Quiz 8

Name:

Problem 1 (5 points): Find the gradient of the function

$$f(x, y) = \arctan \frac{xy}{4} + \ln (x^4 + y^2)$$

at the given point $(2, 2)$.

$$f_x = \frac{y/4}{1 + (\frac{xy}{4})^2} + \frac{4x^3}{x^4 + y^2}, \quad f_x|_{(2,2)} = \frac{2}{4(1+1)^2} + \frac{4(8)}{16+4} = \frac{1}{16} + \frac{8}{5} = \frac{133}{80}$$

$$f_y = \frac{x/4}{1 + (\frac{xy}{4})^2} + \frac{2y}{x^4 + y^2}, \quad f_y|_{(2,2)} = \frac{1}{16} + \frac{1}{5} = \frac{21}{80}$$

$$\nabla f|_{(2,2)} = \left(\frac{133}{80}, \frac{21}{80} \right)$$

Problem 2 (5 points): Find $\partial w/\partial v$ when $u = -1$, $v = 2$ if $w = xy + \ln z$, $x = v^2/u$, $y = u + v$, $z = \cos u$.

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

$$= \frac{2yv}{u} + x + \frac{1}{z}(0)$$

$$= \frac{2v(u+v)}{u} + \frac{v^2}{u}\Bigg|_{(-1,2)} = \frac{2(2)(-1+2)}{-1} + \frac{2^2}{-1}$$

$$= -4 - 4 = -8$$