HW # 27

Section 6.8

$\boxed{384:4}$ $\lim_{x\to 0} \dfrac{\sin x^2}{(\sin x)^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \dfrac{2x\cdot\cos x^2}{2\sin x\cos x}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \dfrac{2x\cdot{}^{-}\sin x^2\cdot 2x + 2\cdot\cos x^2}{-2\sin^2 x + 2\cos^2 x} = \dfrac{2}{2} = 1$

$\boxed{384:11}$ $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \dfrac{2\ln x\cdot\frac{1}{x}}{1}$

$= \lim_{x\to\infty} \dfrac{2\ln x}{x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \dfrac{2\cdot\frac{1}{x}}{1} = \dfrac{2\cdot 0}{1} = 0$

$\boxed{384:12}$ $\lim_{x\to 0} \dfrac{\sin^{-1}x}{e^{2x}-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \dfrac{\frac{1}{\sqrt{1-x^2}}}{2e^{2x}} = \dfrac{1}{2}$

$\boxed{384:13}$ $\lim_{x\to 0} (1-2x)^{1/x} = \text{"}1^{\pm\infty}\text{"}$

$= \lim_{x\to 0} e^{\ln(1-2x)^{1/x}} = \lim_{x\to 0} e^{\frac{1}{x}\ln(1-2x)}$

$= e^{\lim_{x\to 0}\frac{\ln(1-2x)}{x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to 0}\frac{\frac{-2}{1-2x}}{1}} = e^{-2}.$

$\boxed{384:14}$ $\lim_{x\to 0} (1+\sin 2x)^{\csc x} = \text{"}1^{\pm\infty}\text{"}$

$= \lim_{x\to 0} e^{\ln(1+\sin 2x)^{\csc x}} = \lim_{x\to 0} e^{\csc x\cdot\ln(1+\sin 2x)}$

$= \lim_{x\to 0} e^{\frac{\ln(1+\sin 2x)}{\sin x}} = e^{\lim_{x\to 0}\frac{\ln(1+\sin 2x)}{\sin x}}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to 0}\frac{\frac{2\cos 2x}{1+\sin 2x}}{\cos x}} = e^{2}.$

$\boxed{384:15}$ $\lim_{x\to 0^+} (\sin x)^{e^x-1} = "0^0"$

$= \lim_{x\to 0^+} e^{\ln(\sin x)^{e^x-1}} = e^{\lim_{x\to 0^+} (e^x-1)\cdot \ln(\sin x)}$

$= e^{\lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{e^x-1}}} \overset{"\frac{\infty}{\infty}"}{=} e^{\lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{e^x}{(e^x-1)^2}}}$

$= e^{\lim_{x\to 0^+} \frac{(e^x-1)^2\cos x}{e^x \sin x}} = "\frac{0}{0}"$

$= e^{\lim_{x\to 0^+} \frac{-\sin x\cdot (e^x-1)^2\cdot e^x + 2e^x(e^x-1)\cdot \cos x}{e^x\cdot \cos x + e^x\cdot \sin x}}$

$= e^{\frac{0+0}{1+0}} = e^0 = 1.$

$\boxed{384:16}$ $\lim_{x\to 0^+} x^2 \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}}$

$\overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+} \frac{x^2}{-2} = 0$.

$\boxed{384:17}$ $\lim_{x\to 0^+} (\tan x)^{\tan 2x} = "0^0"$

$= \lim_{x\to 0^+} e^{\ln(\tan x)^{\tan 2x}} = \lim_{x\to 0^+} e^{\tan 2x\cdot \ln(\tan x)}$

$= e^{\lim_{x\to 0^+} \frac{\ln(\tan x)}{\cot 2x}} \overset{"\frac{\infty}{\infty}"}{=} e^{\lim_{x\to 0^+} \frac{\sec^2 x}{\tan x}/-2\csc^2 2x}$

$= e^{\lim_{x\to 0^+} \frac{1}{\cos^2 x}\cdot \frac{\cos x}{\sin x}\cdot \frac{\sin^2 2x}{-2}}$

$$= e^{\lim_{x\to 0^+} \frac{1}{\cos x \sin x} \cdot \frac{(2\sin x \cos x)^2}{-2}}$$

$$= e^{\lim_{x\to 0^+} -2\cdot \sin x \cdot \cos x} = e^{-2(0)(1)} = e^0 = 1.$$

$\boxed{384:19}$ $\lim_{x\to\infty} \frac{2^x}{3^x} = \lim_{x\to\infty} \left(\frac{2}{3}\right)^x = 0$.

