Last Name: ____________________, First Name: ____________________

Mat 21C-A02 (6:10 - 7:00pm) Quiz #3 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Find the Taylor polynomial of order 2 generated by $f(x) = 2\cos\frac{3x}{5}$ at $x = \frac{5\pi}{9}$.

Computing the first two derivatives of $f(x)$,

$$f'(x) = -\frac{6}{5}\sin\left(\frac{3x}{5}\right), \qquad f''(x) = -\frac{18}{25}\cos\left(\frac{3x}{5}\right).$$

Evaluating f and its first two derivatives at $x = 5\pi/9$, we have

$$f(5\pi/9) = 2\cos(\pi/3) = 1, \quad f'(5\pi/9) = -\frac{6}{5}\sin(\pi/3) = -\frac{3\sqrt{3}}{5}, \quad f''(5\pi/9) = -\frac{18}{25}\cos(\pi/3) = -\frac{9}{25}.$$

Plugging these values into the formula for second order Taylor polynomial ($a = 5\pi/9$),

$$P_2(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2/2! = 1 - \frac{3\sqrt{3}}{5}(x - 5\pi/9) - \frac{9}{25}(x - 5\pi/9)^2$$

2. (5 points) Find the series interval of convergence.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(4x+2)^n}{\sqrt[3]{3n+2}(-3)^n}$$

Solution We use the ratio test,

$$\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty}\frac{|\sqrt[3]{3n+2}(-3)^n|}{|\sqrt[3]{3(n+1)+2}(-3)^{n+1}|}|4x+2| = \frac{|4x+2|}{3}$$

We want this quantity to be less than one,

$$\frac{|4x+2|}{3} < 1 \Rightarrow |4x+2| < 3 \Rightarrow -3 < 4x+2 < 3 \Rightarrow \frac{-5}{4} < x < \frac{1}{4}.$$

We must check endpoints separately. First, $x = -5/4$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(4(-5/4)+2)^n}{\sqrt[3]{3n+2}(-3)^n} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(-3)^n}{\sqrt[3]{3n+2}(-3)^n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[3]{3n+2}}$$

which is a convergent alternating series. Next, check $x = 1/4$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(4(1/4)+2)^n}{\sqrt[3]{3n+2}(-3)^n} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^n}{\sqrt[3]{3n+2}(-3)^n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[3]{3n+2}(-1)^n} = -\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{3n+2}}$$

which diverges by comparison with a $p-$series. Therefore, the interval of convergence is $[-5/4, 1/4)$.