Name: *Key*   Student ID:_____

For each problem below, explain why the sequence converges or diverges, if it converges find the limit. You may use any theorems which you learned in class.

1. *(5 pts)* $a_n = \frac{2^n - 1}{2^n}$

We can rewrite a_n as $1 - \frac{1}{2^n} = a_n$

Since $\lim\limits_{n \to \infty} 1 = 1$ & $\lim\limits_{n \to \infty} \frac{1}{2^n} = 0$

Then $\lim\limits_{n \to \infty} a_n = \lim\limits_{n \to \infty} 1 - \frac{1}{2^n} = 1 - 0 = 1$

Thus a_n converges to 1

2. *(5 pts)* $a_n = \frac{\cos^2 4n}{2^n}$

Note: $-1 \le \cos 4n \le 1$ for all n thus $0 \le \cos^2 4n \le 1$

therefore $0 = \frac{0}{2^n} \le \frac{\cos^2 4n}{2^n} \le \frac{1}{2^n}$

Since $\lim\limits_{n \to \infty} \frac{1}{2^n} = 0$ & $\lim\limits_{n \to \infty} 0 = 0$

Then by Sandwich Theorem. $\lim\limits_{n \to \infty} a_n = 0$

Thus a_n converges to 0