

CONTEXT-FREE GRAMMARS

Theory of Computation

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1. Construct a context-free grammar generating all the strings in the following language, and no others:

$$\{a^m b^n c^p d^q : m \neq n, p \neq q\}$$

For a context-free grammar it will generate $a^m b^n$ separately from $c^p d^q$ because they do not have any constraints with respect to each other. Therefore the grammar is as follows,

$$G = (V, \Sigma, R, S)$$

$$V = \{S, X, Y, A, B, C, D\} \cup \Sigma$$

$$\Sigma = \{a, b, c, d\}$$

$$R = \left\{ \begin{array}{ll} S \rightarrow XY & \longrightarrow \boxed{\text{Separating the parts}} \\ \left. \begin{array}{l} X \rightarrow_a X_b \\ X \rightarrow A \\ X \rightarrow B \end{array} \right\} & \boxed{\text{Same number of a's and b's}} \\ \left. \begin{array}{l} A \rightarrow a \\ A \rightarrow_a A \end{array} \right\} & \boxed{\text{Additional a on the right}} \\ \left. \begin{array}{l} B \rightarrow b \\ B \rightarrow_b B \end{array} \right\} & \boxed{\text{Additional b on the left}} \\ \left. \begin{array}{l} Y \rightarrow_c Y_d \\ Y \rightarrow C \\ Y \rightarrow D \\ C \rightarrow c \\ C \rightarrow_c C \\ D \rightarrow d \\ D \rightarrow_d D \end{array} \right\} & \boxed{\text{Same Procedure as the First Part}} \end{array} \right.$$