Solutions (Quiz 9, section B04)

Problem 1 (5 points): Find the derivative of the function

$$f(x, y, z) = \mathrm{e}^{xy \sin z}$$

at $P_0(1, 1, \pi/4)$ in the direction $\mathbf{A} = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

solution: First fine the unit vector with the same direction as $\mathbf{A}$.

$$\mathbf{u} = \mathbf{A}/|\mathbf{A}| = (\mathbf{i} + \mathbf{j} + \mathbf{3k})/\sqrt{\mathbf{11}}$$

The directional derivative is given by

$$\nabla f|_{P_0} \cdot \mathbf{u}.$$

First, find ∇f.

$$\nabla f = \left\langle \mathrm{e}^{xy \sin z}\, y \, \sin z, \mathrm{e}^{xy \sin z}\, x \, \sin z, \mathrm{e}^{xy \sin z}\, xy \, \cos z \right\rangle$$

$$\nabla f|_{P_0} = \frac{\sqrt{2}\,\mathrm{e}^{\frac{\sqrt{2}}{2}}}{2} \langle 1, 1, 1 \rangle$$

Hence,

$$\nabla f|_{P_0} \cdot \mathbf{A} = \frac{\sqrt{\mathbf{2}}\,\mathrm{e}^{\frac{\sqrt{\mathbf{2}}}{\mathbf{2}}}}{\mathbf{2}} \langle \mathbf{1}, \mathbf{1}, \mathbf{1} \rangle \cdot \langle \mathbf{1}, \mathbf{1}, \mathbf{3} \rangle / \sqrt{\mathbf{11}} = \frac{\mathbf{5}\sqrt{\mathbf{2}}\,\mathrm{e}^{\frac{\sqrt{\mathbf{2}}}{\mathbf{2}}}}{\mathbf{2}\sqrt{\mathbf{11}}}.$$

Problem 2 (5 points): Find the equation for the tangent plane on the given surface

$$-xy - y^2 - z + 26 = 0$$

at point $P_0(2, 4, 2)$.

solution: Let $f(x, y, z) = -xy - y^2 - z + 26$ and find ∇f.

$$\nabla f = \langle -y, -x - 2y, -1 \rangle$$

$$\nabla f|_{P_0} = \langle -4, -10, -1 \rangle$$

Therefore, the equation for the tangent plane at P_0 is

$$-4(x-2) - 10(y-4) - (z-2) = 0.$$

Or, $4x + 10y + z = 50$.