))]^{n-1}=\Lambda_2$ for $p\in\left[\underline{p_l},\bar{p}_l\right]$, where Λ_1 and Λ_2 are two constants. Thus, for $p\in\left[\underline{p_b},\bar{p}_b\right]\cap\left[\underline{p_l},\bar{p}_l\right]$, we have
$\Lambda_2+(1-q)(p-c_h)(\alpha+(1-\alpha)(1-F_b(p)))^{n-1}=\Lambda_1$. At the same time, since α must be such that sellers are indifferent between learning the cost and not learning in the equilibrium, we must have $\Lambda_2-t=\Lambda_1$. Hence, $(\alpha+(1-\alpha)(1-F_b(p)))^{n-1}=\frac{t}{(1-q)(c_h-p)}$. Since the LHS decreases with p, but the RHS increases with p, contradition.

Fourth, the supports of $F_l(p)$ and $F_b(p)$ are connected. Suppose that they are not and that $\bar{p}_l<\underline{p_b}$, then there exists $\hat{p}\in\left(\bar{p}_l,\underline{p_b}\right)$. A seller's expected profit from quoting $\hat{p}$ must be greater than from quoting $\bar{p}_l$, since the winning probability is the same but the price is higher. At the same time, the seller is indifferent among all $p\in\left[\underline{p_l},\bar{p}_l\right]$. Hence, quoting $\hat{p}$ is a profitable deviation. Contradiction. Hence, we must have $\bar{p}_l\geq\underline{p_b}$. Similarly, we must have $\bar{p}_b\geq\underline{p_l}$. Since the distributions do not overlap, we must have either $\bar{p}_l=\underline{p_b}$. or $\bar{p}_b=\underline{p_l}$.

Last, since $\left[\underline{p_b},\bar{p}_b\right]$ and $\left[\underline{p_l},\bar{p}_l\right]$ do not overlap, if seller i chooses not to learn the cost, then his expected profit is

$$\pi_i=\int_{\underline{p_b}}^{\bar{p}_b} dF_b(p)\begin{pmatrix} q(p-c_l)(1-\alpha)^{n-1}\left(1-\tilde{F}_b(p)\right)^{n-1} \\ +(1-q)(p-c_h)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}\left(1-\tilde{F}_b(p)\right)^{n-1-k}\end{pmatrix};$$

if he chooses to learn the cost, then his expected profit is
$\pi_i=q\int_{\underline{p_l}}^{\bar{p}_l}(p-c_l)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}\left(1-\tilde{F}_l(p)\right)^k dF_l(p)-t$. In a mixed strategy equilibrium, seller i must be indifferent among all choices of p on the support, i.e.,
$q(p-c_l)(1-\alpha)^{n-1}(1-F_b(p))^{n-1}+(1-q)(p-c_h)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(1-F_b(p))^{n-1-k}$ $=\Delta_1$ for $p\in\left[\underline{p_b},\bar{p}_b\right]$. Since competing sellers are identical, they must all earn zero expected profits

in the equilibrium. Hence, we must have

$q(p-c_l)(1-\alpha)^{n-1}(1-F_b(p))^{n-1}+(1-q)(p-c_h)(\alpha+(1-\alpha)(1-F_b(p)))^{n-1}=0$, i.e., $F_b(p)=1-\frac{\alpha}{1-\alpha}\left(\left(\frac{q(p-c_l)}{(1-q)(c_h-p)}\right)^{\frac{1}{n-1}}-1\right)^{-1}$.

