

Solutions

Math 21C Quiz 6

Section: 5:10-6:00 pm, TA: Arpy Mikaelian
Tuesday May 20, 2008

Problem 1

(5 points): Determine if

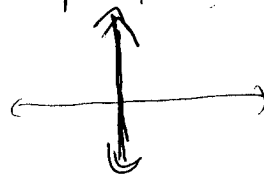
$$\lim_{(x,y) \rightarrow (0,0)} -\frac{x}{\sqrt{x^2 + y^2}}$$

has a limit or not. Give reasons for your answers.

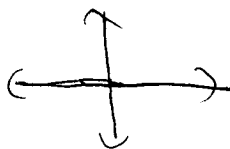
Since plugging in $(x,y) = (0,0)$ gives $\frac{0}{0}$, which is undefined, try

approaching the limit from two different paths.

For example, from $x=0$
 \uparrow
y-axis



and $y=0$
 \uparrow
x-axis:



$$\lim_{(x,y) \rightarrow (0,y)} -\frac{0}{\sqrt{0^2 + y^2}} = -\frac{0}{\sqrt{y^2}} = 0$$

$$\lim_{(x,y) \rightarrow (x,0)} \frac{-x}{\sqrt{x^2 + 0^2}} = \frac{-x}{\sqrt{x^2}} = \frac{-x}{x} = -1$$

Since $0 \neq -1$, the limit does not exist.

It must approach one limit for there to be a limit, no matter what direction you approach from.

Problem 2

(5 points): Find an equation for the level curve of the function

$$f(x, y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n$$

that passes through the point $(1, 2)$.

$$f(1, 2) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = 2 \quad (\text{geometric series})$$

$$2 = \frac{1}{1-\frac{x}{y}}$$

$$1 - \frac{x}{y} = \frac{1}{2}$$

$$y - x = \frac{1}{2}y$$

$$\frac{1}{2}y = x$$

$$\boxed{y = 2x}$$