Nanyang Technological University

Lab 5 Report: Transformations & Motions

CZ2003 Computer Graphics and Visualization

Name: Ernest Ang Cheng Han
Tutorial Group: SSR1
Matric Number: U1921310H
N, M values: [N: 1, M: 10]

1a. Derive a transformation matrix performing rotation by $\frac{\pi}{2}$ about an axis parallel to axis Y and passing through the point with coordinates (M+5, 0, 0).

Transformation Matrix:

1. Rotation by $\frac{\pi}{2}$ about an axis parallel to axis Y
2. Pass through coordinates (M+5, 0, 0), where M = 10

In order to perform the transformation matrix, we will have to:

1. Translate curve to origin from point (15, 0, 0)
2. Rotate the curve by $\frac{\pi}{2}$
3. Translate curve back to point (15, 0, 0) from origin

Hence, transformation matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\frac{\pi}{2}) & 0 & \sin(\frac{\pi}{2}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\frac{\pi}{2}) & 0 & \cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} z + 15 \\ y \\ -x + 15 \\ 1 \end{bmatrix}$$

1b. Apply this matrix to the parametric definitions of the curve obtained in experiment 1 (exercise 3). Derive the transformed definitions $x(u)$,$y(u)$,$z(u)$,$u\in[0,1]$ of the rotated curve and display it.

From experiment 1, exercise 3:

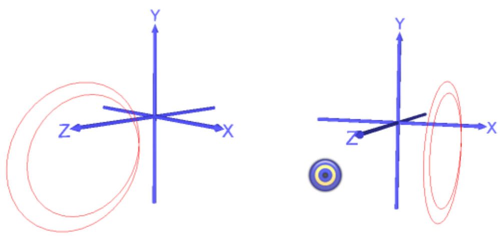
Polar Coordinates:
$r = 1 - (10 + 5)\cos\alpha$, $\alpha\in[0,2\pi]$
$r = 1 - 15\cos\alpha$

Parametric Definition:
$x(u) = (1 - 15\cos(2\pi u))\cos(2\pi u)$
$y(u) = (1 - 15\cos(2\pi u))\sin(2\pi u)$
$z(u) = 0$
$u \in [0,1]$

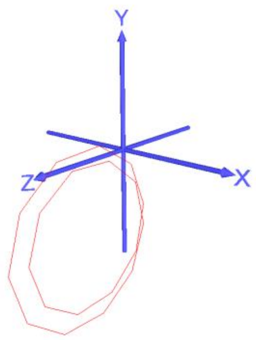
Applying the transformations specified in 1a, we get the following parametric equations:
$x(u) = 15$
$y(u) = (1 - 15\cos(2\pi u))\sin(2\pi u)$
$z(u) = -((1 - 15\cos(2\pi u))\cos(2\pi u) - 15)$
$u \in [0,1]$

Filename: 1b.vrml

Resolution: 70



Resolution: 10



Observations:
The limacon is created from joining multiple lines together between sampling points defined in the formula. The higher the sampling resolution, the higher the number of lines used to create the curve and hence the smoother the resulting shape. Minimum sampling resolution is 70 as any resolution lower would result in a pixelated curve with many uneven edges as observed above

2. Modify the parametric definitions to $x(u,t)$,$y(u,t)$,$z(u,t)$,u,$t\in[0,1]$ so that the rotation of the curve will be displayed as a 5 seconds rotation motion with some deceleration.

Animation of rotation:

1. 5 seconds
2. Deceleration

Deceleration function can be imitated with the following function:

- $f(t) = \frac{\pi t}{2}$

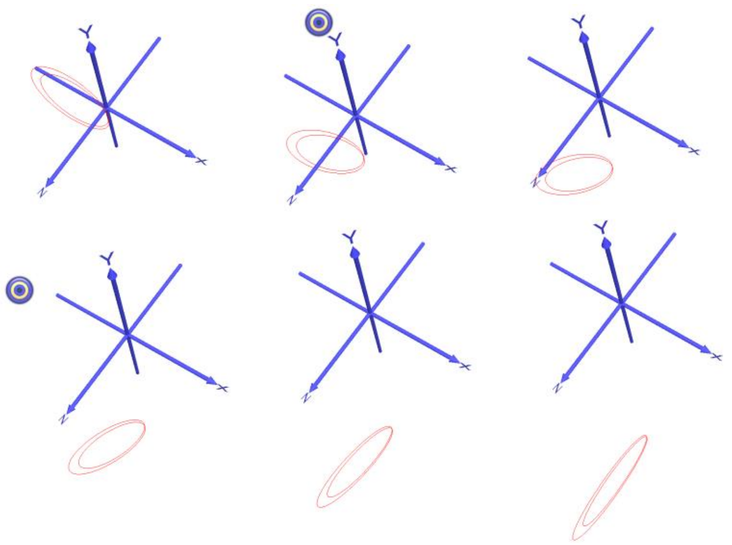
Duration of animation can be set to 5 seconds by changing the INTERVAL parameter to 5

