CS 350 Algorithms and Complexity

Fall 2015

Lecture 6: Exhaustive Search Algorithms

Andrew P. Black

Department of Computer Science

Portland State University

Brute Force

- A straightforward approach, usually based directly on the problem's statement and definitions of the concepts involved
- Examples:

  Computing a^n ($a > 0$, n a nonnegative integer) by repeated multiplication

  Computing $n!$ by repeated multiplication

  Multiplying two matrices following the definition

  Searching for a key in a list sequentially

Closest-Pair Problem

- Find the two closest points in a set of n points (in the two-dimensional Cartesian plane).
- Brute-force algorithm:
  - Compute the distance between every pair of distinct points
    - and return the indices of the points for which the distance is the smallest.

Closest-Pair Brute-Force Algorithm (cont.)

ALGORITHM *BruteForceClosestPoints*(P)

//Finds two closest points in the plane by brute force
//Input: A list P of n $(n \geq 2)$ points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
//Output: Indices $index1$ and $index2$ of the closest pair of points

$dmin \leftarrow \infty$
for $i \leftarrow 1$ **to** $n - 1$ **do**
  for $j \leftarrow i + 1$ **to** n **do**
    $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //$sqrt$ is the square root function
    if $d < dmin$
      $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
return $index1, index2$

Closest-Pair Brute-Force Algorithm (cont.)

ALGORITHM *BruteForceClosestPoints*(P)

//Finds two closest points in the plane by brute force
//Input: A list P of n $(n \geq 2)$ points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
//Output: Indices $index1$ and $index2$ of the closest pair of points
$dmin \leftarrow \infty$
for $i \leftarrow 1$ **to** $n - 1$ **do**
  for $j \leftarrow i + 1$ **to** n **do**
    $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //$sqrt$ is the square root function
    if $d < dmin$
      $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
return $index1, index2$

- Efficiency:

Portland State UNIVERSITY

Closest-Pair Brute-Force Algorithm (cont.)

ALGORITHM *BruteForceClosestPoints*(P)

//Finds two closest points in the plane by brute force
//Input: A list P of n $(n \geq 2)$ points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
//Output: Indices $index1$ and $index2$ of the closest pair of points

$dmin \leftarrow \infty$
for $i \leftarrow 1$ **to** $n - 1$ **do**
  for $j \leftarrow i + 1$ **to** n **do**
    $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //$sqrt$ is the square root function
    if $d < dmin$
      $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
return $index1, index2$

- Efficiency: A: $O(n)$ B: $O(n^2)$ C: $O(\lg n)$ D: $O(n^3)$

Closest-Pair Brute-Force Algorithm (cont.)

ALGORITHM *BruteForceClosestPoints*(P)

//Finds two closest points in the plane by brute force
//Input: A list P of n $(n \geq 2)$ points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
//Output: Indices $index1$ and $index2$ of the closest pair of points
$dmin \leftarrow \infty$
for $i \leftarrow 1$ **to** $n - 1$ **do**
  for $j \leftarrow i + 1$ **to** n **do**
    $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //$sqrt$ is the square root function
    if $d < dmin$
      $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
return $index1, index2$

- Efficiency: A: O(n) B: O(n^2) C: O(lg n) D: O(n^3)
- How to make it faster?

Problem:

ALGORITHM *BruteForceClosestPoints*(P)

//Finds two closest points in the plane by brute force
//Input: A list P of n $(n \geq 2)$ points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
//Output: Indices $index1$ and $index2$ of the closest pair of points
$dmin \leftarrow \infty$
for $i \leftarrow 1$ **to** $n - 1$ **do**
    for $j \leftarrow i + 1$ **to** n **do**
        $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //$sqrt$ is the square root function
        if $d < dmin$
            $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
return $index1$, $index2$

If *sqrt* is 10 x slower than × and +, by how much will *BruteForceClosestPoints* speed up when we take out the *sqrt* ?

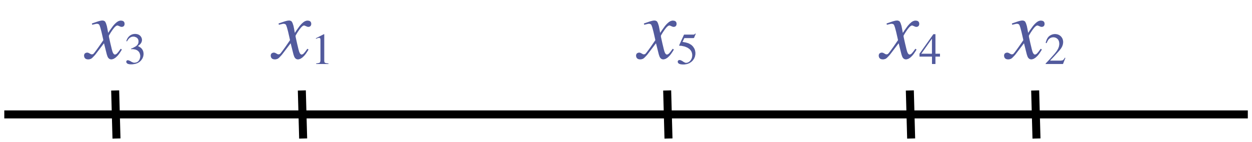
A. ~ 10 times

B. ~ 100 times

C. ~ 1000 times

Problem:

Can you design a more efficient algorithm than the one based on the brute-force strategy to solve the closest-pair problem for n points $x_1, \ldots, x_n$ on the real line?

