Solutions (Quiz 7, section B04)

Problem 1 (5 points):Find the value of $\frac{\partial x}{\partial z}$ at the point $(1,5,-1)$ if the equation

$$z^5x + y\ln x + x^3 + 2 = 2$$

defines x as a function of the two independent variables y and z and the partial derivative exist.

<u>solution:</u> Take partial derivative with respect to z.Then,

$$5z^4x + z^5\frac{\partial x}{\partial z} + y\frac{1}{x}\frac{\partial x}{\partial z} + 3x^2\frac{\partial x}{\partial z} = 0.$$

Solve for $\frac{\partial x}{\partial z}$.

$$\frac{\partial x}{\partial z} = -\frac{5z^4x}{z^5 + \frac{y}{x} + 3x^2}$$

Evaluate the above expression at (1,5,-1) then,

$$\frac{\partial x}{\partial z} = -\frac{5}{7}.$$

Problem 2 (5 points): Find a vector parallel to the line of intersection of the planes $3x + 2y - 4z = -1$ and $-3x - 5z + y = 10$.

<u>solution:</u> Note that the line of intersection of two plane is perpendicular to both normal vectors of the planes. Therefore, the line is parallel to $n_1 \times n_2$, where n_1 and n_2 are the normal vectors of the planes.

$$\begin{aligned} n_1 \times n_2 &= \begin{vmatrix} i & j & k \\ 3 & 2 & -4 \\ -3 & 1 & -5 \end{vmatrix} \\ &= i\begin{vmatrix} 2 & -4 \\ 1 & -5 \end{vmatrix} - j\begin{vmatrix} 3 & -4 \\ -3 & -5 \end{vmatrix} + k\begin{vmatrix} 3 & 2 \\ -3 & 1 \end{vmatrix} \\ &= -6i + 27j + 9k \end{aligned}$$