MAT 21C (Section B03)
Quiz 2

SID :
Name : Solution

Determine whether each series converges or diverges. Give reasons for your answer.

1. (5 points):

$$\sum_{n=0}^{\infty} \frac{\sin^2 n}{6^n}$$

$\frac{\sin^2 n}{6^n} \leq \frac{1}{6^n}$

We know that $\sum_{n=0}^{\infty} \frac{1}{6^n}$ converges. (Geometric series with $a=1$, $r=\frac{1}{6}$)

By the comparison test, $\sum_{n=0}^{\infty} \frac{\sin^2 n}{6^n}$ converges.

2. (5 points):

$$\sum_{n=1}^{\infty} \frac{5(\ln n)^n}{n^n}$$

Use the Root test.

$$\sqrt[n]{\frac{5(\ln n)^n}{n^n}} = \left(\frac{5(\ln n)^n}{n^n}\right)^{1/n} = \frac{5^{1/n} \cdot \ln n}{n}$$

Since $\lim_{n\to\infty} 5^{\frac{1}{n}} = 5^0 = 1$ and $\lim_{n\to\infty} \frac{\ln n}{n} \overset{\text{L'Hospital}}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$,

$$\lim_{n\to\infty} \sqrt[n]{\frac{5(\ln n)^n}{n^n}} = \lim_{n\to\infty} 5^{1/n} \cdot \lim_{n\to\infty} \frac{\ln n}{n} = 1 \cdot 0 < 1$$

Hence, the series converges