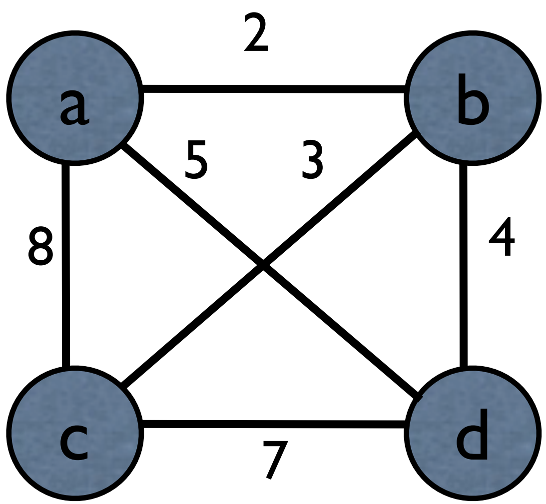


Exhaustive Search

- A brute force solution to a problem involving search for an element with a special property, usually among combinatorial objects such as permutations, combinations, or subsets of a set.
- Method:
  - generate a list of all potential solutions to the problem in a systematic manner (see algorithms in Sec. 4.3)
  - evaluate potential solutions one by one, disqualifying infeasible ones and, for an optimization problem, keeping track of the best one found so far
  - when search ends, announce the solution(s) found

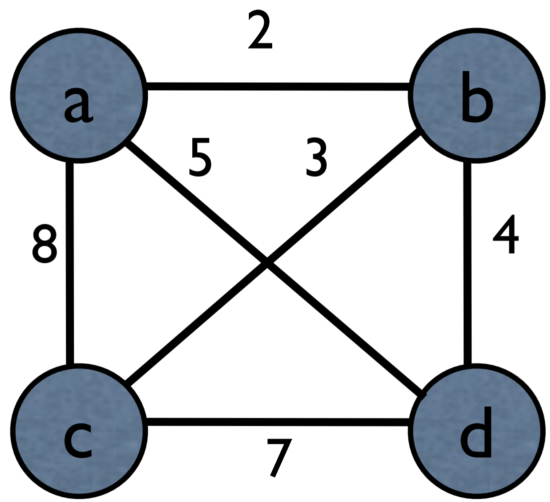
Example 1: Traveling Salesman Problem

- Given *n* cities with known distances between each pair, find the shortest tour that passes through all the cities exactly once before returning to the starting city
- Alternatively: find shortest Hamiltonian circuit in a weighted connected graph
- Example:



TSP by Exhaustive Search

Tour	Cost
a→b→c→d→a	2+3+7+5 = 17
a→b→d→c→a	2+4+7+8 = 21
a→c→b→d→a	8+3+4+5 = 20
a→c→d→b→a	8+7+4+2 = 21
a→d→b→c→a	5+4+3+8 = 20
a→d→c→b→a	5+7+3+2 = 17



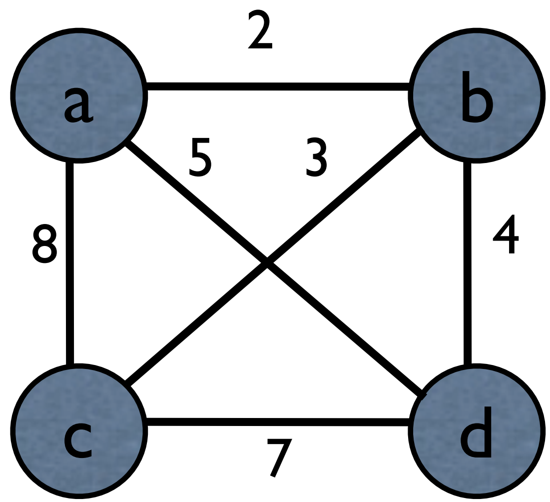
More tours?

Less tours?

Efficiency:

TSP by Exhaustive Search

Tour	Cost
a→b→c→d→a	2+3+7+5 = 17
a→b→d→c→a	2+4+7+8 = 21
a→c→b→d→a	8+3+4+5 = 20
a→c→d→b→a	8+7+4+2 = 21
a→d→b→c→a	5+4+3+8 = 20
a→d→c→b→a	5+7+3+2 = 17



More tours?

Less tours?

