

Math 21A: Calculus

Quiz 2 Solution

(Woei 7/6/04)

This is a closed book, no notes, no calculator quiz. Remember, if a question has English in it, so should the answer.

1. (10 points) Find the derivative of

$$f(x) = \frac{\sqrt{x} \sin^2 x - \cot x}{\sec x}$$

Solution:

Let $f(x) = \frac{h(x)}{g(x)} = \frac{\sqrt{x} \sin^2 x - \cot x}{\sec x}$. Use quotient rule, i.e., $\left(\frac{h}{g}\right)' = \frac{h'g - g'h}{g^2}$. So we need to find h' and g' . For h , the derivative of the difference is the difference of the derivative, so $(\sqrt{x} \sin^2 x)' = \sqrt{x} 2 \sin x \cos x + \frac{1}{2\sqrt{x}} \sin^2 x$ by chain rule and product rule and $(\cot x)' = -\csc^2 x$. So

$$h' = \sqrt{x} 2 \sin x \cos x + \frac{1}{2\sqrt{x}} \sin^2 x + \csc^2 x$$

and

$$g' = \sec x \tan x.$$

Thus

$$f'(x) = \frac{(\sqrt{x} 2 \sin x \cos x + \frac{1}{2\sqrt{x}} \sin^2 x + \csc^2 x) \sec x - \sec x \tan x (\sqrt{x} \sin^2 x - \cot x)}{\sec^2 x}.$$

2. (6 points) Draw the graph of a differentiable function that meets the given condition:

$$\begin{aligned} f'(x) &> 0 && \text{when } x < 1 \\ f'(x) &> 0 && \text{when } x > 3 \\ f'(x) &< 0 && \text{when } 1 < x < 3 \end{aligned}$$

Solution:

