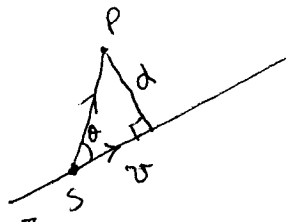
MAT 21C (Section B04)
Quiz 6

Name : Solution

1. (5 points): Find the distance from the point $P(2,1,-1)$ to the line

$$\left.\begin{aligned} x &= 2t \\ y &= 1+2t \\ z &= 2t \end{aligned}\right\}$$ from this, we know that the line is parallel to $v(2,2,2)$.



$d = |\vec{SP}| \cdot |\sin\theta|$

Choose $S(0,1,0)$. Note that $|\vec{SP} \times v| = |\vec{SP}| \cdot |v| \cdot |\sin\theta| \Rightarrow d = \dfrac{|\vec{SP} \times v|}{|v|}$

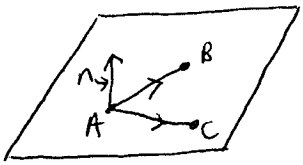
$|v| = \sqrt{4+4+4} = \sqrt{12}$, $\vec{SP} = (2,0,-1)$

$$\vec{SP} \times v = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 2 & 2 & 2 \end{vmatrix} = 2i - 6j + 4k \quad , |\vec{SP} \times v| = \sqrt{4+36+16} = \sqrt{56}$$

Therefore, $d = \dfrac{|\vec{SP} \times v|}{|v|} = \dfrac{\sqrt{56}}{\sqrt{12}} = \boxed{\sqrt{\dfrac{14}{3}}}$

2. (5 points):Find an equation for the plane through $A(-2,-2,5)$, $B(1,2,3)$, and $C(-3,-5,6)$.

First, find the normal vector n of the plane.



n is the cross product of $\vec{AC}$ and $\vec{AB}$.

$\vec{AC} = (-1,-3,1)$, $\vec{AB} = (3,4,-2)$

$$n = \vec{AC} \times \vec{AB} = \begin{vmatrix} i & j & k \\ -1 & -3 & 1 \\ 3 & 4 & -2 \end{vmatrix} = +2i + j + 5k$$

Hence, the plane has an equation $+2x + y + 5z = D$

To find D, plug in any given point into the equation.

Here, I chose $A(-2,-2,5) \Rightarrow (-2)\cdot 2 + (-2) + 5\cdot 5 = D$

$D = 19$

As a result, $\boxed{2x+y+5z = 19}$