Math 113

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1 Quotient Groups and Isomorphism Theorems

Theorem 1.1. *$Nx = Ny \leftrightarrow xy^{-1} \in N$ for group N making right cosets*

- Think of normal subgroups groups as kernels of homomorphisms.
- A subgroup is normal if and only if its left and right cosets are the same for all elements g.

Theorem 1.2. *$\Phi : G \to H$ is a surjective homomorphism, then $H \cong G/\ker\Phi$*

Alternatively, the above can be expressed as:

Note. $\Phi : G \to H$ is a homomorphism, then **image**$(\Phi : G \to H) \cong G/\ker\Phi$

Theorem 1.3. *Let $K \triangleleft G$, $N \triangleleft G$ and $N \subseteq K \subseteq G$ Then $K/N \triangleleft G/N$ and $(G/N)/(K/N) \cong G/K$*

The correspondence theorem below goes in the other direction, saying that we can find a quotient group for any subgroup of of the original quotient group.

Theorem 1.4. *Suppose $T \subseteq G/N$, then there exists some subgroup $H \subseteq G$ such that $N \subseteq H$ and $H/N \cong T$*

Definition 1.5. A group is simple iff it contains no nontrivial proper normal subgroups.

Theorem 1.6. *Theorem of finitely generated abelian groups: Every finitely generated abelian group is isomorphic to a direct sum of cyclic groups of the form:*

$$\mathbb{Z}_{p_1^{n_1}} \oplus \mathbb{Z}_{p_2^{n_2}} \oplus \cdots \oplus \mathbb{Z}_{p_k^{n_k}}$$