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Q1.

1a) 100

b)

A: active

B: non active

C: active

D: non active

c)

slack A: 0

surplus B: 30

slack C: 0

slack D: 100

d) shadow price is the change in objective value per unit increase in the RHS of a constraint

constraint B, D must have zero shadow price

e) constraint C may have positive shadow price

f) constraint A may have negative shadow price

g)

primal:

$$\max\ 2x_1 + 5x_2 + 2x_3 + 5x_4$$

$$\text{s.t}\quad -x_1 - 3x_2 + 2x_4 \geq -5 \quad : p_1$$

$$x_1 + x_2 + x_3 + x_4 \geq 5 \quad : p_2$$

$$4x_2 + 2x_3 + x_4 \leq 10 \quad : p_3$$

$$x_2 \leq 100 \quad : p_4$$

x_1, x_2, x_3, x_4 are free

dual:

$$\min\ -5p_1 + 5p_2 + 10p_3 + 100p_4$$

$$\text{st}\quad -p_1 + p_2 \geq 2$$

$$-3p_1 + p_2 + 4p_3 + p_4 \geq 5$$

$$p_2 + 2p_3 \geq 2$$

$$2p_1 + p_2 + p_3 \geq 5$$

$$p_1, p_2 \leq 0 \quad p_3, p_4 \geq 0$$

Q2.

a) max $4.5x_1 + 8x_2$

s.t. $\frac{1}{5}x_1 + \frac{3}{10}x_2 \leq 90$

$\frac{1}{7}x_1 + \frac{3}{7}x_2 \leq 90$

$x_1 \geq 200$

$x_2 \geq 100$

b) LOP can produce random prices of assembly labor & testing using dual

dual form: let p_1, p_2, p_3, p_4 be dual variables

min $90p_1 + 90p_2 + 200p_3 + 100p_4$

s.t. $\frac{1}{5}p_1 + \frac{1}{7}p_2 + p_3 \geq 4.5$

$\frac{3}{10}p_1 + \frac{3}{7}p_2 + p_4 \geq 8$

$p_1, p_2, \geq 0$

$p_3, p_4 \leq 0$

Q3

Let x be the decision to schedule shows, with $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix}$,

$\boldsymbol{r} = \begin{pmatrix} 6 \\ 10 \\ 9 \\ 4 \\ 5 \\ 2 \\ 6 \\ 7 \\ 8 \end{pmatrix}$, $\boldsymbol{C} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ are revenues and categories, or $\boldsymbol{C} = \begin{pmatrix} \boldsymbol{a'_1} \\ \boldsymbol{a'_2} \\ \boldsymbol{a'_3} \\ \boldsymbol{a'_4} \end{pmatrix}$

Problem: max $r'x - 4z$

s.t $\boldsymbol{a'_1 x} \geq \boldsymbol{a'_2 x}$

$x_7 \geq x_3 + x_4 - 1$

$x_7 + x_9 \leq 1$

$M \times \boldsymbol{a'_4 x} \geq \boldsymbol{a'_3 x} - 1$

$M \times z \geq \boldsymbol{a'_2 x} - 3$ with z as a binary variable

$\sum_{i=1}^{9} x_i = 5$

Q4

Let $A = \{(1,2),(1,3),(1,5),(2,3),(2,6),(2,7),(3,4),(4,7),(4,9),(5,6),(5,7),(6,9),(7,9)\}$,

(a) Max $\sum_{(i,j)\in A} x_{ij}$

s.t $x_{12} + x_{13} + x_{15} = 1$

$x_{12} = x_{23} + x_{27}$

$x_{13} = x_{34}$

$x_{34} = x_{47} + x_{49}$

$x_{15} = x_{56} + x_{57}$

$x_{26} + x_{56} = x_{69}$

$x_{27} + x_{57} = x_{79}$

$x_{69} + x_{79} + x_{49} = 1$

$0 \le x_{12} \le 9, 0 \le x_{13} \le 5, 0 \le x_{15} \le 20, 0 \le x_{23} \le 18,$

$0 \le x_{26} \le 20, 0 \le x_{27} \le 15, 0 \le x_{34} \le 15, 0 \le x_{47} \le 5,$

$0 \le x_{49} \le 16, 0 \le x_{56} \le 14, 0 \le x_{57} \le 5, 0 \le x_{69} \le 6,$

$0 \le x_{79} \le 8$

-3

(a) Find shortest path

A = {1, 2, 3, 4, 5, 6, 7, 9}

min $7x_{12} + 7x_{13} + 8x_{15} + 10x_{26} + 8x_{27} + 11x_{56} + 12x_{57} + 12x_{34} + 13x_{49} + 2x_{79} + 4x_{47} +$

$7x_{49} + 5x_{23}$

s.t. $x_{12} + x_{13} + x_{15} = 1$ (1 arc at start node)

$x_{69} + x_{79} + x_{49} = 1$ (1 arc at end node)

$x_{12} = x_{23} + x_{27} + x_{26}$ (node 2)

$x_{13} = x_{34}$ (node 3)

$x_{34} = x_{47} + x_{49}$ (node 4)

$x_{15} = x_{56} + x_{57}$ (node 5)

$x_{26} + x_{56} = x_{69}$ (node 6)

$x_{57} + x_{47} = x_{79}$ (node 7)

$x_{ij} \in \{0, 1\}$ for $(i, j) \in A$

network opt problem does not require x_{ij} to be binary

-1

4c) min cost flow to satisfy demands at sink node 5, 7, 9

demand: 10, 15, 6

$A = \{1, 2, 3, 4, 5, 6, 7, 9\}$.

