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1.

(a)

$$0.1a + 0.9b = 0.1 \times \begin{pmatrix} 1 \\ 8 \end{pmatrix} + 0.9 \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2.8 \\ 2.6 \end{pmatrix}$$

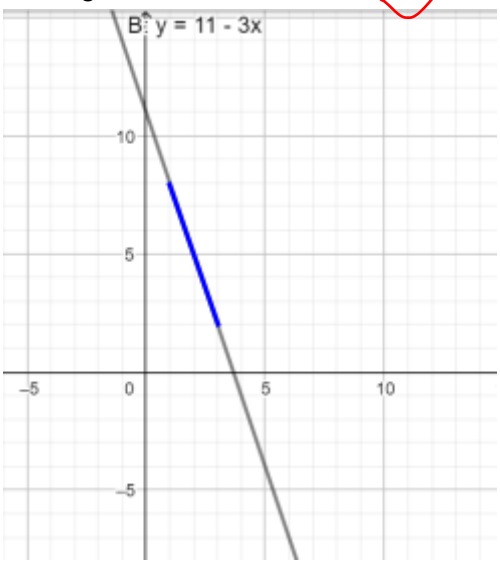
$$a - b = \begin{pmatrix} 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$$

$$a \prime b = \begin{pmatrix} 1 & 8 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 19$$

(b)

$$\lambda a + (1-\lambda)\, b = \lambda \times \begin{pmatrix} 1 \\ 8 \end{pmatrix} + (1-\lambda) \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3-2\lambda \\ 2+6\lambda \end{pmatrix}$$

Plotting all the results of this vector will result in a straight line.



Line segment indicated in blue

2.

(a)

$$AB = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 13 & 3 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix} = \begin{pmatrix} 9 & -12 & 17 \\ 5 & -7 & 10 \\ 2 & -4 & 6 \end{pmatrix}$$

(b) Let:

$$(AB)\prime = \begin{pmatrix} 5 & 0 \\ 13 & 3 \end{pmatrix}\prime = \begin{pmatrix} 5 & 13 \\ 0 & 3 \end{pmatrix}$$

$$B\prime = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}; \quad A\prime = \begin{pmatrix} 1 & 4 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}$$

$$B\prime A\prime = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ -2 & -5 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 13 \\ 0 & 3 \end{pmatrix}$$

Therefore, $(AB)\prime = B\prime A\prime$

3.

(a)

Possible size of matrix A: $m \times 2$

Possible size of matrix B: $m \times 1$

(b)

$$Ax \leq b$$

$$\therefore \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x_1 + x_2 \\ -x_1 + x_2 \\ -x_2 \end{pmatrix} \leq \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

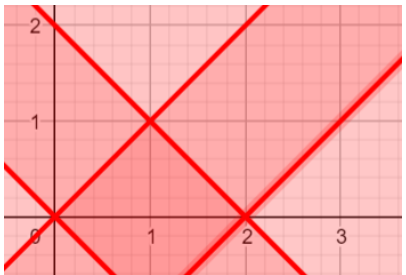
$x_1 + x_2 \leq 3;\ 0 \leq x_2 \leq 1.5;\ 0 \leq x_2 \leq x_1 \leq 3$

(c) The square region could be described with these features:

$$0 \leq x_1 + x_2 \leq 2; 0 \leq x_1 - x_2 \leq 2$$

Hence, we can describe the matrix using this inequality:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$



4.

(a) If $x \in X1$,

$Ax \leq b$

$Ax - b \leq 0$

$Ax - b = -s$, with $s \in R$, $-s \leq 0$

i.e. $Ax - b = -s$, with $s \in R$, $s \geq 0$

Therefore,

$Ax + s = b$

$x \in X2$

Hence, X1 ⊆ X2

Vectors and matrices in bold face

↖ size of s is not correct

−1

(b) If $x \in X_2$, then $Ax + s = b$ for $s \in \mathbb{R}$, $s \geq 0$. Therefore,

$Ax = b - s \leq b - 0 = b$

Hence, $x \in X_1$.

5.

