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Detecting Level Shifts, Temporary Changes and Innovational Outliers in Intervention Analysis

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Abstract

We discuss the extension of the outlier detection procedure originally proposed in Chareka et al. (2006) to detect an additive outlier (AO) to also detect the level shift (LS), the temporary change (TC) and the innovational outlier (IO). The normally distributed statistic is converted to the chi-square statistic which has been shown to converge to the Gumbel distribution. This test which is applicable to a wide range of Gaussian time series is extended using simulated long memory time series data and the Nile river data.

AMS subject classification:
Keywords: additive outlier, level shift, temporary change, innovational outlier, intervention, long memory, extreme values, Gumbel distribution.

1. Introduction

Outlier detection is a major challenge in intervention analysis since it entails determining whether an intervention has actually occurred and what type it is. This is the first stage in this type of analysis pioneered by Box and Tiao(1975)in their quest to solve the Los Angeles pollution problem. Tiao (1983) extended these ideas further but Chang *et al.* (1988)gave a detailed explanation on how to handle an additive outlier (AO) and an innovational outlier(IO). This was extended to the level shift (LS) and temporary change(TC) by Chen and Liu (1993). Also, Kirkendall (1992), Abraham and Chuang (1993) and Trivez (1995) developed robust procedures in the estimation of time series with interventions. The complexity of the robust procedures and limiting distributions of the statistics involved has led to the use of distribution free procedures attributed to Chang et al. (1988) where the data was "short memory" and each time a test is conducted critical values are simulated.

In Chareka et al. (2006) the issue of reliability of existing tests is highlighted. This encompasses how in fractionally integrated series pre-estimation of some model is required before the intervention can be detected and/or removed. The main problem is the estimation of the long memory parameter “d" as given in Granger and Joyeux (1980), Geweke and Porter-Hudak (1983) and others.

At this stage we introduce the various outliers to give the reader a clear picture of each of them before we extend the theory to each of them.

1.1. Additive Outlier (AO)

The Additive Outlier (AO) is usually referred to as a gross error affecting the t^{th} observation as shown in Figure 1. The AO model where $\delta = 0$ in the general form

$$Y_t = Z_t + \frac{\omega}{1-\delta} X_t^{(i)} \tag{1}$$

but with $\delta = 0$ resulting in the model simplifying to

$$Y_t = Z_t + \omega X_t^{(i)} \tag{2}$$

where

$$X_t^{(i)} = \begin{cases} 1, & t = i \\ 0, & otherwise \end{cases} \tag{3}$$

In complete form this is given by

$$Y_t = \frac{\theta(B)}{\phi(B)\alpha(B)} a_t + \omega X_t^{(i)}$$

where $\alpha(B) = (1-B)^{d_1}(1-B^s)^{d_2}$. If we assume that the ARIMA model is $\pi(B) = \frac{\theta(B)}{\phi(B)\alpha(B)}$, the error term $e_t = \omega\pi(B)X_t^{(i)} + a_t$ which is the one time impact as shown in figure.

1.2. Level Shift (LS)

The Level Shift shown in Figure 2 is an abrupt but permanent shift by ω in the series caused by an intervention and takes on the maximum value of $\delta = 1$ in equation 1 so that the model becomes

$$Y_t = Z_t + \frac{\omega}{1-B} X_t^{(i)} \tag{4}$$

The complete Level Shift Model is written as follows,

$$Y_t = \frac{\theta(B)}{\phi(B)\alpha(B)} a_t + \frac{\omega}{1-B} X_t^{(i)}$$

where $X_t^{(i)}$ is as given in equation(eqn3).

The resulting residual model is $e_t = \omega \frac{\pi(B)}{1-B} X_t^{(i)} + a_t$.

