

Name: Key

Student ID: _____

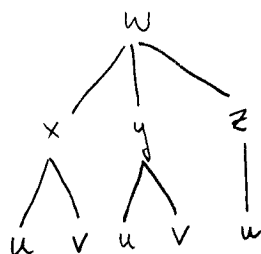
Show all work and justifications to receive full credit.

No Calculators.

1. (5 pts)

Let

$$w = \sin(xy) + \ln(z), \quad x = vu^2, \quad y = uv, \quad z = \cos(u).$$

Find $\frac{\partial w}{\partial u}$ using Chain Rule.

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial u}$$

$$= y \cos(xy) \cdot 2uv + x \cos(xy) \cdot v + \frac{1}{z} \cdot (-\sin u)$$

$$= \cos(xy) (2uvy + xv) - \frac{\sin u}{z}$$

$$= \cos(u^3 v^2) (2u^2 v^2 + u^2 v^2) - \frac{\sin u}{\cos u}$$

$$= 3 \cos(u^3 v^2) u^2 v^2 - \tan u //$$

2. (5 pts)

Find the derivative of the function $f(x, y) = 2x^3 + y^2$ at the point $(-1, 1)$ in the direction $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j}$.

$$D_{\mathbf{u}} f(-1, 1)$$

$$\text{where } \mathbf{u} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\langle 3, -4 \rangle}{\sqrt{3^2 + (-4)^2}} = \frac{\langle 3, -4 \rangle}{5}$$

$$\begin{aligned} \nabla f|_{(-1, 1)} \cdot \mathbf{u} &= \langle 6x^2, 2y \rangle|_{(-1, 1)} \cdot \mathbf{u} = \frac{\langle 6, 2 \rangle \cdot \langle 3, -4 \rangle}{5} \\ &= \frac{10}{5} = 2 // \end{aligned}$$