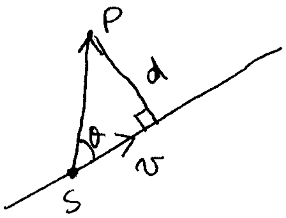
MAT 21C (Section B03)
Quiz 6

Name : Solution

1. (5 points):Find the distance from the point $P(3,-1,4)$ to the line



$$x = 4 - t \qquad y = 3 + 2t \qquad z = -5 + 3t$$

From the equation, we know that the line is parallel to $v(-1,2,3)$

$$d = |\vec{SP}|\,|\sin\theta|$$

Note that $|\vec{SP} \times v| = |\vec{SP}|\,|v|\,|\sin\theta|$. Thus, $d = \dfrac{|\vec{SP} \times v|}{|v|}$

Choose any point S on the line. (I chose $S(4,3,-5)$)

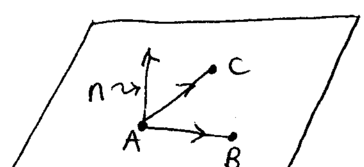
$\vec{SP} = (-1,-4,9)$. and $|v| = \sqrt{1+4+9} = \sqrt{14}$

$$\vec{SP} \times v = \begin{vmatrix} i & j & k \\ -1 & -4 & 9 \\ -1 & 2 & 3 \end{vmatrix} = -30i - 6j - 6k \Rightarrow |\vec{SP} \times v| = \sqrt{900+36+36} = \sqrt{972}$$

Therefore, $d = \dfrac{|\vec{SP} \times v|}{|v|} = \dfrac{\sqrt{972}}{\sqrt{14}} = \boxed{\sqrt{\dfrac{486}{7}}}$

2. (5 points): Find an equation for the plane through $A(-3,6,-1)$, $B(-1,3,-2)$, and $C(2,7,2)$.

First, find the normal vector n of the plane.



n is the cross product of $\vec{AB}$ and $\vec{AC}$.

$\vec{AB} = (2,-3,-1)$ $\qquad \vec{AC} = (5,1,3)$

$$n = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & -3 & -1 \\ 5 & 1 & 3 \end{vmatrix} = -8i - 11j + 17k.$$

Hence, the plane has an equation $-8x - 11y + 17z = D$.
To find D, plug in any given point into the equation.
I chose $A(-3,6,-1)$.

$$-8(-3) - 11\cdot 6 + 17\cdot(-1) = D \Rightarrow D = -59$$

As a result, $\boxed{-8x - 11y + 17z = -59}$.