Quiz 4

Name:

Problem 1 (5 points): Find the Taylor series generated by $f(x) = -x^5 + x^3 - 4x + 4$ at $x = 2$.

Solution:

First we have

$$\begin{aligned} f(2) &= -28 \\ f^1(2) &= -72 \\ f^2(2) &= -148 \\ f^3(2) &= -234 \\ f^4(2) &= -240 \\ f^5(2) &= -120 \\ f^6(2) &= 0 \end{aligned}$$

So the Taylor series at $x = 2$ is

$$-28 - 72(x-2) - \frac{148}{2}(x-2)^2 - \frac{234}{3!}(x-2)^3 - \frac{240}{4!}(x-2)^4 - \frac{120}{5!}(x-2)^5$$

Problem 2 (5 points): Let $f(x) = \sin x$ and $P_2(x)$ the Taylor polynomial of f of order 2 centered $x = 0$. Using Taylor's remainder of order 2, $R_2(x)$, estimate the bound for the error between $f(x)$ and $P_2(x)$ for $|x| < 10^{-2}$.

Solution:

Since the third term of the $\sin(x)$ is 0 we get a better error bound with the 4th derivative. Further more since the 4th drivative of $\sin(x)$ is $-\sin(x)$ we get

$$|R_2(x)| = \left| -\sin(c)\frac{x^4}{4!} \right| \leq \left| \frac{(10^{-2})^4}{4!} \right|$$

So the error bound is $\left| \frac{10^{-8}}{4!} \right|$