Last Name: _____, First Name: _____

**Mat 21C-A02 (6:10 - 7:00pm) Quiz #4 Solutions**

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

**1. (5 points)** Using Euler's Identity write $e^{i\pi} + e^{-i\pi/3}$ in the form $a + bi$.

**Solution** Computing, $e^{i\pi} = \cos\pi + i\sin\pi = -1$, and $e^{-i\pi/3} = \cos(\pi/3) - i\sin(\pi/3) = 1/2 - i\sqrt{3}/2$. The real part of the sum is $-1 + 1/2 = -1/2$ and imaginary part is $-\sqrt{3}/2$. Thus, $a + ib = -1/2 - i\sqrt{3}/2$

**2. (5 points)** Evaluate

$$\lim_{x\to 0} \frac{\ln(1-x) + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}}{11x^5}.$$

**Solution** Use the Taylor series $\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \frac{y^5}{5} - \frac{y^6}{6} + \ldots$, with $y = -x$, to obtain

$$\ln(1-x) = (-x) - \frac{x^2}{2} + \frac{(-x)^3}{3} - \frac{x^4}{4} + \frac{(-x)^5}{5} - \frac{x^6}{6}\cdots$$

which implies

$$\begin{aligned}
\lim_{x\to 0} \frac{\ln(1-x) + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}}{11x^5} &= \lim_{x\to 0} \frac{\left[(-x) - \frac{x^2}{2} + \frac{(-x)^3}{3} - \frac{x^4}{4} + \frac{(-x)^5}{5} - \frac{x^6}{6}\cdots\right] + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}}{11x^5} \\
&= \lim_{x\to 0} \frac{\frac{-x^5}{5} + \frac{-x^6}{6} + \frac{-x^7}{7} + \cdots}{11x^5} \\
&= \lim_{x\to 0} \frac{\frac{-1}{5} + \frac{-x}{6} + \frac{-x^2}{7} + \cdots}{11} \\
&= \frac{-1}{55}.
\end{aligned}$$