Name:

Math 21C Section B05
Thursday 4-5pm
4/10/2008

QUIZ #1

Problem 1 (5 points): Determine if the sequence:

$$a_n = \frac{n! + 2008^{1/n}\, 2008^n}{2008^n}$$

converges or diverges. If it converges, find the limit.

Solution: $a_n = \dfrac{n! + 2008^{1/n} 2008^n}{2008^n} = \dfrac{n! + 2008^{n+1/n}}{2008^n}$

$= \dfrac{n!}{2008^n} + 2008^{1/n}$

Now $2008^{1/n} \to 1$ as $n \to \infty$

and $\dfrac{n!}{2008^n} \to \infty$ as $n \to \infty$ so a_n diverges

why does $\dfrac{n!}{2008^n} \to \infty$ as $n \to \infty$?

for $n > 2008$ we have

$$\frac{n!}{2008^n} = \left(\frac{2008!}{2008^{2008}}\right) \cdot \frac{2009}{2008} \cdot \frac{2010}{2008} \cdots \frac{n}{2008} \geq \text{constant} * \left(\frac{2009}{2008}\right)^{n-2008} \to \infty$$

$n-2008$ times