

Quiz 2 (KEY)

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Determine whether each series converges or diverges. Give reasons for your answer.

Problem 1. (5 points) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^n$

Answer. Apply the Root test to $a_n = \left(\frac{n}{3n+1} \right)^n$.

$$\sqrt[n]{|a_n|} = \frac{n}{3n+1} \rightarrow \frac{1}{3} < 1.$$

Therefore the series converges.

□

Problem 2. (5 points) $\sum_{n=1}^{\infty} n!e^{-n}$

Answer. Apply the Ratio test to $a_n = n!e^{-n}$.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!e^{-(n+1)}}{n!e^{-n}} = \frac{(n+1)n!}{n!} \cdot \frac{e^{-n}e^{-1}}{e^{-n}} = e^{-1}(n+1) \rightarrow \infty$$

Since $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \infty > 1$, the series diverges.

□