Name: Solutions

Math 21C Section B05
Thursday 4-5pm
5/22/2008

QUIZ #6

Problem 1 (5 points): Determine if

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 - y}$$

has a limit or not. Give reasons for your answers.

It does not have a limit since we can make it approach different values by using different curves in the xy-plane.

For instance if we look at the limit behavior along the curve $y = \frac{x^2}{2}$ we have

that $\lim_{\substack{(x,y)\to(0,0) \\ y=\frac{x^2}{2}}} \frac{x^2}{x^2 - y} = \lim_{\substack{(x,y)\to(0,0) \\ y=\frac{x^2}{2}}} \frac{x^2}{\frac{x^2}{2}} = \lim_{\substack{(x,y)\to(0,0) \\ y=\frac{x^2}{2}}} 2 = 2$

but along the curve $y = -x^2$ we have

$\lim_{\substack{(x,y)\to(0,0) \\ y=-x^2}} \frac{x^2}{x^2 - y} = \lim_{\substack{(x,y)\to(0,0) \\ y=-x^2}} \frac{x^2}{2x^2} = \lim_{\substack{(x,y)\to(0,0) \\ y=-x^2}} \frac{1}{2} = \frac{1}{2}$.

So the limit does not exist since the limits along different curves which approach (0,0) do not agree.

Problem 2 (5 points): Find an equation for the level curve of the function

$$f(x,y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n$$

that passes through the point $(1,6)$.

Using the formula for the sum of an infinite geometric series we have

$$f(x,y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n = \frac{1}{1-\frac{x}{y}} = \frac{1}{\frac{y-x}{y}} = \frac{y}{y-x}$$

(Note that this holds if $|\frac{x}{y}|<1$ but that is ok since the sum is divergent otherwise, so it will not be in the domain)

So at $(1,6)$, $f(1,6) = \frac{6}{6-1} = \frac{6}{5}$

So we need to find the level surface

$f(x,y) = \frac{6}{5}$ which is $\frac{y}{y-x} = \frac{6}{5}$

So $5y = 6(y-x)$

$5y = 6y - 6x$

So $y = 6x$ (must exclude $(0,0)$)

To be technical it should be the line $y=6x$ with the origin removed.