Solutions

Math 21C Quiz 3

Section: 5:10-6:00 pm, TA: Arpy Mikaelian
Tuesday April 22, 2008

Problem 1

Find the interval of convergence of the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(5 - \frac{1}{5}x)^n}{n}$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty} \left| \frac{\cancel{(5-\frac{1}{5}x)^{n+1}}}{n+1} \frac{n}{\cancel{(5-\frac{1}{5}x)^n}} \right|$$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{(5-\frac{1}{5}x)n}{n+1} \right| < 1 \Rightarrow |5-\tfrac{1}{5}x| \lim_{n\to\infty} \left(\frac{n+1}{n}\right) < 1$$

$$\Rightarrow |5-\tfrac{1}{5}x| < 1$$

$$\Rightarrow -1 < 5 - \tfrac{1}{5}x < 1$$

$$\Rightarrow -6 < -\tfrac{1}{5}x < -4$$

$$\Rightarrow 6 > \tfrac{1}{5}x > 4$$

$$\Rightarrow 30 > x > 20$$

$$\Rightarrow 20 < x < 30 \, ;$$

Thus: Final Answer

$20 < x \leq 30$

when $x = 20$ we have $\sum_{n=1}^{\infty} \frac{(5-\frac{1}{5}\cdot 20)^n}{n} = \sum_{n=1}^{\infty} \frac{(5-4)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$,

(divergent; harmonic series)

when $x = 30$ we have $\sum_{n=1}^{\infty} \frac{(5-\frac{1}{5}\cdot 30)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, (Convergent: alternating harmonic series)

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