Last Name: _____, First Name: _____

Mat 21C-A02 (6:10 - 7:00pm) Quiz #8 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Determine if the limit below exists or not. Give reasons for your answer.

$$\lim_{(x,y)\to(0,0)} -\frac{x^4}{x^4+y^2}$$

Solution If we approach the origin from the positive $x-$axis, the limit is

$$\lim_{(x,0)\to(0^+,0)} -\frac{x^4}{x^4+0^2} = -1.$$

However, if we approach from the positive $y-$axis, we get

$$\lim_{(0,y)\to(0,0^+)} -\frac{0^4}{0^4+y^2} = 0.$$

Since these two limits do not agree, the limit does not exist.

2. (5 points) Find the value of $\frac{\partial x}{\partial z}$ at the point $(1,-1,-3)$ if the equation

$$x^2z + y\ln x - x^6 + 1 = -3$$

defines x as a function of the two independent variables y and z and the partial derivative exist.

Solution Differentiating the whole equation with respect to z,

$$2xz\left(\frac{\partial x}{\partial z}\right) + x^2\left(\frac{\partial z}{\partial z}\right) + \frac{y}{x}\left(\frac{\partial x}{\partial z}\right) - 6x^5\left(\frac{\partial x}{\partial z}\right) = 0.$$

Plugging in the point $(x,y,z) = (1,-1,-3)$,

$$2(1)(-3)\left(\frac{\partial x}{\partial z}\right) + 1^2\cdot 1 + \frac{-1}{1}\left(\frac{\partial x}{\partial z}\right) - 6(1^5)\left(\frac{\partial x}{\partial z}\right) = (-13)\left(\frac{\partial x}{\partial z}\right) + 1 = 0$$

Solving for $\left(\frac{\partial x}{\partial z}\right)$, we see the answer is

$$\left(\frac{\partial x}{\partial z}\right)\Big|_{(1,-1-3)} = \frac{1}{13}.$$