CONTEXT-FREE LANGUAGES

Theory of Computation

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1. Show that the following language is context-free:

$$\{a^m b^n c^m : n, m \geq 0\} \cup \{c^p b^q a^p : p, q \geq 0\}$$

We will use one of the closure properties, which states that the **union** of two context-free languages is always context-free. It follows to show that,

$$\{a^m b^n c^m : n, m \geq 0\} \rightarrow \textit{ context-free}$$

$$\{c^p b^q a^p : p, q \geq 0\} \rightarrow \textit{ context-free}$$

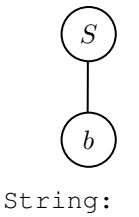
To do this, we will construct a context-free grammars that represent these languages.

The language $\{a^m b^n c^m : n, m \geq 0\}$ can be represented with the following grammar,

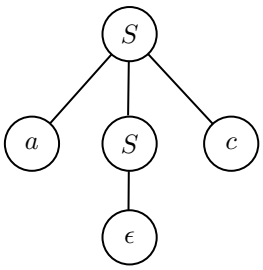
$$S \rightarrow aSc, S \rightarrow b, S \rightarrow bB, B \rightarrow bB, B \rightarrow \epsilon, S \rightarrow \epsilon$$

Let us take some strings in the language, and construct their respective parse trees using the grammar above to confirm they satisfy the conditions imposed by the language:

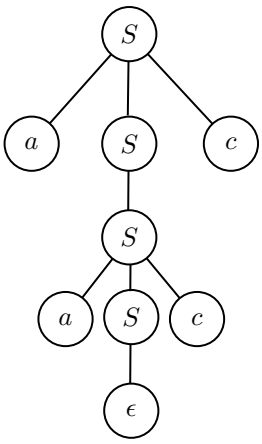
b, ac, aacc, aaabccc, aaabbbccc



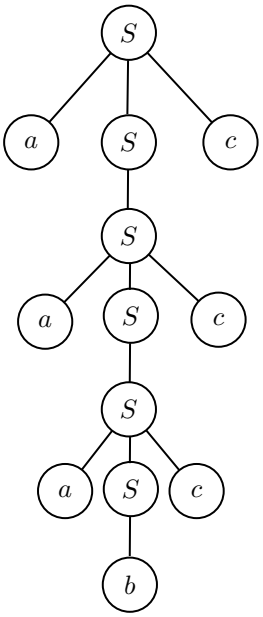
String: b



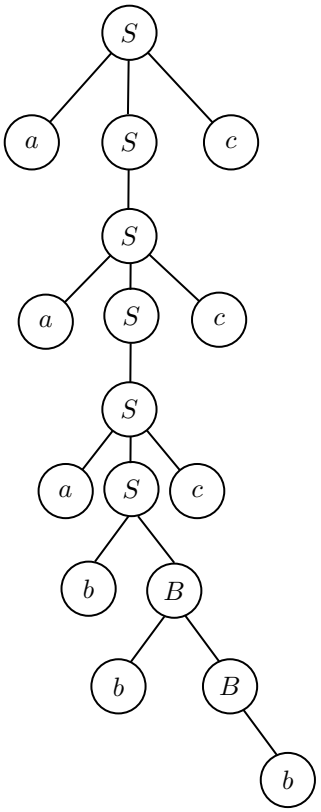
String: ac



String: aacc



String: aaabccc



String: aaabbbccc

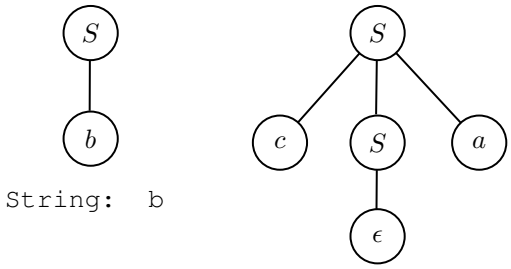
Since we can represent $\{a^m b^n c^m : n, m \geq 0\}$ by a context-free grammar, we can conclude that $\{a^m b^n c^m : n, m \geq 0\}$ is context-free.

Similarly, $\{c^p b^q a^p : p, q \geq 0\}$ can be represented by the following grammar,

$$S \to cSa, S \to b, S \to bB, B \to bB, B \to \epsilon, S \to \epsilon$$

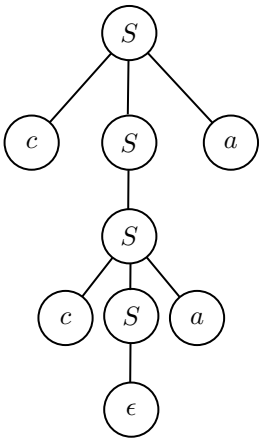
Let us take some strings in the language, and construct their respective parse trees using the grammar above to confirm they satisfy the conditions imposed by the language,

b, ca, ccaa, cccbaaa, cccbbbaaa

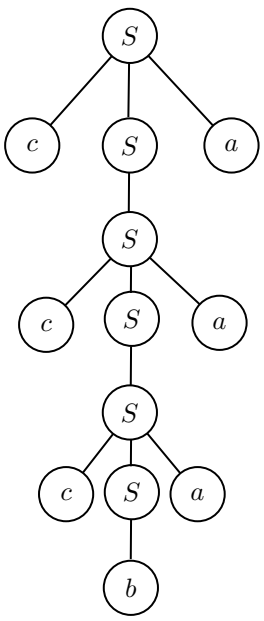


String: b

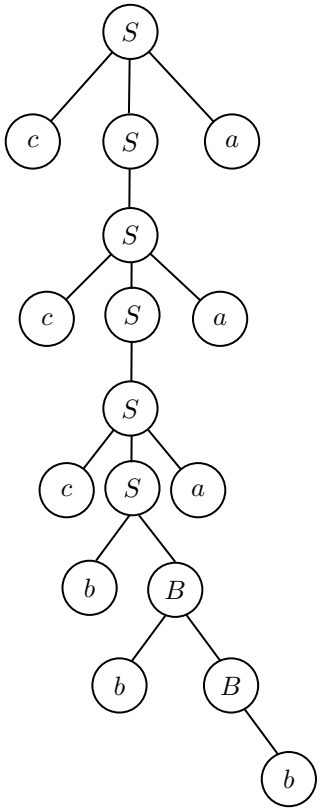
String: ca



String: ccaa



`String: cccbaaa`



`String: cccbbbaaa`

Hence, $\{c^p b^q a^p : p, q \geq 0\}$ is also context-free.

Therefore, the language

$$\{a^m b^n c^m : n, m \geq 0\} \cup \{c^p b^q a^p : p, q \geq 0\}$$

is context-free.