Solutions (Quiz 8, section B03)

Problem 1 (5 points): Find the gradient of the function

$$f(x,y) = \arctan\frac{xy}{5} + \ln\left(x^2+y^4\right)$$

at the given point $(5,1)$.

<u>solution:</u> Recall that $(\arctan x)' = \frac{1}{1+x^2}$.

$$\text{grad } f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{xy}{5}\right)^2}\cdot\frac{y}{5} + \frac{2x}{x^2+y^4}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{xy}{5}\right)^2}\cdot\frac{x}{5} + \frac{4y^3}{x^2+y^4}$$

$$\left.\frac{\partial f}{\partial x}\right|_{(5,1)} = \frac{63}{130} \qquad \left.\frac{\partial f}{\partial y}\right|_{(5,1)} = \frac{17}{26}$$

Finally, $\nabla f = \left(\frac{63}{130}, \frac{17}{26}\right)$ or $\frac{63}{130}i + \frac{17}{26}j$.

Problem 2 (5 points): Find $\partial w/\partial v$ when $u=1$, $v=1$ if $w = xy + \ln z$, $x = v^3/u$, $y = u-v$, $z = \sin u$.

<u>solution:</u> Use the chain rule.

$$\begin{aligned}\frac{\partial w}{\partial v} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}\\ &= y\cdot\frac{3v^2}{u} + x\cdot(-1) + \frac{1}{z}\cdot 0\\ &= (u-v)\cdot\frac{3v^2}{u} - \frac{v^3}{u}\end{aligned}$$

Hence, $\left.\frac{\partial w}{\partial v}\right|_{(1,1)} = -1$.