(a)

Problem: $min\ c'G$

$s.t\ \ nG \geq r$

$G \geq 0$

$n = \begin{pmatrix} 5 & 7 \\ 4 & 2 \\ 2 & 1 \end{pmatrix}$ $r = \begin{pmatrix} 8 \\ 15 \\ 3 \end{pmatrix}$ $G = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}$

$with\ c = \begin{pmatrix} 0.6 \\ 0.35 \end{pmatrix}$, $G = G_1G2, n = 574221, r = 8153$

$\therefore\ G = \begin{pmatrix} 3.75 \\ 0 \end{pmatrix}$

−3

```
from rsome import ro
from rsome import grb_solver as grb

model = ro.Model('LP model')
x = model.dvar()
y = model.dvar()

model.min(0.6*x + 0.35*y)
model.st(5*x + 7*y >= 8)
model.st(4*x + 2*y >= 15)
model.st(2*x + 1*y >= 3)
model.st(y >= 0)
model.st(x >= 0)

model.solve(grb)
```

```
Restricted license - for non-production use only - expires 2023-10-25
Being solved by Gurobi...
Solution status: 2
Running time: 0.0007s
```

```
print('x: {val}'.format(val=x.get()))
print('y: {val}'.format(val=y.get()))
print('Objective: {val}'.format(val=round(model.get(), 2)))
```

```
x: [3.75]
y: [0.]
Objective: 2.25
```

3.75units of G1, 0 units of G2, $2.25 spent in total on G1

(b)

Problem: $min\ cG$

$s.t\ \ nG \geq r$

better to use other letters. easy to be confusing

$G \geq 0$

$$with\ c = \begin{pmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{pmatrix},\ G = \begin{pmatrix} G_1 \\ G_2 \\ \dots \\ G_n \end{pmatrix},\ n = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix},\ r = \begin{pmatrix} r_1 \\ r_2 \\ \dots \\ r_m \end{pmatrix}$$

5b) Decision variables:
qty of food $n = x_n$

Objective:
TT cost $= x_1c_1 + \cdots + x_nc_n$
$= \sum_{i \in n} x_ic_i,\ n \in Z,$
$min\ \sum x_ic_i$

Constraints

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \geq \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix}$$

$N \quad x \quad R$

$N \cdot x \geq R$

6.

Problem:

x_i: No. of employees starting work in Quarter i (i = 1, 2 , 3, 4)

s_i: No. of surplus shoes from Quarter i to Quarter i+1 (i = 1, 2, 3)

Objective: $min \sum_{i=1}^{4} 1500x_i + \sum_{i=1}^{3} 50s_i$

s.t.
$$50(x_1 + x_3 + x_4) = 600 + s_1$$
$$s_1 + 50(x_1 + x_2 + x_4) = 300 + s_2$$
$$s_2 + 50(x_1 + x_2 + x_3) = 800 + s_3$$
$$s_3 + 50(x_2 + x_3 + x_4) = 100$$
$$x_i,\ s_i,\ p_i = 0, p_i = 50$$

where does p_i come from

$$\therefore\ x_1^* = 10,\ x_2^* = x_4^* = 0,\ x_3^* = 2$$
$$\therefore\ s_2^* = 200,\ s_1^* = s_3^* = 0$$

```
# w1, w1, w1
#     w2, w2, w2
# w3      w3, w3
# w4, w4      w4

model = ro.Model("LP model")
w1 = model.dvar()
w2 = model.dvar()
w3 = model.dvar()
w4 = model.dvar()
s1 = model.dvar()
s2 = model.dvar()
s3 = model.dvar()

model.min(1500*w1 + 1500*w2 +1500*w3 +1500*w4 + 50*s1 + 50*s2 + 50*s3)
model.st(50*w1 + 50*w3 + 50*w4 - s1 == 600)
model.st(50*w1 + 50*w2 + 50*w4 - s2 + s1 == 300)
model.st(50*w1 + 50*w2 + 50*w3 - s3 + s2 == 800)
model.st(50*w2 + 50*w3 + 50*w4 +s3 == 100)
model.st(w1 >= 0)
model.st(w2 >= 0)
model.st(w3 >= 0)
model.st(w4 >= 0)
model.st(s1 >= 0)
model.st(s2 >= 0)
model.st(s3 >= 0)
```

```
model.solve(grb)

print('w1: ',w1.get())
print('w2: ',w2.get())
print('w3: ',w3.get())
print('w4: ',w4.get())
print('s1: ',s1.get())
print('s2: ',s2.get())
print('s3: ',s3.get())
print('objective: ',model.get())
```

```
Being solved by Gurobi...
Solution status: 2
Running time: 0.0058s
w1:  [10.]
w2:  [0.]
w3:  [2.]
w4:  [0.]
s1:  [0.]
s2:  [200.]
s3:  [0.]
objective:  28000.0
```

10 employees to start work during Q1

2 employees to start work during Q3

200 surplus shoes to be produced during Q2

Optional Questions

Question 7:

A inverse = B
Hence AB = BA = I

$AB = BA = I$
then $A^{-1} = B$

Question 8a:

A inverse

$$\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$$

Question 8b:

A transpose

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

A transpose inverse

$$\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$$

A inverse transpose

$$\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$$

A inverse transpose = A transpose inverse

Question 9: