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PATH PLANNING FOR COLLISION AVOIDANCE MANEUVER

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ABSTRACT

This paper summarizes the development of an optimal path planning algorithm for collision avoidance maneuver. The goal of the optimal path is to minimize distance to the target vehicle ahead of the host vehicle subject to vehicle and environment constraints. Such path constrained by allowable lateral (centripetal) acceleration and lateral acceleration rate (jerk). Two algorithms with and without lateral jerk limitation, are presented.

The algorithms were implemented in Simulink and verified in CarSim. The results indicate that the lateral jerk limitation increases time-to-collision threshold and leads to a larger distance to the target required for emergency lane change. Collision avoidance path without lateral jerk limitation minimizes the distance to the target vehicle and is suitable for path tracking control in real-time application; however tracking such a path requires very aggressive control.

INTRODUCTION

Collision avoidance control comprises several components: path planning, tracking control, threat assessment, etc. The path tracking performance of an autonomous vehicle depends on good path planning. If the path is easy to follow, then the tracking error will be small. Numerous path representation methods have been suggested in the literature: sinusoidal, exponential, sequence of circular arcs, polynomial splines, clothoids curves, trapezoidal acceleration profile, etc.

Sliding model control and path planning was discussed in [1], where a lane change with piece-wise linear curvatures was considered. Sinusoidal and exponential trajectories were proposed in [2]. Optimal emergency maneuver path was investigated in [3] with different optimality criteria, namely, minimum distance to target or minimum maneuver duration. Sinusoidal trajectory was proposed in [4]. Fifth order polynomials for the path were considered in [5], [6], among which only [5] considered a production type of algorithm, which covers straight and curved road. The use of a fifth-order polynomial guarantees smoothness of the path (for both straight and curved roads), however it does not provide explicit control of the path curvature.

An optimal path for emergency maneuver should satisfy several criteria: minimize the longitudinal gap between the host and the target vehicles; be suitable for path tracking; be simple enough to run in real-time; merge into the target lane (straight or curved) centerline with prescribed degrees of smoothness.

This paper describes approaches to generate an optimal path for collision avoidance maneuver. Without limitation on lateral jerk, the optimal path consists of circular arc and cubic polynomial arc. Such path is continuously-differentiable, but the second derivative exhibits step (jump) discontinuity. On the other hand, with imposed limitation on lateral jerk, the optimal path consists of quadratic and cubic polynomials arcs and is twice continuously-differentiable and therefore does not have discontinuity (trapezoidal acceleration profile, [7]).

PATH PLANNING WITHOUT LATERAL JERK LIMITATIONS

In the context of this paper, the optimal path for a collision avoidance maneuver is the one that avoids obstacle from the closest distance (minimizes the distance from avoidance maneuver starting point to the target vehicle's rear bumper). This path is computed by minimizing the longitudinal distance to the target vehicle subject to vehicle dynamics constraints: maximum allowable lateral acceleration and maximum allowable lateral jerk.

1. Formulation of the problem

It is assumed that the host vehicle is equipped with a forward camera for lane sensing and received data includes left and right positions, heading angles, and curvatures. Thus, the lane marker lateral distance y is a known quadratic function of the longitudinal distance x. Based on the current lane marking position and the lane width, the target lane centerline (left or right adjacent lane) can be represented in the following form:

$$y_{lane} = a_0 + a_1 x + \frac{1}{2} a_2 x^2 \tag{1}$$

where a_0 is the lateral offset (position), a_1 is the heading angle, and a_2 is the curvature (Figure 1). Typically a_0 is close to the lane width. If a_0 is positive, then the emergency maneuver is performed to the left adjacent lane, and if a_0 is negative, then the emergency maneuver is performed to the right adjacent lane.

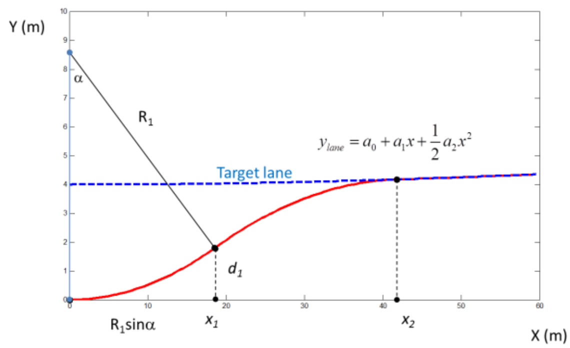

Figure 1. Desired path in vehicle coordinate system.

