Name:

Math 21C Section B05
Thursday 4-5pm
4/24/2008

QUIZ #3

Problem 1 (5 points): Find the interval of convergence for the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(\frac{1}{6}x+4)^n}{n}.$$

let $L = \lim_{n\to\infty} \left| \frac{(\frac{1}{6}x+4)^{n+1}}{n+1} \cdot \frac{n}{(\frac{1}{6}x+4)^n} \right| = \lim_{n\to\infty} \left|\frac{1}{6}x+4\right| \left|\frac{n}{n+1}\right|$

$= \left|\frac{1}{6}x+4\right| \cdot \lim_{n\to\infty} \left|\frac{n}{n+1}\right| = \left|\frac{1}{6}x+4\right|$

By ratio test $f(x)$ converges when $L<1$
diverges when $L>1$

So when $\left|\frac{1}{6}x+4\right| < 1$ $f(x)$ converges,

So $-1 < \frac{1}{6}x+4 < 1$

$-5 < \frac{1}{6}x < -3$

$-30 < x < -18$ so on $(-30,-18)$ $f(x)$ converges,

Still need to check endpoints, $x=-30, -18$.

$f(-30) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ which converges by alternating series test

$f(-18) = \sum_{n=1}^{\infty} \frac{1}{n}$ which diverges. So the complete interval of convergence for $f(x)$ is $\boxed{[-30,-18)}$

Problem 2 (5 points): Does the series

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$$

converge absolutely or converge conditionally or both?

Let $a_n = \frac{\cos(n\pi)}{n\sqrt{n}}$. Now $|a_n| = \frac{|\cos(n\pi)|}{n\sqrt{n}} = \frac{|\cos(n\pi)|}{n^{3/2}} \leq \frac{1}{n^{3/2}}$

The series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges (It's a convergent p-series),

So by the comparison test $\sum_{n=1}^{\infty} |a_n|$ converges,

This means that $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$

is absolutely convergent.

[Note that it cannot be both absolutely convergent and conditionally convergent. The "both" part of the question was designed to test whether one knew this or not.]