## Problem 2

Problem 2 (5 points): Does the series

$$\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$$

converge or diverge? Give reasons for your answer.

Converges by ratio test. (Also converges by other tests)

$$\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\,n!\,3^n} \Rightarrow f(n) = \frac{(n+3)!}{3!\,n!\,3^n}$$

$$\frac{(n+3)!}{3!\,n!\,3^n} = \frac{(n+3)(n+2)(n+1)\cancel{(n!)}}{3!\,\cancel{n!}\,3^n} = \frac{(n+3)(n+2)(n+1)}{6\cdot 3^n}$$

Ration test

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{((n+3)+1)((n+2)+1)((n+1)+1)}{6\cdot 3^{n+1}}}{\dfrac{(n+3)(n+2)(n+1)}{6\cdot 3^n}}$$

$$= \frac{(n+4)\cancel{(n+3)}\cancel{(n+2)}}{\cancel{6}\cdot 3^{n+1}} \cdot \frac{\cancel{6}\cdot \cancel{3^n}}{\cancel{(n+3)}\cancel{(n+2)}(n+1)} = \frac{n+4}{3\cdot(n+1)}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n+4}{3n+1} \text{ by L'Hospital } \cong \frac{1}{3} < 1 \text{ Converges}$$