$$\underline{x}(t)=\begin{pmatrix} x_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{pmatrix}=\begin{pmatrix} \sum_{i=1}^{3} e^{-\alpha_{x_1} t}\cdot C_i\cdot\left|X_{\theta_3}^{(i)}\right|\cdot\cos\left(\omega_{di}\cdot t+\varphi_i+\psi_{x_1}^{(i)}\right) \\ \sum_{i=1}^{3} e^{-\alpha_{\theta_2} t}\cdot C_i\cdot\left|X_{\theta_2}^{(i)}\right|\cdot\cos\left(\omega_{di}\cdot t+\varphi_i+\psi_{\theta_2}^{(i)}\right) \\ \sum_{i=1}^{3} e^{-\alpha_{\theta_3} t}\cdot C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot\cos\left(\omega_{di}\cdot t+\varphi_i+\psi_{\theta_3}^{(i)}\right) \end{pmatrix}$$

$$x_1(t)\big|_{t=0}=x_0 \quad \Leftrightarrow \sum_{i=1}^{3} C_i\cdot\left|X_{\theta_3}^{(i)}\right|\cdot\cos\left(\varphi_i+\psi_{x_1}^{(i)}\right)$$

$$\dot{x}_1\big|_{t=0}=v_0 \Leftrightarrow \frac{\partial}{\partial t}\left[\sum_{i=1}^{3} e^{-\alpha_{x_1} t}\cdot C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot\cos\left(\omega_{di}\cdot t+\varphi_i+\psi_{x_1}^{(i)}\right)\right]\Bigg|_{t=0}=v_0$$

$$\Rightarrow \sum_{i=1}^{3} C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot\left[-\alpha_{x_1}\cdot e^{-\alpha_{x_1} t}\cdot\cos\left(\omega_{di}\cdot t+\varphi_i+\psi_{x_1}^{(i)}\right)-\omega_{di}\cdot e^{-\alpha_{x_1} t}\cdot\sin\left(\omega_{di}\cdot t+\varphi_i+\psi_{x_1}^{(i)}\right)\right]\Bigg|_{t=0}=v_0$$

$$\Rightarrow -\left[\sum_{i=1}^{3} C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot e^{-\alpha_{x_1} t}\cdot\left[\alpha_{x_1}\cdot\cos\left(\omega_{di}\cdot t+\varphi_i+\psi_{x_1}^{(i)}\right)+\omega_{di}\cdot\sin\left(\omega_{di}\cdot t+\varphi_i+\psi_{x_1}^{(i)}\right)\right]\right]\Bigg|_{t=0}=v_0$$

$$\Rightarrow -\sum_{i=1}^{3} C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot\left[\alpha_{x_1}\cdot\cos\left(\varphi_i+\psi_{x_1}^{(i)}\right)+\omega_{di}\sin\left(\varphi_i+\psi_{x_1}^{(i)}\right)\right]=v_0$$

And we have that

$$\cos\left(\varphi_i+\psi_{x_1}^{(i)}+\Gamma\right)=\cos(\Gamma)\cdot\cos\left(\varphi_i+\psi_{x_1}^{(i)}\right)-\sin(\Gamma)\cdot\sin\left(\varphi_i+\psi_{x_1}^{(i)}\right)=\alpha_{x_1}\cdot\cos\left(\varphi_i+\psi_{x_1}^{(i)}\right)+\omega_{di}\sin\left(\varphi_i+\psi_{x_1}^{(i)}\right)$$

$$\Rightarrow \begin{cases} \cos(\Gamma)=\alpha_{x_1} \\ \sin(\Gamma)=-\omega_{di} \end{cases} \qquad \Rightarrow \tan(\Gamma)=-\frac{\omega_{di}}{\alpha_{x_1}} \Leftrightarrow \Gamma=-\arctan\left(\frac{\omega_{di}}{\alpha_{x_1}}\right)$$

$$\Rightarrow -\sum_{i=1}^{3} C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot\left[\alpha_{x_1}\cdot\cos\left(\varphi_i+\psi_{x_1}^{(i)}\right)+\omega_{di}\sin\left(\varphi_i+\psi_{x_1}^{(i)}\right)\right]$$

$$\Rightarrow -\sum_{i=1}^{3}\left[C_i\cdot\left|X_{x_1}^{(i)}\right|\cdot\cos\left(\varphi_i+\psi_{x_1}^{(i)}-\arctan\left(\omega_{di}/\alpha_{x_1}\right)\right)\right]=v_0$$

Where C_i, φ_i are the unknowns of interest