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*Last name:*____________________ *First name:*____________________

1 (5 points): Find a parametrization for the line in which the planes

$$x + y + z = 1 \quad \text{and} \quad x + y = 2$$

intersect.

In order to obtain a parametrization for the line in which the planes intersect we need to first find a normal vector to each plane. A normal vector to the plane $x + y + z = 1$ is

$$\mathrm{n}_1 = \langle 1, 1, 1 \rangle$$

and a normal vector to the plane $x + y = 2$ is

$$\mathrm{n}_2 = \langle 1, 1, 0 \rangle.$$

Taking the vector product of these normal vectors we obtain a vector that is parallel to the direction of the line, i.e.,

$$\mathrm{v} = \mathrm{n}_1 \times \mathrm{n}_2 = \langle -1, 1, 0 \rangle.$$

Finally finding a point on the intersection of the two planes by setting $x = 0$, since we have 2 equations and 3 unknowns, so $y = 2$ and $2 + z = 1$, therefore $z = -1$. Thus $P = (0, 2, -1)$ lies in both planes. The vector line equation is therefore

$$\mathrm{r}(t) = \overrightarrow{OP} + t \cdot \mathrm{v} = \langle 0, 2, -1 \rangle + t\langle -1, 1, 0 \rangle.$$

Hence a parametrization for the line is

$$\begin{aligned} x &= -t, \\ y &= 2 + t, \\ z &= -1. \end{aligned}$$

2 (5 points): Find the function $f(x, y) = \sqrt{9 - x^2 - y^2}$ domain and range and describe its level curve.

The domain of f is all x, y such that $9 - x^2 - y^2 \geq 0 \Leftrightarrow 9 \geq x^2 + y^2$ or all points on and inside the circle of radius 3 centered at $(0, 0)$. Using the domain of the f, we find that the range is $[0, 3]$. Let k be in the range of f, then set $k = \sqrt{9 - x^2 - y^2} \Leftrightarrow k^2 = 9 - x^2 - y^2 \Leftrightarrow x^2 + y^2 = 9 - k^2$ which are circles of radius $\sqrt{9 - k^2}$ centered at $(0, 0)$, except when $k = 3$ which we have a point $(0, 0)$.