Quiz 3

Name:

Problem 1 (2 points): Given $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = \ln 2$. Estimate the magnitude of the error between $\sum_{n=1}^{94}(-1)^{n+1}\frac{1}{n}$ and $\ln(2)$.

Solution:

The magnitude of the error is given by $|(-1)^{95+1}\frac{1}{95}| = \frac{1}{95}$ see Example 1 on page 772 of the text for an explanation.

Problem 2 (8 points): Find the series: $\sum_{n=0}^{\infty}\frac{(x-4)^n}{5^n\ \sqrt{n^2+4}}$ radius and interval of convergence. For what values of x does the series converge absolutely and conditionally?

Solution:

First we use The ratio test to find the interval of convergence and then we test the endpoints.

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{\frac{(x-4)^{n+1}}{5^{n+1}\ \sqrt{(n+1)^2+4}}}{\frac{(x-4)^n}{5^n\ \sqrt{n^2+4}}}\right| &= \lim_{n\to\infty}\left|\frac{(x-4)\ \sqrt{n^2+4}}{5\ \sqrt{(n+1)^2+4}}\right| \\
&= \left|\frac{(x-4)}{5}\right|\sqrt{\lim_{n\to\infty}\frac{n^2+4}{(n+1)^2+4}} \\
&= \left|\frac{(x-4)}{5}\right|.
\end{aligned}$$

Thus $\left|\frac{(x-4)}{5}\right| < 1$ implies that $-1 < x < 9$ and the radius of convergence is 5. Now we test the end points at $x = -1$ we get

$$\sum_{n=0}^{\infty}\frac{(-5)^n}{5^n\ \sqrt{n^2+4}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{\sqrt{n^2+4}}$$

Note that this is an alternating series and the terms are strictly decreasing and positive. Thus by the alternating series theorem the series converges at $x = -1$. Next we test $x = 9$ which give us the following

$$\sum_{n=0}^{\infty}\frac{(5)^n}{5^n\ \sqrt{n^2+4}} = \sum_{n=0}^{\infty}\frac{1}{\sqrt{n^2+4}}.$$

Further more we know that $\sum_{n=0}^{\infty}\frac{1}{n}$ diverges and since

$$\begin{aligned}
\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n^2+4}}}{\frac{1}{n}} &= \sqrt{\lim_{n\to\infty}\frac{n^2}{n^2+4}} \\
&= 1
\end{aligned}$$

Thus by the limit comparison test the series diverges. We now have to show the radius of convergence is 5. Note that the absolute value of the series at $x = -1$ is the same as the series evaluated at $x = 9$ thus the series converges conditionally at $x = -1$ and converges absolutely on the interval $(-1, 9)$. The radius of convergence is $[-1, 9)$.