Name: ___Key___   *Student ID:*_____

For each problem below, explain why the sequence converges or diverges, if it converges find the limit. You may use any theorems which you learned in class.

1. (*5 pts*) $a_n = \left(4 - \frac{1}{4^n}\right)\left(3 + \frac{1}{4^n}\right)$

Recall   if $\lim_{n\to\infty} b_n = K$ & $\lim_{n\to\infty} c_n = L$ then $\lim_{n\to\infty} b_n c_n = KL$

Now $\lim_{n\to\infty} 4 - \frac{1}{4^n} = 4$ & $\lim_{n\to\infty} 3 + \frac{1}{4^n} = 3$

Therefore $\lim_{n\to\infty} a_n = 12$ Thus a_n converges to 12

2. (*5 pts*) $a_n = \frac{\sin^2 4n}{2^n}$

Note : $-1 \leq \sin 4n \leq 1$ $\Longrightarrow$ $0 \leq \sin^2 4n \leq 1$

therefore $0 = \frac{0}{2^n} \leq a_n \leq \frac{1}{2^n}$

Since $\lim_{n\to\infty} \frac{1}{2^n} = 0$ & $\lim_{n\to\infty} 0 = 0$

Then   by   Sandwich Theorem $\lim_{n\to\infty} a_n = 0$

Thus a_n converges   to   0