BC2410 : Prescriptive Analytics

Assignment 2

(Due: 13 Feb. 2022)

1. Consider the two sets in $\mathbb{R}^3$ given by

$$P_1 = \left\{(x, y, z) \in \mathbb{R}^3 : |x| \leq 1, |y| \leq 1, |z| \leq 1\right\},$$

and

$$P_2 = \left\{(x, y, z) \in \mathbb{R}^3 : |x| + |y| + |z| \leq 1\right\}.$$

(a) Show that both sets are polyhedra (15 Marks).

(b) For each set, find the extreme points (10 Marks).

2. Toyz is a discount toy store in Woodland Mall. During the summer and fall, the store must build up its inventory to have enough stock for the Christmas season. To purchase and build up its stock during the months when the revenue is low, the store borrows money from a bank.

Following is the store's projected revenue and liabilities schedule for July through December, where revenues of last month and bills of the coming month are paid at the first of each month.

Month	Revenues	Liabilities
July	\$20,000	\$ 50,000
August	30,000	60,000
September	40,000	50,000
October	50,000	60,000
November	80,000	50,000
December	100,000	30,000

At beginning of July, the store can take out a 6-month loan at 12% interest rate and must be paid back at the beginning of next January. The store can not pay back this loan early. The store can also borrow monthly loan at a rate of 4% interest per month. The store wants to borrow enough money to meet its cash flow needs while minimizing its interest cost.

Formulate and solve the LOP for this problem (15 Marks). Provide the code in Python (15 Marks). You are required to use a "for loop" for constraint 2 to 6.

3. A company must deliver d_i units of its product at the end of the ith month. Material produced during a month can be delivered either at the end of the same month or can be stored as inventory and delivered at the end of a subsequent month; however, there is a storage cost of c_1 dollars per month for each unit of product held in the inventory. The year begins with zero inventory. If the company produces x_i units in month i and x_{i+1} units in month $i+1$, it incurs a cost of $c_2|x_{i+1} - x_i|$ dollars, reflecting the cost of switching to a new production level. Formulate a linear optimization problem whose objective is to minimize the total cost of the production and inventory schedule over a period of twelve months (no need to solve the model). Assume that inventory left at the end of the year has no value and does not incur any storage costs. (25 Marks)

4. Write down the duals of the following optimization problems (20 Marks)

$$\begin{aligned} \max \quad & \sum_{i=1}^{500} c_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{500} x_i = 30 \\ & 0 \leq x_i \leq 1, i = 1, \dots, 500, \end{aligned}$$

$$\begin{aligned} \min \quad & \sum_{i=1}^{500} c_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{500} x_i = 30 \\ & 0 \leq x_i \leq 1, i = 1, \dots, 500. \end{aligned}$$

5. (Optional) The manager of a department store is attempting to decide on the types of ad- vertising the store should use. He has invited representatives from the local radio station, television station, and newspaper to make presentations in which they describe the audiences. The television station representative indicates that a TV commercial, which costs $\$15,000$, would reach 25,000 potential customers. The newspaper representative claims to be able to provide an audience of 10,000 potential costumers at a cost of $\$4,000$ per ad, while the radio station representative says that the audience for one of the station's commercials, which costs $\$6,000$, is 15,000 customers. The breakdown of the audience of the three medias are as follows:

TV	Male	Female
Senior	5,000	5,000
Young	5,000	10,000

Newspaper	Male	Female
Senior	4,000	3,000
Young	2,000	1,000

Radio	Male	Female
Senior	1,500	1,500
Young	4,500	7,500

The store has the following advertisement policy:

(a) Use at least twice as many radio commercials as newspaper ads.

(b) Reach as least 100,000 customers.

(c) Reach at least twice as many young people as senior citizens.

(d) Make sure at least 60% of the audience is female.

Available space limits the number of newspapers ads to seven. The store wants to make the optimal number of each type of advertisement to purchase to minimize total cost.

Formulate and solve the LOP for this problem. Provide the code in Python.

6. (Optional) Consider the following optimization problem

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{500} c_i x_i \\
\text{s.t.} \quad & \sum_{i=1}^{500} x_i = 30 \\
& 0 \leq x_i \leq 1, i = 1, \ldots, 500,
\end{aligned}
$$

for some given parameters, $c_1, \ldots, c_{500}$.

(a) Characterize the extreme points of the problem.

(b) What is the number of extreme points?

(c) Suppose a computer can explore 100 billion extreme points per second. How long would it take to explore all the extreme points (in number of years)? Compare this with the age of the universe, which is roughly 13.8 billions years.

(d) Suppose such a computer weighs 0.5 kg. What would be the total weight of the computers required so that all the extreme points can be explored within a year? Compare this to the weight of the earth. The weight of earth is roughly 6×10^{24} kg.

(e) Propose an effective algorithm to solve the problem.

7. (Optional) Consider the following optimization problem:

$$
\begin{aligned}
\max \quad & 10x_1 + 12x_2 + 12x_3 \\
\text{s.t.} \quad & x_1 + 2x_2 + 2x_3 \leq 20 \\
& 2x_1 + x_2 + 2x_3 \leq 20 \\
& 2x_1 + 2x_2 + x_3 \leq 20 \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}
$$

(a) List all the basic solutions. A basic solution has the same definition of a basic feasible solution, except that it may not necessary be feasible. To do this, pick any three constraints (why 3?) and solve the system of linear equations to get a solution. The solution may or may not be feasible. You can use EXCEL help you solve the system of linear equations. Do a Google search on EXCEL Youtube linear equations.

(b) List the non degenerate the basic feasible solutions.

(c) List the degenerate basic feasible solutions.

(d) What is the optimal solution?