

data $\_a\mathfrak{t}c\_a\mathfrak{L}\ddot{u}\_$ ( $\mathfrak{I}\mathfrak{S} : \text{Cxt}$ ): $\mathfrak{I}\mathfrak{Z} \text{ a}\mathfrak{E}\mathfrak{S}$ Type $\text{a}\mathfrak{E}\mathfrak{S}$ Set where
$\mathfrak{a}\mathfrak{t}c\mathfrak{v} : \{x : \mathbf{V}\}$ $\text{a}\mathfrak{E}\mathfrak{S}$ ( $\mathfrak{p}\mathfrak{a}\mathfrak{L}\mathfrak{L} : x \text{ a}\mathfrak{L}$ )
$\mathfrak{a}\mathfrak{t}c\mathfrak{A}\mathfrak{u} : \{\mathfrak{I}\mathfrak{S} \ \mathfrak{I}\mathfrak{s} : \text{Type}\}\{M \ N : \mathfrak{I}\mathfrak{Z}\}$ $\text{a}\mathfrak{E}\mathfrak{S} \ \mathfrak{I}\mathfrak{S} \ \mathfrak{a}\mathfrak{t}c \ M \ \mathfrak{a}\mathfrak{L}$
$\mathfrak{a}\mathfrak{t}c\mathfrak{A}\mathfrak{Z} : \{x : \mathbf{V}\}\{\mathfrak{I}\mathfrak{S} \ \mathfrak{I}\mathfrak{s} : \text{Type}\}\{M : \mathfrak{I}\mathfrak{Z}\}$ $\text{a}\mathfrak{E}\mathfrak{S} \ \mathfrak{I}\mathfrak{S} \ \mathfrak{a}\mathfrak{Z} \ x \ \mathfrak{a}\mathfrak{L}$

- lemmaWeakening :  $\{x : V\} \{ \hat{I}S : Cxt \} \{M$
- $\hat{a}E_S \ x \ \hat{a}LL \ \hat{I}S \ \hat{a}E_S \ \hat{I}S \ \hat{a}A_Z \ x \ \hat{a}LC \ M \ \hat{a}E_S \ \hat{I}S \ \hat{a}LC \ M$

- lemma#âŁŁ : {x : V}{ÎŒ : Cxt}{M : ÎŒ}
- âŁŁ x # M âŁŁ ÎŒ âŁŁ x âŁŁ M âŁŁ ÎŒ âŁŁ M