Efficiency:

A: $O(n)$
B: $O(n^2)$
C: $O(n^3)$
D: $O((n-1)!)$
E: $O(n!)$

Example 2: Knapsack Problem

- Given *n* items:
  - weights: w_1 w_2 … w_n
  - values: v_1 v_2 … v_n
  - a knapsack of capacity W
- Find most valuable subset of the items that fit into the knapsack
- Example: Knapsack capacity W=16

item	weight	value
1.	2	$20
2.	5	$30
3.	10	$50
4.	5	$10

Knapsack Problem by Exhaustive Search

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	not feasible
{1,2,4}	12	$60
{1,3,4}	17	not feasible
{2,3,4}	20	not feasible
{1,2,3,4}	22	not feasible

item	weight	value
1.	2	$20
2.	5	$30
3.	10	$50
4.	5	$10

Knapsack capacity W=16

Knapsack Problem by Exhaustive Search

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	not feasible
{1,2,4}	12	$60
{1,3,4}	17	not feasible
{2,3,4}	20	not feasible
{1,2,3,4}	22	not feasible

item	weight	value
1.	2	$20
2.	5	$30
3.	10	$50
4.	5	$10

Knapsack capacity W=16

- Efficiency?

  A: $O(n^2)$

  B: $O(2^n)$

  C: O(n!)

  D: O((n-1)!)

Example 3: The Assignment Problem

- There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *p* to job *j* is C[*i*, *j*]. Find an assignment that minimizes the total cost.

	Job 1	Job 2	Job 3	Job 4
Person 1	9	2	7	8
Person 2	6	4	3	7
Person 3	5	8	1	8
Person 4	7	6	9	4

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.
- How many assignments are there …

Example 3: The Assignment Problem

- There are n people who need to be assigned to n jobs, one person per job. The cost of assigning person p to job j is C[i, j]. Find an assignment that minimizes the total cost.

	Job 1	Job 2	Job 3	Job 4
Person 1	9	2	7	8
Person 2	6	4	3	7
Person 3	5	8	1	8
Person 4	7	6	9	4

an *assignment* $\langle a_1, a_2, a_3, a_4 \rangle$ means that person i gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

- How many assignments are there …

Example 3: The Assignment Problem

- There are n people who need to be assigned to n jobs, one person per job. The cost of assigning person p to job j is C[i, j]. Find an assignment that minimizes the total cost.

		Job 1	Job 2	Job 3	Job 4
$\langle a_1,$	Person 1	9	2	7	8
$a_2,$	Person 2	6	4	3	7
$a_3,$	Person 3	5	8	1	8
$a_4\rangle$	Person 4	7	6	9	4

an *assignment* $\langle a_1, a_2, a_3, a_4\rangle$ means that person i gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

- How many assignments are there …

Example 3: The Assignment Problem

- There are n people who need to be assigned to n jobs, one person per job. The cost of assigning person p to job j is C[i, j]. Find an assignment that minimizes the total cost.

		Job 1	Job 2	Job 3	Job 4
⟨2,	Person 1	9	2	7	8
4,	Person 2	6	4	3	7
3,	Person 3	5	8	1	8
1⟩	Person 4	7	6	9	4

an *assignment* $\langle a_1, a_2, a_3, a_4 \rangle$ means that person i gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

- How many assignments are there …

Example 3: The Assignment Problem

- There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *p* to job *j* is C[*i*, *j*]. Find an assignment that minimizes the total cost.

		Job 1	Job 2	Job 3	Job 4
⟨2,	Person 1	9	2	7	8
4,	Person 2	6	4	3	7
3,	Person 3	5	8	1	8
1⟩	Person 4	7	6	9	4

an *assignment* $\langle a_1, a_2, a_3, a_4 \rangle$ means that person *i* gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.
- How many assignments are there …

Example 3: The Assignment Problem

- There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *p* to job *j* is C[*i*, *j*]. Find an assignment that minimizes the total cost.

		Job 1	Job 2	Job 3	Job 4
‹2,	Person 1	9	2	7	8
4,	Person 2	6	4	3	7
3,	Person 3	5	8	1	8
1›	Person 4	7	6	9	4

an *assignment* $\langle a_1, a_2, a_3, a_4 \rangle$ means that person *i* gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.
- How many assignments are there …

Example 3: The Assignment Problem

- There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *p* to job *j* is C[*i*, *j*]. Find an assignment that minimizes the total cost.

		Job 1	Job 2	Job 3	Job 4
⟨2,	Person 1	9	2	7	8
4,	Person 2	6	4	3	7
3,	Person 3	5	8	1	8
1⟩	Person 4	7	6	9	4

an *assignment*
$\langle a_1, a_2, a_3, a_4 \rangle$
means that person *i*
gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

- How many assignments are there …

Example 3: The Assignment Problem

- There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *p* to job *j* is C[*i*, *j*]. Find an assignment that minimizes the total cost.

