

### Quiz 3. Solutions

which of the series converge or diverge?

Give reason for your answers

① 
$$\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$$

Solution: Note that when  $n$  is getting large the  $n$ th-term

$$a_n = \frac{n+1}{n^2\sqrt{n}} \text{ is getting closer to } \frac{1}{n^{3/2}} = b_n$$

Then we may want to compare the series  $\sum a_n$  with the series  $\sum \frac{1}{n^{3/2}} = \sum \frac{1}{n^{3/2}}$  which is a convergent  $p$ -series (with  $p = 3/2 > 1$ )

by limit comparison test

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n^2\sqrt{n}}}{\frac{1}{n^{3/2}}} = \lim_{n \rightarrow \infty} \frac{n+1}{n} = \frac{1+0}{1} = 1 > 0$$

Therefore  $\sum \frac{n+1}{n^2\sqrt{n}}$  (converges).