Similarly, we have $q(p-c_l)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(1-F_l(p))^k=t$ for $p\in[\underline{p_l},\bar{p}_l]$. Hence, $(\alpha-\alpha F_l(p)+(1-\alpha))^{n-1}=\frac{t}{q(p-c_l)}$, i.e., $F_l(p)=\left(1-\left(\frac{t}{q(p-c_l)}\right)^{1/(n-1)}\right)/\alpha$. Since $F_b(\underline{p_b})=0$ by definition, we must have $q(\underline{p_b}-c_l)(1-\alpha)^{n-1}+(1-q)(\underline{p_b}-c_h)=0$. Solving, we obtain $\underline{p_b}=\frac{(1-q)c_h+q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. Similarly, we can obtain $\bar{p}_b=c_h$, $\underline{p_l}=c_l+t/q$ and $\bar{p}_l=c_l+\frac{t}{q(1-\alpha)^{n-1}}$. Since $\bar{p}_b>\underline{p_l}$, we must have $\bar{p}_l=\underline{p_b}$. Hence, $c_l+\frac{t}{q(1-\alpha)^{n-1}}=\frac{(1-q)c_h+q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. From this, we can solve for the information acquisition probability: $\alpha=1-\left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)^{1/(n-1)}$. Plugging it back into $\underline{p_b}$, we obtain $\underline{p_b}=c_h-t/(1-q)$. The expected price is

$$
\begin{aligned}
E(p) &= \alpha^n(1-q)c_h+\alpha^n q\int pd\left(1-(1-F_l(p))^n\right)+(1-\alpha)^n\int pd\left(1-(1-F_b(p))^n\right) \\
&\quad+\sum_{k=1}^{n-1}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}\left((1-q)\int pd\left(1-(1-F_b(p))^{n-k}\right)+q\int pd\left(1-(1-F_l(p))^k\right)\right) \\
&= \alpha^n(1-q)c_h+n\int_0^1\left(\begin{array}{c}(1-\alpha)^n(1-F)^{n-1}\left(q+(1-q)\sum\limits_{k=0}^{n-1}\binom{n-1}{k}\left(\frac{\alpha}{(1-\alpha)(1-F)}\right)^k\right)F_b^{-1}(F) \\ +q\sum\limits_{k=1}^{n}\binom{n-1}{k-1}\alpha^k(1-\alpha)^{n-k}(1-F)^{k-1}F_l^{-1}(F)\end{array}\right)dF \\
&= qc_l+(1-q)c_h+n\alpha t=c_E+n\alpha t.
\end{aligned}
$$

Alternatively, we can derive the result based on the observations that sellers earn zero expected profits and the total surplus equals $v-c_E-ns-n\alpha t$. ■

Proof of Lemma 3. By Lemma 2, $\underline{p_l}=c_l+t/q$, $\bar{p}_l=\underline{p_b}=c_h-t/(1-q)$, $\bar{p}_b=c_h$. The first two results immediately follow. Last, since α decreases with t, $F_b(p)$ must increase with t, implying that blind bids for a smaller t first-order stochastically dominates blinds bids for a larger t. ■

Lemma 4 *For given values of s and t, let $\psi(n)=\psi(n,s,t)=(\alpha t+s)n$ and $\gamma=-\ln\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}$, $\alpha=1-\left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)^{1/(n-1)}$, hence $\frac{1}{n-1}\gamma=-\ln(1-\alpha)$, i.e., $z=-\ln(1-\alpha)$, Solution is: $-e^{-z}+1$.*

(i) $\psi(n)$ monotonically increases with n on $(1,\infty)$ if $1+s/t\geq e^{\gamma-2}(4/\gamma-1)$; otherwise,

(ii.a) if $s=0$, then $\psi(n)$ is unimodal on $(1,\infty)$;

(ii.b) If $s > 0$, then $\psi(n)$ has two critical points on $(1, \infty)$. Denote them by $\{n_1, n_2\}$, where $n_1 < n_2$, $\psi(n)$ is maximized at n_1 and minimized at n_2. If $1 + s/t < e^{-\gamma}(2\gamma + 1)$ and $\gamma < 1.256\,4$, then $n_1 < 2 < n_2$; if $1 + s/t \in \left(e^{-\gamma}(2\gamma + 1), e^{\gamma-2}(4/\gamma - 1)\right)$ and $\gamma < 1$, then $n_1 < n_2 \leq 2$; if $1 + s/t \in \left(e^{-\gamma}(2\gamma + 1), e^{\gamma-2}(4/\gamma - 1)\right)$ and $\gamma > 1$, then $2 < n_1 < n_2$.

(iii) if $n \to \infty$, then $\psi(n) - (ns + \gamma t) \to 0$.