$\min\ 7x_{12} + 7x_{13} + 8x_{15} + 10x_{26} + 8x_{27} + 11x_{56} + 12x_{57} + 12x_{34} + 13x_{49} + 2x_{79} + 4x_{47} +$

$7x_{49} + 5x_{23}$

s.t. $30 = x_{12} + x_{13} + x_{15}$ (node 1)

$x_{12} = x_{23} + x_{27} + x_{26}$ (node 2)

$x_{13} = x_{34}$ (node 3)

$x_{34} = x_{47} + x_{49}$ (node 4)

$x_{15} = x_{56} + x_{57} + 10$ (node 5)

$x_{26} + x_{56} = x_{69}$ (node 6)

$x_{57} + x_{47} = x_{79} + 15$ (node 7)

$x_{69} + x_{79} + x_{49} = 5$ (node 9)

(arc capacities)

$x_{12} \leq 9$ $x_{49} \leq 16$

$x_{15} \leq 20$ $x_{56} \leq 16$

$x_{13} \leq 5$ $x_{57} \leq 5$

$x_{23} \leq 18$ $x_{69} \leq 6$

$x_{26} \leq 20$ $x_{79} \leq 8$

$x_{27} \leq 15$

$x_{34} \leq 16$

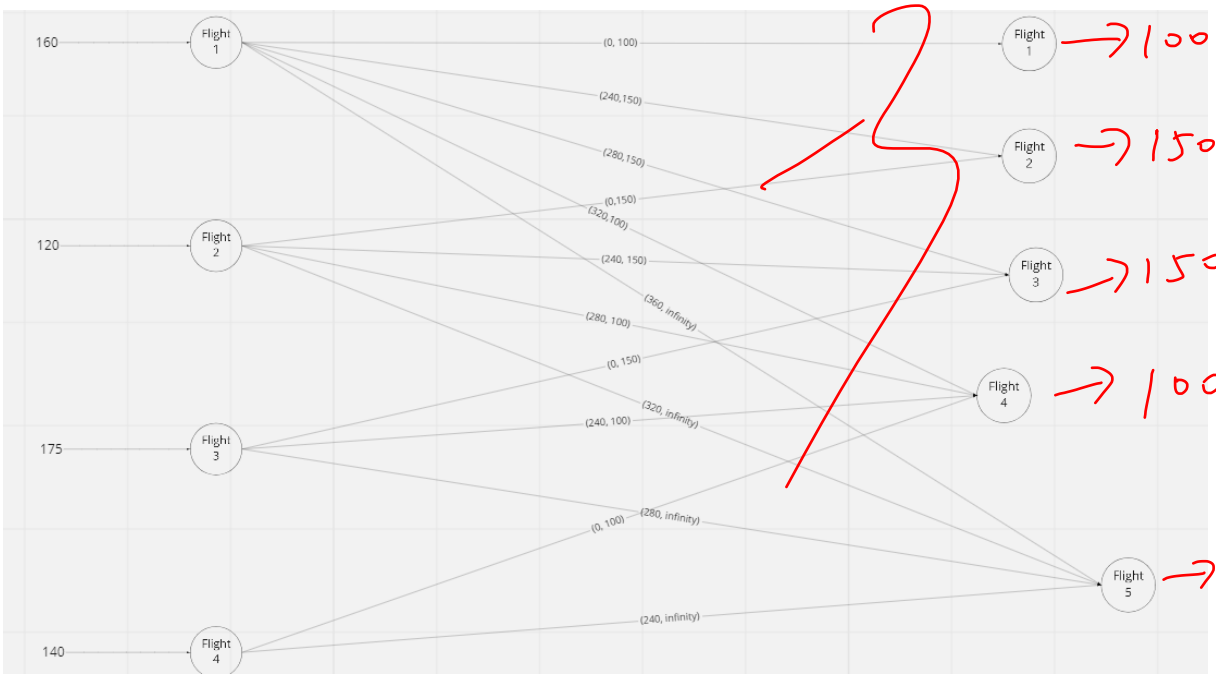
$x_{47} \leq 5$

$x_{ij} \geq 0 \quad \forall (i,j) \in A$ (non negativity)

A is not correctly defined

$A = \{(1,2), \dots, (6,9)\}$.

Q5



Min $240(x_{12} + x_{23} + x_{34} + x_{45}) + 280(x_{13} + x_{24} + x_{35}) + 320(x_{14} + x_{25}) + 360x_{15}$

s.t $160 = x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$

$120 = x_{22} + x_{23} + x_{24} + x_{25}$

$175 = x_{33} + x_{34} + x_{35}$

$140 = x_{44} + x_{45}$

$0 \le x_{11} \le 100, 0 \le x_{12} \le 150, 0 \le x_{13} \le 150, 0 \le x_{14} \le 100, 0 \le x_{15}$
$0 \le x_{22} \le 150, 0 \le x_{23} \le 150, 0 \le x_{24} \le 100, 0 \le x_{25}$
$0 \le x_{33} \le 150, 0 \le x_{34} \le 100, 0 \le x_{35}$
$0 \le x_{44} \le 100, 0 \le x_{45}, 0 \le x_{55}$