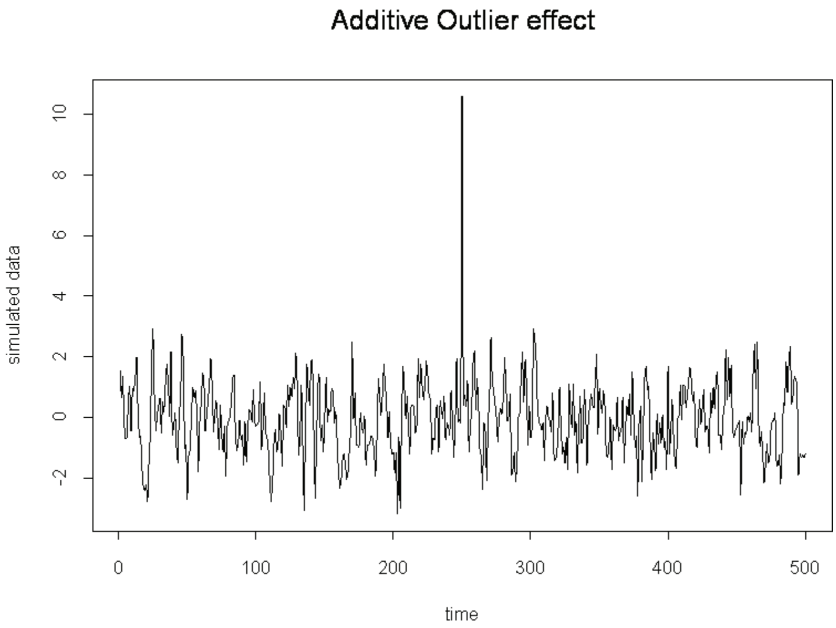

Figure 1: Additive Outlier Effect.

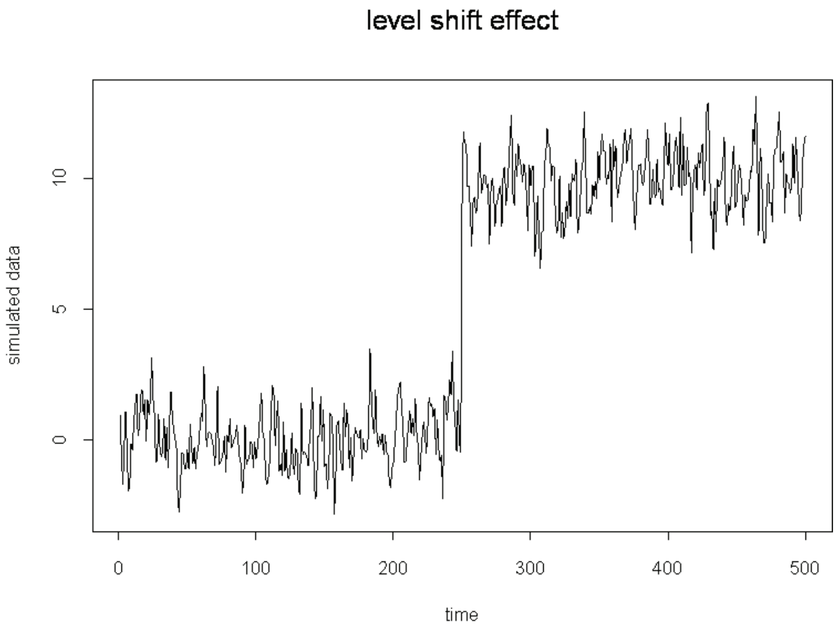

Figure 2: Level Shift.

1.3. Temporary Change (TC)

The intervention which occurs when $0 < \delta < 1$ is called a temporary change because it is not a one time effect like the additive outlier but results in an effect ω at time t_1

