Solutions (Quiz 8, section B04)

Problem 1 (5 points): Find the gradient of the function

$$f(x,y) = \arctan \frac{xy}{6} + \ln\left(x^4 + y^2\right)$$

at the given point $(2,3)$.

solution: Recall that $(\arctan x)' = \frac{1}{1+x^2}$.

$$\text{grad } f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{xy}{6}\right)^2} \cdot \frac{y}{6} + \frac{4x^3}{x^4+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{xy}{6}\right)^2} \cdot \frac{x}{6} + \frac{2y}{x^4+y^2}$$

$$\left.\frac{\partial f}{\partial x}\right|_{(2,3)} = \frac{153}{100} \qquad \left.\frac{\partial f}{\partial y}\right|_{(2,3)} = \frac{61}{150}$$

Finally, $\nabla f = \left(\frac{153}{100}, \frac{61}{150}\right)$ or $\frac{153}{100}i + \frac{61}{150}j$.

Problem 2 (5 points): Find $\partial w/\partial v$ when $u = 1$, $v = 2$ if $w = xy + \ln z$, $x = v^4/u$, $y = v - u$, $z = \tan u$.

solution: Use the chain rule.

$$\begin{aligned}
\frac{\partial w}{\partial v} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} \\
&= y \cdot \frac{4v^3}{u} + x + \frac{1}{z} \cdot 0 \\
&= (v-u) \cdot \frac{4v^3}{u} + \frac{v^4}{u}
\end{aligned}$$

Hence, $\left.\frac{\partial w}{\partial v}\right|_{(1,2)} = 48$.