Solutions

Math 21C Quiz 4

Section: 5:10-6:00 pm, TA: Arpy Mikaelian
Tuesday April 29, 2008

Problem 1

(5 points): Find the Taylor series at $x = 0$ for the function

$$f(x) = \sin\left(\frac{\pi x}{2}\right).$$

$$f'(x) = \frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)$$

$$f''(x) = -\frac{\pi^2}{4}\sin\left(\frac{\pi x}{2}\right)$$

$$\sin\left(\frac{\pi x}{2}\right) = \sin(0) + \frac{\pi}{2}\cos(0)(x-0) + \frac{\left(-\frac{\pi^2}{4}\sin(0)\right)(x-0)^2}{2!} + \frac{\left(-\frac{\pi^3}{8}\right)(\cos(0))(x-0)^3}{3!} + \cdots$$

$$= 0 + \frac{\pi}{2}x + 0 - \frac{\pi^3}{48}x^3$$

(Could also substitute $\frac{\pi x}{2}$ for x in Taylor series for $\sin(x)$)

Thus: $\sin\left(\frac{\pi x}{2}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi x}{2}\right)^{2k+1}}{(2k+1)!}$

See p. 798 for more details