Solutions (Quiz 7, section B03)

Problem 1 (5 points):Find the value of $\frac{\partial y}{\partial z}$ at the point $(2, 1, -1)$ if the equation

$$z^4 y + x \ln y - y^3 + 1 = 1$$

defines y as a function of the two independent variables x and z and the partial derivative exist.

solution: Take partial derivative with respect to z.Then,

$$4z^3 y + z^4 \frac{\partial y}{\partial z} + x \frac{1}{y} \frac{\partial y}{\partial z} - 3y^2 \frac{\partial y}{\partial z} = 0.$$

Solve for $\frac{\partial y}{\partial z}$.

$$\frac{\partial y}{\partial z} = -\frac{4z^3 y}{z^4 + \frac{x}{y} - 3y^2}$$

However, the denominator becomes zero at (2,1,-1). Hence, $\frac{\partial y}{\partial z}$ does not exist at (2,1,-1).

Problem 2 (5 points): Find a vector parallel to the line of intersection of the planes $-4x + y + 6z = 11$ and $2x + 3z - y = 4$.

solution: Note that the line of intersection of two plane is perpendicular to both normal vectors of the planes. Therefore, the line is parallel to $n_1 \times n_2$, where n_1 and n_2 are the normal vectors of the planes.

$$\begin{aligned} n_1 \times n_2 &= \begin{vmatrix} i & j & k \\ -4 & 1 & 6 \\ 2 & -1 & 3 \end{vmatrix} \\ &= i \begin{vmatrix} 1 & 6 \\ -1 & 3 \end{vmatrix} - j \begin{vmatrix} -4 & 6 \\ 2 & 3 \end{vmatrix} + k \begin{vmatrix} -4 & 1 \\ 2 & -1 \end{vmatrix} \\ &= 9i + 24j + 2k \end{aligned}$$