The path is defined in the host vehicle coordinate system with *x* pointing forward and *y* pointing to the left, and the origin at the vehicle center of gravity (CG). The tangent line to the path coincides with the vehicle centerline, so the desired vehicle heading along the path is the angle between the axis *X* and the tangent to the path.

Assuming that the vehicle speed is *V* and the maximum allowable lateral acceleration is a_{ymax}, the minimal allowable path radius of curvature is $R_1 = \frac{V^2}{a_{y\max}}$. For the path planning purpose, it is assumed that the vehicle is a mass point with combined lateral and longitudinal accelerations satisfying the friction ellipse concept. Typical path radius of curvature ranges between 30 and 300 meters and is a function of the speed and allowable maximum lateral acceleration.

Referring to Figure1, the optimal path consists of three parts defined by the breaking points $x=x_1$ and $x=x_2$. The first part ($0 \le x \le x_1$) is to avoid collision (steer away from the target object in front), the second part ($x_1 < x \le x_2$) is to position the host vehicle in the target lane, and the third part ($x_2 < x$) is to follow the target lane until the driver regains control.

The first part is a circular arc with radius R_1. The second part is a parabolic curve. A circular arc can be also used for the second part, but the algebraic computations become more difficult and a closed form solution for the curved road is not possible. However it should be noted that for our application the difference between a circular arc and its parabolic approximation is very small due to the fact that the turning radius is much larger than the lane width (collision avoidance maneuver is done at sufficiently high speed).

All three parts should be matched at the breaking points with prescribed degrees of smoothness. For example, we require that the path is continuous and has continuous derivative. The second derivative, on the other hand, will have jump discontinuities or infinite jerk, at the breaking points x_2 and x_3. Such path is an aggressive one. In reality, the lateral jerk will be limited by the vehicle inertia properties and chassis actuator capabilities, so the vehicle may not exactly follow the planned path. Insisting on tracking such a path will require very aggressive control.

The following expression for the path is considered:

$$y = \begin{cases} (R_1 - \sqrt{R_1^2 - x^2})\,sign(a_0), \; x \le x_1 \\ y_1 - \frac{1}{2} k (x - x_1)^2 + b(x - x_1), x_1 < x \le x_2 \\ a_0 + a_1 x + \frac{1}{2} a_2 x^2, x > x_2 \end{cases} \tag{2}$$

where

$$x_1 = R_1 \sin\alpha \,, \; \alpha = \arccos\,(1 - \frac{d_1}{R_1}) \,,$$

$$y_1 = d_1 sign(a_0) \,, \; b = \tan\alpha$$

$$k = \frac{(a_1 + a_2 x_1 - b)^2}{2(a_0 - d_1 + a_1 x_1 + \frac{1}{2} x_1^2)} - a_2 \,, \;\; x_2 = x_1 + \Delta x \,,$$

$$\Delta x = \frac{2(a_0 - d_1 + a_1 x_1 + \frac{1}{2} x_1^2)}{a_1 + a_2 x_1 - b} \tag{3}$$

Here Δx is the longitudinal distance corresponding to the second part of the maneuver. Based on the measured lane lateral offset a_0, lane heading angle a_1, lane curvature a_2, longitudinal velocity *V*, predefined a_{ymax} and d_1, Formula (2) determines the lateral coordinate *y* as a function of the longitudinal distance *x*.

The heading angle can be obtained from (2) by differentiating with respect to x:

$$\frac{dy}{dx} = \begin{cases} \frac{x}{\sqrt{R_1^2 - x^2}} sign(a_0), \; x \le x_1 \\ y_1 - k(x - x_1) + b, x_1 < x \le x_2 \\ a_1 + a_2 x, x > x_2 \end{cases} \tag{4}$$

The curvature (signed) of the path can be calculated according to the curvature formula for a plane curve:

$$\kappa = \frac{y''}{(1 + y'^2)^{3/2}} \tag{5}$$

where the prime denotes differentiation with respect to x.

Figures 2 shows an example of the collision avoidance path, for the following case: $V=80$ km/h, $a_{ymax}=8.0$ m/sec^2, ($R_1=61.7$ m), $d_1=1.8$ m, $a_0=3.6$ m; $a_1=0$, $a_2=0.002$ (lane radius of curvature is 500 m).

