

# Solutions

① Find  $\frac{\partial w}{\partial u}$

$$w = xy + \ln z, \quad x = \frac{v^2}{u}, \quad y = u+v, \quad z = \cos u$$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial u}$$

$$= y \cdot -\frac{v^2}{u^2} + x \cdot 1 + \frac{1}{z} (-\sin u)$$

$$= -(u+v) \left( \frac{v^2}{u^2} \right) + \frac{v^2}{u} - \frac{\sin u}{\cos u}$$

$$= -\frac{(u+v)v^2}{u^2} + \frac{v^2}{u} - \tan u$$

② Find the derivative of the function  $P_0$  in the direction  $A$   
 $f(x,y) = x^2 + 2y^2$ ,  $P_0(-1,1)$ ,  $\vec{A} = 3\vec{i} - 4\vec{j}$

$$\vec{u} = \frac{\vec{A}}{\|\vec{A}\|} = \frac{(3, -4)}{\sqrt{9+16}} = \left( \frac{3}{5}, -\frac{4}{5} \right) = \frac{1}{5}(3, -4)$$

$$D_{\vec{u}} f|_{P_0} = \nabla f|_{P_0} \cdot \vec{u} = (2x, 4y)|_{P_0} \cdot \left( \frac{3}{5}, -\frac{4}{5} \right)$$

$$= \frac{1}{5}(-2, 4) \cdot (3, -4) = \frac{1}{5}(-6 - 16)$$

$$= \underline{\underline{-\frac{22}{5}}}$$