Name: Solution

Math 21C Section B05
Thursday 4-5pm
6/5/2008

QUIZ #8

Problem 1 Let $f(x,y) = 4xe^y$.
(A) (4 points): Find the rate of change at $P(2,0)$ in the direction from P to $Q(1/2,2)$.
(B) (4 points): In what direction does f have the maximum rate of change? What is the value of the maximum rate?
(C) (2 points): What is the flat direction of $D_u f(2,0)$ or when $D_u f(2,0) = 0$.

(A) $\vec{PQ} = \langle \frac{1}{2}, 2\rangle - \langle 2, 0\rangle = \langle -\frac{3}{2}, 2\rangle$ $\quad |\vec{PQ}| = \sqrt{(\frac{3}{2})^2 + 2^2} = \sqrt{\frac{9}{4}+4} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

So the unit vector in the direction of $\vec{PQ}$ (going from P to Q)
is $u = \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{\langle -\frac{3}{2}, 2\rangle}{5/2} = \frac{2}{5}\langle -\frac{3}{2}, 2\rangle = \langle -\frac{3}{5}, \frac{4}{5}\rangle = -\frac{3}{5}i + \frac{4}{5}j$,

$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\rangle = \langle 4e^y, 4xe^y\rangle$, so $(\nabla f)_{(2,0)} = \langle 4e^0, 4\cdot 2e^0\rangle = \langle 4, 8\rangle$

Now the rate of change at $(2,0)$ in the u direction is
the directional derivative in the direction of u at $(2,0)$
which is $\left(\frac{df}{ds}\right)_{u,(2,0)} = (\nabla f)_{(2,0)} \cdot u = \langle 4, 8\rangle \cdot \langle -\frac{3}{5}, \frac{4}{5}\rangle$

$= \frac{-12}{5} + \frac{32}{5} = \frac{20}{5} = \boxed{4}$

So $\boxed{4 \text{ is the rate of change at } (2,0) \text{ in } u \text{ direction}}$

$\boxed{\text{B and C on back}}$

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(B) From part A) we found $\nabla f = \langle 4e^y, 4xe^y\rangle$ and $(\nabla f)_{(2,0)} = \langle 4,8\rangle$. The gradient is the direction of the maximal rate of change, and its length is this maximum rate of change,

So f has maximal rate of change in $\langle 4,8\rangle$ direction, and this maximum rate is $|\nabla f_{(2,0)}| = |\langle 4,8\rangle| = \sqrt{4^2+8^2} = \sqrt{16+64} = \sqrt{80}$

(or equal $\sqrt{80} = 4\sqrt{5}$)

(C) The flat direction is the direction perpendicular (or normal) to $\nabla f_{(2,0)} = \langle 4,8\rangle$. So $\langle -8,4\rangle$ will be normal to $\langle 4,8\rangle$

Since $\langle -8,4\rangle \cdot \langle 4,8\rangle = -32+32 = 0$.

so $\langle -8,4\rangle$ (or $\langle 8,-4\rangle$) is the flat direction

2