TA: Ernest Woei *February 17, 2011*

*Last name:*________________ *First name:*________________

1 (5 points): Let $\mathbf{u} = \left\langle 2, -4, \sqrt{5} \right\rangle$ and $\mathbf{v} = \left\langle -2, 4, \sqrt{5} \right\rangle$. Find the vector $\text{proj}_{\mathbf{v}}\mathbf{u}$. Find the angle between $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}$.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} = \frac{2(-2) + -4(4) + \sqrt{5}(\sqrt{5})}{(-2)^2 + 4^2 + (-\sqrt{5})^2}\mathbf{v} = -\frac{3}{5}\mathbf{v} = \left\langle \frac{6}{5}, -\frac{12}{5}, -\frac{3\sqrt{5}}{5} \right\rangle,$$

The vectors $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u}$ are perpendicular to each other, so the angle is $\frac{\pi}{2}$.

2 (5 points): Find the distance from the point $(1, 2, 3)$ to the line

$$\begin{aligned} x &= 5 + 3t, \\ y &= 5 + 4t, \\ z &= -3 - 5t. \end{aligned}$$

Let $S = (1, 2, 3)$. Choose a point P on the line defined above, e.g., let $t = 0$, then $P = (5, 5, -3)$. Then $\overrightarrow{PS} = \langle -4, -3, 6 \rangle$. The vector parallel to the line is $\mathbf{v} = \langle 3, 4, -5 \rangle$. Thus the distance between the line and S is

$$d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} = \frac{|\langle -9, -2, -7 \rangle|}{|\mathbf{v}|} = \sqrt{\frac{67}{25}}.$$