VIBRATION ANALYSIS AND VIBROACOUSTICS

MODULE 1: VIBRATION ANALYSIS (PROF. STEFANO ALFI)

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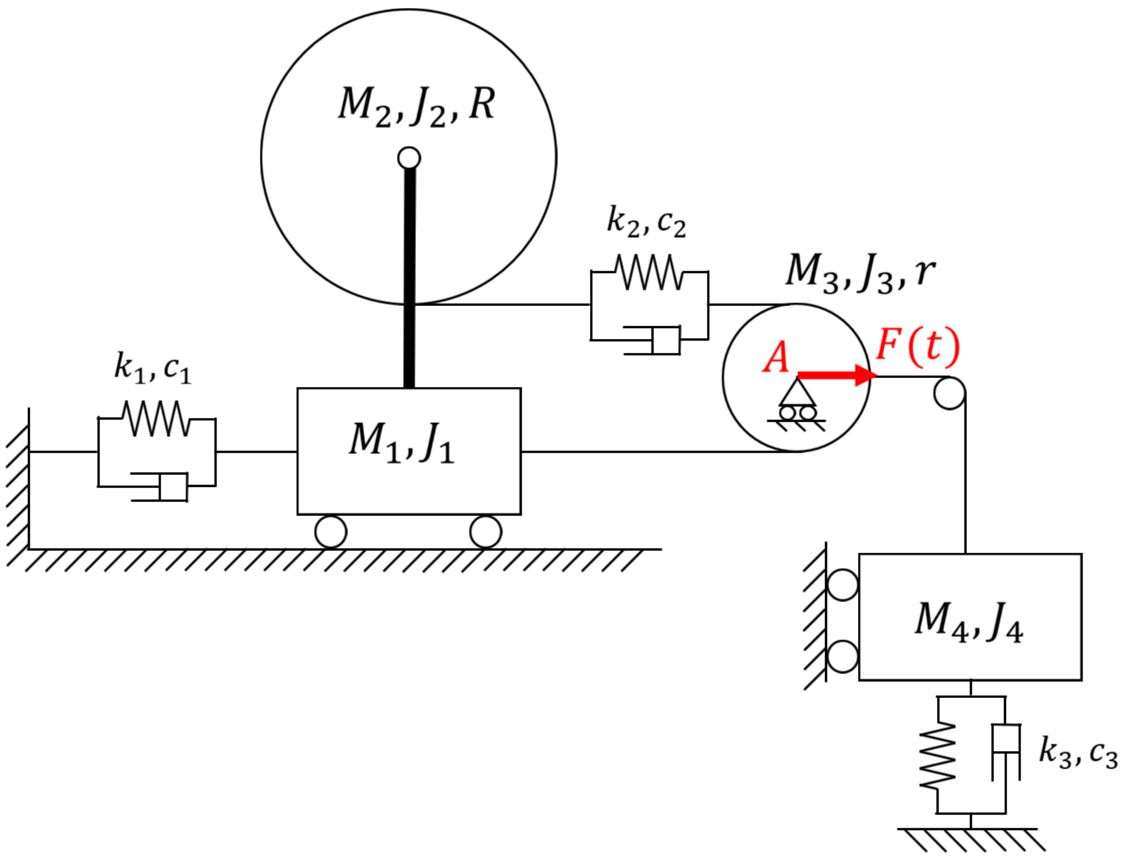
<u>Assignment 2</u>: Dynamics of n-dof systems

The system to be studied consists of 4 rigid bodies. A disk (M_2, J_2, R) is constrained through a hinge to a massless vertical beam which is in turn rigidly connected to a mass (M_1) that slides in the horizontal plane. An inextensible rope connects the periphery of the disk (M_2, J_2, R) to another disk (M_3, J_3, r) that can translate horizontally and can be rolled back and forth by the rope, which then connects it to the translating mass. The centre of the latter disk is rigidly connected to a mass (M_4, J_4) whose horizontal motion and rotation are constrained. Springs (k_1, k_2, k_3) and dampers (c_1, c_2, c_3) realise together with the rigid bodies the mechanical system depicted below.



