

Last name: _____

First name: _____

1 (5 points): Let

$$w = \sin(xy) + \ln z, \quad x = vu^2, \quad y = uv, \quad z = \cos u.$$

Find $\frac{\partial w}{\partial u}$ using Chain Rule.

$$\begin{aligned} \frac{\partial w}{\partial u} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial u} \\ &= \cos(xy)y \cdot 2uv + \cos(xy)x \cdot v + \frac{1}{z} \cdot -\sin u \\ &= 2u^2v^2 \cos(u^3v^2) + u^2v^2 \cos(u^3v^2) - \tan u \end{aligned}$$

2 (5 points): Let $f(x, y) = 6xe^y$. Find the rate of change at $P(2, 0)$ in the direction from P to $Q(1/2, 2)$.

The rate of change of f in the direction $\overrightarrow{PQ} = \langle -\frac{3}{2}, 2 \rangle$ is given by $D_{\mathbf{u}}f(P)$ where $\mathbf{u} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \langle -\frac{3}{5}, \frac{4}{5} \rangle$.

Therefore,

$$D_{\mathbf{u}}f|_{(2,0)} = \nabla f|_{(2,0)} \cdot \mathbf{u} = \langle 6e^y, 6xe^y \rangle|_{(2,0)} \cdot \mathbf{u} = \langle 6, 12 \rangle \cdot \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle = 6.$$