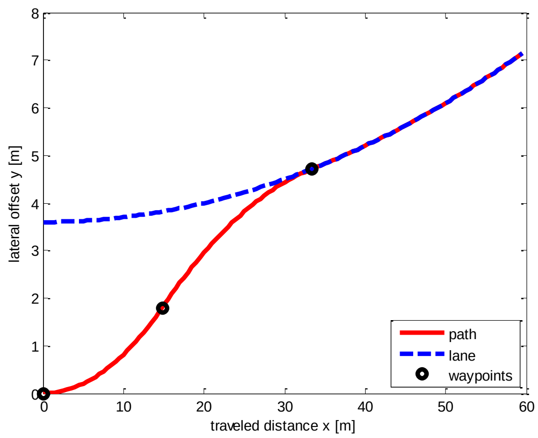

Figure 2. Collision avoidance path

2. Planned maneuver duration

The planned duration of the maneuver can be computed as $T=T_1+T_2$, where T_1 is the time for the first part of the maneuver (steer away) and T_2 is the time for the second part (counter steer and positioning in the target lane). T_1 and T_2 can be computed as:

$$T_1 = x_1/V\ ,\ T_2 = \Delta x/V \tag{6}$$

In (6) longitudinal component of the motion only is considered. Strictly speaking, the duration should be the ratio of the path length and absolute velocity (which is assumed constant), but in a collision avoidance maneuver, the lateral motion is much smaller than the longitudinal motion. Since the lateral offset d_1 is much smaller than the radius R_1, the following approximate value for T_1 can be obtained:

$$x_1 = \sqrt{R_1^2 - (R_1 - d_1)^2} \approx \sqrt{2d_1R_1}$$

$$T_1 \approx \sqrt{2d_1R_1}\,/V = \sqrt{2d_1R_1/V^2} = \sqrt{\frac{2d_1}{a_{y\max}}} \tag{7}$$

Let us assume for simplicity, that $a_1=a_2=0$, then expression for T_2 and T can be obtained using longitudinal distances x_1 and Δx corresponding to the first and the second part of the maneuver defined in (3):

$$T_2 = \frac{\Delta x}{V} \approx \frac{2(a_0-d_1)}{\sqrt{2d_1a_{y\max}}} = \sqrt{\frac{2d_1}{a_{y\max}}}\frac{a_0-d_1}{d_1} \tag{8}$$

$$T = T_1 + T_2 = \sqrt{\frac{2d_1}{a_{y\max}}}\left(1+\frac{a_0-d_1}{d_1}\right) = a_0\sqrt{\frac{2}{d_1a_{y\max}}} \tag{9}$$

It is seen that the durations T_1 and T do not depend on velocity, but mainly depend on lane width, target vehicle width, and maximum allowable lateral acceleration. For $a_0=3.6$ m, $d_1=1.8$ m, $a_{ymax}=8.0$ m/sec^2, the duration of the first part of maneuver is about $T_1=0.67$ sec and the duration of the whole maneuver is about $T=1.49$ sec.

In general, the duration of the emergency maneuver for this example is about 1.5-2 sec and depends only slightly on the velocity.

3. Maneuver feasibility

The path defined by the Formulas (2)-(4) exist only when certain conditions apply:

- the end point of the maneuver x_2 should be after the counter steer point x_1, or $\Delta x \ge 0$;
- the curvature of the second part should be finite and opposite to the curvature of the first part;
- the absolute value of the curvature of the second part should be less than or equal to the absolute value of the curvature of the first part.

Let us analyze these conditions. Assume that $a_0>0$ for definiteness. Expression for Δx becomes unbounded if $b = a_1 + a_2x_1$. This condition has a clear geometric meaning: the slope of the path and the slope of the lane are equal at the counter steer point $x=x_1$. Thus a necessary condition for the path to exist is that the slope of the lane at $x=x_1$ is less than the slope of the path at the same point x_1, i.e., $b > a_1 + a_2x_1$. In other words, the tangent to the path should intersect the tangent to the lane at some point to the right of x_1.

Curvature k in (3) becomes unbounded if

$$a_0 + a_1x_1 + \frac{1}{2}x_1^2 = d_1 \tag{10}$$

The left hand side of (10) is the lane lateral coordinate at $x=x_1$ which means that the lane passes through the counter steer point (x_1, d_1), i.e. $y_{lane}(x_1) = d_1$ Therefore, another necessary condition for the path to exist is that the lane lateral offset at the counter steer point is greater than d_1, in other words, the lane is above the path when $x= x_1$.

