PUSH-DOWN AUTOMATA

Theory of Computation

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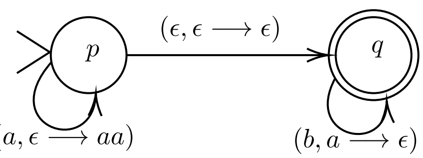
1. Consider the push-down automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$ where $K = \{p, q\}, \Sigma = \{a, b\}, s = p, F = q$, and Δ contains the following transitions:

$$((p, a, \epsilon), (p, aa)), ((p, \epsilon, \epsilon), (q, \epsilon)), ((q, b, a), (q, \epsilon)).$$

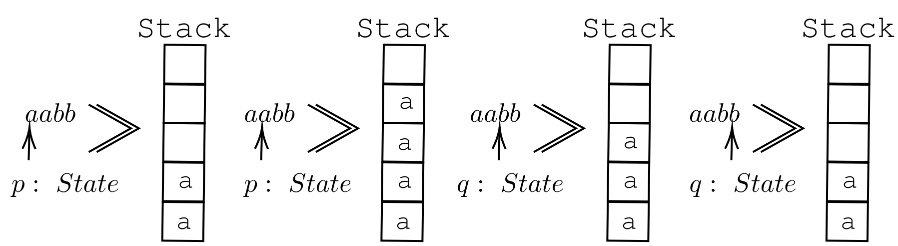
   (a) Is the string $aabb$ accepted by M?

   Answer: NO

   The PDA described by the exercise goes as follows,



   Therefore we have,



   According to the definition, M accepts a string $w \in \Sigma^*$ if and only if $(s, w, \epsilon) \vdash^* (q, \epsilon, \epsilon)$ for some state $q \in F$. In other words, in order for $aabb$ to be accepted by this PDA, it must reach the (q, ϵ, ϵ) configuration (final state, end of input, and empty stack).

   Therefore, even though the string $aabb$ reaches the accepting state q, the stack is not empty at the end of the computation sequences, so this string is NOT accepted by this automaton.

(b) Is the string ba accepted by M?

Answer: NO

There are no transitions involving b in the initial state p, so by taking into consideration that ϵ reaches the final state q, then we have the following:

$$q(b, \epsilon \to N/A)$$

There is not an a to pop from the stack, so the operation fails to proceed, and the string ba is not accepted by the automaton.

(c) Give an informal description of the set of all strings accepted by M.

Answer:

M accepts any number of $a's$ followed by $b's$, such that the number of $b's$ is twice the number of $a's$, including the empty string. Formally:

$$L(M) = \{a^n b^m : 2n = m, m \geq 0, n \geq 0\}$$