Quiz 5 (KEY)

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Spring 2008

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Let $\mathbf{u} = \langle 1, -3, 2 \rangle$ and $\mathbf{v} = \langle 6, 1, -1 \rangle$.

Problem 1. (5 points) Find the angle between the **u** and **v**. Do not simplify.

Answer. Recall

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} \qquad \Longleftrightarrow \qquad \theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right).$$

Compute

$$\begin{aligned} \mathbf{u}\cdot\mathbf{v} &= (1)(6) + (-3)(1) + (2)(-1) \\ &= 1 \end{aligned}$$

$$\begin{aligned} |\mathbf{u}| &= \sqrt{1^2 + (-3)^2 + 2^2} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} |\mathbf{v}| &= \sqrt{6^2 + 1^2 + (-1)^2} \\ &= \sqrt{38} \end{aligned}$$

So then

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{14\cdot 38}}\right)$$

□

Problem 2. (5 points) Find $\mathrm{proj}_{\mathbf{u}}\mathbf{v}$.

Answer. Recall the projection of **v** onto **u** is the vector

$$\mathrm{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|^2}\mathbf{u}.$$

By the computations solving Problem 1,

$$\mathrm{proj}_{\mathbf{u}}\mathbf{v} = \frac{1}{\left(\sqrt{14}\right)^2}\mathbf{u} = \tfrac{1}{14}\langle 1, -3, 2\rangle.$$

□