Solutions (Quiz 9, section B03)

Problem 1 (5 points): Find the derivative of the function

$$f(x, y, z) = x^3yz$$

at $P_0(2, 1, -1)$ in the direction $\mathbf{A} = 2\mathbf{i} - \mathbf{k}$.

<u>solution:</u> First fine the unit vector with the same direction as $\mathbf{A}$. $\mathbf{u} = \mathbf{A}/|\mathbf{A}| = (\mathbf{2i} - \mathbf{k})/\sqrt{\mathbf{5}}$
The directional derivative is given by

$$\nabla f|_{P_0} \cdot \mathbf{u}.$$

First, find ∇f.

$$\nabla f = \left\langle 3x^2yz, x^3z, x^3y \right\rangle$$

$$\nabla f|_{P_0} = \langle -12, -8, 8 \rangle$$

Hence,

$$\nabla f|_{P_0} \cdot \mathbf{A} = \langle \mathbf{-12}, \mathbf{-8}, \mathbf{8} \rangle \cdot \frac{\mathbf{1}}{\sqrt{\mathbf{5}}} \langle \mathbf{2}, \mathbf{0}, \mathbf{-1} \rangle = \mathbf{-32}/\sqrt{\mathbf{5}}.$$

Problem 2 (5 points): Find the equation for the tangent plane on the given surface

$$x^2 + 2xy + z^2 = 56$$

at point $P_0(5, 3, 1)$.

<u>solution:</u> Let $f(x, y, z) = x^2 + 2xy + z^2$ and find ∇f.

$$\nabla f = \langle 2x + 2y, 2x, 2z \rangle$$

$$\nabla f|_{P_0} = \langle 16, 10, 2 \rangle$$

Therefore, the equation for the tangent plane at P_0 is

$$16(x - 5) + 10(y - 3) + 2(z - 1) = 0.$$

Or, $16x + 10y + 2z = 112$.