MAT 21C (Section B04)
Quiz 5

Name : Solution

1. (5 points):The following equation

$$x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$$

generates a sphere in 3D. Find the center and radius of that sphere.

Complete the squares

$$x^2-2x+1-1 + y^2+4y+4-4 + z^2-6z+9-9 + 10 = 0$$

add & subtract add & subtract

$$\Rightarrow (x-1)^2+(y+2)^2+(z-3)^2-1-4-9+10=0$$

$$\Rightarrow (x-1)^2+(y+2)^2+(z-3)^2 = 4 = 2^2$$

Center : $(1,-2,3)$ radius : 2

2. (5 points): Let $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$. Compute

$$\text{proj}_{\mathbf{u}}\mathbf{v}.$$

$$\text{proj}_u V = \left(\frac{u \cdot v}{|u|^2}\right) u$$

$$|u|^2 = 3^2 + 2^2 + (-3)^2 = 9 + 4 + 9 = 22$$

$$u \cdot v = 3\cdot(-2) + 2\cdot 4 + (-3)\cdot(-1) = -6 + 8 + 3 = 5$$

Hence, $\text{proj}_u V = \frac{5}{22}(3\bar{i} + 2\bar{j} - 3k)$

or $\frac{5}{22}(3, 2, -3)$