MAT 21C (Section B03)
Quiz 1

SID :
Name : Solution (5 pm)

1. (5 points): Given $a_1 = 5$ and the recursion formula

$$a_{n+1} = a_n + \frac{1}{5^n}$$

for the remaining terms of the sequence. Determine if the sequence converges or diverges. If it converges, determine its limit. If it diverges, give reason why.
Hint: Write out the first few terms without simplifying.

$a_1 = 5$

$a_2 = 5 + \frac{1}{5}$

$a_3 = 5 + \frac{1}{5} + \frac{1}{5^2}$

$\vdots$

$a_{n+1} = 5 + \frac{1}{5} + \cdots + \frac{1}{5^n}$

Since $\frac{1}{5} + \cdots + \frac{1}{5^n} = \frac{1}{5} \cdot \frac{1 - (\frac{1}{5})^n}{1 - \frac{1}{5}} = \frac{1}{4}(1 - (\frac{1}{5})^n)$

$a_{n+1} = 5 + \frac{1}{4}(1 - (\frac{1}{5})^n)$

$\lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty}\left[5 + \frac{1}{4}(1 - \underset{0}{\underbrace{(\frac{1}{5})^n}}) \right]$

$= 5 + \frac{1}{4}$

The sequence converges to $5 + \frac{1}{4}$.

$\left(= \frac{21}{4} \right)$

2. (5 points): Determine if the series

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{5n} \right)^n$$

converges or diverges. Give reasons for your answer.

Note : $\lim_{n \to \infty} \left(1 + \frac{x}{n} \right)^n = e^x$

$\lim_{n \to \infty} \left(1 - \frac{1}{5n} \right)^n = \lim_{n \to \infty} \left(1 + \frac{(-\frac{1}{5})}{n} \right)^n = e^{-\frac{1}{5}} \neq 0$.

Hence, the series diverges.