Name: Solutions

Math 21C Section B05
Thursday 4-5pm
5/1/2008

QUIZ #4

Problem 1 (5 points): Find the Taylor series at $x = 0$ for the function

$$\sin x^{2/3}/\sqrt{3}.$$

This does not have a Taylor series since

$$\frac{d}{dx}\left(\sin x^{2/3}/\sqrt{3}\right) = \frac{1}{\sqrt{3}}\cos(x^{2/3}) \cdot \frac{2}{3}x^{-1/3}$$

$$= \frac{2}{3\sqrt{3}\,x^{1/3}}\cos(x^{2/3})$$

and this is undefined at $x=0$, so there is not a Taylor series.

Problem 2 (5 points): How close is the approximation $\sin x = x$ when $|x| < 10^{-6}$?

This is asking approximatlly how small is the remainder $-\frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = R_2(x)$

Taylor expansion $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

Thm 23 on p. 797 says that

$$|R_n(x)| \leq M \frac{|x|^{n+1}}{(n+1)!}$$

where M bounds $f^{(n+1)}(t)$ for all t between 0 and x.

So for $n=2$ we have

$$|R_2(x)| \leq M \frac{|x|^3}{3!}$$

where $|f^{(3)}(t)| = |-\cos t| \leq 1 = M$

main part

So by Thm 23

$$|R_2(x)| \leq \frac{|x|^3}{3!} < \frac{|10^{-6}|^3}{6} = \frac{10^{-18}}{6} = \frac{1}{6 \cdot 10^{18}}$$

This would all you would have to show