Last Name: _____, First Name: _____

Mat 21C-A03 (5:10 - 6:00pm) Quiz #5 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Find the angle between the two vectors $\langle -1, 1, -1\rangle$ and $\langle 2, 2, -2\rangle$.

Solution Let $\vec{u}$ be the first vector given and $\vec{v}$ the second. First, we'll need to compute the dot product $\vec{u}\cdot\vec{v} = (-1)(2)+(1)(2)+(-1)(-2) = 2$. We also need the magnitude of each vector, $|\vec{u}| = \sqrt{(-1)^2+1^2+(-1)^2} = \sqrt{3}$, and $|\vec{v}| = \sqrt{2^2+2^2+(-2)^2} = \sqrt{12}$. Using the formula below will yield the angle

$$|\vec{u}||\vec{v}|\cos\theta = \vec{u}\cdot\vec{v} \quad \Rightarrow \quad \theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \arccos\left(\frac{2}{\sqrt{3\cdot 12}}\right) = \arccos\frac{1}{3} \quad (\approx 70.5287794^\circ.)$$

2. (5 points) Find the equation of the sphere if one of its diameters has endpoints $(-1, 1, -1)$ and $(2, 2, -2)$.

Solution A sphere is uniquely determined by its radius and center. To find the former, we compute half the distance between these two points (diameter).

$$r = \frac{d}{2} = \frac{1}{2}\sqrt{(2-(-1))^2+(2-1)^2+(-2-(-1))^2} = \frac{\sqrt{9+1+1}}{2} = \frac{\sqrt{11}}{2}.$$

To find the center, we need to find the coordinate of the midpoint of the given endpoints.

$$\text{center} = \frac{(2+(-1), 2+1, -2+(-1))}{2} = \frac{(1,3,-3)}{2} = \left(\frac{1}{2}, \frac{3}{2}, -\frac{3}{2}\right).$$

Plug these values into the general formula for a sphere

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2+\left(z+\frac{3}{2}\right)^2 = \frac{11}{4}.$$