

Quiz 6 Solutions

Problem 1 (5 points): Find the distance from the point $P(-1, 4, 3)$ to the line $x = 10 + 4t$, $y = -3$, $z = 4t$.

The point $S = \langle 10, -3, 0 \rangle$ is on the line, and the direction V of the line is $\langle 4, 0, 4 \rangle$.
 $\overrightarrow{PS} = \langle 10 - (-1), -3 - 4, 0 - 3 \rangle = \langle 11, -7, -3 \rangle$

$$\overrightarrow{PS} \times V = \begin{vmatrix} i & j & k \\ 11 & -7 & -3 \\ 4 & 0 & 4 \end{vmatrix} = i(-28 - 0) - j(44 - (-12)) + k(0 - (-28)) = -28i - 56j + 28k$$

$$\frac{|\overrightarrow{PS} \times V|}{|V|} = \frac{\sqrt{28^2 + 56^2 + 28^2}}{\sqrt{16 + 16}} = \frac{\sqrt{28^2 + 2^2 28^2 + 28^2}}{4\sqrt{2}} = \frac{28\sqrt{6}}{4\sqrt{2}} = 7\sqrt{3}$$

Problem 2 (5 points): Find an equation for the plane through $A(4, -6, 2)$, $B(7, -3, 5)$, and $C(1, -9, -2)$.

$$\begin{aligned} \overrightarrow{AB} &= \langle 7 - 4, -3 + 6, 5 - 2 \rangle = \langle 3, 3, 3 \rangle \\ \overrightarrow{AC} &= \langle 1 - 4, -9 + 6, -2 - 2 \rangle = \langle -3, -3, -4 \rangle \end{aligned}$$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 3 & 3 & 3 \\ -3 & -3 & -4 \end{vmatrix} = i(-12 + 9) - j(-12 + 9) + k(0) = -3i + 3j$$

Therefore, the equation for the plane is given by:

$$\begin{aligned} \langle x - 4, y + 6, z - 2 \rangle \cdot \vec{n} &= -3(x - 4) + 3(y + 6) = 0 \\ -3x + 3y &= -30 \end{aligned}$$

$$-x + y = -10$$