MAT 21C (Section B03)
Quiz 1

SID :
Name : Solution (6pm)

1. (5 points): Given $a_1 = 6$ and the recursion formula

$$a_{n+1} = a_n + \frac{1}{6^n}$$

for the remaining terms of the sequence. Determine if the sequence converges or diverges. If it converges, determine its limit. If it diverges, give reason why.
Hint: Write out the first few terms without simplifying.

$a_1 = 6$

$a_2 = 6 + \frac{1}{6}$

$a_3 = 6 + \frac{1}{6} + \frac{1}{6^2}$

$\vdots$

$a_{n+1} = 6 + \frac{1}{6} + \cdots + \frac{1}{6^n}$

Since $\frac{1}{6} + \cdots + \frac{1}{6^n} = \frac{1}{6} \cdot \frac{1-(\frac{1}{6})^n}{1-\frac{1}{6}} = \frac{1}{5}\left(1-(\frac{1}{6})^n\right)$,

$a_{n+1} = 6 + \frac{1}{5}\left(1-(\frac{1}{6})^n\right)$.

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\Big[6 + \frac{1}{5}\big(1-\underbrace{(\tfrac{1}{6})^n}_{0}\big)\Big]$$

$= 6 + \frac{1}{5}$

The sequence converges to $6 + \frac{1}{5}$ $\left(= \frac{31}{5}\right)$

2. (5 points): Determine if the series

$$\sum_{n=1}^{\infty}\left(1 - \frac{1}{6n}\right)^n$$

converges or diverges. Give reasons for your answer.

Note : $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$

$$\lim_{n\to\infty}\left(1-\frac{1}{6n}\right)^n = \lim_{n\to\infty}\left(1+\frac{(-\frac{1}{6})}{n}\right)^n = e^{-\frac{1}{6}} \neq 0$$

Hence, the series diverges.