Solutions

Math 21C Quiz 2

Section: 5:10-6:00 pm, TA: Arpy Mikaelian
Tuesday April 15, 2008

Problem 1

Problem 1 (5 points): Does the series

$$\sum_{n=1}^{\infty} \frac{8\tan^{-1}(n)}{1+n^2}$$

converge or diverge? Give reasons for your answer.

Converges by Integral Test:

$$\sum_{n=1}^{\infty} \frac{8\tan^{-1}(n)}{1+n^2} = 8 \cdot \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{1+n^2}$$

$$f(x) = \frac{8\tan^{-1}(x)}{1+x^2} \qquad \int_1^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_1^b f(x)\,dx$$

$$= \lim_{b\to\infty} 8\int_1^b \frac{\tan^{-1}(x)}{1+x^2}\,dx \qquad \text{let } u = \tan^{-1}(x) \quad du = \frac{1}{1+x^2}dx$$

$$\Rightarrow \lim_{b\to\infty} 8\int_1^b \frac{\tan^{-1}(x)dx}{1+x^2} = 8\cdot\lim_{b\to\infty}\int_1^b u\,du = 8\cdot\lim_{b\to\infty}\left[\frac{u^2}{2}\right]_1^b$$

$$= \frac{8}{2}\cdot\lim_{b\to\infty}\left[(\tan^{-1}(b))^2 - \frac{1}{2}\right] = 4\left[\left(\frac{\pi}{2}\right)^2 - \frac{1}{2}\right]$$

$$= \pi^2 - 2$$

Since $\lim_{b\to\infty}\int_1^b f(x)\,dx$ converges, then so does the given series.