Math 21C **Quiz 4**

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1 (5 points): Compute the 6th derivative of

$$f(x) = \frac{\cos(3x^2) - 1}{x^2}$$

at $x = 0$.

Note that directly computing the 6th derivative takes quite a bit of work. We know the Maclaurin series of $\cos x$ and that differentiating this Maclaurin series reduces to taking derivatives of monomials. First, the Maclaurin series of

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots,$$

therefore the Maclaurin series of

$$-1 + \cos(3x^2) = -1 + 1 - \frac{3^2 x^4}{2!} + \frac{3^4 x^8}{4!} - \frac{3^6 x^{12}}{6!} + \ldots \tag{1}$$

Dividing (1) by x^2 we have

$$f(x) = -\frac{3^2 x^2}{2!} + \frac{3^4 x^6}{4!} - \frac{3^6 x^{10}}{6!} + \ldots$$

Taking 6 derivatives of the above result in

$$f^{(6)}(x) = \frac{3^4 6!}{4!} - \frac{3^6 \frac{10!}{4!} x^4}{6!} + \ldots$$

Therefore $f^{(6)}(0) = 3^4 \cdot 6 \cdot 5$.

2 (5 points): Using Euler's Identity write $\mathrm{e}^{i\pi} + \mathrm{e}^{\mathrm{i}\pi/4}$ in the form $a + b\mathrm{i}$.

Euler's Identity:

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

Thus

$$\mathrm{e}^{i\pi} + \mathrm{e}^{\mathrm{i}\pi/4} = \cos\pi + \mathrm{i}\sin\pi + \cos\pi/4 + \mathrm{i}\sin\pi/4 = \left(-1 + \frac{\sqrt{2}}{2}\right) + \mathrm{i}\frac{\sqrt{2}}{2}$$