Quiz 2 Solutions

Problem 1 (5 points): Determine if the series $\sum_{n=0}^{\infty} \frac{\sin^2(n)}{4^n}$ converges or diverges. Give reasons for your answer.

Since $0 \leq \frac{\sin^2(n)}{4^n} \leq \frac{1}{4^n}$, and $\sum_{n=0}^{\infty} \frac{1}{4^n}$ converges, being a geometric series, it follows from the direct comparison test that $\sum_{n=0}^{\infty} \frac{\sin^2(n)}{4^n}$ converges.

Problem 2 (5 points): Determine if the series $\sum_{n=1}^{\infty} \frac{7(\ln n)^n}{n^n}$ converges or diverges. Give reasons for your answer.

By the root test, $\sum_{n=1}^{\infty} \frac{7(\ln n)^n}{n^n}$ converges if $\lim_{n\to\infty} \left(\frac{7(\ln n)^n}{n^n}\right)^{\frac{1}{n}} = \lim_{n\to\infty} 7^{\frac{1}{n}} \frac{\ln n}{n} < 1$. But $\lim_{n\to\infty} 7^{\frac{1}{n}} = 7^0 = 1$ and $\lim_{n\to\infty} \frac{\ln n}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$, so that $\lim_{n\to\infty} 7^{\frac{1}{n}} \frac{\ln n}{n} = 0 < 1$. Therefore, $\sum_{n=1}^{\infty} \frac{7(\ln n)^n}{n^n}$ converges.