Quiz 6 (KEY)

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Problem 1. (5 points)

Determine if following limit exists. Give reasons for your answers.

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4 + y^2}$$

Answer. Consider the limit along the y-axis (which is all points (x, y) such that $x = 0$)

$$\lim_{(0,y)\to(0,0)} f(x, y) = \lim_{(0,y)\to(0,0)} \frac{0}{y^2} = 0.$$

But along the path determined by the equation $x = \sqrt{y}$ where $y \geq 0$,

$$\lim_{(\sqrt{y},y)\to(0,0)} f(x, y) = \lim_{(\sqrt{y},y)\to(0,0)} \frac{(\sqrt{y})^4}{(\sqrt{y})^4 + y^2} = \frac{1}{2}.$$

Since $\frac{1}{2} \neq 0$,

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4 + y^2} \quad \text{does not exist.}$$

$\square$

Problem 2. (5 points)

Find an equation for the level curve of the function

$$f(x, y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n$$

that passes through the point $(1, 3)$.

Answer. At $(x, y) = (1, 3)$, $\sum_{n=0}^{\infty} \frac{1}{3^n} = \frac{1}{1-1/3}$. Hence, the level curve is all (x, y) such that

$$f(x, y) = \frac{3}{2}.$$

$\square$