Last Name: ____________________, First Name: ____________________

Mat 21C-A02 (6:10 - 7:00pm) Quiz #2 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic device for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n+5^n}$$

Solution First, rewrite the series as

$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n+5^n} = \sum_{n=1}^{\infty} a_n = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n}{n+5^n} = 2\sum_{n=1}^{\infty}(-1)^{n+1}u_n, \quad u_n = \frac{2^n}{n+5^n}.$$

Next, we use the alternating series test (see textbook),

$$\begin{aligned} u_n &= \frac{2^n}{n+5^n} > 0, \quad \forall n. \\ u_{n+1} &= \frac{2^{n+1}}{n+1+5^{n+1}} < \frac{2^n}{n+5^n} = u_n, \quad \forall n. \\ \lim_{n\to\infty} u_n &= 0. \end{aligned}$$

Therefore, the series converges. To see whether it converges absolutely or conditionally, we compare this series to a more familiar series.

$$u_n = \frac{2^n}{n+5^n} < \frac{2^n}{5^n} = b_n.$$

But $\sum b_n$ is a convergent geometric series. Hence, $\sum |a_n| = \sum u_n$ converges by the comparison test. Therefore, the series $\sum a_n$ is absolutely convergent.

2. (5 points) Use Ratio or Root test to determine whether the series below converge or diverge. Justify your answer.

$$\sum_{n=1}^{\infty} \left(\frac{-1+3n^2}{2n^2}\right)^n$$

Solution We use the root test (see textbook) with $a_n = \left(\frac{-1+3n^2}{2n^2}\right)^n$ and compute

$$\rho = \lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{-1+3n^2}{2n^2} = \lim_{n\to\infty} \frac{-1/2n^2 + 3/2}{1} = 3/2.$$

Since $\rho > 1$, the ratio test implies the series diverges.