The curvature of the second part will be opposite to the curvature of the first part when

$$\frac{(a_1 + a_2x_1 - b)^2}{2(a_0 - d_1 + a_1x_1 + \frac{1}{2}x_1^2)} > a_2 \tag{11}$$

The numerator of the left side in (11) is the square of the difference between the path slope and the lane slope at $x=x_1$, and denominator is the lane lateral offset at the counter steer point x_1. Condition (11) means that the difference between the path slope and the lane slope at $x=x_1$ should be sufficiently large to compensate for the lane curvature. Condition (11) automatically holds if a_1 is negative, so the lane is curved in the same direction as the second part of the path.

The lateral acceleration in the second part of the maneuver should not exceed a_{ymax}, so the last necessary condition is $k \le 1/R_1$ which can be rewritten using (3) as

$$\frac{(a_1 + a_2 x_1 - b)^2}{2(a_0 - d_1 + a_1 x_1 + \frac{1}{2} x_1^2)} > a_2 + \frac{1}{R_1} \tag{12}$$

The meaning of (12) is analogous to (11) in that the difference between the path slope and the lane slope at $x=x_1$ should be sufficiently large to compensate for the lane curvature and maximum lateral acceleration restrictions.

4. Path planning parameter d_1 and TTC threshold

The path planning algorithm described above depends on parameter d_1, which defines the lateral displacement at the counter steer point. What is the best way to choose this parameter? In a collision avoidance maneuver the host vehicle must move laterally at least the distance equal to the average of the host vehicle width (h_h) and the target vehicle width (h_t).

To minimize the distance to the target at the start of the emergency maneuver, parameter d_1 should be as large as possible. However, the value of d_1 is bounded by the host and target vehicle widths and the vehicle ability to perform the second part of the maneuver (feasibility condition). One simple way to choose d_1 is

$$d_1 = \min(\frac{1}{2} a_0, h)\,, \quad h = \frac{h_h + h_t}{2} \tag{13}$$

Condition (13) reflects the fact that the host vehicle needs to move laterally at least h meters, however if h is more than the half of the lane width, then d_1 is equal to the half of the lane width.

A different approach for choosing d_1 is discussed in the next paragraph.

Let x_h be the longitudinal distance that the host vehicle needs to travel to move laterally h meters, then the time to collision (TTC) threshold can then be defined as:

$$Th = \frac{x_h + \Delta}{V} + \tau \tag{14}$$

Here Δ is the safety margin in meters and τ accounts for the steering actuator delay. Distance x_h can be derived using path representation (2) by equating y to h. Let us assume for simplicity that $a_1=a_2=0$, then TTC threshold becomes

$$Th = \left[a_0 - \sqrt{(a_0 - d_1)(a_0 - h)}\right]\sqrt{\frac{2}{d_1 a_{y\max}}} + \frac{\Delta}{V} + \tau \tag{15}$$

Conservatively, we can use the duration of the complete maneuver and not the duration of the maneuver required to steer away from the target vehicle. In this case Formula (9) defines TTC threshold as

$$Th = a_0\sqrt{\frac{2}{d_1 a_{y\max}}} + \frac{\Delta}{V} + \tau \tag{16}$$

5. Path planning with maximum allowable lateral acceleration in both phases.

The lateral acceleration during the second part (after counter steer) of the emergency maneuver has been constrained before not to be more than a_{ymax} only. However, the most aggressive emergency maneuver corresponds to the case when the lateral acceleration in both parts equals to a_{ymax} (with opposite signs). In this case, parameter d_1 is not a path design parameter but will depend on lane parameters, velocity, and a_{ymax}.

Consider the path planning problem in which the path curvature is always at the maximum allowable level. The curvature of the second part of the path is determined in (3). By equating k in the equation to $1/R_1$ we obtain

$$k = \frac{(a_1 + a_2 x_1 - b)^2}{2(a_0 - d_1 + a_1 x_1 + \frac{1}{2} x_1^2)} - a_2 = \frac{1}{R_1} \tag{17}$$

where $d_1 = \sqrt{R_1^2 - (R_1 - x_1)^2} \approx \dfrac{x_1^2}{2R_1}$ and $b = \dfrac{x_1}{\sqrt{R_1^2 - x_1^2}} \approx \dfrac{x_1}{R_1}$.