$\boxed{384:20}$ $\lim_{x\to\infty} \frac{2^x + x}{3^x} = \lim_{x\to\infty} \left(\frac{2}{3}\right)^x + \lim_{x\to\infty} \frac{x}{3^x}$

$$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} 0 + \lim_{x\to\infty} \frac{1}{3^x \cdot \ln 3} = \text{"}\frac{1}{\infty}\text{"} = 0.$$

$\boxed{384:21}$ $\lim_{x\to\infty} \frac{\log_2 x}{\log_3 x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}} \cdot \log_2 e}{\cancel{\frac{1}{x}} \cdot \log_3 e} = \frac{\log_2 e}{\log_3 e}$

$\boxed{384:22}$ $\lim_{x\to 1} \frac{\log_2 x}{\log_3 x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{\cancel{\frac{1}{x}} \cdot \log_2 e}{\cancel{\frac{1}{x}} \cdot \log_3 e} = \frac{\log_2 e}{\log_3 e}.$

$\boxed{384:23}$ $\lim_{x\to\infty} \left(\frac{1}{x} - \frac{1}{\sin x}\right) = \lim_{x\to\infty} \frac{1}{x} - \lim_{x\to\infty} \frac{1}{\sin x}$

$= 0 - \lim_{x\to\infty} \frac{1}{\sin x}$ does not exist.

$\boxed{384:24}$ $\lim_{x\to\infty} \left(\sqrt{x^2+3} - \sqrt{x^2+4x}\right) \frac{\left(\sqrt{x^2+3} + \sqrt{x^2+4x}\right)}{\left(\sqrt{x^2+3} + \sqrt{x^2+4x}\right)}$

$$= \lim_{x\to\infty} \frac{(\cancel{x^2}+3) - (\cancel{x^2}+4x)}{\sqrt{x^2+3} + \sqrt{x^2+4x}} = \lim_{x\to\infty} \frac{\frac{3}{x} - 4}{\sqrt{1+\frac{3}{x^2}} + \sqrt{1+\frac{4}{x}}} = \frac{0-4}{2}$$

$$= -2$$

$\boxed{384:25}$ $\displaystyle\lim_{x\to\infty} \frac{x^2+3\cos 5x}{x^2-2\sin 4x}$

$$= \lim_{x\to\infty} \frac{1+\frac{3\cos 5x}{x^2}}{1-\frac{2\sin 4x}{x^2}} = \frac{1+0}{1-0} = 1 .$$

$\boxed{384:26}$ $\displaystyle\lim_{x\to\infty} \frac{e^x-\frac{1}{x}}{e^x+\frac{1}{x}}$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{xe^x}}{1+\frac{1}{xe^x}} = \frac{1-0}{1+0} = 1 .$$

$\boxed{384:29}$ $\displaystyle\lim_{x\to\infty} \frac{\sin x}{4+\sin x} = \lim_{x\to\infty} \frac{1}{1+4\csc x}$

does not exist since $\displaystyle\lim_{x\to\infty} 4\csc x$ does not exist.

$\boxed{384:38}$ $\displaystyle\lim_{x\to 0} \frac{xe^x\cos^2 6x}{e^{2x}-1}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{e^x\cos^2 6x + xe^x\cos^2 6x - 12xe^x\cos 6x\cdot\sin 6x}{2e^{2x}}$$

$$= \frac{1+0-0}{2} = \frac{1}{2} .$$

$\boxed{384:45}$ $\displaystyle\lim_{x\to 0}\left(\frac{1}{1-\cos x} - \frac{2}{x^2}\right) = \text{"}\infty - \infty\text{"}$

$$= \lim_{x\to 0} \frac{x^2-2(1-\cos x)}{x^2(1-\cos x)}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2x - 2\sin x}{x^2 \sin x + 2x(1-\cos x)}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2 - 2\cos x}{x^2\cos x + 2x\sin x + 2x\sin x + 2(1-\cos x)}$$

$$= \lim_{x\to 0} \frac{2-2\cos x}{x^2\cos x + 4x\sin x - 2\cos x + 2}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2\sin x}{-x^2\sin x + 2x\cos x + 4x\cos x + 4\sin x + 2\sin x}$$

$$= \lim_{x\to 0} \frac{2\sin x}{-x^2\sin x + 6x\cos x + 6\sin x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2\cos x}{-x^2\cos x - 2x\sin x - 6x\sin x + 6\cos x + 6\cos x}$$

$$= \frac{2}{0-0-0+6+6} = \frac{1}{6}.$$

$\boxed{384:46}$ $$\lim_{x\to 0} \frac{\sin^{-1}x}{\tan^{-1}2x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{\frac{2}{1+4x^2}} = \frac{1}{2}$$

$\boxed{384:53}$ $$\lim_{x\to 0} \left(\frac{1+2^x}{2}\right)^{1/x} = \text{"}1^{\pm\infty}\text{"}$$

$$= \lim_{x\to 0} e^{\ln\left(\frac{1}{2}+2^{x-1}\right)^{1/x}} = \lim_{x\to 0} e^{\frac{\ln\left(\frac{1}{2}+2^{x-1}\right)}{x}}$$

$$= e^{\lim_{x\to 0} \frac{\ln\left(\frac{1}{2}+2^{x-1}\right)}{x}} = e^{\lim_{x\to 0} \frac{2^{x-1}\ln 2}{\frac{1}{2}+2^{x-1}}}$$

$$= e^{\frac{1}{2}\ln 2} = e^{\ln\sqrt{2}} = \sqrt{2}.$$

$\boxed{384:56}$ $f(x) = (1+x)^{1/x}$ for $x > -1$, $x \neq 0$;

