Quiz 8 (KEY)

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Problem 1. Let $f(x, y) = 6xe^y$.
(a) **(4 points)** Find the rate of change at $P(2, 0)$ in the direction from P to $Q(1/2, 2)$.
(b) **(4 points)** In what direction does f have the maximum rate of change? What is the value of the maximum rate?
(c) **(2 points)** What is the flat direction of $D_u f(2, 0)$ or when $D_u f(2, 0) = 0$.

Answer.

(a) We are asked to compute the directional derivative $D_{\mathbf{u}} f$ in the direction $\vec{PQ} = \langle 3/2, 2\rangle$ and evaluate $D_{\mathbf{u}} f$ at P. Set $\mathbf{u} = \frac{\vec{PQ}}{|\vec{PQ}|}$.

$$\mathbf{u} = \frac{\vec{PQ}}{\sqrt{\frac{9}{4}+4}} = \frac{2}{5}\vec{PQ} = \langle \tfrac{3}{5}, \tfrac{4}{5}\rangle.$$

Recall that a directional derivative of a function f at the point P in the direction $\mathbf{u}$ is the dot product of the gradient $\nabla f|_P$ of f with the vector $\mathbf{u}$, where the gradient is the vector $\nabla f = \langle f_x, f_y\rangle$. If $f(x, y) = 6xe^y$, then

$$f_x = 6e^y, \quad f_y = 6xe^y, \qquad \text{so that} \quad \nabla f = \langle 6e^y, 6xe^y\rangle.$$

The gradient ∇f evaluated at P is

$$\nabla f|_P = \langle 6, 12\rangle.$$

Hence,

$$D_{\mathbf{u}} f(2, 0) = \nabla f|_P \cdot \mathbf{u} = \langle 6, 12\rangle \cdot \langle \tfrac{3}{5}, \tfrac{4}{5}\rangle = \tfrac{66}{5}.$$

(b) The direction of maximum rate of change for the function f at the point P is

$$\mathbf{v} = \nabla f|_P = 6\langle 1, 2\rangle.$$

The maximum rate of change is the magnitude of $\mathbf{v}$.

$$\text{maximum rate of change} = 6\sqrt{5}.$$

(c) We are asked for a direction $\mathbf{u} = \langle u_1, u_2\rangle$ such that $D_{\mathbf{u}} f(2, 0) = 0$, or equivalently

$$6\langle 1, 2\rangle \cdot \langle u_1, u_2\rangle = 0.$$

After dividing both sides by 6 and computing the dot product, we have that $u_1 + 2u_2 = 0$, or that $u_1 = -2u_2$. Here only direction that matters. A convenient choice is then $u_2 = 1$ or $u_2 = -1$ so that $\mathbf{u} = \langle -2, 1\rangle$ or $\mathbf{u} = \langle 2, -1\rangle$ is correct.

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