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*Last name:*____________________ *First name:*____________________

1 (5 points): Determine if the limit below exists or not. Give reasons for your answer.

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y}$$

The limit does not exist. Consider the following curves $x = 0$ and $y = x^2$. Along the path $x = 0$ we have

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y} = \lim_{(0,y)\to(0,0)} \frac{0^2+y}{y} = \lim_{y\to 0} 1 = 1.$$

Along the path $y = x^2$ we have

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y} = \lim_{(x,x^2)\to(0,0)} \frac{x^2+x^2}{x^2} = \lim_{x\to 0} 2 = 2.$$

Since the limit along two different paths are not the same, thus the limit does not exist.

2 (5 points): Find the value of $\frac{\partial x}{\partial z}$ at the point $(1, 5, -1)$ if the equation

$$z^5 x + y\ln x + x^3 + 2 = 2$$

defines x as a function of the two independent variables z and y and the partial derivative exist.

Doing implicit differentiation on the above equation

$$\begin{aligned}
&\frac{\partial}{\partial z}\left(z^5 x + y\ln x + x^3 + 2 = 2\right)\\
&\frac{\partial}{\partial z} z^5 x + y\frac{\partial}{\partial z}\ln x + \frac{\partial}{\partial z}x^3 + \frac{\partial}{\partial z}2 = \frac{\partial}{\partial z}2\\
&5z^4 x + \frac{\partial x}{\partial z}z^5 + \frac{y}{x}\frac{\partial x}{\partial z} + 3x^2\frac{\partial x}{\partial z} = 0
\end{aligned}$$

Substituting in $(x, y, z) = (1, 5, -1)$ we have

$$5 - \frac{\partial x}{\partial z} + 5\frac{\partial x}{\partial z} + 3\frac{\partial x}{\partial z} = 0$$

which implies

$$\frac{\partial x}{\partial z} = -\frac{5}{7}.$$