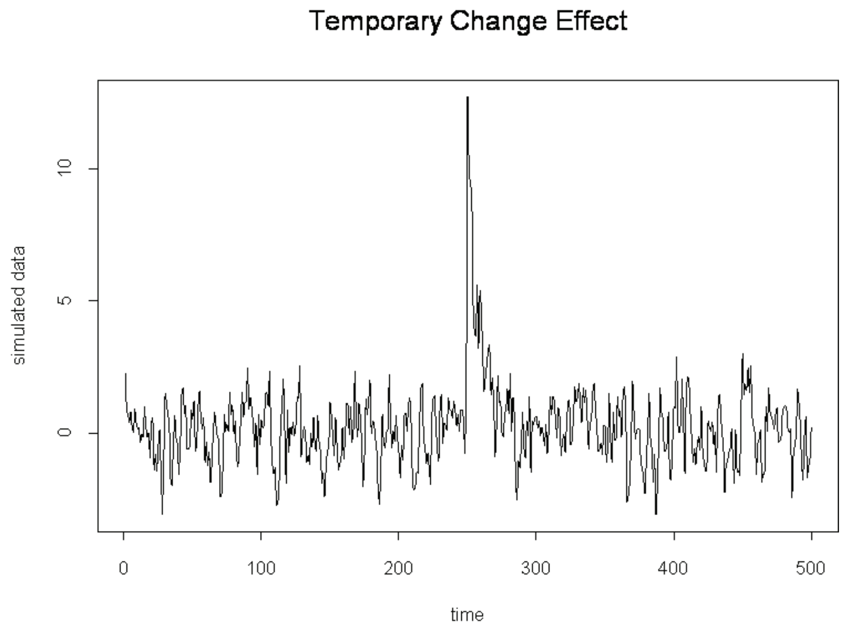

Figure 3: Temporary Change Effect.

and dies out gradually. Chen and Liu (1993) differentiate these outliers clearly and the model for a TC is given as follows

$$Y_t = Z_t + \frac{\omega}{1-\delta B} X_t^{(i)} \tag{5}$$

where $X_t^{(i)}$ is given in Equation(eqn3). The complete TC model can be written as follows:

$$Y_t = \frac{\theta(B)}{\phi(B)\alpha(B)} a_t + \frac{\omega}{1-\delta B} X_t^{(i)}$$

The effect of a Temporary Change are shown in Figure 3.

1.4. Innovational Outlier (IO)

An innovational outlier(IO) is an extraordinary shock at time t influencing Y_t,$Y_{t+1}\dots$ through the dynamic system $\pi(B) = \dfrac{\theta(B)}{\phi(B)\alpha(B)}$. The complete IO outlier model can be written as follows:

$$Y_t = \frac{\theta(B)}{\phi(B)\alpha(B)} (\omega X_t^{(i)} + a_t) \tag{6}$$

where as usual, $X_t^{(i)}$ is as given in Equation (eqn3).

For a stationary ARMA, an IO will produce a temporary effect since the π_j's decay rapidly to zero. Now, for a strongly dependent time series, Chen and Liu (1993) believe that an IO may produce a level shift for various types of ARIMA models.

The effect of an Innovational Outlier is shown in Figure 4.

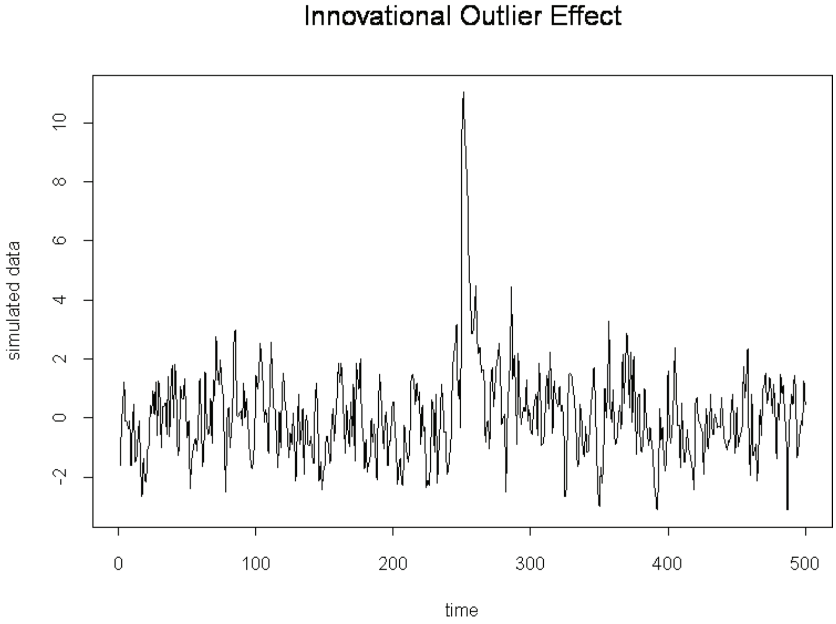

Figure 4: Innovational Outlier Effect.

