VIBRATION ANALYSIS AND VIBROACOUSTICS

MODULE 1: VIBRATION ANALYSIS (PROF. STEFANO ALFI)

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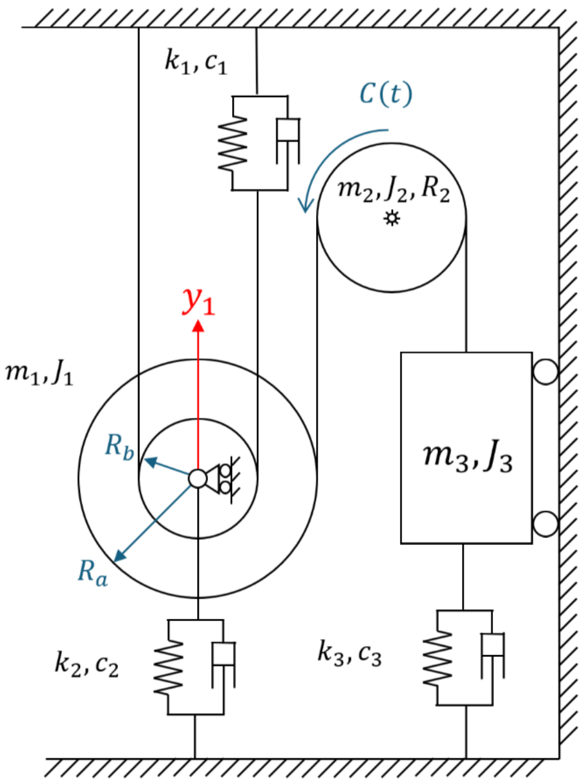
<u>Assignment 1</u>: Dynamics of 1-dof systems

The mechanical system in figure lies on the vertical plane and it is represented in its steady-state equilibrium position. The system consists of 3 rigid bodies. The first one (M_1, J_1) is in turn composed by two disks welded together (of radius R_a and R_b respectively) and connected to a roller, which enables the body to rotate and move vertically, but not sideways. An inextensible rope is tied around the smaller disc. This rope is connected to the ground on one side and to the vertical spring and damper (k_1, c_1). A second set composed by a vertical spring and damper (k_2, c_2) connect the centre or body one and the ground.
The second body is a disc of characteristics (M_2, J_2, R_2) which is hinged on its axis. An inextensible string connects it to the outer disc of the first body. Moreover, the string winds itself around the second body and continues down to the third body (M_3, J_3), which in turn translates vertically. A third set of spring and damper (k_3, c_3) connects the third body to the ground. Finally, a torque $C(t)$ acts on the second disc. The systems' parameters are summarised in the following Table.



Parameter	Value
M_1 $[Kg]$	15
M_2 $[Kg]$	10
M_3 $[Kg]$	5
J_1 $[Kgm^2]$	10
J_2 $[Kgm^2]$	8
J_3 $[Kgm^2]$	10
R_a $[m]$	0.80
R_b $[m]$	0.50
R_2 $[m]$	0.60
c_1 $[Ns/m]$	30
c_2 $[Ns/m]$	40
c_3 $[Ns/m]$	50
$k_1 [N/m]$	2600
k_2 $[N/m]$	2000
k_3 $[N/m]$	1600

Data regarding the external torque C is summarized in the following table:

Parameter	Value	Parameter	Value	Parameter	Value
A $[Nm]$	25	f $[Hz]$	1	φ $[rad]$	$\pi/3$
A_i $[Nm]$	25	f_i $[Hz]$	[0.5, 20]	φ $[rad]$	0
B_1 $[Nm]$	50	f_1 $[Hz]$	0.5	φ_1 $[rad]$	$\pi/2$
B_2 $[Nm]$	5.55	f_2 $[Hz]$	1.5	φ_2 $[rad]$	$-\pi/2$
B_3 $[Nm]$	2.00	f_3 $[Hz]$	2.5	φ_3 $[rad]$	$\pi/2$

According to the data defined in the above table, it is requested to compute:

1) Equations of motion:
    a. Write the equation of motion for small vibrations about the represented configuration considering that the system is in its static equilibrium position.
    b. Evaluate the **natural frequency** of the system.
    c. Compute the **adimensional damping ratio** and the **damped frequency**.

2) Free motion of the system:
    a. Plot and comment the free motion of the system starting from generic non null initial conditions.
    b. Considering the same initial condition, evaluate and comment the free motion of the system when the system is characterised by an adimensional damping ratio 4 times that obtained in 1.c.
    c. Considering the same initial condition, evaluate and comment the free motion of the system when the system is characterised by an adimensional damping ratio 23 times that obtained in 1.c.

3) Forced motion of the system:
    a. Plot and comment the Frequency Response Function *H(Ω)* diagrams (magnitude and phase) resulting from conditions reported in (2.a), (2.b) and (2.c).
    b. Starting from the general initial condition and the system characteristics defined in (2.b), evaluate the complete time response of the system to the torque applied to the disk, considering that the torque is harmonic of the form:
    $$C(t) = A\,cos(2\pi f t + \varphi)$$
    c. Considering the harmonic torque:
    $$C(t) = A_i\,cos(2\pi f_i\,t + \varphi)$$
    for the two cases $f_i = f_1$ and $f_i = f_2$, compute the displacement considering the force statically applied and compare the time trend of the steady-state response with respect to the force.

    d. Evaluate the steady-state response of the system to the torque applied to the disk, considering that the periodic force consists of the superposition of three harmonic contributions of the form:
    $$C(t) = \sum_{k=1}^{3} B_k\,cos(2\pi f_k\,t + \varphi_k)$$
    For both the torque and the steady-state response of the system, plot:

    - the time histories of the signals;
    - the spectra of the signals.