Math 21A: Calculus
Quiz 3 Solution
(Woei 7/22/04)

This is a closed book, no notes, no calculator quiz. Remember, if a question has English in it, so should the answer.

1. (10 points) The volume of a spherical balloon is decreasing at a rate of 2 cubic inches per second. How fast does the radius change when the volume is 1 cubic foot.

Solution: The volume of a spherical balloon with radius r inches is $\frac{4}{3}\pi r^3$ cubic inches. We want to find the rate of change of the radius, i.e. $\frac{dr}{dt}$, since we have our relation between volume and radius then all we need to do is differentiate with respect to time. So $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, since the volume of the spherical balloon is decreasing then $\frac{dV}{dt} = -2\ \frac{in^3}{sec}$, so $-2\ \frac{in^3}{sec} = 4\pi r^2 \frac{dr}{dt}$. It remains to find what r is. Since we want to find the rate of change of the radius when the volume of the spherical balloon is $1\ ft^3 = 12^3\ in^3$, then $12^3\ in^3 = \frac{4}{3}\pi r^3$, so $r = 12\sqrt[3]{\frac{3}{4\pi}}\ in$. Thus $\frac{dr}{dt} = \frac{-1}{2\pi\left(\frac{3}{4\pi}\right)^{\frac{2}{3}}12^2}\frac{in}{sec}$.

2. (10 points) Estimate $\log_2(2.1)$. Assume $\ln 2 = .693$ and $\frac{1}{13.86} = 0.0721$. Hint: $f(x) = \log_2 x$

Solution: Using the hint $f(x) = \log_2 x$ and the formula below

$$f(a+\Delta x) \approx f(a) + f'(a)\Delta x$$

where $a = 2$ and $\Delta x = 0.1$. We get

$$f(2.1) \approx f(2) + f'(2)\cdot\Delta x = 1 + \frac{1}{\ln 2}\cdot\frac{1}{2}\cdot\frac{1}{10} = 1.0721$$

3. (4 points) Extra Credit (All or Nothing). Compute

$$\lim_{\Delta x\to 0}\left(1+\frac{\Delta x}{2x}\right)^{\frac{x}{\Delta x}}$$

Solution: It is in your notes.

$$\lim_{\Delta x\to 0}\left(1+\frac{\Delta x}{2x}\right)^{\frac{x}{\Delta x}} = \sqrt{e}$$