2. The Outlier Detection Procedure

The outlier detection procedure proposed in Chareka et al. (2006) is based on the statistic

$$T = \max\left\{|t_n(1)|, \ldots, |t_n(n)|\right\} = \max\left\{\frac{|\hat{\omega}_0(1)|}{s_n(1)}, \ldots, \frac{|\hat{\omega}_0(n)|}{s_n(n)}\right\} \tag{7}$$

where $s_n(i)$ is an estimate of the standard error of $\hat{\omega}_0(i)$, $\hat{\omega}_0(i) = Y_i - \bar{Y}_i(n)$ is the estimated intervention at time $t = i$ and $\bar{Y}_i(n)$ is the the sample mean of $Y_1, Y_2 \cdots Y_n$ a time series with observation Y_i omitted.

This statistic is normally distributed and is squared to give the χ^2 a Gamma distribution which is in the domain of attraction of the Gumbel distribution an extreme value distribution of Type I with normalising constants
$d_n = \sqrt{2\ln(n)} - \dfrac{\ln(\ln(n)) + \ln(4\pi)}{2\sqrt{2\ln(n)}}$, $c_n = 1/\sqrt{2\ln(n)}$ and $d_n = 2\ln(n) - \ln(\ln(n)) - 2\ln\Gamma(\pi)$ and $c_n = 2$, respectively. The maximum domain of attraction of the Gumbel is shown to some extent in Leadbetter et al. (1983) and in greater detail in Embrechts et al. (1997). The main result of that paper was the theorem proving the convergence to the Gumbel. This theorem is given below:

Theorem 2.1. Let $\{Y_t\}$ be a time series satisfying the AO model

$$Y_t = \omega_0 X_t^{(i)} + Z_t$$

Assume that the stationary component of the model $\{Z_t\}$ is a Gaussian time series with mean 0 and autocovariance function $\{\gamma_z(k)\}$ such that

$$\lim_{n\to\infty} \gamma_z(n)ln(n) = 0$$
$$\sup_{h\geq 1}\{\gamma_z(h)\} < \gamma_z(0)\ and$$
$$\sum_{k=1}^{\infty} \frac{|\gamma_z(k)|}{k^{\delta}} < \infty \text{ for some } \delta,\ 0 \leq \delta < 1$$

For any realization $Y_1, Y_2, \cdots, Y_n$ of this time series, let $C_n = \dfrac{T_{nn}^2 - d_n}{c_n}$ as defined in equations above. Then under $H_0: \omega_0 = 0$ the statistic C_n satisfies

$$C_n \xrightarrow{D} \Lambda(x) = exp(-e^{-x}),\ \ as\ n \to \infty,$$

where D signifies convergence in distribution.

The proof in Chareka et al. (2006) consists of verifying whether the conditions $D(u_n)$ and $D'(u_n)$ are satisfied by the time series since they fall into the categories of sequences mentioned above. This is done by means of lemmas and theorems such as results contained in lemma 4.4.7 of Embrechts et al. (1997 p. 217) or the Normal Comparison Lemma in Leadbetter and Rootzen (1988) or Leadbetter et al. (1983). The stationary sequences described here show that Gaussian stationary time series have the same extremal behavior as the Gaussian i.i.d. sequences under Berman's condition as well as theorems and lemmas used to prove the convergence such as “squeeze theorems" including Slutsky theorem from Chung (1968).

The hypothesis tested is

$$H_0: \omega_0 = 0 \quad \text{against} \quad H_1: \omega_0 \neq 0$$

which is based on $Y_1, \cdots, Y_n$ a realization of a time series $\{Y_t\}$ satisfying for example the AO model which as before is given by

$$Y_t = \omega_0 X_t^{(i)} + Z_t$$

The intervention/outlier parameter ω_0 can be estimated using various methods like the maximum likelihood and least Squares methods. The least squares estimate of ω_0 if the intervention is at time $t = i$ is $\hat{\omega}_{0i} = \dfrac{\sum_{t=1} Y_t X_t^{(i)}}{\sum_{t=1}^{n} X_t^{(i)2}} = Y_i = Z_i$ on simplification and in view of the conditions of $X_t^{(i)}$. Here the test of hypothesis entails comparing

$$C_n = \frac{T_{nn}^2 - d_n}{c_n}$$

the statistic obtained with an appropriate critical value from the Gumbel distribution. The largest $T_i^2(n)$statistic is considered an outlier at the α significance if the C_n value exceeds the $1-\alpha$ quantile of the Gumbel distribution. This has been shown to be true for the additive outlier (AO).

