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1 (5 points): Find the power series $f(x)$ interval of convergence.

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(5x-2)^n}{\sqrt[4]{4n-2}(-2)^n}$$

We use the Absolute Ratio Test to determine the power series interval of convergence. Let

$$a_n = (-1)^{n+1} \frac{(5x-2)^n}{\sqrt[4]{4n-2}(-2)^n},$$

then

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+2}(5x-2)^{n+1}}{\sqrt[4]{4(n+1)-2}(-2)^{n+1}} \frac{\sqrt[4]{4n-2}(-2)^n}{(-1)^{n+1}(5x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{5x-2}{-2} \sqrt[4]{\frac{4n-2}{4n+2}} \right| = \frac{|5x-2|}{2}.$$

For the power series to converge, we need x such that $\frac{|5x-2|}{2} < 1$ or equivalently $0 < x < \frac{4}{5}$. Since the Absolute Ratio Test is inconclusive when $\frac{|5x-2|}{2} = 1$ we need to check if the series converges when $x = 0$ and $x = \frac{4}{5}$. Plugging in $x = 0$,

$$f(0) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{4n-2}},$$

which converges by the Alternating Series Test. For $x = \frac{4}{5}$,

$$f\left(\frac{4}{5}\right) = -\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{4n-2}}$$

which diverges by Limit Comparison Test to the p-series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}.$$

Therefore the interval of convergence for the power series $f(x)$ is $0 \leq x < \frac{4}{5}$.

2 (5 points): Find the Taylor polynomial of order 2 generated by $f(x) = 2\sin\frac{3x}{7}$ at $a = \frac{7\pi}{9}$.

The Taylor polynomial of order 2 generated by $f(x) = 2\sin\frac{3x}{7}$ and centered at $a = \frac{7\pi}{9}$ is

$$P_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 = \sqrt{3} + \frac{3}{7}\left(x - \frac{7\pi}{9}\right) - \frac{9\sqrt{3}}{98}\left(x - \frac{7\pi}{9}\right)^2.$$