Proof. We show in (i) that $\psi(n)$ is monotonic if t is small, then show in (ii) that for a large t, $\psi(n)$ may be unimodal or have two critical points, depending on whether $s = 0$. We also show the asymopotic behavior of $\psi(n)$ in (iii).

First of all, we show that the monotonic behavior of $\psi(n)$ can be analyzed by studying $\phi(z, \gamma)$, defined below in $(A.1)$. Let $z = \gamma/(n-1)$, we have $n = \gamma/z + 1$, $\alpha = 1 - \exp(-z)$, $\frac{\partial\gamma}{\partial t} < 0$, and $\frac{\partial}{\partial n} z = -\frac{\gamma}{(n-1)^2} = -\frac{z^2}{\gamma}$. Thus, $\psi(n)$ can be rewritten as $((1 - \exp(-z))t + s)n = t\left(\frac{(1+s/t)\exp z - 1}{z}\right)(\gamma + z)\exp(-z)$. Hence, $\psi'(n) = tz\exp(-z)\left(\frac{(1+s/t)\exp z - 1}{z} - \frac{\gamma+z}{\gamma}\right)$. Let

$$\phi(z, \gamma) = \gamma \frac{(1 + s/t)\exp z - 1}{z(z + \gamma)}. \tag{A.1}$$

Hence, $\psi'(n) \lesseqgtr 0$ as long as $\phi(z, \gamma) \lesseqgtr 1$. Using L'Hôpital's rule (Estrada and Pavlovic 2017), we can verify that $\phi(z, \gamma)$ is convex in z, increases with γ, and $\lim_{z\to 0} \phi(z, \gamma) \geq 1$ (This is because $\lim_{z\to 0} \phi(z, \gamma) = +\infty$ if $s > 0$ and $\lim_{z\to 0} \phi(z, \gamma) = 1$ if $s = 0$).

To establish (i), we show that $\phi(z, \gamma) \geq 1$ for all $z > 0$ if $1 + s/t \geq e^{\gamma-2}(4/\gamma - 1)$. Since $\phi_z(z, \gamma) = \gamma \frac{(\gamma+2z)+e^z\left(-2z-\gamma+z\gamma+z^2\right)(1+s/t)}{z^2(z+\gamma)^2}$, $\phi(z, \gamma)$ is minimized when

$$(1 + s/t)\exp z = \frac{\gamma + 2z}{\gamma + 2z - z\gamma - z^2}. \tag{A.2}$$

Since $\phi(z, \gamma)$ is convex in z, $(A.2)$ has a unique solution. Denote it by z^*. Hence, $\min_z \phi(z, \gamma) = \phi(z^*, \gamma) = \gamma \frac{(1+s/t)\exp z^* - 1}{z^*(z^*+\gamma)} = \frac{\gamma}{\gamma+2z^*-z^*\gamma-z^{*2}}$, which equals 1 if $z^* = 2 - \gamma$. Since z^* solves $(A.2)$, $\min_z \phi(z, \gamma) = 1$ if and only if $(A.2)$ holds and $z = 2 - \gamma$, i.e., $(1 + s/t)\exp(2 - \gamma) = 4/\gamma - 1$. Denote its solution by γ^*. Thus, $\min_z \phi(z, \gamma^*) = 1$. Since $\phi_\gamma(z, \gamma) > 0$, we must have $\min_z \phi(z, \gamma) \geq 1$ for all $\gamma \geq \gamma^*$ by the Envelope Theorem. Therefore, $\psi'(n) \geq 0$ if $\gamma \geq \gamma^*$, i.e., $1 + s/t \geq e^{\gamma-2}(4/\gamma - 1)$.

(ii) By contrast, if $\gamma < \gamma^*$, i.e., t is large, then $\psi(n)$ is non-monotonic. The critical point(s) of $\psi(n)$ can be found by solving $\phi(z, \gamma) = 1$.

$(ii.a)$ If $s = 0$, then $\gamma^* = 2$. Since $\phi(z,\gamma)$ is convex in z, $\lim_{z\to 0}\phi(z,\gamma) = 1$, and $\lim_{z\to 0}\phi_z(z,\gamma) = \frac{\gamma-2}{2\gamma} < 0$, there is a unique solution of z for $\phi(z,\gamma) = 1$ on $(0,\infty)$. Denote it by $\hat{z}$. $\psi'(n) > 0$ if and only if $z > \hat{z}$. Thus $\hat{n} = \gamma/\hat{z} + 1$ must be the unique critical point on $(1,\infty)$ and $\psi'(n) \gtreqless 0$ if $n \lesseqgtr \hat{n}$, i.e., $\psi(n)$ is unimodal.

$(ii.b)$ If $s > 0$, then $\gamma^* < 2$, since $e^{\gamma-2}(4/\gamma - 1)$ decreases with γ. For $\gamma < \gamma^*$, $e^{\gamma-2}(4/\gamma - 1) > 1 + s/t$, there are two solutions of z for $\phi(z,\gamma) = 1$ on $(0,\infty)$. Denote them by z_1 and z_2, where $z_1 > z_2$. Thus, $\psi'(n) < 0$ if and only if $z \in (z_2, z_1)$. This implies that $n_i = \gamma/z_i + 1$, $i = 1, 2$, must be the only two critical points on $(1,\infty)$. Since $n_1 < n_2$ and $\psi'(n) < 0$ on (n_1, n_2), it must be true that n_1 (locally) maximizes $\psi(n)$ and n_2 minimizes $\psi(n)$. Next we identify the locations of the critical points n_1 and n_2 in relation to $n = 2$. If $s/t < e^{-\gamma}(2\gamma+1) - 1$, then $\phi(z,\gamma)|_{z=\gamma} = \frac{(1+s/t)\exp\gamma - 1}{2\gamma} < 1$. Note that $e^{-\gamma}(2\gamma+1) > 1$ if and only if $\gamma < 1.2564$. Since $\phi(z,\gamma)$ is convex in z and $\phi(z_1,\gamma) = \phi(z_2,\gamma) = 1$, we must have $z_2 < \gamma < z_1$, i.e., $n_1 < 2 < n_2$. By contrast, if $s/t > e^{-\gamma}(2\gamma+1) - 1$, then $\phi(z,\gamma)|_{z=\gamma} > 1$. It is easy to verify $e^{-\gamma}(2\gamma+1) < e^{\gamma-2}(4/\gamma - 1)$ if $\gamma \neq 1$ and $e^{-\gamma}(2\gamma+1) = e^{\gamma-2}(4/\gamma-1) = 3/e$ if $\gamma = 1$. Since $\phi_z(z,\gamma)|_{z=\gamma} = 0$ when $\gamma = 1$ and $\phi_{z\gamma}(z,\gamma) > 0$, we must have $\phi_z(z,\gamma)|_{z=\gamma} \gtreqless 0$ when $\gamma \gtreqless 1$. Thus, if $\gamma < 1$, we have $\phi_z(z,\gamma)|_{z=\gamma} < 0$, implying that $\gamma < z_2 < z_1$, i.e., $n_1 < n_2 < 2$; otherwise, if $\gamma > 1$, then $z_2 < z_1 < \gamma$, i.e., $2 < n_1 < n_2$. Last, if $\gamma = 1$, then we cannot have $e^{-\gamma}(2\gamma+1) < 1 + s/t < e^{\gamma-2}(4/\gamma - 1)$.

(iii) This is because $\psi(n) = (\alpha t + s)n$ and $\lim_{n\to\infty}\alpha n = \lim_{z\to 0}(1 - e^{-z})\frac{\gamma+z}{z} = \lim_{z\to 0} e^{-z}\frac{(z+\gamma)^2}{\gamma} = \gamma$. ∎

Proof of Proposition 2. If $s = 0$, by Lemma 4 $(ii.a)$, we only need to compare $\psi(2)$ and $\psi(\infty)$. By Lemma 4 (iii), $\lim_{n\to\infty}\psi(n) = t\gamma$, whereas $\psi(2) = 2t\left(1 - \left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)\right)$. Since $1 - \exp(-\gamma) < \gamma/2$ for $\gamma > 1.594$, we have $\psi(2) < \lim_{n\to\infty}\psi(n)$ if and only if $\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)} < \exp(-1.594)$, i.e., $t < \frac{q(1-q)}{4.92(1-q)+q}(c_h - c_l)$. The reason why $s = 0$ has to be considered as a special case is due its different asymptotic behavior: $\lim_{n\to\infty}\psi'(n) = s$ if $s > 0$, but $\psi'(n) < 0$ and $\lim_{n\to\infty}\psi'(n) = 0$ if $s = 0$.

If $s > 0$, by Lemma 4 $(ii.b)$, there are four possibilities:

(i) $\psi(n)$ is increasing on $(1,\infty)$ when $\gamma > \gamma^*$, i.e., $\frac{t}{c_h - c_l} \le \frac{q(1-q)}{(1-q)e^{\gamma^*}+q}$. Hence, $n^o = 2$.

(ii) $n_1 < 2 < n_2$ when $1 + s/t < e^{-\gamma}(2\gamma+1)$. Since $\psi(n)$ is decreasing on the interval of $[2, n_2]$, $\psi(n_2) < \psi(2)$. Therefore, $n^o = n_2 > 2$.

(iii) $n_1 < n_2 < 2$ when $1 + s/t \in \left(e^{-\gamma}(2\gamma+1), e^{\gamma-2}(4/\gamma - 1)\right)$ and $\gamma < 1$. Since $\psi(n)$ is increasing on the interval of (n_2, ∞), $\psi(n_2) < \psi(2) < \lim_{n\to\infty}\psi(n)$. In addition, $n^o \geq 2$, so we must have $n^o = 2$.

(iv) $2 < n_1 < n_2$ when $1 + s/t \in \left(e^{-\gamma}(2\gamma+1), e^{\gamma-2}(4/\gamma - 1)\right)$ and $\gamma > 1$. Since $\psi(n)$ increases on $[2, n_1]$ and then decreases on $[n_1, n_2]$, $n^o = \arg\min_{n\in\{2,n_2\}}\psi(n)$. Since $\psi(2) = \left(\frac{(1+s/t)\exp z - 1}{z}\right)(\gamma + z)\exp(-z)\,|_{z=\gamma} = 2\left(1 + s/t - e^{-\gamma}\right)$ and $\psi(n_2) = \left(\frac{(1+s/t)\exp z - 1}{z}\right)(\gamma + z)\exp(-z)$ where $\gamma\frac{(1+s/t)\exp z - 1}{z(z+\gamma)} = 1$, $n^o > 2$ if $2\left(1 + s/t - e^{-\gamma}\right) > \frac{(z+\gamma)^2}{\gamma}\exp(-z)$.

Therefore, in order for $n^o > 2$, if $\gamma < 1$, i.e., $\frac{t}{c_h - c_l} > \frac{q(1-q)}{(1-q)e+q}$, then we must have $s/t < e^{-\gamma}(2\gamma + 1) - 1$; if $\gamma > 1$, i.e., $\frac{t}{c_h - c_l} < \frac{q(1-q)}{(1-q)e+q}$, then we must have $s/t < e^{\gamma-2}(4/\gamma - 1) - 1$ and $2\left(1 + s/t - e^{-\gamma}\right) > \frac{(z^*+\gamma)^2}{\gamma}\exp(-z^*)$, where $z^* is$ the solution to $(A.2)$. ■

Proof of Proposition 3. (i) Suppose that n^o changes to $n^{o\prime}$ and n_2 changes to n_2' when s increases to $s' > s$. To prove that n^o is (weakly) decreasing in s, we need to show $n^{o\prime} \leq n^o$.

If $s = 0$, by Lemma 4 $(ii.a)$, $n^o = \arg\min_{n\in\{2,\infty\}}\psi(n, 0, t)$ and $n^{o\prime} = \arg\min_{n\in\{2,n_2'\}}\psi(n, s', t)$. We only need to prove that $n^{o\prime}$ cannot be $n_2' > 2$ when $n^o = 2$, i.e., $\psi(n_2', s', t) > \psi(2, s', t)$ when $\psi(n_2', 0, t) > \psi(2, 0, t)$, but this is true because $\psi(n_2', s', t) = \psi(n_2', 0, t) + n_2's' > \psi(2, 0, t) + 2s' = \psi(2, s', t)$.