2.1. Extension to the other outliers

Theorem 1 discussed above can be applied to the rest of the interventions/outliers i.e. the LS,the TC, the IO as explained below.

2.1.1 Level Shift (LS)

Recall from equation 4 that the level shift is

$$Y_t = \frac{\omega_0 X_t^(i)}{1-B} + Z_t$$

assuming $\mu = 0$ and since $\delta = 1$. This becomes

$$\begin{aligned} Y_t - Y_{t-1} &= \omega_0 X_t^{(i)} + Z_t - Z_{t-1} && (8) \\ \Rightarrow \Delta Y_t &= \omega_0 X_t^{(i)} + \Delta Z_t && (9) \end{aligned}$$

This implies differencing the series which is a transformation. Thus if $\omega_0 = 0$ then $\Delta Y_t = \Delta Z_t$. Now

$$\begin{aligned} \hat{\omega_0} &= \frac{\sum_{t=2}^{n} \Delta Y_i X_t(i)}{\sum_{t=2}^{n} X_t^{(i)}} \quad for\ i = 2 && (10) \\ &= \Delta Y_i && (11) \\ &= \Delta Z_i && (12) \end{aligned}$$

This is true since $X_t^{(i)}$

$$X_t^{(i)} = \begin{cases} 1, & t = i \\ 0, & otherwise \end{cases} \quad (13)$$

Then by theorem (1)

$$d_n = 2\ln(n-1) - \ln(\ln(n-1)) - 2\ln\Gamma(\pi)$$

$$c_n = 2$$

$$\Rightarrow C_n = \frac{T_{nn} - d_n}{c_n} \quad \Rightarrow \Lambda(x) \quad (14)$$

Implying convergence in distribution to the Gumbel i.e $C_n \overset{D}{\to} e^{-e^{-x}}$.

2.2. Temporary Change (TC)

The extension of the theorem to Temporary Change is possible with the application of the invariance principle. Suppose that as before we assume that the mean $\mu = 0$ then the following arguement holds.

$$Y_t = \frac{\omega X_t^{(i)}}{1-\delta B} + Z_t \tag{15}$$

This can be expressed as

$$\begin{aligned} Y_t &= (1-\delta)\mu + \omega X_t^{(i)} + \delta Y_{t-1} + Z_t - \delta Z_{t-1} \\ &= \omega X_t^{(i)} + \delta Y_{t-1} + Z_t - \delta Z_{t-1} \\ &= \omega X_t^{(i)} + Z_t^* \end{aligned}$$

Now

$$Y_t = \omega X_t^{(i)} + Z_t^* \tag{16}$$

where $Z_t^* = \delta Y_{t-1} + Z_t - \delta Z_{t-1}$. It is now easy to see that under $H_0 : \omega_0 = 0$, $Z_t^* = Z_t$. Thus, the test statistic for testing $H_0 : \omega_0 = 0$ based on the least squares estimate of ω_0 has the same limiting distribution as the test statistic for testing $H_0 : \omega_0 = 0$ in the AO model

$$Y_t = \omega_0 X_t^{(i)} + Z_t$$

This means that by theorem 1 but with

$$d_n = 2\ln(n-1) - \ln(\ln(n-1)) - 2\ln\Gamma(\pi)$$

$$c_n = 2$$

the statistic C_n converges in distribution to the Gumbel.

