

COUNTING SORT

Algorithms Analysis

github.com/erngv

1. Illustrate the operation of counting sort on the array $A = (3, 5, 7, 2, 4, 3, 5, 8, 1)$. Show the contents of the array C after the completion of steps 3 and 4 and again after the completion of steps 5 and 6. Show the contents of arrays B and C after each iteration of steps 7, 8, and 9.

	1	2	3	4	5	6	7	8	9
A	3	5	7	2	4	3	5	8	1

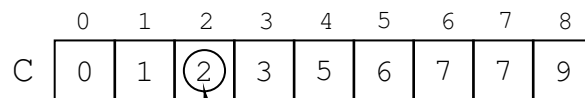
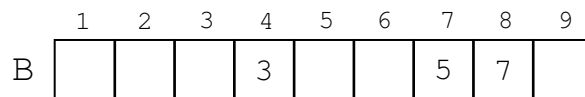
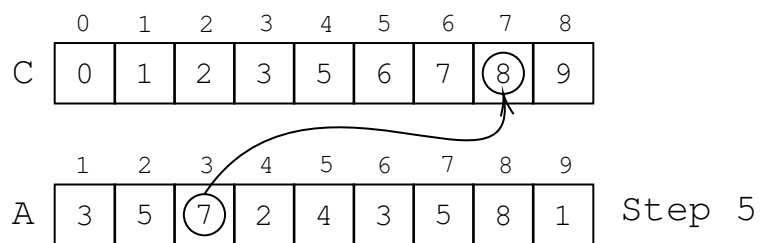
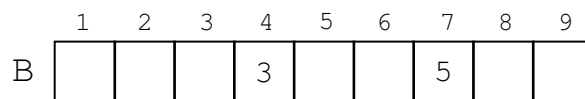
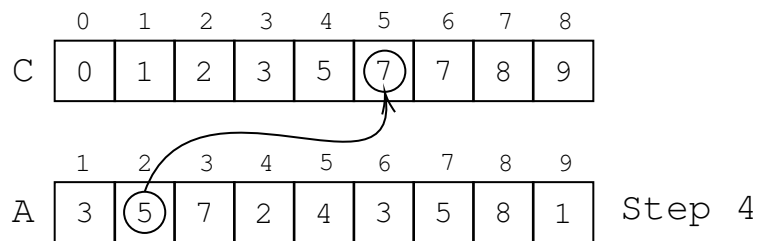
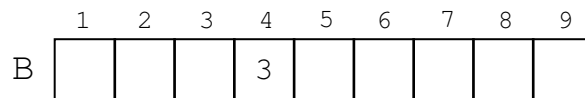
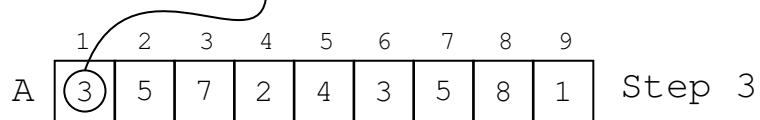
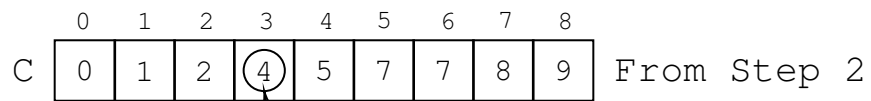
Step 1: Count each element in the given array A and place the count at the appropriate index.

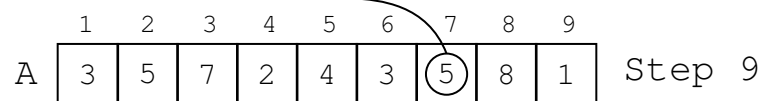
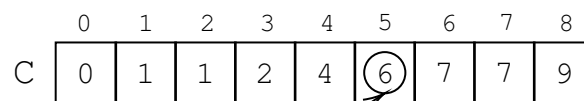
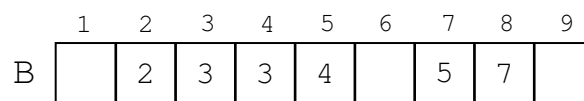
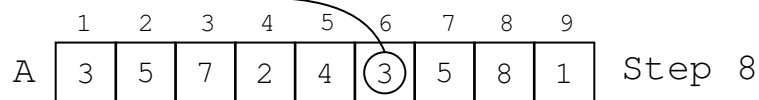
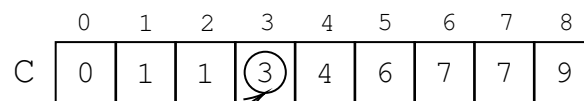
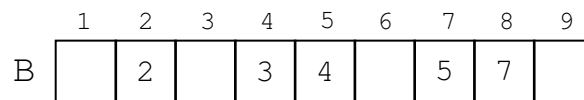
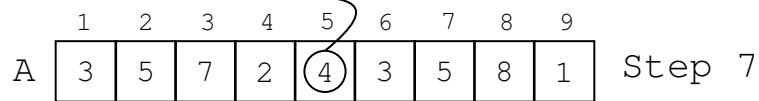
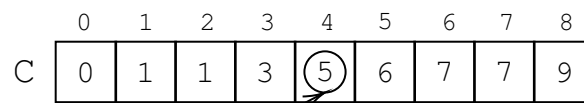
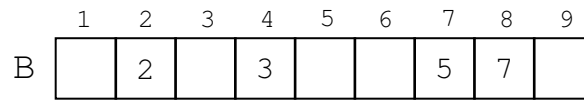
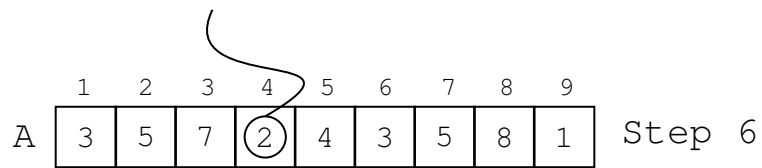
	0	1	2	3	4	5	6	7	8
C	0	1	1	2	1	2	0	1	1

