

# Math 21A: Calculus

## Quiz 1 Solution

(Woei 6/24/04)

This is a closed book, no notes, no calculator quiz. Remember, if a question has English in it, so should the answer.

1. (10 points) Compute

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$$

**Solution:**

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} \cdot \frac{\sqrt{x+3} + \sqrt{3}}{\sqrt{x+3} + \sqrt{3}} = \\ &= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+3} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+3} + \sqrt{3}} = \frac{1}{2\sqrt{3}} \end{aligned}$$

2. (8 points) Graph  $f(x) = \frac{(x+1)^2}{(x-1)(x-2)}$

**Solution:**

**Symmetry:** None, i.e. neither even nor odd, since  $f(-3) \neq f(3)$

**Intercepts:**

Y-intercept:  $f(0) = \frac{1}{2}, (0, \frac{1}{2})$

X-intercept: Since  $f(x)$  is in reduced form, so the only x-intercept is when  $(x+1)^2 = 0 \Rightarrow x = -1, (-1, 0)$ .

**Asymptotes:**

Horizontal Asymptotes (H.A.):

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1,$$

so H.A. at  $y = 1$ .

Vertical Asymptotes (V.A.):

$$@x = 1 \text{ \& } x = 2$$

Now check what happens when  $x \rightarrow 1$  from the left and right side of 1 and similarly for  $x = 2$ .

$$\lim_{x \rightarrow 1^-} f(x) = \infty,$$

since  $x \rightarrow 1^-$  means  $x$  is approaching 1 from the left hand side, so  $x < 1$  thus  $x - 1 < 0$ . The quotient  $\frac{(x+1)^2}{x-2} \rightarrow -4$  as  $x \rightarrow 1^-$ , then  $\lim_{x \rightarrow 1^-} f(x) = \infty$ .

Similarly,

$$\lim_{x \rightarrow 1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

The graph looks like:

