

Name: Key

Student ID: \_\_\_\_\_

**No Calculators.****Show all work and justifications to receive full credit.**

1. (4 pts) Find the angle between the two planes  $8x + 8y + 8z = 8$  and  $4x - 11y + 6z = -9$ . You do not need to simplify your answer.

To find the angle between two planes is equivalent to finding the angle between the normals to the plane.  $\langle 8, 8, 8 \rangle$  or

$\langle 1, 1, 1 \rangle = n_1$  is the normal to the plane  $8x + 8y + 8z = 8$  &

$\langle 4, -11, 6 \rangle = n_2$  is the normal to the plane  $4x - 11y + 6z = -9$ .

$$\text{Thus } \cos \theta = \frac{n_1 \cdot n_2}{\|n_1\| \|n_2\|} = \frac{-1}{\sqrt{3} \sqrt{173}} = \frac{-1}{\sqrt{519}} \Rightarrow$$

$$\theta = \cos^{-1} \left( \frac{-1}{\sqrt{519}} \right)$$

2. (4 pts) Find the domain and range of  $f(x, y) = \frac{1}{\sqrt{8-x^2-y^2}}$ .

The domain  $(f) = \{(x, y) \mid 8 - x^2 - y^2 > 0\} = \{(x, y) \mid x^2 + y^2 < 8\}$   
 = points in the circle of radius  $\sqrt{8}$  centered at the origin

range  $(f) = \left[ \frac{1}{\sqrt{8}}, \infty \right)$  since the largest  $8 - x^2 - y^2$  is 8 & smallest is close to 0.

3. (2 pts) Describe the level curve  $1 = z = f(x, y)$  where  $f(x, y)$  is defined in problem 2.

$$1 = \frac{1}{\sqrt{8-x^2-y^2}} \Rightarrow \sqrt{8-x^2-y^2} = 1 \Rightarrow 8-x^2-y^2 = 1 \Rightarrow$$

$x^2 + y^2 = 7$  which is a circle of radius  $\sqrt{7}$  centered at the origin.