If $s > 0$, by Lemma 4 $(ii.b)$, $n^o = \arg\min_{n\in\{2,n_2\}}\psi(n, s, t)$ and $n^{o\prime} = \arg\min_{n\in\{2,n_2'\}}\psi(n, s', t)$. By the Envelope theorem,

$$\text{(A.3)} \qquad \operatorname{sign}\frac{dn_2}{ds} = \operatorname{sign} - \frac{\partial^2}{\partial n\partial s}\psi(n, s, t) = (-),$$

since $\frac{\partial^2}{\partial n\partial s}\psi(n, s, t) = 1$. Hence, $n_2' < n_2$. Suppose that $n^o = n_2$, then $n^{o\prime}$ will be greater than n^o only if $n_2' > n^o$, but $n_2' < n_2 = n^o$ by $(A.3)$. Suppose that $n^o = 2$, then $n^{o\prime}$ will be greater than n^o only if $n_2' > 2$, i.e., $\psi(2, s, t) < \psi(n_2, s, t)$ and $\psi(2, s', t) > \psi(n_2', s', t)$, but $\psi(2, s', t) = \psi(2, s, t) + 2(s' - s) < \psi(n_2, s, t) + 2(s' - s) \leq \psi(n_2, s, t) + n_2'(s' - s) < \psi(n_2', s, t) + n_2(s' - s) = \psi(n_2', s', t)$, contradiction.

(ii) is immediate from Proposition 2. ■

Proof of Proposition 4. (i) By Lemma 4, $\varphi(s,t) = \min\{\psi(2), \psi(n_2), \psi(\infty)\}$. By the Envelope Theorem, $\frac{d}{ds}\psi(n_2) = n_2 > 0$. At the same time, $\frac{d}{ds}\psi(2) = 2 > 0$ and $\frac{d}{ds}\psi(\infty) = n > 0$. Therefore, $\frac{\partial}{\partial s}\varphi(s,t) > 0$.

(ii) First, $\frac{d}{dt}\psi(2) = \frac{\partial}{\partial t}\left(1 - \frac{t(1-q)}{q((1-q)-t)}\right)t = \frac{t^2-2t(1-q)+q(1-q)^2}{q(1-q-t)^2}$, $\frac{d^2}{dt^2}\psi(2) = \frac{\partial^2}{\partial t^2}\left(1 - \frac{t(1-q)}{q((1-q)-t)}\right)t = -\frac{2}{q}\frac{(1-q)^3}{(1-q-t)^3} < 0$, so $\psi(2)$ increases with t if and only if $t < 1-q-\sqrt{(1-q)^3}$. Second, by the Envelope Theorem, $\frac{d}{dt}\psi(n_2) = \frac{\partial}{\partial t}(1-\exp(-\gamma/(n_2-1)))tn_2 = -n_2\left(e^{-\frac{\gamma}{n_2-1}} - 1\right)\frac{\partial\gamma}{\partial t} < 0$, i.e., $\psi(n_2)$ decreases with t. Third, by Lemma 4 (iii), $\frac{d^2}{dt^2}\psi(\infty) = \frac{\partial^2}{\partial t^2}\left(-t\ln\frac{t(1-q)}{q((1-q)-t)}\right) = -\frac{1}{t}\frac{(1-q)^2}{(1-q-t)^2} < 0$. If $s > 0$, by Lemma 4 $(ii.b)$, $\varphi(s,t) = \min\{\psi(2), \psi(n_2)\}$, Since $\psi(2)$ and $\psi(n_2)$ are both quasi-concave, $\varphi(s,t)$ must be quasi-concave in t. If $s = 0$, by Lemma 4 $(ii.a)$, $\varphi(s,t) = \min\{\psi(2), \psi(\infty)\}$. Since $\psi(2)$ and $\psi(\infty)$ are both concave in t, $\varphi(s,t)$ must be concave in t. Last, since $\varphi(s,0) = \varphi(s,\bar{t})$, where $\bar{t} = q(1-q)(c_h - c_l)$, $\varphi(s,t)$ must be unimodal in t. ■