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Exercise 1:

Let $\boldsymbol{x} = (x, y, z)$.

(a) Regarding set $P1$: $\{(x, y, z) \in \mathbb{R}^3 : |x| \leq 1, |y| \leq 1, |z| \leq 1\}$, i.e. $\begin{cases} -1 \leq x \leq 1, x \in \mathbb{R} \\ -1 \leq y \leq 1, y \in \mathbb{R} \\ -1 \leq z \leq 1, z \in \mathbb{R} \end{cases}$

Hence, P1 can be expressed as: $P1 = \{\boldsymbol{x} \mid \boldsymbol{A_1 x} \geq \boldsymbol{b_1}\}$

with $\boldsymbol{A}_1 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}, \boldsymbol{b_1} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$

Regarding set $P2$: $\{(x, y, z) \in \mathbb{R}^3 : |x| + |y| + |z| \leq 1\}$, i.e. $\pm x \pm y \pm z \leq 1$

for $x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}$

Hence, P2 can be expressed as: $P2 = \{\boldsymbol{x} \mid \boldsymbol{A_2 x} \geq \boldsymbol{b_2}\}$

with $\boldsymbol{A}_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \\ -1 & -1 & -1 \end{pmatrix}, \boldsymbol{b_2} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$

Therefore, both sets are polyhedra.

Additionally, below are the plots of these 2 polyhedra.

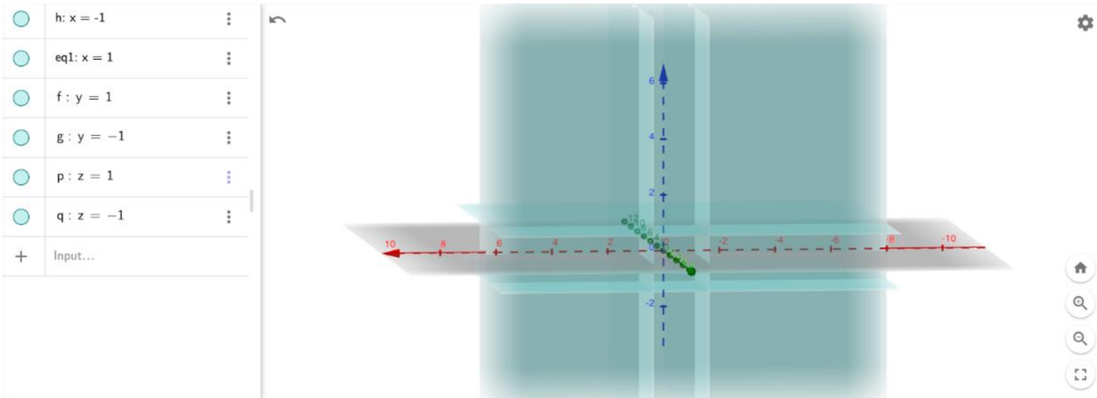

Figure 1: Polyhedron P1

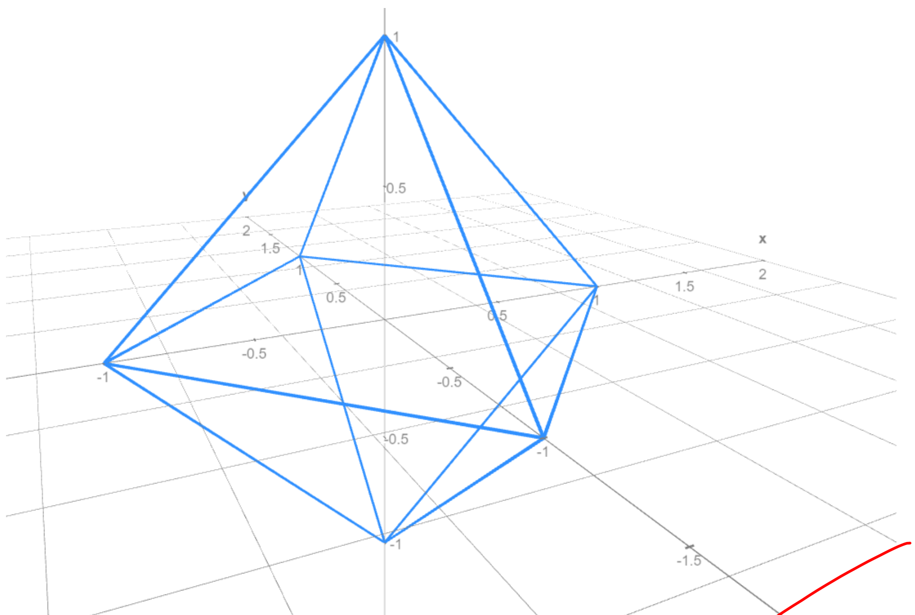

Figure 2: Polyhedron P2

(b) Regarding set P1: The extreme points satisfy $\{(x, y, z) \in \mathbb{R}^3 : |x| = 1, |y| = 1, |z| = 1\}$, which are: $\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\end{pmatrix}, \begin{pmatrix}-1\\-1\\-1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\-1\end{pmatrix}$

Regarding set P2: The extreme points satisfy $\{(x, y, z) \in \mathbb{R}^3 : |x| + |y| + |z| = 1\}$, or subsets such as $\{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1;\ x, y, z \geq 0\}$, which are:
$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}0\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}$

Exercise 2:

Let L and M_i for I = 1,…,6 are amounts of a 6-month loan and 6 monthly loans respectively.

