Numerical Methods (ENUME 2019) – Project
Assignment C: Solving ordinary differential equations

1. Develop a program for solving the following differential equation:

$$4y'' + 4y' + 5y = 0 \text{ for } t \in [0,10], \ y(0) = 4 \text{ and } y'(0) = -2$$

   by means of the implicit Lobatto IIID order 4 method defined by the following Butcher table:

$$
\begin{array}{c|ccc}
0 & \dfrac{1}{6} & 0 & -\dfrac{1}{6} \\[2mm]
\dfrac{1}{2} & \dfrac{1}{12} & \dfrac{5}{12} & 0 \\[2mm]
1 & \dfrac{1}{2} & \dfrac{1}{3} & \dfrac{1}{6} \\[2mm]
\hline
& \dfrac{1}{6} & \dfrac{2}{3} & \dfrac{1}{6}
\end{array}
$$

   Compare the solution, obtained by means of this program for the constant integration step $h = 0.01$, with the solution, obtained by means of the MATLAB operator ***ode45***. Optimise the parameters ***RelTol*** and ***AbsTol*** of the latter in such a way as to get the most accurate solution $\dot{\mathbf{y}}(t;h)$ (to be used as a reference in the following sections).

2. Carry out a systematic investigation of the dependence of the accuracy of the solution $\hat{\mathbf{y}}(t; h)$ on the integration step h. Use the following accuracy indicators for this purpose:

$$\delta_2(h) = \frac{\left\| \hat{\mathbf{y}}(t; h) - \dot{\mathbf{y}}(t, h) \right\|_2}{\left\| \dot{\mathbf{y}}(t, h) \right\|_2} \quad \text{(the root-mean-square error)}$$

$$\delta_\infty(h) = \frac{\left\| \hat{\mathbf{y}}(t; h) - \dot{\mathbf{y}}(t, h) \right\|_\infty}{\left\| \dot{\mathbf{y}}(t, h) \right\|_\infty} \quad \text{(the maximum error)}$$

   Make the graphs $\delta_2(h)$ and $\delta_\infty(h)$.

3. Repeat the systematic investigation, defined in Section 3, for the implicit Euler method. Add the curves representative of $\delta_2(h)$ and $\delta_\infty(h)$ to the graph obtained in Section 2.