$$\begin{aligned} \text{lemma}\hat{a}\hat{t}\hat{c}\hat{i}\hat{c}\hat{M} &: \{ \hat{I}\check{C}: \hat{l}\hat{c} \} \{ \hat{I}\hat{S} \hat{I}\check{T}: \text{Cxt} \} \{ M: \hat{I}\hat{Z} \} \{ \hat{I}\hat{s}: \text{Type} \} & \text{lemma}\hat{a}\hat{t}\hat{c}\hat{a}\hat{U}\hat{z} &: \{ \hat{I}\hat{S}: \text{Cxt} \} \{ \hat{I}\hat{s}: \text{Type} \} \{ M \ N: \hat{I}\hat{Z} \} \\ \text{aE}\hat{S} \ \hat{I}\hat{S} \ \hat{a}\hat{t}\hat{c} \ M \ \hat{a}\hat{L}\hat{U} \ \hat{I}\hat{s} \ \text{aE}\hat{S} \ \hat{I}\check{C} \ \hat{a}\hat{L}\hat{U} \ \hat{I}\hat{S} \ \hat{a}\hat{G}\hat{A} \ \hat{I}\hat{T} \ \hat{a}\hat{G}\hat{C} \ M \ \text{aE}\hat{S} \ \hat{I}\hat{T} \ \hat{a}\hat{t}\hat{c} \ M \ \text{aE}\hat{S} \ \hat{I}\hat{S} \ \hat{a}\hat{L}\hat{U} \ \hat{I}\hat{s} \ \text{aE}\hat{S} \ M \ \hat{a}\hat{U}\hat{z} \ N \ \text{aE}\hat{S} \ \hat{I}\hat{S} \ \hat{a} & \\ & & \text{lemma}\hat{a}\hat{t}\hat{c}\hat{a}\hat{U}\hat{z} \ \{ \hat{I}\hat{S} \} \ (\hat{a}\hat{t}\hat{c}\hat{A}\hat{U} \ . \{ M = \hat{A}\hat{E}\hat{Z} \ x \ M \} \ \{ N \} \ (\hat{a}\hat{t}\hat{c}\hat{A}\hat{E}\hat{Z} \ \{ x & \\ & & = \text{lemma}\hat{a}\hat{t}\hat{c}\hat{i}\hat{c}\hat{M} \ \{ \hat{I}\hat{z} \ \hat{a}\hat{L}\hat{z} + (x, N) \} \ \{ \hat{I}\hat{S} \ \hat{a}\hat{A}\hat{Z} \ x \ \hat{a}\hat{L}\hat{U} \ \hat{I}\hat{s} \} \ \{ \hat{I}\hat{S}, x: \hat{I}\hat{s} \hat{a}\hat{L}\hat{c} \ M: \hat{I}\hat{s} & \\ & & \ (\text{lemma}\hat{a}\hat{G}\hat{A} \ (\text{lemma}\hat{I}\hat{z}\hat{a}\hat{L}\hat{z} + \hat{a}\hat{G}\hat{A} \ \hat{I}\hat{S} \hat{a}\hat{L}\hat{c} \ N: & \end{aligned}$$
$$\text{lemma } \{M = v \ x\} \quad \hat{I}S \hat{a} L c x : \hat{I}S = \hat{I}S \hat{a} L c x : \hat{I}S$$
$$\begin{aligned} & \text{lemma}\hat{a}\hat{t}\hat{c}\hat{l}\hat{z} \{M = \hat{A}\hat{E}z \ x \ M\} \ (\hat{a}\hat{t}\hat{c}\hat{A}\hat{E} \ \hat{I}\hat{S}, y: \hat{I}\hat{s}\hat{a}\hat{L}\hat{c}\hat{M}\hat{z}\hat{a}\hat{L}\hat{z}+x, y: \hat{I}\hat{s}) \\ & \quad \text{with lemma}\hat{a}\hat{t}\hat{c}\hat{l}\hat{C}\hat{M} \ \hat{I}\hat{S}, y: \hat{I}\hat{s}\hat{a}\hat{L}\hat{c}\hat{M}\hat{z}\hat{a}\hat{L}\hat{z}+x, y: \hat{I}\hat{s} \ (\text{lemma}\hat{f}\hat{z}\hat{a}\hat{L}\hat{z}+\hat{a}\hat{G}\hat{A}y \ \{x = x\} \ \{\hat{I}\hat{G} \ (\hat{I}\hat{z}, \hat{A}\hat{E}z \ x \ M)\} \ \{M\}) \\ & \dots \mid \hat{I}\hat{S}, x: \hat{I}\hat{s}\hat{a}\hat{L}\hat{c}\hat{M}\hat{z}\hat{a}\hat{L}\hat{z}+x, y: \hat{I}\hat{z}\hat{a}\hat{L}\hat{z}y, x: \hat{I}\hat{s} \text{rewrite lemma}\hat{a}\hat{L}\hat{z}+\hat{I}\hat{z} \ (\text{lemma-}\hat{I}\hat{G}\hat{I}\hat{z} \ (\hat{A}\hat{E}z \ x \ M)) \\ & = \hat{a}\hat{t}\hat{c}\hat{A}\hat{E} \ (\text{lemma}\hat{a}\hat{t}\hat{c}\hat{l}\hat{z} \ \hat{I}\hat{S}, x: \hat{I}\hat{s}\hat{a}\hat{L}\hat{c}\hat{M}\hat{z}\hat{a}\hat{L}\hat{z}+x, y: \hat{I}\hat{z}\hat{a}\hat{L}\hat{z}y, x: \hat{I}\hat{s}) \end{aligned}$$
$$\begin{aligned} \text{lemma}\hat{a}\hat{t}\hat{c}\hat{l}\hat{s} &: \{ \hat{I}\hat{S} : \text{Cxt} \} \{ \hat{I}\hat{s} : \text{Type} \} \{ M N : \hat{\mathbb{I}}\hat{z} \} \text{with } M \hat{a}\hat{L}\hat{Z} \hat{I}\hat{z} \mid \text{lemma}\hat{a}\hat{t}\hat{c}\hat{l}\hat{c}\hat{M} \hat{I}\hat{S}\hat{a}\hat{L}\hat{c}\hat{M} (\text{lemma}\hat{a}\hat{G}\hat{A} \text{ lemma}\hat{I}\hat{z}\hat{a}\hat{G}\hat{A}) \mid \text{lemma}\hat{M}\hat{a}\hat{L}\hat{i}\hat{j} \\ &\quad \text{aE}\hat{S} \ M \hat{a}\hat{L}\hat{i}\hat{j}\hat{s} \ N \text{aE}\hat{S} \ \hat{I}\hat{S} \ \hat{a}\hat{t}\hat{c} \ M \hat{a}\hat{L}\hat{U} \ \hat{I}\hat{s} \ (\text{aE}\hat{S} \ \hat{a}\hat{L}\hat{Z} \ \hat{I}\hat{z}) \ N \hat{a}\hat{L}\hat{U} \hat{I}\hat{s} \ \hat{N}\hat{a}\hat{L}\hat{Z}\hat{I}\hat{z} \mid \text{refl} \\ &= \text{lemma}\hat{a}\hat{t}\hat{c}\hat{l}\hat{z} \ \hat{I}\hat{S}\hat{a}\hat{L}\hat{c}\hat{N}\hat{a}\hat{L}\hat{Z}\hat{I}\hat{z} \end{aligned}$$