

Problem 2

Does the series

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

converge absolutely or converge conditionally or both?

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} &= \frac{\cos(\pi)}{1} + \frac{\cos(2\pi)}{2} + \frac{\cos(3\pi)}{3} + \dots \\ &= -1 + \frac{1}{2} + \frac{-1}{3} + \dots \end{aligned}$$

\Rightarrow alternating harmonic series

\Rightarrow Convergent

$$\sum_{n=1}^{\infty} \left| \frac{\cos(n\pi)}{n} \right| = 1 + \frac{1}{2} + \frac{1}{3} + \dots$$

\Rightarrow harmonic series

\Rightarrow divergent

So this series converges, but does not converge absolutely, so it converges conditionally.