MAT 21C (Section B04)
Quiz 2

SID :
Name : Solution

Determine whether each series converges or diverges. Give reasons for your answer.

1. (5 points):

$$\sum_{n=0}^{\infty} \frac{\sin^2 n}{5^n}$$

$\frac{\sin^2 n}{5^n} \leq \frac{1}{5^n}$

We know that $\sum_{n=0}^{\infty} \frac{1}{5^n}$ converges (Geometric series with $a=1, r=\frac{1}{5}$).

By the Comparison test, $\sum_{n=0}^{\infty} \frac{\sin^2 n}{5^n}$ converges.

2. (5 points):

$$\sum_{n=1}^{\infty} \frac{6(\ln n)^n}{n^n}$$

Use the Root test.

$$\sqrt[n]{\frac{6(\ln n)^n}{n^n}} = \left(\frac{6(\ln n)^n}{n^n}\right)^{1/n} = \frac{6^{1/n} \cdot \ln n}{n}$$

Since $\lim_{n\to\infty} 6^{\frac{1}{n}} = 6^0 = 1$ and $\lim_{n\to\infty} \frac{\ln n}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$,
(↑ L'Hospital)

$$\lim_{n\to\infty} \sqrt[n]{\frac{6(\ln n)^n}{n^n}} = \lim_{n\to\infty} 6^{\frac{1}{n}} \cdot \lim_{n\to\infty} \frac{\ln n}{n} = 1 \cdot 0 = 0 < 1$$

Hence, the series converges.