Problem 2

(5 points): Find an equation for the level curve of the function

$$f(x,y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n$$

that passes through the point $(1,2)$.

$$f(1,2) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = 2 \quad \text{(Geometric Series)}$$

$$2 = \frac{1}{1-\frac{x}{y}}$$

$$1 - \frac{x}{y} = \frac{1}{2}$$

$$y - x = \frac{1}{2}y$$

$$\frac{1}{2}y = x$$

$$\boxed{y = 2x}$$