Quiz 8

(A) $\nabla f(x,y) = (7e^y, 7xe^y) \Rightarrow \nabla f(2,0) = (7, 14)$

$\vec{PQ} = (\frac{1}{2} - 2, 2 - 0) = (-\frac{3}{2}, 2)$

$\Rightarrow \vec{u} = \frac{(-\frac{3}{2}, 2)}{\sqrt{(-\frac{3}{2})^2 + 2^2}} = \frac{(-\frac{3}{2}, 2)}{\frac{5}{2}} = (-\frac{3}{5}, \frac{4}{5})$

rate of change $= \nabla f(2,0) \cdot \vec{u} = -\frac{21}{5} + \frac{56}{5} = 7$

(B) $|\nabla f(2,0)| = \sqrt{7^2 + 14^2} = 7\sqrt{5}$ (max value)

$\vec{u} = \frac{\nabla f(2,0)}{7\sqrt{5}} = \frac{(7,14)}{7\sqrt{5}} = (\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ (direction)

(C) $\vec{u} = (a, b)$. $\vec{u} \cdot \nabla f(2,0) = 0$

$\Rightarrow 7a + 14b = 0 \Rightarrow a = -2b$

$\Rightarrow \vec{u} = (-2b, b)$

so $\vec{u} = (-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$. $(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}})$