Solutions : Quiz 8

a) $\vec{PQ} = (-\frac{3}{2}, 2) \implies V = -\frac{3}{2}i + 2j$

Direction of V is obtained by dividing v by its length:

$$|v| = \sqrt{(-\frac{3}{2})^2 + (2)^2} = \sqrt{\frac{9}{4}+4} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

$$u = \frac{V}{|v|} = \frac{-\frac{3}{2}i}{\frac{5}{2}} + \frac{2j}{\frac{5}{2}} = \frac{3}{5}i + \frac{4}{5}j$$

$$f_x = 5e^y, \quad f_y = 5xe^y, \quad \nabla f \Big|_{(2,0)} = 5e^0 i + 5 \cdot 2e^0 j$$

$$= 5i + 10j$$

rate of change: $\nabla f \Big|_{(2,0)} \cdot u = (5i + 10j) \cdot (-\frac{3}{5}i + \frac{4}{5}j)$

$$= -3 + 8 = \boxed{5}$$

b) f has maximum rate of change in direction of

$$\nabla f = 5i + 10j$$

rate of change is $|\nabla f| = \sqrt{(5)^2 + (10)^2} = \sqrt{125}$

$\implies$ Value of maximum rate $= \sqrt{125}$