Solutions

Math 21C Quiz 7

Section: 5:10-6:00 pm, TA: Arpy Mikaelian
Tuesday May 27, 2008

Problem 1

(5 points): Find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ if the equation

$$x^2 z + y \ln x - x^6 + 1 = 0$$

defines x as a function of the two independent variables y and z and the partial derivative exists.

Use Implicit Differentiation (p.g. 972)

$$\frac{\partial}{\partial z}\left[x^2 z + y\ln x - x^6 + 1 = 0\right]$$

$$= \frac{\partial}{\partial z}(x^2 z) + \frac{\partial}{\partial z}(y \ln x) - \frac{\partial}{\partial z}(x^6) + \frac{\partial}{\partial z}(1) = 0$$

$$= \overbrace{2x\frac{\partial x}{\partial z} z + 1 \cdot x^2}^{\text{multiplication rule for derivating}} + \frac{y}{x}\frac{\partial x}{\partial z} - 6x^5\frac{\partial x}{\partial z} = 0$$

$$\Rightarrow \quad x^2 = -2x\frac{\partial x}{\partial z} z - \frac{y}{x}\frac{\partial x}{\partial z} + 6x^5\frac{\partial x}{\partial z}$$

$$x^2 = \frac{\partial x}{\partial z}\left(-2xz - \frac{y}{x} + 6x^5\right)$$

$$\frac{x^2}{-2xz - \frac{y}{x} + 6x^5} = \frac{\partial x}{\partial z} = \frac{1^2}{-2(1 \cdot -3) - \left(\frac{-1}{1}\right) + 6 \cdot 1^5}$$

$$= \boxed{\frac{1}{13}}$$