Last Name: ____________________, First Name: ____________________

Mat 21C-A03 (5:10 - 6:00pm) Quiz #2 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic device for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Use Ratio or Root test to determine whether the series below converge or diverge. Justify your answer. Let $a_1 = \frac{1}{3}$ and $a_{n+1} = \dfrac{2n-1}{3n+5}a_n$.

$$\sum_{n=1}^{\infty} a_n$$

Solution Using the ratio test (see textbook), we need to compute

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2n-1}{3n+5} = \lim_{n\to\infty} \frac{2-1/n}{3+5/n} = \frac{2}{3} < 1.$$

Since $\rho < 1$, then the ratio test says the series converges.

2. (5 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$$

Solution Notice that we can write $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} u_n$. Using the alternating series test (see textbook),

$$\begin{aligned}
u_n &= \frac{1}{\sqrt{n}} > 0, \quad \forall n. \\
u_{n+1} &= \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} = u_n, \quad \forall n. \\
\lim_{n\to\infty} u_n &= 0.
\end{aligned}$$

Therefore, the series converges. However, consider the series $\sum |a_n| = \sum u_n$. This series diverges by the $p-$series test

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}.$$

Since $p < 1$, the series diverges.

Therefore, $\sum a_n$ converges, but $\sum |a_n|$ diverges. This is the definition of conditionally convergent.