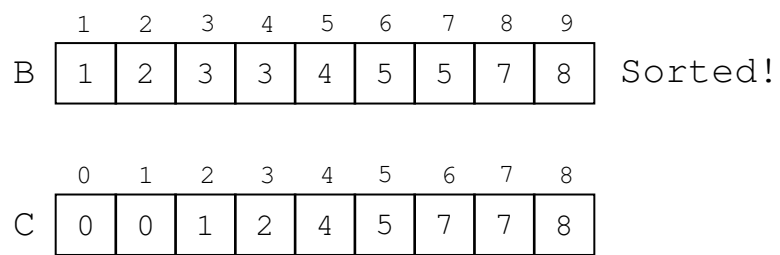
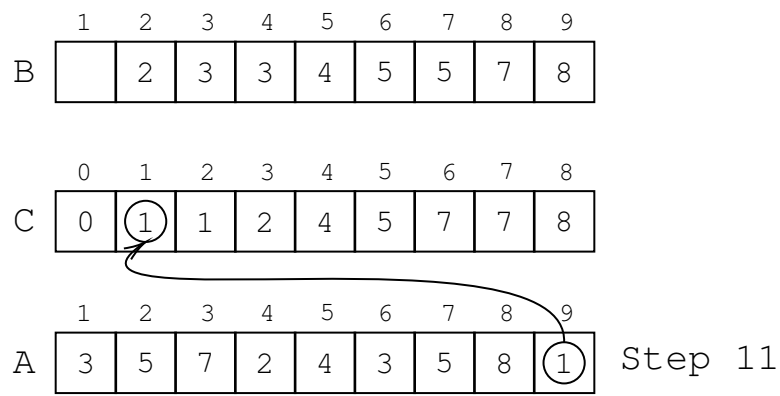
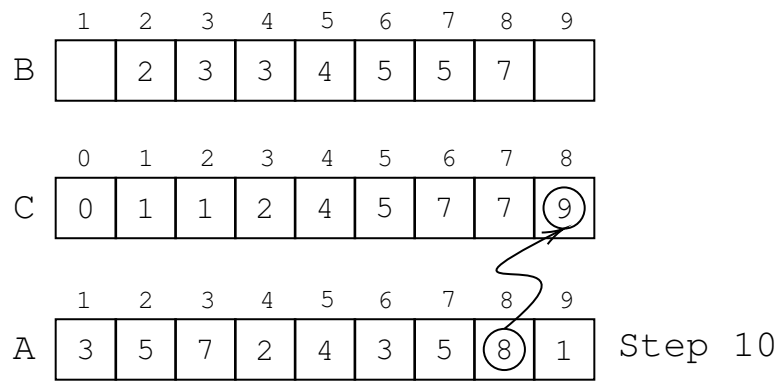
Step 2: Modify the count array C by adding the previous counts.

	0	1	2	3	4	5	6	7	8
C	0	1	2	4	5	7	7	8	9

Since there are 9 elements in the given array, we create an array B with 9 entries to sort the elements of array A . Each of the 9 entries in array B represents the places in the updated count array. Therefore, for the remaining steps (3–11), we will place the numbers in their correct sorted position (in array B) and decrease the count by one.







2. Let X be a random variable that is equal to the number of times a 5 is rolled in three rolls of a fair 5-sided die with integers 1 through 5 on the sides. What is $E[X^2]$? What is $E^2[X]$, that is $(E[X])^2$? Justify your answers briefly.

Let us define n as number of times the die is rolled, p as probability of getting a 5, and X as the number of times a 5 is rolled.

Since we are looking at the number of successes in a sequence of n independent experiments, X follows a binomial distribution, with $n = 3$, $p = \frac{1}{5}$ and $X = 0, 1, 2, 3$.

Probability mass function: $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$

- $P(X = 0) = \binom{3}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^3 = \frac{64}{125}$
- $P(X = 1) = \binom{3}{1} \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^2 = \frac{48}{125}$
- $P(X = 2) = \binom{3}{2} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^1 = \frac{12}{125}$
- $P(X = 3) = \binom{3}{3} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^0 = \frac{1}{125}$

Therefore, applying the expected value definition:

$$E[X] = \sum_k k \cdot P(X = k) = \left(0 \cdot \frac{64}{125}\right) + \left(1 \cdot \frac{48}{125}\right) + \left(2 \cdot \frac{12}{125}\right) + \left(3 \cdot \frac{1}{125}\right)$$

$$E[X] = \frac{75}{125} = 0.6$$

$$E^2[X] = (E[X])^2 = \left(\frac{75}{125}\right)^2 = (0.6)^2 = \frac{5625}{15625} = \mathbf{0.36}$$

$$E[X^2] = \sum_k k^2 \cdot P(X = k) = \left(0^2 \cdot \frac{64}{125}\right) + \left(1^2 \cdot \frac{48}{125}\right) + \left(2^2 \cdot \frac{12}{125}\right) + \left(3^2 \cdot \frac{1}{125}\right)$$

$$E[X^2] = 0 + \frac{48}{125} + \frac{48}{125} + \frac{9}{125} = \frac{105}{125} = \mathbf{0.84}$$