Math 21C - Section B01 - Quiz 1 **SOLUTION**
E. Kim

Problem 1: Determine if the sequence $\frac{2008^n+n!\sqrt[n]{n}}{n!}$ converges or diverges. If it converges, find the limit.

Solution: Consider first the sequence

$$a_n = \frac{2008^n}{n!},$$

which converges to 0, by the comment titled "Factorial Notation" in the margin on page 738 in the textbook.

Consider next the sequence

$$b_n = \frac{n!\sqrt[n]{n}}{n!} = \sqrt[n]{n}.$$

The sequence b_n converges to 1 by a fact in the book[1].

Let $A = 0$ and $B = 1$. Since $a_n \to A$ and $b_n \to B$, the sequence $a_n + b_n$, which is the original sequence in the question, converges to $A + B = 1$ by the Sum Rule[2].

Problem 2: Write the binary number $0.\overline{011} = 0.011011011\ldots$ as a rational number. Hint: Recall the binary number 0.011 is $0 * 2^{-1} + 1 * 2^{-2} + 1 * 2^{-3}$ in decimal.

Solution: As the hint indicates, we note that

$$0.\overline{011} = 0 * 2^{-1} + 1 * 2^{-2} + 1 * 2^{-3} + 0 * 2^{-4} + 1 * 2^{-5} + 1 * 2^{-6} + \cdots$$

We can rewrite this by separating the terms in two three groups, and considering them separately. First, consider the ones where you have 2 to the power negative 1, 4, 7, 10, and so on. Second, consider the ones where you have negative powers 2, 5, 8, and so on. Finally, the third set is with negative powers 3,6,9,12 and so on. Then, by rearranging the terms, we have the three groups (represented by the three sets of parentheses) as follows:

$$\begin{aligned}
0.\overline{011} &= 0\left(\frac{1}{2^1} + \frac{1}{2^4} + \cdots\right) + 1\left(\frac{1}{2^2} + \frac{1}{2^5} + \cdots\right) + 1\left(\frac{1}{2^3} + \frac{1}{2^6} + \cdots\right) \\
&= 0 + \frac{1}{2^2} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n + \frac{1}{2^3} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \\
&= 0 + \left(\frac{1}{2^2} \cdot \frac{1}{1 - \frac{1}{2}}\right) + \left(\frac{1}{2^3} \cdot \frac{1}{1 - \frac{1}{2}}\right).
\end{aligned}$$

---

[1] Namely, Part 2 of Theorem 5.

[2] If you read the Sum Rule in Theorem 1 carefully, it is required that A and B are real numbers. In particular, that means that neither of the two constituent sequences a_n or b_n should diverge in order to use the Sum Rule as stated in the book.