To display the function, we will manipulate the “ParametricMotion.wrl” file and replace the x1, y1, z1 equations with the respective original Limacon parametric equations and replace the x2, y2, z2 equations with the respective transformed Limacon parametric equations.

Hence we get the final equation:

```
definition "
function parametric_x(u,v,w,t)
        {x1=cos(2*pi*u)*(1-15*cos(2*pi*u));x2=15;return x1+(x2-x1)*sin(pi/2*t);}
function parametric_y(u,v,w,t)
        {y1=sin(2*pi*u)*(1-15*cos(2*pi*u));y2=sin(2*pi*u)*(1-15*cos(2*pi*u));return
         y1+(y2-y1)*sin(pi/2*t);}
function parametric_z(u,v,w,t)
        {z1=0;z2=-cos(2*pi*u)*(1-15*cos(2*pi*u))+15;return z1+(z2-z1)*sin(pi/2*t);}"
```

Filename: 2.vrml
Resolution: 70



Observations:
Resolution remains the same as question 1 at 70 since we are using the same type of curve Limacon, any lower will again result in a pixelated curve with many uneven edges.

3a. With reference to Table 3, convert to
$x(u,v)$, $y(u,v)$, $z(u,v)$, $u,v \in [0,1]$ definitions of surfaces M and (N+M) and display them.

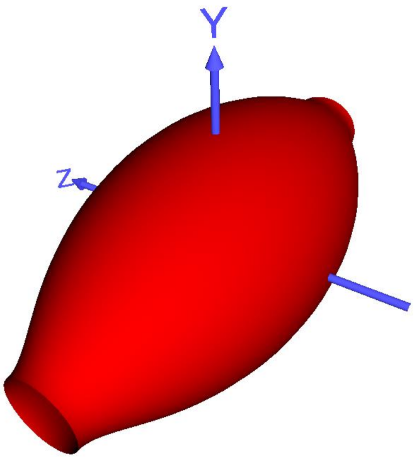
Surface 10 (M = 10):
x=0.5((ϕ–0.5sin(ϕ))–3)
y=0.5cos(4aπ)(1-0.5cos(ϕ))
z=0.5sin(4aπ)(1-0.5cos(ϕ))
0≤ a ≤ 0.5
0≤ ϕ ≤ 2π

Replace a with 0.5u,
Replace ϕ with 2πv

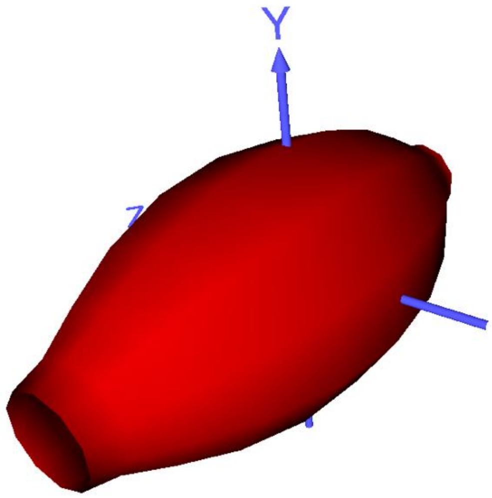
x=0.5((2πv–0.5sin(2πv))–3)
y=0.5cos(4(0.5u)π)(1-0.5cos(2πv))
z=0.5sin(4(0.5u)π)(1-0.5cos(2πv))
u,v ϵ [0,1]

Filename: 3aSurface10.vrml

Resolution: 40



Resolution: 10



Observation:
Minimum Resolution for this surface would be 40. Any lower value in resolution would result in a surface with many clear and uneven edges which renders a rough surface as shown above.

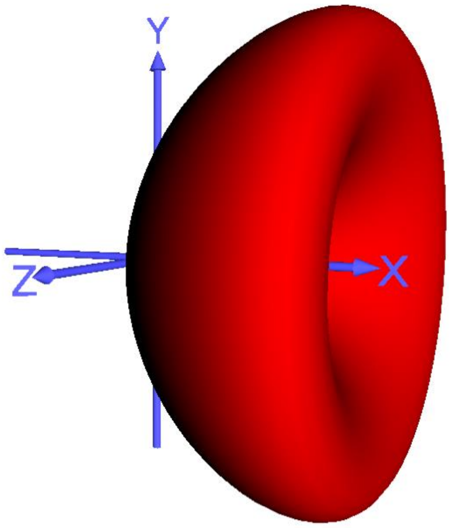
Surface 11 (N = 1, M = 10):
$x=\frac{2.5\varphi}{(1+\varphi^3)}$
$y=\frac{2.5(\cos\theta)\varphi^2}{(1+\varphi^3)}$
$z=\frac{2.5(\sin\theta)\varphi^2}{(1+\varphi^3)}$
0≤ θ ≤ 2π
0≤ ϕ ≤ 2π

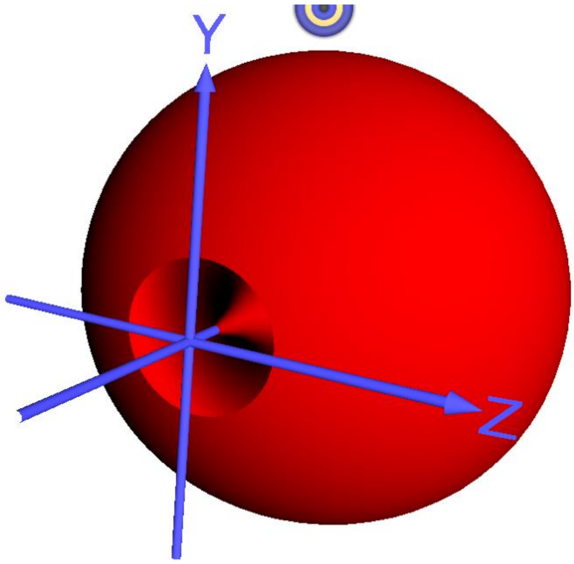
Replace θ with 2πu,
Replace ϕ with 2πv

$$x=\frac{2.5(2\pi v)}{(1+2\pi v^3)}$$
$$y=\frac{2.5(\cos(2\pi u))2\pi v^2}{(1+2\pi v^3)}$$
$$z=\frac{2.5(\sin(2\pi u))2\pi v^2}{(1+2\pi v^3)}$$
$u,v \in [0,1]$

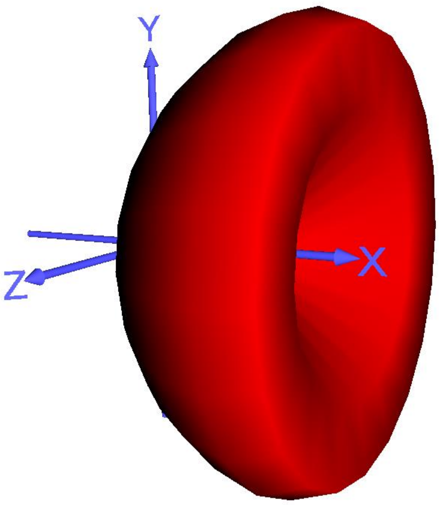
Filename: 3aSurface11.vrml

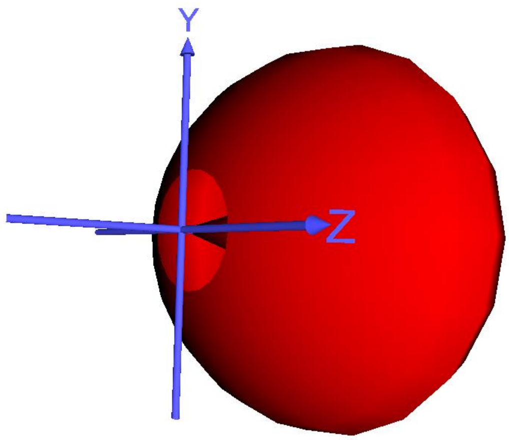
Resolution: 70





Resolution: 20





Observation:

Minimum Resolution for this surface would be 70. Any lower value in resolution would result in a surface with many clear and uneven edges which renders a rough surface as shown above.

