Math 21C - Section B01 - Quiz 5 **SOLUTION**
E. Kim

For both problems, let $\mathbf{u} = \langle -2, 1, 2 \rangle$ and $\mathbf{v} = \langle 1, 3, -1 \rangle$.

Problem 1: Find the angle between the $\mathbf{u}$ and $\mathbf{v}$. Do not simplify.

Solution:

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|} = \frac{(-2)(1)+(1)(3)+(2)(-1)}{\left(\sqrt{(-2)^2+(1)^2+(2)^2}\right)\left(\sqrt{(1)^2+(3)^2+(-1)^2}\right)}$$

Thus,

$$\theta = \cos^{-1}\left(\frac{(-2\times 1)+(1\times 3)+(2\times -1)}{\left(\sqrt{(-2)^2+1^2+(2)^2}\right)\left(\sqrt{1^2+3^2+(-1)^2}\right)}\right)$$

Problem 2: Find $\text{proj}_{\mathbf{u}}\,\mathbf{v}$.

Solution:

$$\begin{aligned}
\text{proj}_{\mathbf{u}}\,\mathbf{v} &= \frac{\mathbf{v}\cdot\mathbf{u}}{|\mathbf{u}|^{\mathbf{2}}}\mathbf{u} \\
&= \frac{(1\times -2)+(3\times 1)+(-1\times 2)}{\left(\sqrt{(-2)^2+1^2+2^2}\right)^2}\ \langle -2,1,2\rangle \\
&= \frac{-1}{\left(\sqrt{9}\right)^2}\ \langle -2,1,2\rangle \\
&= \frac{-1}{9}\langle -2,1,2\rangle \\
&= \left\langle \frac{2}{9}, -\frac{1}{9}, -\frac{2}{9}\right\rangle
\end{aligned}$$

One can alternatively write final answer as: $\frac{2}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} - \frac{2}{9}\mathbf{k}$ instead of the above.

Remarks

- In problem 2, the roles of $\mathbf{v}$ and $\mathbf{u}$ are switched from the book's "normal" formula.