Problem 2 (5 points): Does the series

$$\sum_{n==2}^{\infty} \frac{n}{(\ln n)^n}$$

converge or diverge? Give reasons for your answer.

let $a_n = \frac{n}{(\ln n)^n}$.

Lets apply the ratio test which says that for $\sum_{n=2}^{\infty} a_n$, we have converges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$,

diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$,

and inconclusive results if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$.

So $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{(\ln n)^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{\ln n}$

$= \left(\lim_{n\to\infty} \frac{1}{\ln n}\right) \cdot \left(\lim_{n\to\infty} \sqrt[n]{n}\right) = 0 \cdot 1 = 0$.

So by ratio test $\boxed{\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \frac{n}{(\ln n)^n} \text{ converges}}$

Name: Solutions

Math 21C Section B05
Thursday 4-5pm
4/17/2008

QUIZ #2

Problem 1 (5 points): Does the series

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$$

converge or diverge? Give reasons for your answer.

Let $a_n = \frac{1}{n\sqrt[n]{n}} = \frac{1}{n^{1+1/n}}$

and let $b_n = \frac{1}{n}$.

We know the series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

Now $\frac{a_n}{b_n} = \frac{1/n^{1+1/n}}{1/n} = \frac{n}{n^{1+1/n}} = \frac{1}{n^{1/n}} = \frac{1}{\sqrt[n]{n}}$

and we have that $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} \sqrt[n]{n}} = \frac{1}{1} = 1$

So by limit comparison test, since $\sum_{n=1}^{\infty} b_n$ diverges,

we have that $\boxed{\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}} \text{ diverges}}$