Thus Equation (17) is a quadratic equation with respect to x_1. Solution takes the following form:

$$x_1 = R_1 \begin{cases} \dfrac{a_1 + \sqrt{\frac{a_1^2}{2}\left(1 + a_2 R_1\right) + \frac{a_0}{R_1}\left[1 - \left(a_2 R_1\right)^2\right]}}{1 - a_2 R_1}, & a_0 > 0 \\[2ex] \dfrac{-a_1 + \sqrt{\frac{a_1^2}{2}\left(1 - a_2 R_1\right) - \frac{a_0}{R_1}\left[1 - \left(a_2 R_1\right)^2\right]}}{1 + a_2 R_1}, & a_0 < 0 \end{cases} \tag{18}$$

If $a_2=0$ then

$$x_1 = R_1\left(a_1 + \sqrt{\frac{a_1^2}{2} + \frac{a_0}{R_1}}\right) \tag{19}$$

Lane curvature a_2 is much smaller than the path curvature, i.e., $\left|a_2\right| << 1/R_1$ so expression (19) captures the main effect. Parameter d_1 in this case is equal to

$$d_1 = \frac{R_1}{2}\left(a_1 + \sqrt{\frac{a_1^2}{2} + \frac{a_0}{R_1}}\right)^2 \tag{20}$$

In the case of a straight lane and a straight position of the host vehicle ($a_1=0$, $a_2=0$), Formula (20) simplifies to $d_1 = \dfrac{a_0}{2}$ as expected.

Feasibility conditions take the following form:

- discriminant of the quadratic equation for x_1 must be nonnegative, i.e.

$$\frac{a_1^2}{2}\left(1 + sign(a_0) a_2 R_1\right) + \frac{|a_0|}{R_1}\left[1 - \left(a_2 R_1\right)^2\right] \ge 0 \tag{21}$$

- and

$$x_1 > 0, x_2 > x_1$$

6. Implementation of path planning algorithm and path distortion due to coordinate transform

A path tracking control system for collision avoidance maneuver is based on the vehicle motion relative to the lane marking. It requires lateral displacement y and heading angle ψ relative to the lane marking in the vehicle reference frame. If Y_{CG} and ψ_{CG} are the lateral offset and heading angle of the vehicle CG in the ground reference frame, and Y_{lane} and ψ_{lane} are the lateral offset and heading angle of the lane marking in the ground reference frame, then the relative lateral displacement and heading angle in ground reference frame OXY are (see Figure 3):

$$\Delta Y = Y_{CG} - Y_{lane},\ \Delta\psi = \psi_{CG} - \psi_{lane} \tag{22}$$

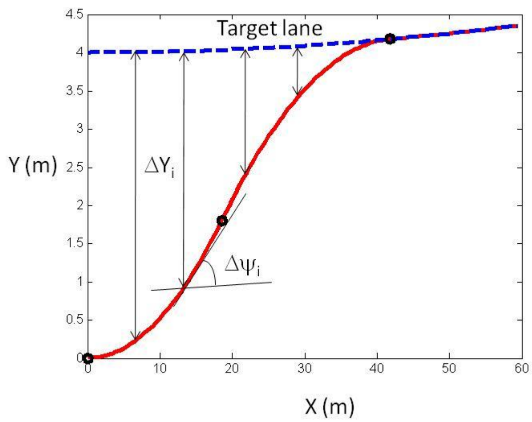

Figure 3. Practical implementation of the path

The path planning procedure described above was done in ground reference frame with the origin at the start of the maneuver. The vehicle local reference frame coincides with this ground reference frame only at the start of the maneuver. Position (X, Y, ψ) of the vehicle with respect to the ground reference frame fixed at the start of the maneuver can be estimated using kinematic equations:

$$\begin{aligned} \dot{X} &= V_x \cos\psi - V_y \sin\psi \\ \dot{Y} &= V_x \sin\psi + V_y \cos\psi \\ \dot{\psi} &= r \end{aligned} \tag{23}$$

where V_x and V_y are longitudinal and lateral velocities in vehicle reference frame, and r is the yaw rate. Discretized equations (23) will define an estimator for X, Y, and ψ. Since this estimator needs to run only during the emergency maneuver (1-2 sec), the integration error will not build up. Estimated X will determine the current point on the path (X, ΔY), as well as all subsequent values of ΔY and $\Delta\psi$.

