TA: Ernest Woei *February 10, 2011*

*Last name:*____________________ *First name:*____________________

1 (5 points): Find the angle between the two vectors $\langle 1, -1, 1\rangle$ and $\langle 3, 1, -2\rangle$.

Let $\mathbf{a} = \langle 1, -1, 1\rangle$ and $\mathbf{b} = \langle 3, 1, -2\rangle$. Then the angle, θ, between $\mathbf{a}$ and $\mathbf{b}$ is

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right),$$

since $\mathbf{a}\cdot\mathbf{b} = 1\cdot 3 + -1\cdot 1 + 1\cdot -2 = 0$, thus

$$\theta = \cos^{-1}(0) = \frac{\pi}{2}.$$

2 (5 points): Find the equation of the sphere if one of its diameters has endpoints $(1, -1, 1)$ and $(3, 1, -2)$.

Let $P1 = (1, -1, 1)$ and $P2 = (3, 1, -2)$. Since one of its diameter has endpoints $P1$ and $P2$, then the center, C, of the sphere lies on the midpoint of the line segment between $P1$ and $P2$. Thus

$$C = \left(\frac{1+3}{2}, \frac{-1+1}{2}, \frac{1-2}{2}\right) = \left(2, 0, -\frac{1}{2}\right).$$

To determine the radius of the sphere we need calculate the length of the diameter and divide that by 2. The length of the diameter,

$$d = \sqrt{(3-1)^2 + (1--1)^2 + (-2-1)^2} = \sqrt{17},$$

therefore the radius of the sphere, $r = \frac{\sqrt{17}}{2}$. Finally, the equation of the sphere is

$$(x-2)^2 + y^2 + \left(z + \frac{1}{2}\right) = \frac{17}{4}.$$