Quiz 8 Solution for A05.

1、 $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2-y}$

Ans: If the limit exists, then $(x(t), y(t)) = (0, t)$ and $(x'(t), y'(t)) = (t, 0)$, $\lim\limits_{t\to 0} \dfrac{x^2(t)}{x^2(t)-y(t)} = \lim\limits_{t\to 0} \dfrac{(x'(t))^2}{(x'(t))^2-y'(t)}$

However: $\lim\limits_{t\to 0} \dfrac{x^2(t)}{x^2(t)-y(t)} = \lim\limits_{t\to 0} \dfrac{0}{0-t} = 0$

and $\lim\limits_{t\to 0} \dfrac{(x'(t))^2}{(x'(t))^2-y'(t)} = \lim\limits_{t\to 0} \dfrac{t^2}{t^2-0} = 1$

$0 \neq 1$

Therefore, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2-y}$ doesn't exist.

2、 $y^7x + z\ln x + x^3 - 1 = -1$ at $(1, -1, 3)$, find $\dfrac{\partial x}{\partial y}$.

$$\frac{\partial(y^7x + z\ln x + x^3 - 1)}{\partial y} = \frac{\partial(-1)}{\partial y} = 0$$

$$= \frac{\partial(y^7x)}{\partial y} + \frac{\partial(z\ln x)}{\partial y} + \frac{\partial x^3}{\partial y}$$

$$= 7y^6x + y^7\frac{\partial x}{\partial y} + z\frac{1}{x}\cdot\frac{\partial x}{\partial y} + 3x^2\cdot\frac{\partial x}{\partial y} = 0 \quad \text{—— ①}$$

plugin $x=1$, $y=-1$, $z=3$ in ①, we get

$$7(-1)^6\cdot 1 + (-1)^7\frac{\partial x}{\partial y} + 3\cdot\frac{1}{1}\cdot\frac{\partial x}{\partial y} + 3\cdot 1^2\cdot\frac{\partial x}{\partial y} = 0$$

Solve this equation, we have $\dfrac{\partial x}{\partial y} = -\dfrac{7}{5}$

12