Quiz 7

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Spring 2008

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Problem 1. (5 points)
Find the value of $\partial x/\partial z$ at the point $(1,-1,-3)$ if the equation

$$x^3 z + y\ln x - x^5 + 2 = 0$$

defines x as a function of the two independent variables y and z and the partial derivative exists.

Answer. The first term $x^3 z$ requires the product rule. The second term does not require the product rule. As an example, observe by the chain rule $\frac{\partial}{\partial z}x^3 = 3x^2\frac{\partial x}{\partial z}$.

Apply $\frac{\partial}{\partial z}$ to $x^3 z + y\ln x - x^5 + 2 = 0$.

$$3x^2 z\frac{\partial x}{\partial z} + x^3 + \frac{y}{x}\frac{\partial x}{\partial z} - 5x^4\frac{\partial x}{\partial z} = 0.$$

Solve for $\frac{\partial x}{\partial z}$.

$$\frac{\partial x}{\partial z} = \frac{-x^3}{3x^2 z + y/x - 5x^4}.$$

Evaluate at the point $(x,y,z) = (1,-1,-3)$.

$$\left.\frac{\partial x}{\partial z}\right|_{(x,y,z)=(1,-1,-3)} = \frac{-1}{-9-1-5}$$

□

Problem 2. (5 points)
Find $\partial w/\partial u$ when $u = -1$, $v = 2$ if $w = xy + \ln z$ and

$$\begin{aligned} x &= v^2/u, \\ y &= u + v, \\ z &= \cos u. \end{aligned}$$

Answer. By the chain rule

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}.$$

Since $\frac{\partial w}{\partial x} = y$, $\frac{\partial w}{\partial y} = x$ and $\frac{\partial w}{\partial z} = \frac{1}{z}$

$$\begin{aligned} \frac{\partial w}{\partial u} &= y\frac{\partial x}{\partial u} + x\frac{\partial y}{\partial u} + \frac{1}{z}\frac{\partial z}{\partial u} \\ &= (u+v)\frac{\partial}{\partial u}\left(\frac{v^2}{u}\right) + \frac{v^2}{u}\frac{\partial}{\partial u}(u+v) + \frac{1}{\cos u}\frac{\partial}{\partial u}\cos u \\ &= -(u+v)\frac{v^2}{u^2} + \frac{v^2}{u} - \tan u \\ &= -\frac{v^3}{u^2} - \tan u. \end{aligned}$$

Evaluate at $(u,v) = (-1,2)$: $\frac{\partial w}{\partial u}(-1,2) = -8 - \tan(-1)$. □