MAT 21C (Section B03)
Quiz 5

Name : Solution

1. (5 points): The following equation

$$x^2 + y^2 + z^2 + 4x - 6y + 2z + 5 = 0$$

generates a sphere in 3D. Find the center and radius of that sphere.

Complete the squares

$$x^2+4x+\underbrace{4-4}_{\text{add \& subtract}} + y^2-6y+\underbrace{9-9}_{\text{add and subtract}} + z^2+2z+\underbrace{1-1} + 5 = 0$$

$$\Rightarrow (x+2)^2 + (y-3)^2 + (z+1)^2 - 4 - 9 - 1 + 5 = 0$$

$$\Rightarrow (x+2)^2 + (y-3)^2 + (z+1)^2 = 9 = 3^2$$

Center : $(-2, 3, -1)$ radius : 3

2. (5 points):Let $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. Compute

$$\text{proj}_{\mathbf{u}}\mathbf{v}.$$

$$\text{proj}_u v = \left(\frac{u \cdot v}{|u|^2}\right) u$$

$$u \cdot v = 2(-1) + 2 \cdot 3 + (-1)(-2) = -2 + 6 + 2 = 6$$

$$|u|^2 = 2^2 + 2^2 + 1^2 = 4 + 4 + 1 = 9$$

Hence, $\text{proj}_u v = \frac{6}{9}(2\vec{i} + 2\vec{j} - k) = \frac{2}{3}(2\vec{i} + 2\vec{j} - k)$

or $\frac{2}{3}(2, 2, -1)$