2.3. Innovational Outlier

Similarly, for the innovational outlier the invariance principle can also be applied as follows,

$$Y_t = \frac{\theta(B)}{\phi(B)\alpha(B)}(a_t + \omega_0 X_t^{(i)})$$

where a_t is white noise. For simplification suppose that $\alpha(B) = 1$ and $\theta(B) = 1$. Then

$$1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p = a_t + \omega_0 X_t^{(i)}$$

Implying that

$$\begin{aligned} Y_t &= \phi_1 B Y_{t-1} + \phi_2 B Y_{t-2} + \cdots + \phi_p^p + a_t + \omega_0 X_t^{(i)} \\ &= Z_t^* + \omega_0 X_t^{(i)} \end{aligned}$$

where

$$Z_t^* = \phi_1 B Y_{t-1} + \phi_2 B^2 Y_{t-2} + \cdots + \phi_p B^p Y_{t-p} + a_t$$

which is an autoregressive process $AR(p)$.

When $\omega_0 = 0$ then $Y_t = Z_t^*$ since the least squares estimator yields

$$\hat{\omega} = Y_i = Z_i$$

when $X_t^{(i)}$ is as given in equation (7). This means that $Y_t = Z_t$ for all t and this implies that $Z_t^* = Z_t$. By theorem 1 the statistic C_n converges in distribution to the Gumbel. i.e. $\Lambda(x)$, with norming constants $d_n = 2\ln(n-p) - \ln(\ln\ (n-p) - 2\ln\Gamma(\pi)$, $c_n = 2$ This can be extended to more complex cases of the ARMA and FARMA.

2.4. General Transfer Function

There are other outliers which can take up the form of the general transfer function model and it can be shown that this also converges in distribution to the Gumbel by theorem 1. We have applied this to a case where one intervention or outlier exists but with several δs then following is true.

$$\begin{aligned} Y_t &= \frac{\omega(B)}{\delta(B)} X_t^{(i)} + Z_t \\ &= \frac{\omega_0}{1 - \delta_1 B - \delta_2 B^2 - \cdots \delta_r B^r} X_t^{(i)} + Z_t \\ &= \delta_1(Y_{t-1} - Z_{t-1}) + \delta_2(Y_{t-2} - Z_{t-2}) + \cdots \delta_r(Y_{t-r} - Z_{t-r}) + Z_t \\ &= Z_t^* + \omega_0 X_t^i \end{aligned}$$

where $Z_t^* = Z_t + \delta_1(Y_{t-1} - Z_{t-1} + \cdots + \delta_r(Y_{t-r} - Z_{t-r})$.

If $\omega_0 = 0$ then $Y_t = Z_t$ for all t. This implies that $Z_t* = Z_t$. This is true since $X_t^{(i)}$ is as given in Equation (7) and the least squares estimator

$$\begin{aligned} \hat{\omega}_0 &= \frac{\sum_{t=r}^n Y_t X_t^{(i)}}{\sum_{t-r}^n X^{(i)2}} \quad i \geq r \\ &= Y_i = Z_i^* \\ &\Rightarrow Z_t^* = Z_t \end{aligned}$$

Then the statistic C_n converges in distribution to the Gumbel.

In this case $d_n = 2\ln(n-r) - \ln(\ln(n-r)) - ln\Gamma(\pi)$ and $c_n = 2$.

3. Simulation Study of the Outlier Detection Procedure

We conducted simulations consisting of simulations of critical values for different outliers, simulations of different sizes of outliers and conducting detection tests as well as determining the power of the outlier detection procedure. This was done to show how how the procedure works.

Table 1: Critical Values for Level Shift using (r=1000,AR=0.2,MA=0.3)

	α	$d=0$	$d=0.1$	$d=0.2$	$d=0.3$	$d=0.4$
$n=200$	10%	2.2476	2.3822	2.1423	2.1884	2.0470
	5%	2.9384	2.9541	2.9377	2.9304	2.6631
	1%	4.7360	4.5172	4.9400	4.6858	3.7196
$n=300$	10%	2.4496	2.4322	2.2043	2.0833	1.9731
	5%	3.3289	3.1390	2.8427	2.7883	2.6615
	1%	4.7813	4.4327	4.1325	4.2346	4.1592
$n=400$	10%	2.5257	2.4074	2.2236	2.2010	1.9399
	5%	3.1257	3.1939	2.7509	2.7891	2.7299
	1%	4.8819	4.6416	4.6615	4.1372	4.3755
$n=500$	10%	2.3199	2.3522	2.3297	2.0930	2.1501
	5%	3.1271	2.8283	2.8417	2.7948	2.7566
	1%	5.1649	4.7859	4.2018	4.7931	4.4904
$n=1000$	10%	2.1093	2.9067	2.2622	2.1939	2.0615
	5%	2.7874	2.9067	2.8145	2.7742	2.4854
	1%	4.3935	4.4377	4.7806	4.2161	4.2576

