

$$(2) \sum \frac{n 2^n (n+1)!}{3^n n!}$$

Solution: we will use Ratio test. Let $a_n = \frac{n 2^n (n+1)!}{3^n n!}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1) 2^{n+1} (n+2)!}{3^{n+1} (n+1)!} \cdot \frac{3^n n!}{n 2^n (n+1)!} = \frac{(n+1) 2 (n+2)}{3 (n+1) n} = \frac{2}{3} \frac{n+2}{n}$$

$$\text{Then } \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2}{3} \frac{n+2}{n} = \frac{2}{3} \left(\frac{1+0}{1} \right) = \frac{2}{3} < 1$$

Therefore $\sum \frac{n 2^n (n+1)!}{3^n n!}$ (converge)