Quiz 1 Solutions

Problem 1 (5 points): Given $a_1 = 7$ and the recursion formula $a_{n+1} = a_n + \frac{1}{7^n}$ for the remaining terms of the sequence, determine if the sequence converges or diverges. If it converges, determine its limit. If it diverges, give reason why. Hint: Write out the first few terms without simplifying.

Note that $a_n = 7 + \sum_{i=0}^{n-1} \left(\frac{1}{7}\right)^{i+1} = 7 + \frac{1}{7} \sum_{i=0}^{n-1} \left(\frac{1}{7}\right)^i$. This is a geometric series with one term added on, which therefore converges to $7 + \frac{1}{7} \frac{1}{1-\frac{1}{7}} = 7 + \frac{1}{7}\frac{7}{6} = 7 + \frac{1}{6}$

$$7 + \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \frac{1}{7^5} = 7.1667$$
$$7 + \frac{1}{6} = 7.1667$$

Problem 2 (5 points): Determine if the series $\sum_{n=1}^{\infty} \left(1 - \frac{1}{7n}\right)^n$ converges or diverges. Give reasons for your answer.

The series diverges by the nth term test, because $\lim_{n\to\infty} \left(1 - \frac{1}{7n}\right)^n = \lim_{n\to\infty} \left(\left(1 + \frac{1}{-7n}\right)^{-7n}\right)^{-\frac{1}{7}} = e^{-\frac{1}{7}} \neq 0$. This limit also follows from part 5 of theorem 5, which states that $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$, with $x = -\frac{1}{7}$ in this case.