3.1. Critical Value Simulation for different Outliers

The critical values obtained using SPLUS are shown in table 1. An assumption that there are no outliers is made then simulations are conducted. This is based on an estimate of the statistic $C_n = \dfrac{T^2_{nn} - d_n}{c_n}$ with the norming constants.

$$d_n = 2ln(n) - \ln(\ln(n)) - 2\ln(\Gamma(\pi)) \text{ and} \tag{17}$$

$$c_n = 2 \tag{18}$$

for an additive outlier and these vary for different types of outliers as given in the previous section.

The resulting critical values are very close are very close to the Gumbel critical values for the 10%, 5% and 1% level of significance i.e 2.2504, 2.9702 and 4.6102. Table 1 obtained using differenced data for level shift detection is typical of the simulated critical values.This is true for different types of outliers, different models regardless of whether the long memory parameter values are negative or positive within the range $d = 0$ to 0.4. Samples of size 200, 300, 400, 500 and 1000 are used for brevity and a detailed analysis of the tables can be found in the thesis.

3.2. Simulated Outlier Detection

We need stress the point that the position of the outlier i.e. point $t = i$ is not known. This outlier C_n is tested for significance using the hypotheses $H_0 : \omega_0 = 0$ versus $H_1 : \omega_0 \neq 0$ as stated before. An observation corresponding to the maximum $T_i^2(n)$ is considered an outlier at the α level of significance if the C_n statistic exceeds the $1-\alpha$ quantile of the Gumbel distribution given by $q_\alpha = -ln[-ln(1-\alpha)]$.

For illustration purposes, different outliers of different sizes $\omega = 5\sigma$ and 10σ are introduced in a long memory time series of $n = 500$ at point 250 using SPLUS and conclusions are made. A brief description of the test per outlier is given below.

(i) **Additive Outlier**: Additive outliers of size 10σ and 5σ are introduced at point 250 and the resulting test statistics are, $\hat{C}_n = 35.462$ and $\hat{C}_n = 6.4056$ respectively. These are greater than the critical values of the Gumbel 2.2504, 2.9702 and 4.6102 implying the rejection of H_0 at all levels of significance.

(ii) **Level Shift**: Level shifts of similar sizes as above are introduced at point 250 and the resulting test statistics occurring at point 249 due to differencing are $\hat{C}_n = 17.5840$ and $\hat{C}_n = 5.44756$ respectively. These are also greater than the critical values at all levels of significance implying the rejection of H_0.

(iii) **Temporary Change**: Introducing temporary changes of sizes similar to the ones introduced in (i) above yields test statistics, $\hat{C}_n = 15.6914$ and $\hat{C}_n = 4.53248$ respectively. This implies that we reject H_0 for the first value and do not reject H_0 for the second value.

(iv) **Innovational Outlier**: Introduction of innovational outliers of similar sizes as above results in $\hat{C}_n = 46.0815$ and $\hat{C}_n = 9.0108$ which are both greater than the critical values at all levels of significance, implying the rejection of H_0.

Clearly, whenever an outlier is exists it will be detected but the problem paused by the long memory parameter d may affect the detection.

3.3. Power of the Outlier Detection Test

Ideally the power of the test based on the C_n statistic which is the probability of correctly identifying an intervention should be equal to 1. Its empirical estimate is the fraction of the time the statistic exceeds the given quantile of the Gumbel distribution. Thus it is obtained from the Gumbel distribution formula as a probability which is

$$P(x) = e^{-e^{-x}}$$

where x is known. If a level of significance of 0.05 is used then the power should converge to 0.05 for a case where the intervention $\omega = 0$.

