Last Name: ____________________, First Name: ____________________

Mat 21C-A03 (5:10 - 6:00pm) Quiz #7 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Find the point in which the line

$$\begin{aligned} x &= 1 - t \\ y &= 3t, \\ z &= 1 + t, \end{aligned}$$

meets the plane

$$2x - y + 3z = 6.$$

Solution Following examples 8 and 9 in section 12.5 on page 869, plug each of the components in the line into the plane

$$2(1-t) - (3t) + 3(1+t) = -2t + 5 = 6 \quad \Rightarrow \quad t = -\frac{1}{2}.$$

Substituting this t into the parametrized line, the point is $\frac{1}{2}(3, -3, 1)$.

2. (5 points) Find a parametrization for the line in which the planes

$$x - 2y + 4z = 2 \quad \text{and} \quad x + y - 2z = 5$$

intersect.

Solution Following example 10 on the same page, take the cross product of the two normal vectors $\vec{n}_1 = \langle 1, -2, 4 \rangle$ and $\vec{n}_2 = \langle 1, 1, -2 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 1 & 1 & -2 \end{vmatrix} = 6\hat{j} + 3\hat{k} = 3\langle 0, 2, 1 \rangle.$$

Next, we need to find a point on both planes. Since there are three variables and two unknowns, we may arbitrarily set any variable to be any constant. The simplest computations happen when we set $z = 0$. From the first plane, this implies $x = 2 + 2y$. Plugging this into the equation from the second plane, we see that $(2 + 2y) + y = 5$ or $y = 1$. So then $x = 4$ and a point on both planes is $(3, 2, 0)$. Plugging this into the equation of a parametrized line,

$$\vec{r}(t) = \langle 4, 1, 0 \rangle + t\langle 0, 2, 1 \rangle, \quad -\infty < t < \infty$$

describes where the two planes intersects. (Notice I have dropped the constant 3 in the vector because $3t$ also captures the entire real line as t runs from negative infinity to positive infinity.)