

Name: Key

Student ID: _____

No Calculators.

1. (7 pts) The estimate $\sqrt{1+x} = 1 + \frac{x}{2}$ is used when x is small. Estimate the error when $|x| < 0.1$.

Hint: $\left(\frac{9}{10}\right)^{1/2} \approx 1.17$

The Taylor Polynomial of order 1 centered at $x=0$ is

$$P_1(x) = \sum_{k=0}^1 \frac{f^{(k)}(0) x^k}{k!} \quad \text{where} \quad f^{(1)}(x) = \frac{1}{2\sqrt{1+x}}$$

So $P_1(x) = 1 + \frac{1}{2}x$ therefore $R_1(x) = \frac{f^{(2)}(c) x^2}{2!}$

where $f^{(2)}(x) = -\frac{1}{4(1+x)^{3/2}}$ and c is between 0 & x

if $|x| < \frac{1}{10}$ then $|R_1(x)| = \frac{1}{4} \left| \frac{1}{(1-c)^{3/2}} \right| |x|^2 \leq \frac{1}{4} |x|^2 \frac{12}{10}$
 $= \frac{1}{30} |x|^2 < \frac{1}{3000}$ is the estimate of the error. largest when $c = -\frac{1}{10}$

2. (3 pts) Find the distance between points P_1 and P_2 . $P_1(3, 4, 5)$ and $P_2(2, 3, 4)$.

$$\sqrt{(3-2)^2 + (4-3)^2 + (5-4)^2} = \sqrt{3}$$