Quiz 5 Solutions

Problem 1 (5 points): The following equation $x^2+y^2+z^2+6x-2y+4z+5=0$ generates a sphere in 3D. Find the center and radius of that sphere.

$x^2+y^2+z^2+6x-2y+4z+5=(x+3)^2-9+(y-1)^2-1+(z+2)^2-4+5=(x+3)^2+(y-1)^2+(z+2)^2-9$

Therefore, the equation above is equivalent to:
$(x+3)^2+(y-1)^2+(z+2)^2=9$

Therefore, the sphere described has center $(-3,1,-2)$ and radius 3.

Problem 2 (5 points): Let $\mathbf{u}=2\mathbf{i}-\mathbf{j}+5\mathbf{k}$ and $\mathbf{v}=-4\mathbf{i}+4\mathbf{j}+\mathbf{k}$. Compute $proj_{\mathbf{u}}\mathbf{v}$.

$proj_{\mathbf{u}}\mathbf{v}=\left(\frac{\mathbf{v}\cdot\mathbf{u}}{|\mathbf{u}|^2}\right)\mathbf{u}=\frac{(-8-4+5)}{4^2+1^2+5^2}\mathbf{u}=-\frac{7}{42}\mathbf{u}=-\frac{14}{42}\mathbf{i}+\frac{7}{42}\mathbf{j}-\frac{35}{42}\mathbf{k}$