Math 21C - Section B01 - Quiz 7 **SOLUTION**
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Problem 1: Find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ if the equation

$$x^5 z + y \ln x - x^3 + 4 = 0$$

defines x as a function of the two independent variables y and z and the partial derivative exists.

Solution: In the manner prescribed on page 988 of the textbook[1], we should define

$$F(x, y, z) = x^5 z + y \ln x - x^3 = 4.$$

Note that the question is about $\partial x/\partial z$, so the roles of "x" and "z" are reversed from the "usual" order. Then, by the discussion on page 988, we need to compute

$$\frac{\partial x}{\partial z} = -\frac{F_z}{F_x}.$$

First, the partial derivative of F with respect to z is

$$F_z = x^5.$$

The partial derivative of F with respect to x is

$$F_x = 5x^4 z + \frac{y}{x} - 3x^2.$$

Thus,

$$\frac{\partial x}{\partial z} = -\frac{F_z}{F_x} = -\frac{x^5}{5x^4 z + \frac{y}{x} - 3x^2}.$$

We need to evaluate this at $(x, y, z) = (1, -1, -3)$, so our final result is

$$\left.\frac{\partial x}{\partial z}\right|_{(x,y,z)=(1,-1,-3)} = -\frac{(1)^5}{5 \cdot (1)^4 \cdot (-3) + \frac{-1}{1} - 3 \cdot (1)^2}.$$

Problem 2: Find $\partial w/\partial u$ when $u = -1$, $v = 2$ if $w = xy + \ln z$, $x = v^2/u$, $y = u + v$, $z = \sin u$.

Solution: Though completely optional to draw, I note first the dependencies in the dependency diagram. w depends on x, y, and z. x depends on v and u. y depends on v and u. z depends on u.

[1] See the section in blue titled "Three-Variable Implicit Differentiation"

By the chain rule,

$$
\begin{aligned}
\frac{\partial w}{\partial u} &= \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial u} \\
&= \left[\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u}\right] + \left[\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u}\right] + \left[\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial u}\right] \\
&= [(y)(v^2\cdot(-u^{-2}))] + [(x)\cdot(1)] + [(z^{-1})\cdot(\cos u)].
\end{aligned}
$$

New we need to evaluate at the point $(u, v) = (-1, 2)$. When $u = -1$ and $v = 2$, then $x = -4$, $y = 1$, and $z = \sin(-1)$. By substituting $-1, 2, -4, 1$, and -1 for u, v, x, y, and z (respectively), we get

$$
\left.\frac{\partial w}{\partial u}\right|_{(u,v)=(-1,2)} = \left[(1)\cdot\left(2^2\cdot\frac{-1}{(-1)^2}\right)\right] + [(-4)(1)] + \left[\frac{\cos(-1)}{\sin(-1)}\right].
$$

Remarks

- Don't forget to substitute for the point that we care about at the end.
- Be careful with the "simple stuff".... a lot of careless mistakes on substitutions, arithmetic, etc. (all non-calculus things). Sometimes, a z was confused for a 2. These things were graded leniently on this quiz, but you can imagine that final exam grading is not quite as nice.
- I think a lot of you are doing very well in understanding the concepts... GREAT!!
- However, try hard to work on organizing your solutions. They should ideally read from top to bottom with clarity of your logic (the order of presentation is important). If you need to do scratch work, use the back side. (Same comment applies to the final exam.)