Quiz 9 Solution

Problem 1 (5 points): Find the derivative of the function $f(x,y,z) = \ln(1+xyz)$ at $P_0(2,3,1)$ in the direction $A = -i + j$

$\text{grad}\,(f) = f_x i + f_y j + f_z k = \frac{yz}{1+xyz} i + \frac{xz}{1+xyz} j + \frac{xy}{1+xyz} k$

Thus the directional derivative in that direction A is given by:

$\text{grad}\,(f) \cdot \frac{-i+j}{|-i+j|} = \text{grad}\,(f) \cdot \left(-\sqrt{2}i + \sqrt{2}j\right) = -\sqrt{2}\frac{yz}{1+xyz} + \sqrt{2}\frac{xz}{1+xyz}$

Problem 2 (5 points): Find the equation for the tangent plane on the given surface $y^2 - 2xy - z = 1$ at point $P_0(1,4,7)$.

Take $f(z,y,z) = y^2 - 2xy - z - 1$. Then:

$f_x = -2y$
$f_y = 2y - 2x$

$f_z = -1$

Hence the tangent plane is given by:
$f_x(1,4,7)(x-1) + f_y(1,4,7)(y-4) + f_z(1,4,7)(z-7) = 0$

$-8(x-1) + 6(y-4) - (z-7) = 0$