The lateral control algorithm requires the path to be defined in the vehicle coordinate system. Therefore, ΔY needs to be converted into the vehicle reference frame. Relative (path with respect to lane) angle $\Delta\psi$ is the same in the vehicle reference frame and the ground reference frame.

Correction to the lateral offset can be computed as

$$\Delta y_i = \frac{\Delta Y_i}{\cos\psi_i} \tag{24}$$

The algorithm for path generation can now be formulated as follows:

- Estimate position X_O of the vehicle CG in ground reference frame fixed at the start of the maneuver
- Define set points x_i in vehicle reference frame ($i=1,2,\ldots,n$)
- Find corresponding points X_i ($X_i=X_O+x_i\cos\psi$))
- Find ΔY_i and $\Delta\psi_i$ corresponding to X_i (path in ground reference frame)
- Determine Δy_i using (24)
- Δy_i and $\Delta\psi_i$ forms a path point

The sequence of pairs $\{\Delta y_i,\ \Delta\psi_i\}$ constitutes the path used by the path tracking control.

Correction (24) is an approximation. Exact transformation of the path from the ground reference frame into the vehicle reference frame is not possible in real time because of the nonlinear nature of the algebraic problem. However, (24) still provides a very good approximation since the heading angle during the emergency maneuver is not large.

PATH PLANNING WITH LATERAL JERK LIMITATIONS

The lateral acceleration limit implicitly imposes a limit on the path curvature. For a given vehicle speed V (in m/sec) the relationship between the path curvature and the maximum lateral acceleration a_{ymax} is

$$\kappa = \frac{a_{y\max}}{V^2} \tag{25}$$

where κ is the trajectory curvature. Assuming that the vehicle velocity does not change much during the maneuver, the relationship between the lateral jerk and trajectory curvature derivative with respect to longitudinal distance x can be derived from (25) by differentiating both sides with respect to time and dividing both sides by V:

$$\frac{d\kappa}{dx} = \frac{\eta}{V^3} \tag{26}$$

where $\eta = da_y/dt$ is the lateral acceleration rate or jerk. The limitation on lateral jerk also imposes a limitation on the derivative of the curvature with respect to the longitudinal distance x, in other words,

$$\kappa \le \frac{a_{y\max}}{V^2} \tag{27}$$

$$\left|\frac{d\kappa}{dx}\right| \le \frac{\eta_{\max}}{V^3} \tag{28}$$

The optimal path for this problem will have piecewise linear curvatures as illustrated in Figure 4. In the beginning, the curvature increases linearly with the maximum rate until it reaches its maximum value at X_1, then it remains constant until X_2, then it linearly decreases until it reaches its negative maximum value at X_3; it then stays constant until X_4, and finally linearly increases until it reaches the target lane curvature at X_5.

The optimal path is defined in vehicle reference frame and described with the function $y = y(x)$. The signed curvature of the path has the following form:

$$\kappa = \frac{y''}{\left(1 + y'^2\right)^{3/2}} \tag{29}$$

where prime denotes differentiation with respect to x. For a vehicle emergency maneuver (which is done at sufficiently high speed) y' is much smaller than 1, therefore we can approximately assume the curvature to be equal to the second derivative y'' of the path $y(x)$ with respect to x. Consequently, the second derivative of the optimal path has the form shown in Figure 4. In order to get the path $y=y(x)$, we need to twice integrate the path curvature in Figure 4 with respect to x. The resulting function $y(x)$ is a combination of cubic and quadratic functions separated by the break points x_1-x_5. These break points can be determined using matching conditions for $y(x)$, its derivative $y'(x)$, and its second derivative $y''(x)$ at the matching points (break points).

