Quiz 6 The coldest winter in America is summer in San Francisco — Mark Twain

First Name: Frederic Last Name: Chopin.

Problem 1 (5 points): Find the distance from the point $(3, -1, 7)$ to the line

$$x = 2 + 2t,$$
$$y = 1 + 6t,$$
$$z = 3.$$

$A = (3, -1, 7)$ $\quad l = \{(x, y, z) \mid x = 2+2t,\ y = 1+6t,\ z = 3\}$

$$[d(A,l)]^2 = [(2+2t)-3]^2 + [(1+6t)-(-1)]^2 + [3-7]^2$$

We want to find minal value of $d(A,l)$, but it suffices to find minimal value of $[d(A,l)]^2$.

By taking derivative, we have when $t = -\frac{1}{4}$, $[d(A,l)]^2 = \frac{37}{2}$, attaining its minimum, so $\min d(A,l) = \sqrt{\frac{37}{2}} = \frac{\sqrt{74}}{2}$

Problem 2 (5 points): Let $\mathbf{u} = \langle \frac{3}{5}, 0, \frac{4}{5} \rangle$ and $\mathbf{v} = \langle 5, 12, 0 \rangle$.
Find the vector $\text{proj}_{\mathbf{u}}\mathbf{v}$.
Find the angle between $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}$.

(i) $\text{proj}_u v = \left(v \cdot \frac{u}{\|u\|}\right) \cdot \frac{u}{\|u\|} = \left(\frac{v \cdot u}{\|u\|^2}\right) \cdot u = \langle \frac{9}{5}, 0, \frac{12}{5} \rangle$

(ii) $\text{proj}_v u = \left(\frac{u \cdot v}{\|v\|^2}\right) \cdot v$

so $\text{proj}_v u \cdot (u - \text{proj}_v u)$

$$= \left(\frac{u \cdot v}{\|v\|^2}\right) v \cdot \left(u - \frac{u \cdot v}{\|v\|^2} \cdot v\right)$$

$$= \left(\frac{u \cdot v}{\|v\|^2}\right) v \cdot u - \frac{(u \cdot v)}{\|v\|^2} \cdot \left(\frac{u \cdot v}{\|v\|^2}\right) \cdot v \cdot v$$

$$= \left(\frac{u \cdot v}{\|v\|^2}\right) v \cdot u - \left(\frac{u \cdot v}{\|v\|^2}\right) \frac{u \cdot v}{\|v\|^2} \cdot \|v\|^2$$

$$= \frac{(u \cdot v)^2}{\|v\|^2} - \frac{(u \cdot v)^2}{\|v\|^2} = 0;$$

So the angle is $\frac{\pi}{2}$, or, $90°$