		Job 1	Job 2	Job 3	Job 4
⟨2,	Person 1	9	2	7	8
4,	Person 2	6	4	3	7
3,	Person 3	5	8	1	8
1⟩	Person 4	7	6	9	4

an *assignment* $\langle a_1, a_2, a_3, a_4 \rangle$ means that person *i* gets job a_i

- Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

- How many assignments are there …

Assignment Problem by Exhaustive Search

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
etc.	

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
etc.	

How many assignments are there?

Assignment Problem by Exhaustive Search

- Consider the problem in terms of the *Cost Matrix C*

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
etc.	

How many assignments are there?

A: $O(n)$ B: $O(n^2)$ C: $O(n^3)$ D: $O(n!)$

Convex Hulls

- What is a Convex Hull?

A. A bad design for a boat
B. A good design for a boat
C. A set of points without any concavities
D. None of the above

Convex Hulls

- What is a Convex Set?

A. A bad design for a boat
B. A good design for a boat
C. A set of points without any concavities
D. None of the above

Convex Hulls

- What is a Convex Set?

Convex Hulls

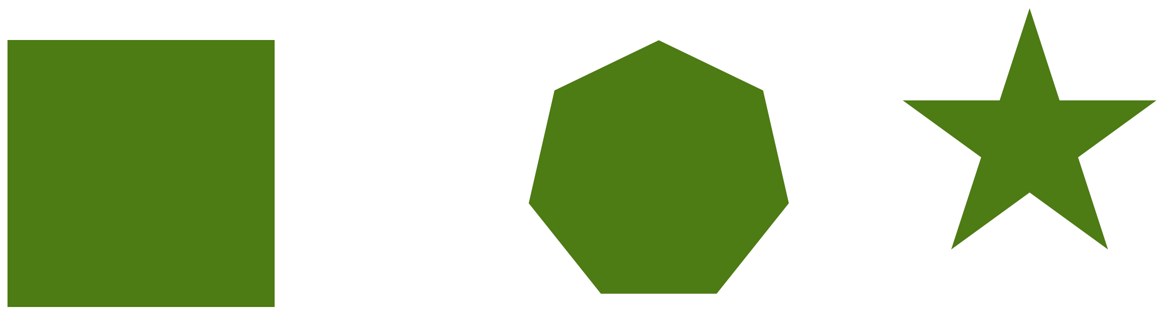
- What is a Convex Set?

Convex Hulls

- What is a Convex Set?

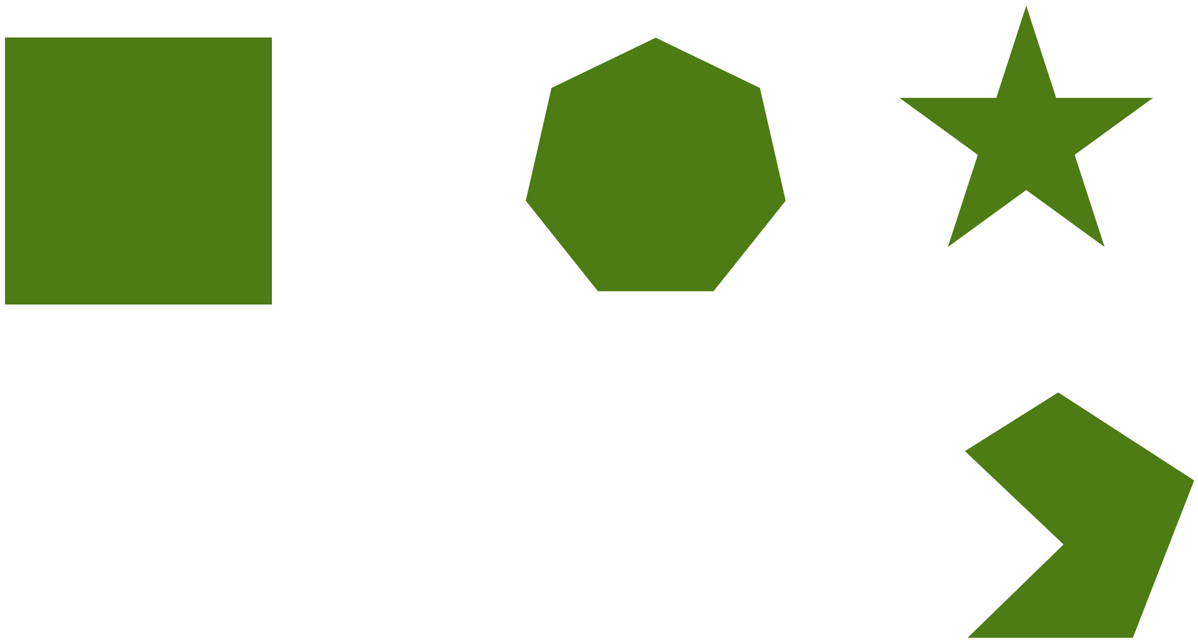
Convex Hulls

- What is a Convex Set?



