Last Name: _____,   First Name: _____

Mat 21C-A03 (5:10 - 6:00pm) Quiz #4 Solutions

You have 15 minutes to do the following problems. Justify all solutions. You may not use any electronic devices for the duration of the quiz. Answers without support will receive no credit.

1. (5 points) Evaluate

$$\lim_{x \to 0} \frac{e^x - \frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} - \frac{x^2}{2} - x - 1}{11x^6}.$$

Solution First notice $5! = 120$, $4! = 24$, $3! = 6$. Next use the Taylor expansion of the exponential

$$\lim_{x \to 0} \frac{e^x - \frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} - \frac{x^2}{2} - x - 1}{11x^6} = \lim_{x \to 0} \frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots\right) - \frac{x^5}{120} - \frac{x^4}{24} - \frac{x^3}{6} - \frac{x^2}{2} - x - 1}{11x^6}$$

$$= \lim_{x \to 0} \frac{\frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \cdots}{11x^6}$$

$$= \lim_{x \to 0} \frac{\frac{1}{6!} + \frac{x}{7!} + \frac{x^2}{8!} + \cdots}{11}$$

$$= \frac{1}{(6!)11} = \frac{1}{7920}.$$

2. (5 points) Using Euler's Identity write $e^{i\pi/3} + e^{i\pi}$ in the form $a + bi$.

Solution Computing, $e^{i\pi} = \cos \pi + i \sin \pi = -1$, and $e^{i\pi/3} = \cos(\pi/3) + i \sin(\pi/3) = 1/2 + i\sqrt{3}/2$. The real part of the sum is $-1 + 1/2 = -1/2$ and imaginary part is $\sqrt{3}/2$. Thus, $a + ib = -1/2 + i\sqrt{3}/2$