Math 21C - Section B01 - Quiz 8 **SOLUTION**
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**Problem 1:** Let $f(x,y) = 3xe^y$.

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**Part A:** Find the rate of change at $P(2,0)$ in the direction from $P$ to $Q(1/2,2)$.

**Solution:** The rate of change is the value of the directional derivative of $f$. We need to compute a directional derivative. The vector that points in the right direction goes from $P$ to $Q$, so the components of this vector are $\langle \frac{1}{2} - 2, 2 - 0\rangle = \langle -\frac{3}{2}, 2$. This vector needs to have length one, so we normalize it and use

$$\mathbf{u} = \frac{\langle -\frac{3}{2}, 2\rangle}{\sqrt{(-3/2)^2 + 2^2}}.$$

We need the gradient of $f$, which is $\nabla f(x,y) = \langle 3e^y, 3xe^y\rangle$. We evaluate this at $P = (2,0)$ to get $\nabla f(2,0) = \langle 3e^0, 3 \cdot 2e^0$.

Now we have everything we need to compute the directional derivative (also known as the [instantaneous rate of change]) of $f$:

$$D_u f = \nabla f(P) \cdot u = \langle 3e^0, 3 \cdot 2e^0\rangle \cdot \frac{\langle -\frac{3}{2}, 2\rangle}{\sqrt{(-3/2)^2 + 2^2}}.$$

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**Part B:** In what direction does $f$ have the maximum rate of change? What is the value of the maximum rate?

**Solution:** A function $f$ has its maximum rate of change in the direction given by the gradient. That is, we should report the vector $\nabla f(P)$, which we computed in part A, only we should give the length one version of the vector:

$$\frac{\langle 3e^0, 3 \cdot 2e^0\rangle}{\sqrt{(3e^0)^2 + (3 \cdot 2e^0)^2}}$$

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**Part C:** What is the flat direction of $D_u f(2,0)$ or when $D_u f(2,0) = 0$.

**Solution:** Find any vector prependicular to $\nabla f(P)$ (of length one). If we labeled our answer vector $\langle a, b\rangle$, then we need to solve the system $\langle 3e^0, 3 \cdot 2e^0\rangle \cdot \langle a, b\rangle = 0$ and $a^2 + b^2 = 1^2$.