Last Name: _____, First Name: _____

Mat 21C-A02 (6:10 - 7:00pm) Quiz #7 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Find a parametrization for the line in which the planes

$$5x - 2y = 11 \quad \text{and} \quad 4y - 5z = -17$$

intersect.

Solution Following example 10 on the same page, take the cross product of the two normal vectors $\vec{n}_1 = \langle 5, -2, 0\rangle$ and $\vec{n}_2 = \langle 0, 4, -5\rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -2 & 0 \\ 0 & 4 & -5 \end{vmatrix} = 10\hat{j} + 25\hat{j} + 20\hat{k} = 5\langle 2, 5, 4\rangle.$$

Next, we need to find a point on both planes. Since there are three variables and two unknowns, we may arbitrarily set any variable to be any constant. The simplest computations happen when we set $y = 0$. From the first plane, this implies $x = 11/5$ and from the second plane we have $z = 17/5$. Plugging this into the equation of a parametrized line,

$$\vec{r}(t) = \frac{1}{5}\langle 11, 0, 17\rangle + t\langle 2, 5, 4\rangle, \quad -\infty < t < \infty$$

describes where the two planes intersects. (Notice I have dropped the constant 5 in the vector because $5t$ also captures the entire real line as t runs from negative infinity to positive infinity.)

2. (5 points) Find the point in which the line

$$\begin{aligned} x &= 2 \\ y &= 3 + 2t, \\ z &= -2 - 2t, \end{aligned}$$

meets the plane

$$6x + 3y - 4z = -12.$$

Solution Following examples 8 and 9 in section 12.5 on page 869, plug each of the components in the line into the plane

$$6(2) + 3(3 + 2t) + 4(2 + 2t) = 29 + 14t = -12 \quad \Rightarrow \quad t = -\frac{41}{14}.$$

Substituting this t into the parametrized line, the point is $\frac{1}{7}(14, -20, 27)$.