Quiz 2

Name:

Determine whether each series converges or diverges. Give reasons for your answer.

Problem 1 (5 points): $\sum_{n=0}^{\infty} \frac{\sin^2 n}{7^n}$

Solution:

First note that $0 \leq \sin^2(n) \leq 1$, further more we know that

$$\sum_{n=0}^{\infty} \frac{1}{7^n}$$

is a geometric series with $r = 1/7$ so it converges. Finally since $\frac{\sin^2(n)}{7^n} \leq \frac{1}{7^n}$ for all n then by the comparison test $\sum_{n=0}^{\infty} \frac{\sin^2 n}{7^n}$ converges.

Problem 2 (5 points): $\sum_{n=1}^{\infty} \frac{4(\ln n)^n}{n^n}$

Solution:

We will try to use the nth root test

$$
\begin{aligned}
\lim_{n \to \infty} \left(4 \left(\frac{\ln(n)}{n} \right)^n \right)^{1/n} &= \lim_{n \to \infty} (4)^{1/n} \lim_{n \to \infty} \frac{\ln(n)}{n} \\
&= 1 \lim_{n \to \infty} \frac{1/n}{1} \\
&= 0
\end{aligned}
$$

and since $0 < 1$ by the nth root test we know the series converges.