

**Problem 2 (5 points):** Write the binary number

$$0.\overline{110} = 0.110110110\dots$$

as a rational number.

(Hint: Recall the binary number 0.110 is  $1 * 2^{-1} + 1 * 2^{-2} + 0 * 2^{-3}$  in decimal.)

Solution:  $0.\overline{110} = 0.110110110\dots$

$$= \frac{1}{2} + \frac{1}{2^2} + \frac{0}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{0}{2^6} + \dots$$

$$= \sum_{i=0}^{\infty} \frac{1}{2^{3i+1}} + \sum_{i=0}^{\infty} \frac{1}{2^{3i+2}}$$

$$= \frac{1}{2} \sum_{i=0}^{\infty} \frac{1}{(2^3)^i} + \frac{1}{2^2} \sum_{i=0}^{\infty} \frac{1}{(2^3)^i}$$

$$= \frac{1}{2} \left( \frac{1}{1 - \frac{1}{8}} \right) + \frac{1}{2^2} \left( \frac{1}{1 - \frac{1}{8}} \right)$$

$$= \left( \frac{1}{2} + \frac{1}{4} \right) \left( \frac{1}{\frac{7}{8}} \right) = \left( \frac{3}{4} \right) \left( \frac{8}{7} \right) = \boxed{\frac{6}{7}}$$