Last Name: ______________________________, First Name: ______________________________

Mat 21C-A03 (5:10 - 6:00pm) Quiz #1 SOLUTIONS

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic device for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Determine if the series converges or diverges. Justify your answer.
$\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$

Solution Let $a_n = (n/(3n+1))^n$ and observe all the terms all positive. We can compare this to a more familiar series,

$$a_n \leq \left(\frac{n}{3n}\right)^n = \frac{1}{3^n} = b_n.$$

Notice $\sum_{n=1}^{\infty} b_n$ is a convergent geometric series. Therefore, the series $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$ converges by the comparison test.

2. (5 points) Determine if the series converges or diverges. If it converges, determine the sum for the series in its simplest form. Justify your answer.
$\sum_{n=2}^{\infty}\frac{51\cdot\sqrt{8}^n+2^n 33}{4^n 3}$

Solution The sum can be split, then computed using geometric series formulas as follows:

$$\begin{aligned}
\sum_{n=2}^{\infty}\frac{51\cdot\sqrt{8}^n+2^n 33}{4^n 3} &= \frac{51}{3}\sum_{n=2}^{\infty}\frac{\sqrt{8}^n}{4^n} + \frac{33}{3}\sum_{n=2}^{\infty}\frac{2^n}{4^n} \\
&= 17\sum_{n=2}^{\infty}\left(\frac{\sqrt{8}}{4}\right)^n + 11\sum_{n=2}^{\infty}\left(\frac{1}{2}\right)^n \\
&= 17\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n + 11\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \\
&= \frac{17}{2}\frac{1}{1-\frac{1}{\sqrt{2}}} + \frac{11}{4}\frac{1}{1-\frac{1}{2}} = \frac{45+17\sqrt{2}}{2}.(\approx 34.5208153)
\end{aligned}$$

Therefore, the series converges.