M_1 $[kg]$	$M_2 [kg]$	J_2 $[kg\ m^2]$	$M_3 [kg]$	J_3 $[kg\ m^2]$	$M_4 [kg]$	$R [m]$	r $[m]$
5	4	7	5	6	2	0.8	0.6
k_1 $[N/m]$	c_1 $[Ns/m]$	k_2 $[N/m]$	c_2 $[Ns/m]$	k_3 $[N/m]$	c_3 $[Ns/m]$		
300	0.5	350	2	50	10		
A_1 $[N]$	A_2 $[N]$	f_1 $[Hz]$	f_2 $[Hz]$	f_0 $[Hz]$			
15	7	0.15	0.75	0.10			
$x_{1,0}$ $[m]$	$\theta_{2,0}$ $[rad]$	$\theta_{3,0}$ $[rad]$	$\dot{x}_{1,0}$ $[m/s]$	$\dot{\theta}_{2,0}$ $[rad/s]$	$\dot{\theta}_{3,0}$ $[rad/s]$		
0.1	$\pi/12$	$-\pi/12$	1	0.5	2		

According to the data defined in the above table, it is requested to:

1) Equations of motion and system matrices:
    a. Write the equations of motion for small vibrations around the represented configuration considering that the system is in its equilibrium position.
    b. Evaluate the eigenfrequencies and corresponding eigenvectors in case of **undamped** and **damped** system.
    c. Assuming Rayleigh damping, evaluate α and β to approximate the generalized damping matrix [C*] to be of the form α[M]+β[K].
2) Free motion of the system (considering the Rayleigh damping as in 1.c)
    a. Plot and comment the free motion of the system starting from the initial conditions reported in the table, being x_1 , θ_2 , θ_3 the displacement of mass M_1, and the rotation of disks 2 and 3 (the initial conditions for generic independent variables can be obtained through kinematic relations).
    b. Impose particular initial conditions so that only one mode contributes to the free motion of the system.

3) Forced motion of the system (considering the Rayleigh damping as in 1.c)
    a. Plot and comment the elements of the Frequency Response Matrix *H(Ω)*.
    b. Plot the co-located FRF of point A at the centre of the disk 3 (co-located meaning the FRF between the displacement in A and a force applied in A)
    c. Plot the co-located FRF between the rotation of the disk of radius r and the torque applied onto the disk itself.
    d. Starting from the initial condition defined in (2.a), evaluate the complete time response of the system for the three degrees of freedom to the horizontal force in point A, considering that the force is harmonic of the form:

$$F(t) = A_1\, cos(2\pi f_1\, t) + \; A_2\, cos(2\pi f_2\, t)$$

    e. Evaluate the steady-state response of the system of horizontal displacement of point A to the horizontal force applied in A considering that the force is a periodic triangular wave, with fundamental frequency f_0, of the form:

$$F(t) = \frac{8}{\pi^2}\sum_{k=0}^{4}(-1)^k \frac{\sin\big((2k+1)\, 2\pi f_0\, t\big)}{(2k+1)^2}$$

4) Modal approach (considering the Rayleigh damping as in 1.c)
    a. Derive the equations of motion in modal coordinates and plot the elements of the corresponding Frequency Response Matrix $H_q(\Omega)$.
    b. Reconstruct the co-located FRF of the displacement of point A employing the modal approach and compare with the one obtained using physical coordinates in (3.b).
    c. Reconstruct the co-located FRF between the rotation of the disk of radius r and the torque applied onto the disk employing a modal approach (showing the contribution of the different modes) and compare with the one obtained using physical coordinates in (3.c).
    d. Compute the steady state amplitude of response for the three degrees of freedom when excited by a horizontal force applied in A. Compare the system response with the one obtained considering only the first mode of vibration, for the two following cases:
        i. a harmonic force $F(t) = A_1\, cos(2\pi f_1\, t)$;
        ii. a harmonic force $F(t) = A_2\, cos(2\pi f_2\, t)$.

OPTIONAL

i. Give a qualitative graphical representation of the mode shapes.