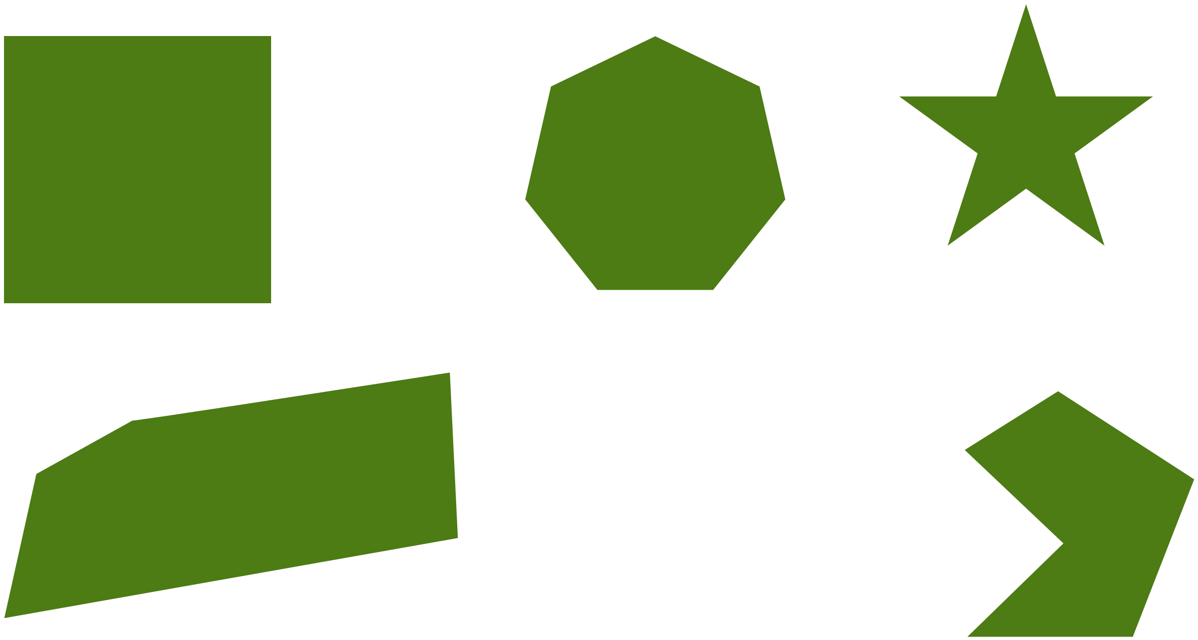
Convex Hulls

- What is a Convex Set?



