Name: Solutions

Math 21C Section B05
Thursday 4-5pm
5/8/2008

QUIZ #5

Problem 1 (5 points): Let $\mathbf{u} = <4, -1, 2>$ and $\mathbf{v} = <1, 2, 3>$.
Find the angle between the $\mathbf{u}$ and $\mathbf{v}$. Do not simplify.

$$\cos\theta = \frac{u \cdot v}{|u||v|} = \frac{u}{|u|} \cdot \frac{v}{|v|}$$

Either approach is fine

So $u \cdot v = \langle 4,-1,2\rangle \cdot \langle 1,2,3\rangle = 4\cdot 1 + -1\cdot 2 + 2\cdot 3 = 4-2+6 = 8$

$|u| = |\langle 4,-1,2\rangle| = \sqrt{4^2+(-1)^2+2^2} = \sqrt{16+1+4} = \sqrt{21}$

$|v| = |\langle 1,2,3\rangle| = \sqrt{1^2+2^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$

So $\cos\theta = \dfrac{8}{\sqrt{21}\cdot\sqrt{14}}$

answer for quiz

So $\boxed{\theta = \cos^{-1}\left(\dfrac{8}{\sqrt{21}\cdot\sqrt{14}}\right)}$ (remember do not simplify)

Note for those who like simplifying:
we can simplify further to $\theta = \cos^{-1}\left(\dfrac{8}{7\sqrt{6}}\right)$
if we wanted to

v

u

Problem 2 (5 points): Let $\mathbf{u} =< 4, -1, 2 >$ and $\mathbf{v} =< 1, 2, 3 >$ (same as before).

Find $\text{proj}_{\mathbf{u}}\mathbf{v}$.

$$\text{proj}_u v = \left(\frac{u}{|u|} \cdot v\right)\frac{u}{|u|} = \left(\frac{u \cdot v}{|u|^2}\right) u$$

This is just a side note explanation

Note: $\text{proj}_u v$ is telling us what part of v points in the u direction. The dot product (which gives a scalar) $\left(\frac{u}{|u|} \cdot v\right)$ tells us what scalar amount v points in the u direction. Multiplying a scalar amount by some unit vector gives us a vector of the scalar length pointing in the unit vector direction. So we scalar multiply $\left(\frac{u}{|u|} \cdot v\right)$ by the unit vector $\frac{u}{|u|}$ (which points in the u direction) to get $\left(\frac{u}{|u|} \cdot v\right)\frac{u}{|u|}$.

Solution: $u \cdot v = 8$ $\quad |u|^2 = 21$ (from previous page)

so $\frac{u \cdot v}{|u|^2} = \frac{8}{21}$

so $\text{proj}_u v = \left(\frac{u \cdot v}{|u|^2}\right) u = \boxed{\frac{8}{21} \langle 4, -1, 2 \rangle = \left\langle \frac{32}{21}, -\frac{8}{21}, \frac{16}{21} \right\rangle}$

Either answer correct