3b. Define parametrically using x(u,v,t), y(u,v,t), z(u,v,t), u,v,t∈[0,1] a swing (back and forth) morphing transformation between surfaces M and (N+M). The morphing animation has to take 5 seconds and has be done with a uniform speed.

Animation of rotation:

1. 5 seconds
2. Swing back and forth uniformly

Deceleration function can be imitated with the following function:

- $f(t) = 1 - \mathrm{fabs}(1 - 2 * \mathrm{t}))$

Duration of animation can be set to 5 seconds by changing the INTERVAL parameter to 5

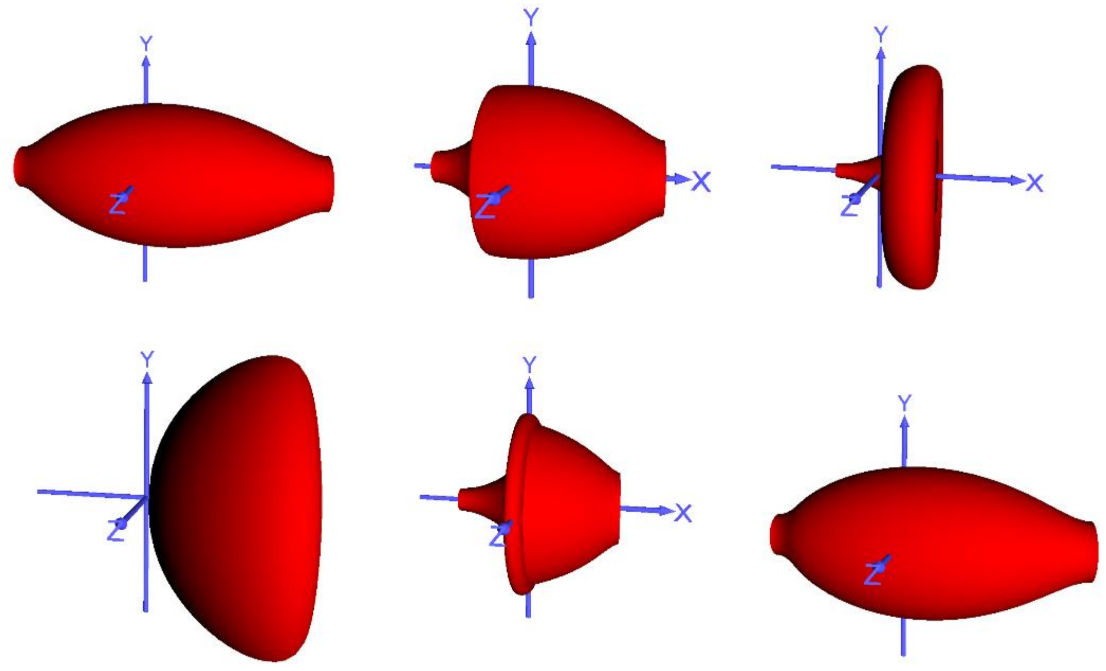
To display the function, we will manipulate the “ParametricMotion.wrl” file and replace the x1, y1, z1 equations with the respective parametric equations of Surface 10 and replace the x2, y2, z2 equations with the respective parametric equations of Surface 11.

```
definition "
function parametric_x(u,v,w,t)
{x1 = 0.5*((2*pi*v-0.5*sin(2*pi*v))-3);x2 = (2.5*(2*pi*v))/(1+(2*pi*v)^3);return x1*(1-(1-fabs(1-2*t))) + x2*(1-fabs(1-2*t));}

function parametric_y(u,v,w,t)
{y1 = 0.5*cos(4*(0.5*u)*pi)*(1-0.5*cos(2*pi*v));y2 =
(2.5*((2*pi*v)^2)*(cos(2*pi*u)))/(1+((2*pi*v)^3));return y1*(1-(1-fabs(1-2*t))) + y2*(1-fabs(1-2*t));}

function parametric_z(u,v,w,t)
{z1 = 0.5*sin(4*(0.5*u)*pi)*(1-0.5*cos(2*pi*v));z2 =
(2.5*((2*pi*v)^2)*(sin(2*pi*u)))/(1+((2*pi*v)^3)); return z1*(1-(1-fabs(1-2*t))) + z2*(1-fabs(1-2*t)); }"
```

Filename: 3b.vrml
Resolution: 70



Observations:
Resolution used will be the higher of the 2 resolutions of surface 10 and 11, in this case it would be 11 at 70. Any lower value in terms of resolution will again result in a pixelated surface with many uneven edges.