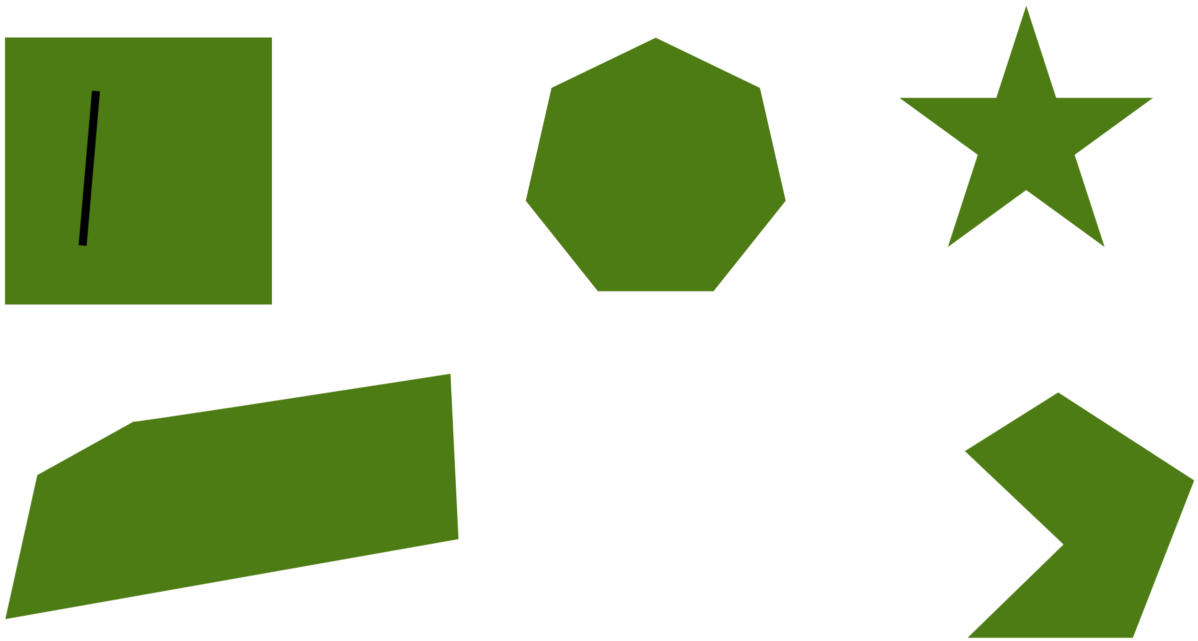
Convex Hulls

- What is a Convex Set?



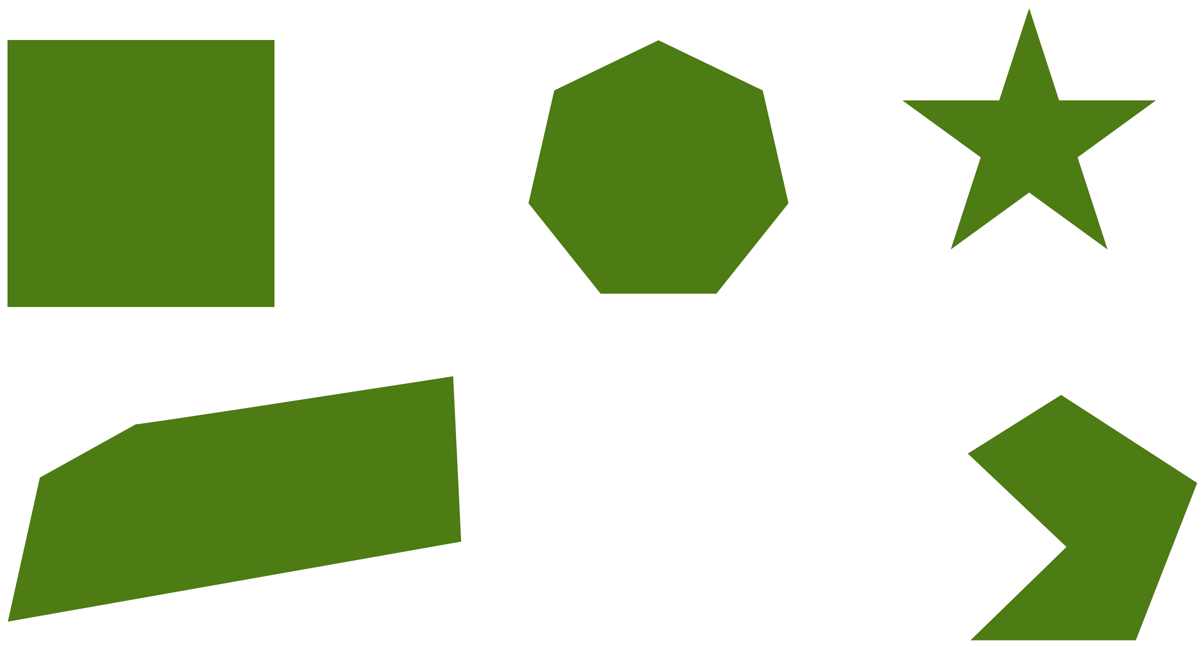
Convex Hulls

- What is a Convex Set?



