Quiz 8 Solutions

Problem 1 (5 points): Find the gradient of the function $f(x,y) = \arctan \frac{xy}{7} + \ln\left(x^4 + y^2\right)$ at the given point $(1,7)$.

$$f_x = \frac{1}{1+\left(\frac{xy}{7}\right)^2} \frac{y}{7} + \frac{4x^3}{x^4+y^2} \rightarrow \frac{1}{1+\left(\frac{7}{7}\right)^2} \frac{7}{7} + \frac{4}{1+49} = \frac{7}{14} + \frac{4}{50} = \frac{29}{50}$$

$$f_y = \frac{1}{1+\left(\frac{xy}{7}\right)^2} \frac{x}{7} + \frac{2y}{x^4+y^2} \rightarrow \frac{1}{1+\left(\frac{7}{7}\right)^2} \frac{1}{7} + \frac{14}{1+49} = \frac{1}{14} + \frac{14}{50} = \frac{123}{350}$$

Therefore, $\operatorname{grad}(f)(1,7) = \frac{29}{50} i + \frac{123}{350} j$.

Problem 2 (5 points): Find $\frac{\partial w}{\partial v}$ when $u = 2$, $v = 2$ if $w = xy + \ln z$, $x = \frac{v^5}{u}$, $y = -u - v$, $z = -\cos u$.

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} = y\left(\frac{5v^4}{u}\right) + x\,(-1) + \frac{1}{z}\,(0) = (-u-v)\left(\frac{5v^4}{u}\right) - \frac{v^5}{u}$$

Therefore, $\frac{\partial w}{\partial v}(2,2) = (-2-2)\left(\frac{5\left(2^4\right)}{2}\right) - \frac{2^5}{2} = -4\,(40) - 16 = -176$