MUSIC AND
ACOUSTIC ENGINEERING

SOUND ANALYSIS, SYNTHESIS AND PROCESSING

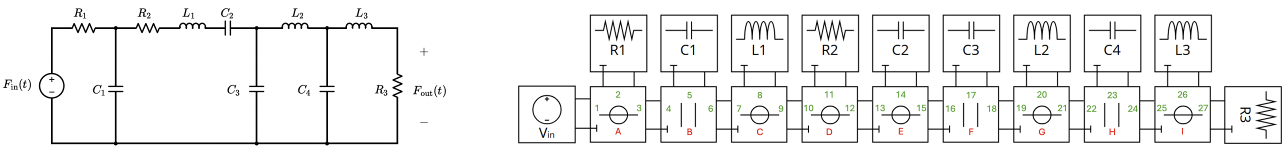
Module 2 - Sound Synthesis and Spatial Processing

OUALI Ernest — 10984484
ABOUELAZM Youssef — 10960436

POLITECNICO
MILANO 1863

1 WDF Structure

In this assignment, we are tasked to implement the Wave Digital Filter equivalent of a given MEMS loudspeaker electric scheme:



(a) Reference circuit: MEMS loudspeaker (b) WDF design of the MEMS loudspeaker.

Figure 1.0.1: (a) Mechanical-domain equivalent circuit; (b) corresponding wave-digital filter structure.

Our WDF system is composed of:

- 9 nodes that are the three-port junctions: 6 series and 3 parallel junctions
- 11 linear one-port elements, including the non-adaptable ideal voltage source.

The root of our system is the ideal voltage source as it is the only non-adaptable element.

1.1 Adaptation Conditions

Ports facing linear elements:

At ports facing physical linear elements, the adaptation impedance coincide with the element values, we have for resistors, inductors, and capacitors respectively:

$$Z_R = R, \quad Z_L = \frac{2L}{T_s}, \quad Z_C = \frac{T_s}{2C}, \qquad T_s = \frac{1}{F_s} = \frac{1}{192 \cdot 10^3} \text{ the sampling period [s]}$$

This leads us to:

$$\begin{aligned} &Z_2 = R_1, \quad && Z_5 = \frac{T_s}{2C_1}, \quad && Z_8 = \frac{2L_1}{T_s}, \quad && Z_{11} = R_2, \quad && Z_{14} = \frac{T_s}{2C_2}, \\ &Z_{17} = \frac{T_s}{2C_3}, \quad && Z_{20} = \frac{2L_2}{T_s}, \quad && Z_{23} = \frac{T_s}{2C_4}, \quad && Z_{26} = \frac{2L_3}{T_s}, \quad && Z_{27} = R_3 \end{aligned}$$

Series Adaptors:

For series adaptors,i.e. blocks A, C, D, E, G, I, we have a consistent adaptation scheme where the left port of those junctions is always being adapted. Therefore, for all the blocks mentioned above, we have :

$$Z_{\text{Left}} = Z_{\text{Top}} + Z_{\text{Right}} \qquad \Rightarrow \qquad Z_i = Z_{i+1} + Z_{i+2}, \quad \forall i \in \{1, 7, 10, 13, 19, 25\}$$

Parallel Adaptors:

For parallel adaptors,i.e. blocks B, F, H, we have a consistent adaptation scheme where the left port of those junctions is always being adapted. Therefore, for all the blocks mentioned above, we have :

$$Z_{\text{Left}} = \frac{Z_{\text{Top}} \cdot Z_{\text{Right}}}{Z_{\text{Top}} + Z_{\text{Right}}}$$

Based on our WDF scheme, we observe the following relation for the adapted port of each parallel junction:

$$Z_i = \frac{Z_{i+1} \cdot Z_{i+2}}{Z_{i+1} + Z_{i+2}}, \quad \forall i \in \{4, 16, 22\}$$

1.2 Connected junctions

For all junctions connected to another one, their port impedances where the connection occurs must be equal. For example, junctions A and B satisfy $Z_3 = Z_4$. For our given WDF system, we observe

$$Z_{3p} = Z_{3p+1}, \quad \forall p \in \{1, 2, 3, 4, 5, 6, 7, 8\}$$

1.3 Scattering matrices of junctions

For each junction $X \in \{A, B, ..., I\}$ we have the following scattering relation:

$$\underline{b} = \underline{\underline{S}}_X \cdot \underline{a}$$

Where $\underline{b}$ is the vector of reflected waves, $\underline{\underline{S}}_X$ the scattering matrix and $\underline{a}$ the vector of incident waves.

For the series and parallel junctions, we have, respectively:

$$\underline{\underline{S}}_X^{\text{series}} = \underline{\underline{I}} - 2 \cdot \underline{\underline{Z}}_X \cdot \underline{\underline{B}}^T \cdot \left(\underline{\underline{B}} \cdot \underline{\underline{Z}}_X \cdot \underline{\underline{B}}^T\right)^{-1} \cdot \underline{\underline{B}}, \qquad \underline{\underline{S}}_X^{\text{para}} = 2 \cdot \underline{\underline{Q}}^T \cdot \left(\underline{\underline{Q}} \cdot \underline{\underline{Z}}_X^{-1} \cdot \underline{\underline{Q}}^T\right)^{-1} \cdot \underline{\underline{Q}} \cdot \underline{\underline{Z}}^{-1} - \underline{\underline{I}}$$

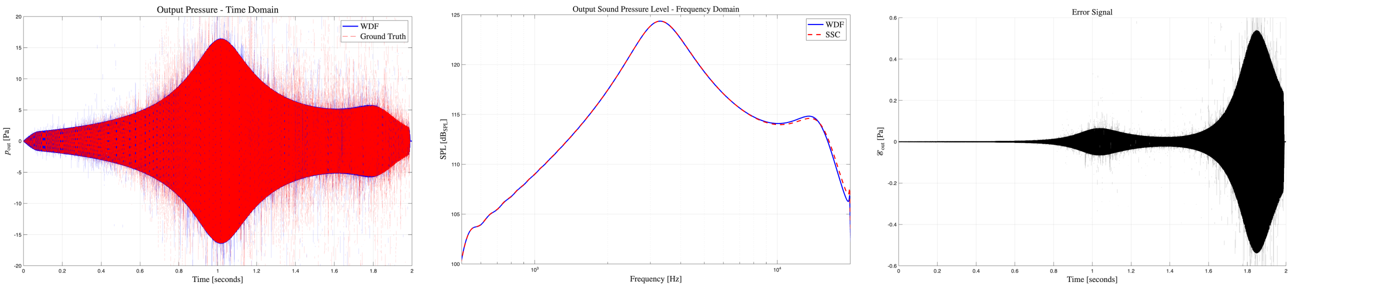
Where $\underline{\underline{Z}}_X = \text{diag}\left(Z_X^{\text{Left}}, Z_X^{\text{Top}}, Z_X^{\text{Bottom}}\right)$ the diagonal matrix of port impedances of each junction. E.g $\underline{\underline{Z}}_A = \text{diag}\left(Z_1, Z_2, Z_3\right)$. $\underline{\underline{B}}$ and $\underline{\underline{Q}}$ are respectively the fundamental tie-set and cut-set matrices specific to the topological junction we are considering. As we rely here on simple 3 ports junctions, all currents and voltages follow the same direction, thus, we obtain:

$$\underline{\underline{B}} = [1, 1, 1], \quad \underline{\underline{Q}} = [1, 1, 1]$$

Note that we kept the matrix convention for writing the above quantities while they end up being simple vectors. To be more rigorous, one might write $\underline{B}$, $\underline{Q}$.

2 Results

After performing the computational flow of the system (forward scan + scattering at the root + backward scan), we obtain the following results.



(a) Output pressure (time domain). (b) Output pressure (frequency domain). (c) Error signal (time domain).

Figure 2.0.1: Obtained results

The implementation of the computational flow can be found in the matlab script `SSSP_HW2_ABOUELAZM_OUALI.m` with intensive comments to help understand the process.

The computed MSE is 0.0119 [Pa^2] which seems to be a satisfying low value. At low frequencies, corresponding to the early portion of the swept-sine input, the model reproduces the reference circuit almost perfectly: the instantaneous error is virtually 0, and the SPL curve of the frequency response overlap exactly.

However, as frequency increases (starting from around 10 kHz, close to the middle of the simulation ($t \approx 1s$), a small discrepancy appears: the instantaneous pressure error grows to modest values, though this has minimal impact on the frequency response of the WDF system that slightly differs from the ground truth.

Since the error at each time-step is simply the numerical difference between the WDF output and the Simscape reference, it is highly sensitive to any phase misalignment. Listening tests may help determine the perceived error in the WDF signal, considering that human ears are largely insensitive to phase shifts (absent interference effects), so even small phase mismatches can produce non-negligible point-by-point errors without altering the perceived sound.