

Quiz 3 Solution

Problem 1.

$$a_n = \frac{(7 - \frac{1}{3}x)^n}{n}, \quad \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(7 - \frac{1}{3}x)^{n+1}/(n+1)}{(7 - \frac{1}{3}x)^n/n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(7 - \frac{1}{3}x) n}{n+1} \right| = |7 - \frac{1}{3}x| < 1 \implies 18 < x < 24$$

When $x=18$, $\sum_{n=1}^{+\infty} \frac{1}{n}$ NOT converge

When $x=24$, $\sum_{n=1}^{+\infty} (-1)^n \frac{1}{n}$ Converge

Therefore. interval of convergence : $18 < x \leq 24$

Problem 2

$$a_n = \frac{\sin(\frac{2n+1}{2}\pi)}{n}, \quad \text{So } a_1 = \frac{1}{1}, a_2 =$$

$$\text{So } a_1 = \frac{1}{1} \quad a_2 = \frac{-1}{2} \quad a_3 = \frac{1}{3} \quad a_4 = \frac{-1}{4} \quad a_5 = \frac{1}{5} \dots\dots$$

$$\text{i.e. } a_n = (-1)^{n+1} \frac{1}{n}$$

$$\sum_{n=1}^{+\infty} |a_n| = \sum_{n=1}^{+\infty} \frac{1}{n} \quad \text{NOT converge}$$

$$\sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{1}{n} \quad \text{Converge}$$

Therefore. conditionally converge, NOT absolutely converge