Table 2 is an illustration of the power of the test for the different outliers. It shows the frequency(denoted as Count) with which the location of an outlier is correctly detected,the probability(denoted as Prob) of correctly detecting the outlier and the size of the outlier in the form of the statistic $\hat{C}_n$. It must be noted that the size of the error and the underlying model affects the observations because of the fluctuations that are inherent in long memory time series. The underlying model used is $AR1 = 0.2, MA = 0.3$ for 1000 replications.

Table 2: Power of the Test using(r=1000,AR1=0.2,MA=0.3 C=2.2504 d=0.4)

	ω		3σ			4σ			5σ	
AO	n	300	400	500	300	400	500	300	400	500
	$Count$	480	479	432	760	740	697	917	913	895
	$Prob$	0.999	0.939	0.996	0.922	0.931	0.993	0.996	0.962	0.999
	C_n	11.958	2.778	5.996	2.504	2.652	4.998	5.526	3.253	6.715
LS	$Count$	245	214	202	477	449	447	745	765	758
	$Prob$	0.989	0.977	0.978	0.901	0.965	0.986	0.999	0.977	0.999
	C_n	4.549	3.766	3.783	2.265	3.333	6.748	5.831	3.747	6.698
TC	$Count$	286	254	229	571	561	524	841	821	840
	$Prob$	0.559	0.974	0.478	0.985	0.854	0.999	1.000	1.000	0.998
	C_n	0.541	3.622	0.304	4.201	1.846	7.359	11.746	13.832	6.132
IO	$Count$	294	263	243	572	548	511	847	831	842
	$Prob$	0.788	0.754	0.601	0.970	0.982	0.947	1.000	1.000	1.000
	C_n	1.433	1.263	0.675	3.470	3.985	2.906	7.760	10.2073	11.676

In the table the power of the detection test is shown when different outliers of size $\omega = 3\sigma$,4σ and 5σ are introduced using the 90% critical value of 2.2504. It is clear that the probability of correctly detecting an outlier is high as long as the outlier $\hat{C}_n$ is significantly different from the critical value 2.2504 but it is low if the particular observation is not necessarily an outlier.

The AO occurs more frequently as shown by the frequency of occurrence which is generally higher than that of the other outliers. The level shift occurs less often as shown by the low frequency. It should be stressed however, that when any of the outliers are present and depending on their size they can be detected with a probability close to 1.

3.4. Application to Real Life Data (Nile data)

The outlier detection procedure is applied to real life data, in this case the Nile data which is a typical long memory data set consisting of water levels measured at Roda Gauge near Cairo for the years 622-1281 Beran (1994 p. 237–239). The hypotheses of

$$H_0 : \omega_0 = 0 \ \ (there\ is\ no\ outlier)$$

versus

$$H_1 : \omega_0 \neq 0 \ \ (thereisanoutlier)$$

are tested as before for all the four types of outliers on the raw data comparing the resulting C_n statistics to the Gumbel critical values.

Testing for a level shift which involves differencing and results in a significant observation being detected at $n = 257$ of the differenced series with $C_n = 6.239$ which is significant at 0.10, 0.05 and 0.01 with a power of 0.998. In the real series this point occurs at $n = 258$ and is due to a drop of the level from 1340 to 959 with the rather low level lasting for another year. The shift is temporary and a plot of the series shows a rather slight shift probably due to the concentrated data. It must be noted that the procedure is more convenient and sensitive since it detected the shift which had also been detected using the procedure by Chang et al. (1988). However, the advantage here is that the statistic has a distribution and more inference is now possible.

Tests for the additive outlier, the temporary change and the innovational outlier yields insignificant values because of the fluctuations inherent in the underlying long memory model. The proposed solution mentioned before requires preestimation of the model and when this is done the two additive outliers are detected.

4. Conclusion

We have extended the use of the Gumbel distribution in detecting level shifts, temporary changes and innovational outliers. The observation in Chareka et al. (2006) that only large outliers will be detected from the raw data is reaffirmed by our ability to detect only the level shift in the Nile data set because the shift is very significant. We do not detect any other outlier type except after preestimation but then if the outliers are not significant in the raw data they may not significantly affect the results. These results tally with those presented in Matarise's unpublished DPhil thesis. Further research needs to be conducted to determine these effects.

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