COUNTING SORT

Algorithms Analysis

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1. Illustrate the operation of counting sort on the array $A = (3, 5, 7, 2, 4, 3, 5, 8, 1)$. Show the contents of the array C after the completion of steps 3 and 4 and again after the completion of steps 5 and 6. Show the contents of arrays B and C after each iteration of steps 7, 8, and 9.

	1	2	3	4	5	6	7	8	9
A	3	5	7	2	4	3	5	8	1

Step 1: Count each element in the given array A and place the count at the appropriate index.

	0	1	2	3	4	5	6	7	8
C	0	1	1	2	1	2	0	1	1

Step 2: Modify the count array C by adding the previous counts.

	0	1	2	3	4	5	6	7	8
C	0	1	2	4	5	7	7	8	9

Since there are 9 elements in the given array, we create an array B with 9 entries to sort the elements of array A. Each of the 9 entries in array B represents the places in the updated count array. Therefore, for the remaining steps (3−11), we will place the numbers in their correct sorted position (in array B) and decrease the count by one.

	0	1	2	3	4	5	6	7	8	
C	0	1	2	4	5	7	7	8	9	From Step 2

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 3

	1	2	3	4	5	6	7	8	9
B				3					

	0	1	2	3	4	5	6	7	8
C	0	1	2	3	5	7	7	8	9

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 4

	1	2	3	4	5	6	7	8	9
B				3			5		

	0	1	2	3	4	5	6	7	8
C	0	1	2	3	5	6	7	8	9

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 5

	1	2	3	4	5	6	7	8	9
B				3			5	7	

	0	1	2	3	4	5	6	7	8
C	0	1	2	3	5	6	7	7	9

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 6

	1	2	3	4	5	6	7	8	9
B		2		3			5	7	

	0	1	2	3	4	5	6	7	8
C	0	1	1	3	5	6	7	7	9

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 7

	1	2	3	4	5	6	7	8	9
B		2		3	4		5	7	

	0	1	2	3	4	5	6	7	8
C	0	1	1	3	4	6	7	7	9

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 8

	1	2	3	4	5	6	7	8	9
B		2	3	3	4		5	7	

	0	1	2	3	4	5	6	7	8
C	0	1	1	2	4	6	7	7	9

	1	2	3	4	5	6	7	8	9	
A	3	5	7	2	4	3	5	8	1	Step 9

	1	2	3	4	5	6	7	8	9
B		2	3	3	4	5	5	7	

	0	1	2	3	4	5	6	7	8
C	0	1	1	2	4	5	7	7	9

	1	2	3	4	5	6	7	8	9
A	3	5	7	2	4	3	5	8	1

Step 10

	1	2	3	4	5	6	7	8	9
B		2	3	3	4	5	5	7	8

	0	1	2	3	4	5	6	7	8
C	0	1	1	2	4	5	7	7	8

	1	2	3	4	5	6	7	8	9
A	3	5	7	2	4	3	5	8	1

Step 11

	1	2	3	4	5	6	7	8	9
B	1	2	3	3	4	5	5	7	8

Sorted!

	0	1	2	3	4	5	6	7	8
C	0	0	1	2	4	5	7	7	8

2. Let X be a random variable that is equal to the number of times a 5 is rolled in three rolls of a fair 5-sided die with integers 1 through 5 on the sides. What is $E[X^2]$? What is $E^2[X]$, that is $(E[X])^2$? Justify your answers briefly.

Let us define n as number of times the die is rolled, p as probability of getting a 5, and X as the number of times a 5 is rolled.

Since we are looking at the number of successes in a sequence of n independent experiments, X follows a binomial distribution, with $n = 3$, $p = \frac{1}{5}$ and $X = 0, 1, 2, 3$.

Probability mass function: $\mathrm{P}(X = k) = \binom{n}{k} \, p^k \, (1-p)^{n-k}$

- $\mathrm{P}(X = 0) = \binom{3}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^3 = \frac{64}{125}$
- $\mathrm{P}(X = 1) = \binom{3}{1} \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^2 = \frac{48}{125}$
- $\mathrm{P}(X = 2) = \binom{3}{2} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^1 = \frac{12}{125}$
- $\mathrm{P}(X = 3) = \binom{3}{3} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^0 = \frac{1}{125}$

Therefore, applying the expected value definition:

$$E[X] = \sum_k k \cdot P(X = k) = \left(0 \cdot \frac{64}{125}\right) + \left(1 \cdot \frac{48}{125}\right) + \left(2 \cdot \frac{12}{125}\right) + \left(3 \cdot \frac{1}{125}\right)$$

$$E[X] = \frac{75}{125} = 0.6$$

$$\boldsymbol{E^2[X]} = (E[X])^2 = \left(\frac{75}{125}\right)^2 = (0.6)^2 = \frac{5625}{15625} = \mathbf{0.36}$$

$$E[X^2] = \sum_k k^2 \cdot P(X = k) = \left(0^2 \cdot \frac{64}{125}\right) + \left(1^2 \cdot \frac{48}{125}\right) + \left(2^2 \cdot \frac{12}{125}\right) + \left(3^2 \cdot \frac{1}{125}\right)$$

$$\boldsymbol{E[X^2]} = 0 + \frac{48}{125} + \frac{48}{125} + \frac{9}{125} = \frac{105}{125} = \mathbf{0.84}$$