Quiz 3 (KEY)

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Problem 1. (5 points)
Find the interval of convergence of the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(\frac{1}{4}x + 6)^n}{n}.$$

Answer. Apply the ratio test to $a_n(x) = \frac{(\frac{1}{4}x+6)^n}{n}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(\frac{1}{4}x + 6)^{n+1}}{n+1} \cdot \frac{n}{(\frac{1}{4}x + 6)^n}\right| \longrightarrow \left|\tfrac{1}{4}x + 6\right|$$

Convergence by the ration test requires the limit of $\left|\frac{a_{n+1}}{a_n}\right|$ to be strictly less than one.

$$\left|\tfrac{1}{4}x + 6\right| < 1 \iff -1 < \tfrac{1}{4}x + 6 < 1 \iff -28 < x < -20$$

Test the end points $x = -28$ and $x = -20$ to determine the interval of convergence. Notice $x = -20$ implies $a_n(x) = \frac{1}{n}$ which defines the Harmonic series. And $x = -28$ implies $a_n(x) = (-1)^n \frac{1}{n}$.

The interval of convergence is $[-28, -20)$ or $-28 \le x < -20$. $\square$

Problem 2. (5 points)
Does the following series converge absolutely or converge conditionally or both?

$$\sum_{n=1}^{\infty} \frac{\sin \frac{(2n-1)\pi}{2}}{n\sqrt{n}}$$

Answer. For all positive integers n

$$\sin\left(\frac{(2n-1)\pi}{2}\right) = \sin\left((2n-1)\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1.$$

So

$$a_n = \frac{1}{n\sqrt{n}} = \frac{1}{n^{3/2}}$$

which defines a p-series with $p = \frac{3}{2} > 1$.

The series converges absolutely. $\square$