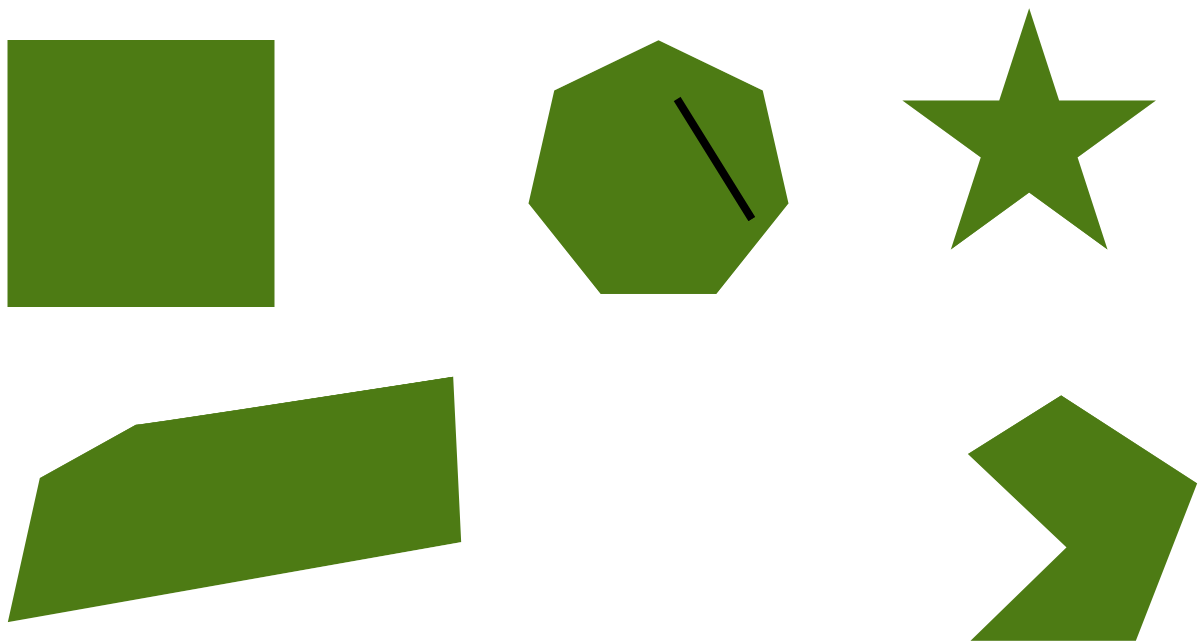
Convex Hulls

- What is a Convex Set?



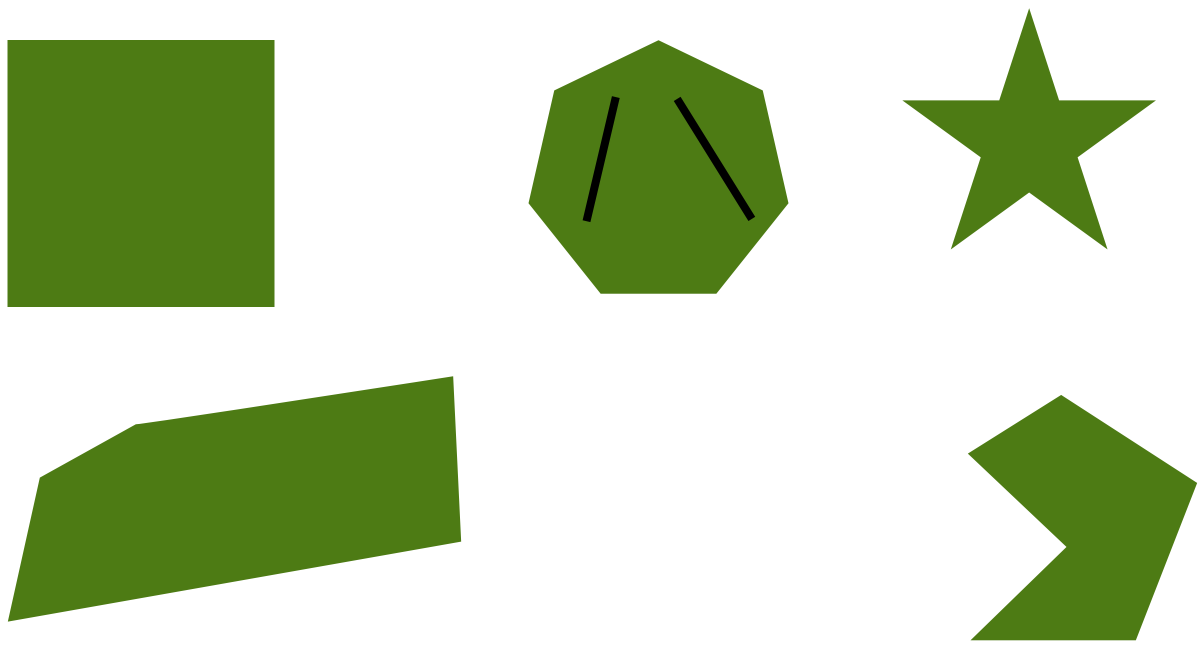
Convex Hulls

- What is a Convex Set?



