Name: Solutions

Math 21C Section B05
Thursday 4-5pm
5/29/2008

QUIZ #7

Problem 1 (5 points): Find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ if the equation

$$x^6 z + y \ln x - x^2 + 5 = 0$$

defines x as a function of the two independent variables y and z and the partial derivative exists.

Differentiate both sides partially with respect to z

$$6x^5 \cdot \frac{\partial x}{\partial z} \cdot z + x^6 \cdot 1 + \frac{y}{x} \cdot \frac{\partial x}{\partial z} - 2x \cdot \frac{\partial x}{\partial z} = 0$$

$$\frac{\partial x}{\partial z}\left(6x^5 z + \frac{y}{x} - 2x\right) = -x^6$$

$$\frac{\partial x}{\partial z} = \frac{-x^6}{6x^5 z + y/x - 2x}$$

So

$$\left.\frac{\partial x}{\partial z}\right|_{(1,-1,-3)} = \frac{-1^6}{6 \cdot 1^6(-3) + {-1}/1 - 2(1)} = \frac{-1}{-18-1-2} = \frac{-1}{-21} = \boxed{\frac{1}{21}}$$

Problem 2 (5 points): Find $\partial w/\partial v$ when $u = -1$, $v = 2$ if $w = xy + \ln z$, $x = v^3/u$, $y = u + v$, $z = \cos u$.

Two ways to do the problem:

(1) Substitute x, y, z values into the expression $w = xy + \ln z$ to get

$$w(u,v) = \frac{v^3}{u}\cdot(u+v) + \ln(\cos u) = v^3 + \frac{v^4}{u} + \ln(\cos u)$$

so $$\frac{\partial w}{\partial v} = 3v^2 + \frac{4v^3}{u}$$

(2) Alternatively we can apply the chain rule to get

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial v}$$

$$= y\cdot\frac{3v^2}{u} + x\cdot 1 + \frac{1}{z}\cdot 0 = (u+v)\cdot\frac{3v^2}{u} + \frac{v^3}{u}$$

$$= 3v^2 + \frac{3v^3}{u} + \frac{v^3}{u} = 3v^2 + \frac{4v^3}{u}$$

Either way we find that $\dfrac{\partial w}{\partial z} = 3v^2 + \dfrac{4v^3}{u}$

Thus at $u=-1$, $v=2$ we have

$$\frac{\partial w}{\partial z}\Big|_{(u=-1,\, v=2)} = 3(2)^2 + \frac{4(2)^3}{-1} = 3\cdot 4 + \frac{4\cdot 8}{-1} = 12 - 32 = \boxed{-20}$$