

Solutions

Math 21C Quiz 1

Section: 5:10-6:00 pm, TA: Arpy Mikaelian
Tuesday April 8, 2008

Problem 1

(5 points): Determine if the sequence $\frac{n\sqrt[n]{n} + \ln n}{n}$ converges or diverges. If it converges, find the limit.

$$\frac{n \cdot \sqrt[n]{n} + \ln n}{n} = \frac{n \cdot (n)^{1/n} + \ln n}{n}$$

$$= \frac{n \cdot 1 + \ln n}{n} \quad (\text{by Thm. 5})$$

$$= \frac{n}{n} + \frac{\ln n}{n}$$

$$= 1 + \frac{\ln n}{n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n\sqrt[n]{n} + \ln n}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{\ln n}{n} \right)$$

$$= 1 + 0, \quad (\text{since } \lim_{n \rightarrow \infty} \ln n = 1)$$

$$= 1$$

\Rightarrow Converges to 1