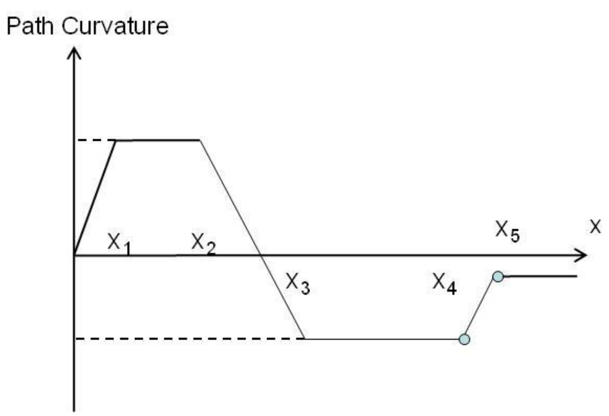

Figure 4. Path curvature as a function of distance traveled

On each of these subintervals path $y(x)$ can be described as

$$y_1 = \frac{a_1 x^3}{6},\ x \in [0, x_1],\quad a_1 = \frac{\eta}{V^3},\quad x_1 = \frac{V a_{y\max}}{\eta} \tag{30}$$

$$y_2 = \frac{1}{2} b_2 (x - x_1)^2 + c_2 (x - x_1) + d_2,\ x \in [x_1, x_2]$$

$$y_3 = \frac{1}{6} a_3 (x - x_2)^3 + \frac{1}{2} b_3 (x - x_2)^2 + c_3 (x - x_2) + d_3,\ x \in [x_2, x_3]$$

$$y_4 = \frac{1}{2} b_4 (x - x_3)^2 + c_4 (x - x_3) + d_4,\ x \in [x_3, x_4]$$

$$y_5 = \frac{1}{6} a_5 (x - x_4)^3 + \frac{1}{2} b_5 (x - x_4)^2 + c_5 (x - x_4) + d_5,\ x \in [x_4, x_5]$$

$$y = y_{lane}(x),\ x \ge x_5$$

The matching conditions (continuity of the function y(x) and its first and second derivatives) at the break points x_1, x_2, x_3, x_4 can be described as

$$d_2 = \frac{1}{6} a_1 x_1^3,\ c_2 = \frac{1}{2} a_1 x_1^2,\ b_2 = a_1 x_1,\ x = x_1 \tag{31}$$

$$d_3 = \frac{1}{2} b_2 \left(x_2 - x_1\right)^2 + c_2 \left(x_2 - x_1\right) + d_2, \qquad c_3 = b_2 (x_2 - x_1) + c_2,$$

$$b_3 = b_2,\ a_3 = -a_1,\ x = x_2$$

$$d_4 = \frac{1}{6} a_3 (x_3 - x_2)^3 + \frac{1}{2} b_3 \left(x_3 - x_2\right)^2 + c_3 \left(x_3 - x_2\right) + d_3,$$

$$c_4 = \frac{1}{2} a_3 (x_3 - x_2)^2 + b_3 (x_3 - x_2) + c_3,$$

$$b_4 = a_3 (x_3 - x_2) + b_3,\ x = x_3$$

$$d_5 = \frac{1}{2} b_4 \left(x_4 - x_3\right)^2 + c_4 \left(x_4 - x_3\right) + d_4, \qquad c_5 = b_4 (x_4 - x_3) + c_4,$$

$$b_5 = b_4,\ a_5 = a_1,\ x = x_4$$

At $x=x_5$ the path should match the target lane. Assume that the lane (more precisely lane centerline) is described by a quadratic function (in vehicle reference frame):

$$y_{lane} = \frac{1}{2} \beta x_5^2 + \gamma x_5 + \delta \tag{32}$$

where β is the lane curvature, γ is the lane heading, and δ is the lane lateral offset. Then the matching conditions at $x=x_5$ are

$$\frac{1}{6} a_5 (x_5 - x_4)^3 + \frac{1}{2} b_5 (x_5 - x_4)^2 + c_5 (x_5 - x_4) + d_5 = \frac{1}{2} \beta x_5^2 + \gamma x_5 + \delta$$

$$\frac{1}{2} a_5 (x_5 - x_4)^2 + b_5 (x_5 - x_4) + c_5 = \beta x_5 + \gamma \tag{33}$$

$$a_5 (x_5 - x_4) + b_5 = \beta$$

In (33) coefficients a_5, b_5, c_5, d_5 should be expressed in terms of a_1 and x_1 using conditions (31). Equations (33) are nonlinear algebraic equations for finding x_2, x_4, x_5.

Using the last expression in (33), x_5 can be expressed in terms of x_4 (linear relation) as

$$x_5 = x_4 + \frac{\beta + b_2}{a_1} = x_4 + \frac{\beta V^3 + V a_{\max}}{\eta} \tag{34}$$

Using the second equation in (33), x_4 can be expressed in terms of x_2 (linear relation). Eventually first expression in (33) becomes a quadratic equation with respect to x_2 and can be explicitly solved. The coefficients of this quadratic equation were obtained using the Matlab symbolic toolbox and are not presented here due to their complicated form.

