Last Name: __________________________________, First Name: __________________________________

Mat 21C-A02 (6:10 - 7:00pm) Quiz #1 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic device for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Determine if the series converges or diverges. Justify your answer.

$\sum_{n=-1}^{\infty} \frac{(-3)^{n+2}75+\sqrt{5}^{n+3}105}{4^{n+2}15}$

Solution The series can be split up and simplified using the geometric series

$$\begin{aligned}
\sum_{n=-1}^{\infty} \frac{(-3)^{n+2}75+\sqrt{5}^{n+3}105}{4^{n+2}15} &= \sum_{n=0}^{\infty} \frac{5\cdot(-3)^{n+1}+7\cdot\sqrt{5}^{n+2}}{4^{n+1}} \\
&= \frac{-15}{4}\sum_{n=0}^{\infty}\frac{(-3)^n}{4^n} + \frac{35}{4}\sum_{n=0}^{\infty}\frac{\sqrt{5}^n}{4^n} \\
&= \frac{-15}{4}\cdot\frac{1}{1+3/4} + \frac{35}{4}\cdot\frac{1}{1-\sqrt{5}/4} \\
&= \frac{35}{4-\sqrt{5}} - \frac{15}{7}.(\approx 17.6991773)
\end{aligned}$$

Therefore, the series converges.

2. (5 points) Determine if the series converges or diverges. If it converges, determine the sum for the series in its simplest form. Justify your answer.

$\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

Solution Notice each term, say $a_n = (n\ln(n))^{-1}$, in the series is positive. Let $f(x) = x\ln(x)$. Then we have $f(n) = a_n$ and $f(x)$ is a continuous, decreasing function on $[2,\infty)$. The hypotheses for using the integral test are satisfied, so the series and the integral of $f(x)$ on the interval $[2,\infty)$ converge and diverge together. Using a u-substitution, with $u = \ln(x)$, $du = dx/x$, the integral is

$$\int_2^{\infty}\frac{dx}{x\ln(x)} = \int_{\ln(2)}^{\infty}\frac{du}{u} = \lim_{b\to\infty}\int_{\ln(2)}^{b}\frac{du}{u} = \lim_{b\to\infty}\ln(b)-\ln(\ln(2))$$

The last limit diverges to positive infinity, therefore the series diverges.