Therefore, the problem is as following:

$$min\ L \times 0.12 + 0.04 \times \sum_{i=1}^{6} M_i$$

$$s.t\ L + M_1 + 20000 - s_1 = 50000$$

$$s_1 + M_2 + 30000 - s_2 = 60000 + M_1 \times 1.04$$

$$s_2 + M_3 + 40000 - s_3 = 50000 + M_2 \times 1.04$$

$$s_3 + M_4 + 50000 - s_4 = 60000 + M_3 \times 1.04$$

$$s_4 + M_5 + 80000 - s_5 = 50000 + M_4 \times 1.04$$

-2

$$s_5 + M_6 + 100000 - s_6 = 30000 + M_5 \times 1.04$$

$$L \times 1.12 + 1.04 \times M_6 \leq s_6$$ no need to spend all the surplus

with each line being the cashflow of each month and s_i are excess money of month i, for i = 1,…,6

$L \geq 0$, $M_i, s_i \geq 0$ $i = 1, \ldots, 6$

-2

```
rev = [20000,30000,40000,50000,80000,100000]
lia = [50000,60000,50000,60000,50000,30000]

#Claim a model
e2 = ro.Model()

#Define dv
L = e2.dvar() #6-month loan @ 12% interest rate
M = e2.dvar((6)) #Monthly loan @ 4% interest rate/month
s = e2.dvar((6)) #Excess cash each month

#Define obj
e2.min(L*0.12 + 0.04*M.sum())

#Define constraints
e2.st(L + M[0] + rev[0] - s[0] == lia[0]) #July
for i in range(5): #August, September, October, November, December
    e2.st(s[i] + M[i+1] + rev[i+1] - s[i+1] - 1.04*M[i] == lia[i+1])
e2.st(L*1.12 + M[5]*1.04 - s[5] == 0) #January
e2.st(L >= 0, M >= 0, s >= 0)

#Solve the model
e2.solve(grb)

#Obtain solution
print("The amount of 6-month loan and monthly loan to be borrowed are {} and {} respectively".format(L.get(), M.get()))
print("The minimum interest cost is: {}".format(e2.get()))
```

no need the brackets.

−2

You put sign restrictions here, but you forgot to put in your model.

```
Set parameter Username
Academic license - for non-commercial use only - expires 2023-01-03
Being solved by Gurobi...
Solution status: 2
Running time: 0.0283s
The amount of 6-month loan and monthly loan to be borrowed are [60000.] and [     0.      0.  10000.  20400.      0. 289600.] r
espectively
The minimum interest cost is: 19999.999999999993
```

Exercise 3:

Let

d_i: Required units to deliver at the end of the ith month, i = 1,…,12

x_i: No. of units produced in the ith month, i = 1,…,12

$c_2|x_{i+1} - x_i|$: Cost of switching to a new production level from the ith month to (i+1)th

r_i: Remainder left from ith month's production to be delivered in the (i+1)th month, i = 1,…,11

$c_1 r_i$: Storage cost for units held in inventory of the ith month

Problem: $min\ c_2 \sum_{i=1}^{12} a_i + c_1 \sum_{i=1}^{11} r_i$

$s.t\ x_1 - r_1 = d_1$

$x_{i+1} + r_i - r_{i+1} = d_{i+1}$ for i = 1,…,11

$x_1 \le a_1, -x_1 \le a_1$

$x_{i+1} - x_i \le a_i$ for i = 2,…,12

what is x_{13} ?

−5

$-x_{i+1} + x_i \leq a_i$ for i = 2,…,12 why you need this -2

$r_i \leq d_{i+1}$ for i = 1,…,11

$r_i \geq 0$ for i = 1,…,11

$a_i, x_i \geq 0$ for i = 1,…,12

Exercise 4:

Primal	Dual
Max $\sum_{i=1}^{500} c_i x_i$ s.t $\sum_{i=1}^{500} x_i = 30: p_1$ $x_1 \leq 1: p_2$ $x_2 \leq 1: p_3$ … $x_{500} \leq 1: p_{501}$ $x_i \geq 0$ for i = 1,…,500	Min $30p_1 + p_2 + \ldots + p_{501}$ s.t $p_1 + p_2 \geq c_1: x_1 \geq 0$ $p_1 + p_3 \geq c_2: x_2 \geq 0$ $p_1 + p_4 \geq c_3: x_3 \geq 0$ … $p_1 + p_{501} \geq c_{500}: x_{500} \geq 0$ p_1 free $p_2, \ldots, p_{501} \geq 0$
Min $\sum_{i=1}^{500} c_i x_i$ s.t $\sum_{i=1}^{500} x_i = 30: p_1$ $x_1 \leq 1: p_2$ $x_2 \leq 1: p_3$ … $x_{500} \leq 1: p_{501}$ $x_i \geq 0$ for i = 1,…,500	Max $30p_1 + p_2 + \ldots + p_{501}$ s.t $p_1 + p_2 \leq c_1: x_1 \geq 0$ $p_1 + p_3 \leq c_2: x_2 \geq 0$ $p_1 + p_4 \leq c_3: x_3 \geq 0$ … $p_1 + p_{501} \geq c_{500}: x_{500} \geq 0$ why this constraint is different to others? p_1 free $p_2, \ldots, p_{501} \leq 0$

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