As an illustrative example, let the target lane centerline have the following representation in the vehicle coordinate system:

$$\delta = 3.6[m], \gamma = -0.1[rad], \beta = 0.001[1/m]$$

Figure 5 shows the collision avoidance path and the break points which were obtained as

$x_1=4.5$ m, $x_2=8.7$ m, $x_3=17.8$ m, $x_4=28.5$ m, $x_5=33.5$ m

For this scenario: $Vx=100$ kph, $a_{ymax}= 8.0$ m/sec^2, η = 49 m/sec^3 (or 1 g per 200 msec).

Path planning algorithm was verified using Carsim and Matlab Simulink simulation software. Integrated model includes

Carsim test vehicle model and EPS actuator model. Model predictive control algorithm was employed for directing the vehicle along the planned path. Figure 6 shows collision avoidance path and vehicle CG trajectory during collision avoidance maneuver. Speed, maximum lateral acceleration, and maximum lateral jerk values are the same as in Figure 5. The maximum tracking error in Figure 6 is less than 10 cm.

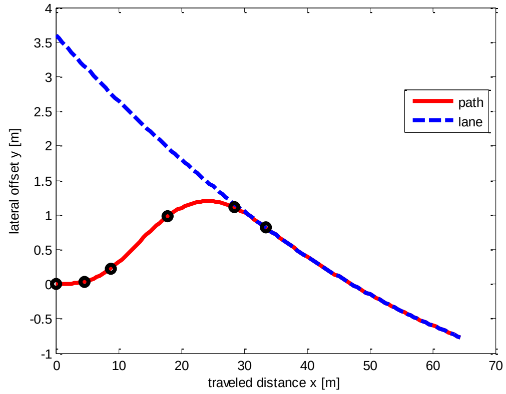

Figure 5. Collision avoidance path y=y(x)

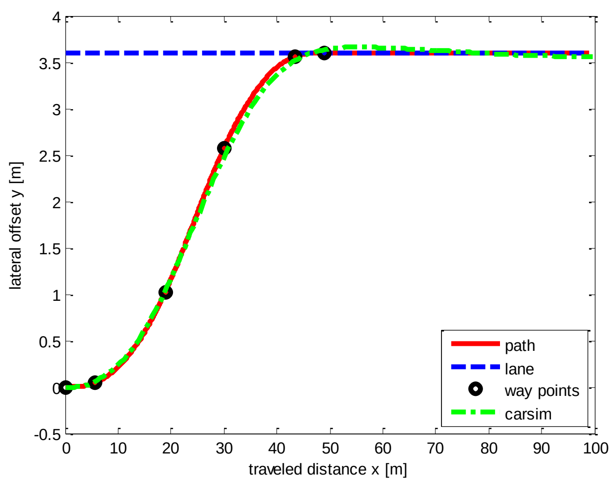

Figure 6. Path tracking using Carsim simulation

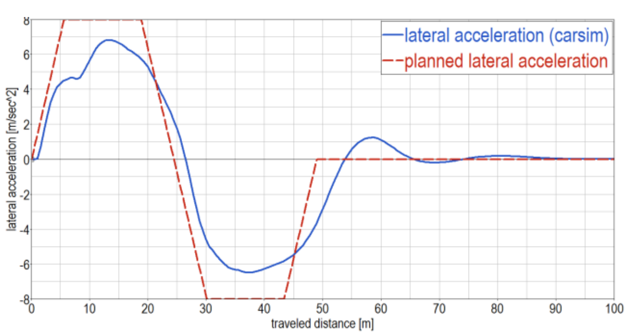

Figure 7. Lateral acceleration profile

Figure 7 shows planned lateral acceleration profile with lateral jerk limitation, and the actual lateral acceleration profile from carsim simulation.

SUMMARY

A path planning algorithm for collision avoidance maneuver is proposed. An optimal path minimizes distance to the target vehicle and depends on the allowable lateral (centripetal) acceleration level and allowable lateral acceleration rate (jerk). Two variants are presented: with and without the lateral jerk limitation. The algorithm was successfully verified in CarSim.

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