TA: Ernest Woei *January 13, 2011*

*Last name:*____________________ *First name:*____________________

1 (5 points): Determine if the infinite series converge or diverge. If it converge, determine the sum for the series in its simplest form. Justify your answer.

$$\sum_{n=2}^{\infty} \frac{8 \cdot 3^{n+1} + (-4)^n 16}{5^n 4}$$

A simplification of the term in the sum shows that

$$\begin{aligned}
\sum_{n=2}^{\infty} \frac{8 \cdot 3^{n+1} + (-4)^n 16}{5^n 4} &= 6 \sum_{n=2}^{\infty} \left(\frac{3}{5}\right)^n + 4 \sum_{n=2}^{\infty} \left(\frac{-4}{5}\right)^n = 6 \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^{n+2} + 4 \sum_{n=0}^{\infty} \left(\frac{-4}{5}\right)^{n+2} \\
&= 6 \left(\frac{3}{5}\right)^2 \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n + 4 \left(\frac{-4}{5}\right)^2 \sum_{n=0}^{\infty} \left(\frac{-4}{5}\right)^n \\
&= 6 \left(\frac{3}{5}\right)^2 \frac{1}{1 - \frac{3}{5}} + 4 \left(\frac{-4}{5}\right)^2 \frac{1}{1 + \frac{4}{5}} = \frac{307}{45}
\end{aligned}$$

so the series converge to $\frac{307}{45}$.

2 (5 points): Determine if the series converge or diverge. Justify your answer.

$$\sum_{n=1}^{\infty} \frac{1}{1 + \ln n}$$

We use Limit Comparison Test to determine the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1 + \ln n}$. Note the Harmonic series, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, diverge. So, let $a_n = \frac{1}{1+\ln n}$ and $b_n = \frac{1}{n}$. Note $a_n, b_n > 0$ for all $n \geq 1$ to satisfy the assumption of the Limit Comparison Test. By taking, $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{1 + \ln n} \overset{\text{L'Hôpital}}{=} \lim_{n\to\infty} n = \infty$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverge, then $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1 + \ln n}$ diverge.