$\lim_{x\to 0} (1+x)^{1/x} = e$; and

$\lim_{x\to\infty} \ln(1+x)^{1/x} = \lim_{x\to\infty} \frac{\ln(1+x)}{x}$

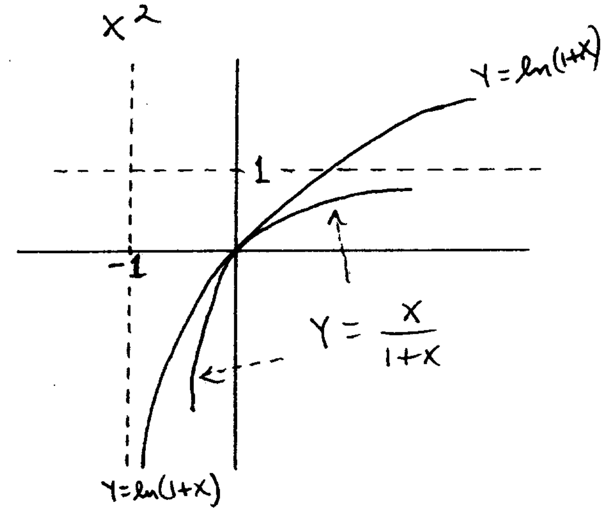
$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{1}{1+x} = 0$ so $\lim_{x\to\infty} (1+x)^{1/x} = 1$;

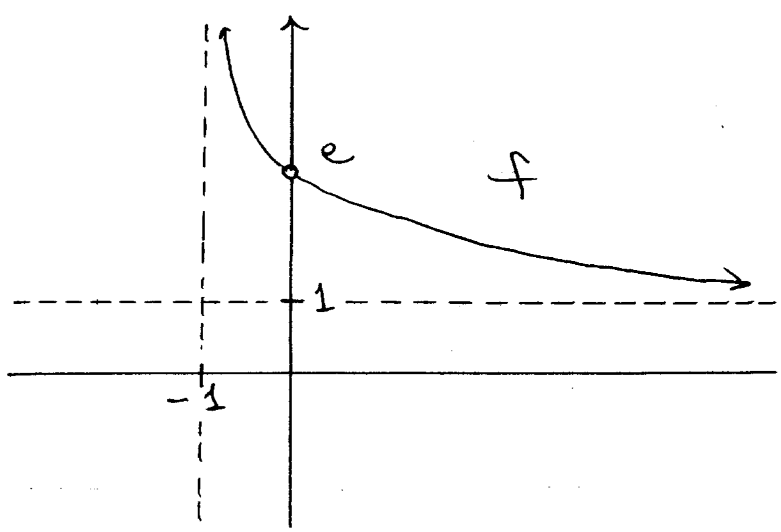
$\ln y = \frac{\ln(1+x)}{x} \xrightarrow{D} \frac{1}{y} y' = \frac{x \cdot \frac{1}{1+x} - \ln(1+x)}{x^2} \rightarrow$

$y' = (1+x)^{1/x} \cdot \frac{\frac{x}{1+x} - \ln(1+x)}{x^2}$

< 0 for $x > -1, x \neq 0$;

$\lim_{x\to -1^+} (1+x)^{1/x} = (0^+)^{-1} = +\infty$

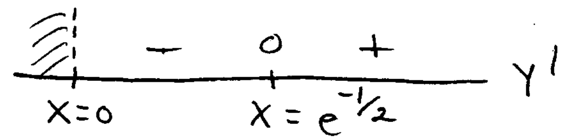




$\boxed{384:58}$ $Y = x^2 \ln x \quad , \quad x > 0$

$Y' = x^2\left(\frac{1}{x}\right) + 2x \ln x = x + 2x \ln x$

$= x(1 + 2\ln x) = 0$

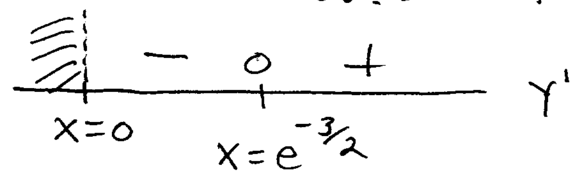


$Y = \frac{-1}{2}e^{-1}$

abs. min

$Y'' = 1 + 2x\left(\frac{1}{x}\right) + 2\ln x$

$= 3 + 2\ln x = 0$



$Y = \frac{-3}{2}e^{-3}$

infl. pt.

$Y = 0 \rightarrow x = 1$

Y is $\uparrow$ for $x > e^{-1/2}$,

Y is $\downarrow$ for $0 < x < e^{-1/2}$,

Y is $\cup$ for $x > e^{-3/2}$,

Y is $\cap$ for $0 < x < e^{-3/2}$

$$\lim_{x \to 0^+} x^2 \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} \overset{\text{"}\frac{-\infty}{\infty}\text{"}}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$= \lim_{x \to 0^+} \frac{-1}{2}x^2 = 0$$