Problem 2

(5 points): Write the binary number $0.\overline{010} = 0.010010010\ldots$ as a rational number. Hint: Recall the binary number 0.010 is $0 * 2^{-1} + 1 * 2^{-2} + 0 * 2^{-3}$ in decimal.

Alternate (shorter) solution

$0.010010010\ldots = \frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \cdots$

$= \frac{1}{2^2} \sum_{n=0}^{\infty} \frac{1}{2^{3n}}$, (check this)

$\Rightarrow a = \frac{1}{2^2}$, $r = \frac{1}{2^3}$

$= \frac{1}{2^2} \sum_{n=0}^{\infty} \left(\frac{1}{2^3}\right)^n$

$\Rightarrow \frac{a}{1-r} = \frac{\frac{1}{2^2}}{1-\frac{1}{2^3}} = \frac{\frac{1}{4}}{1-\frac{1}{8}}$

$= \frac{\frac{1}{4}}{\frac{7}{8}}$

$= \frac{1}{4} \cdot \frac{8^2}{7}$

$= \boxed{\frac{2}{7}}$