Math 113

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1 Rings

Definition 1.1. A ring is defined under the following :

- Closure under addition.
- Associative addition.
- Commutative addition.
- Additive zero element
- Additive inverse element
- Closure under multiplication
- Associative multiplication
- Multiplication is distributive over addition.

Definition 1.2. An integral domain is a Commutative ring with identity $1_R \neq 0_R$ that satisfies: Whenever $a, b \in R$ and $ab = 0_R$, then $a = 0_R$ or $b = 0_R$.

Definition 1.3. We can classify a subset of a ring as a subring if:

- S is closed under addition
- S is closed under multiplication
- 0_R is in S
- if $a \in S$, then $-a \in S$

Definition 1.4. A field is a commutative ring with identity $1_R \neq 0_R$ such that every nonzero element has a multiplicative inverse.