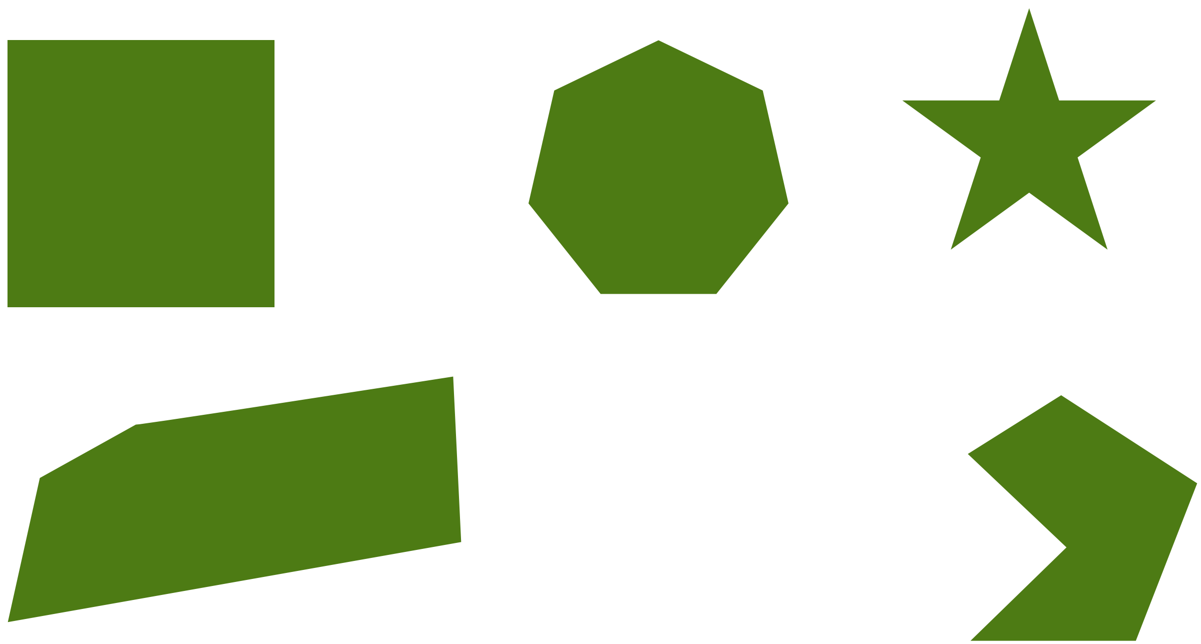
Convex Hulls

- What is a Convex Set?



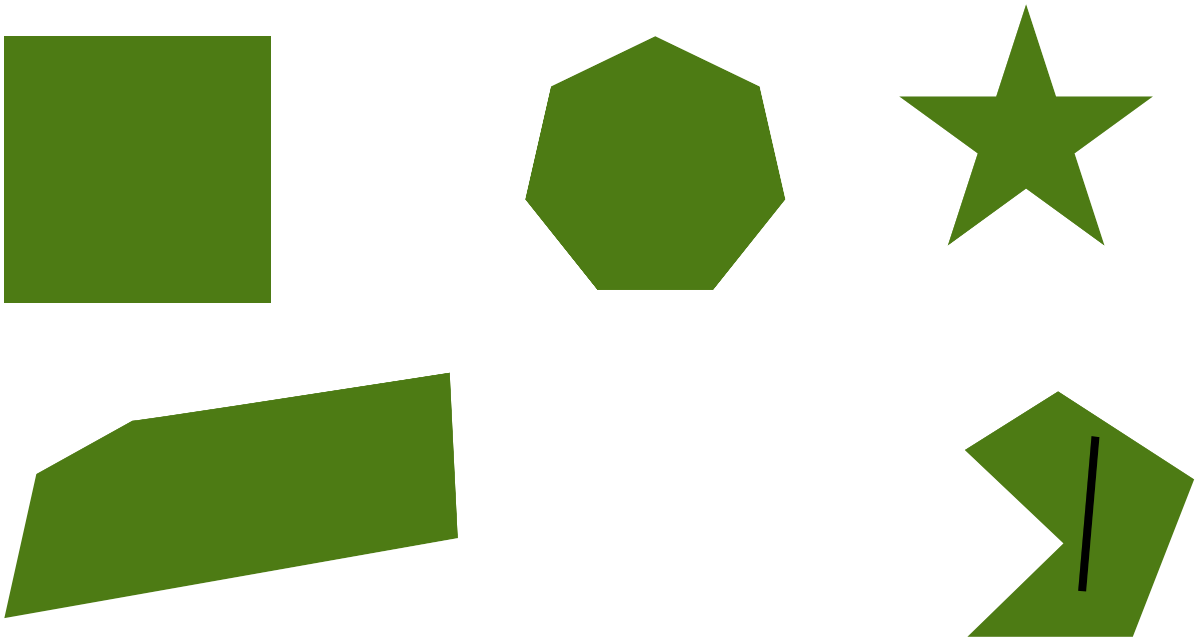
Convex Hulls

- What is a Convex Set?



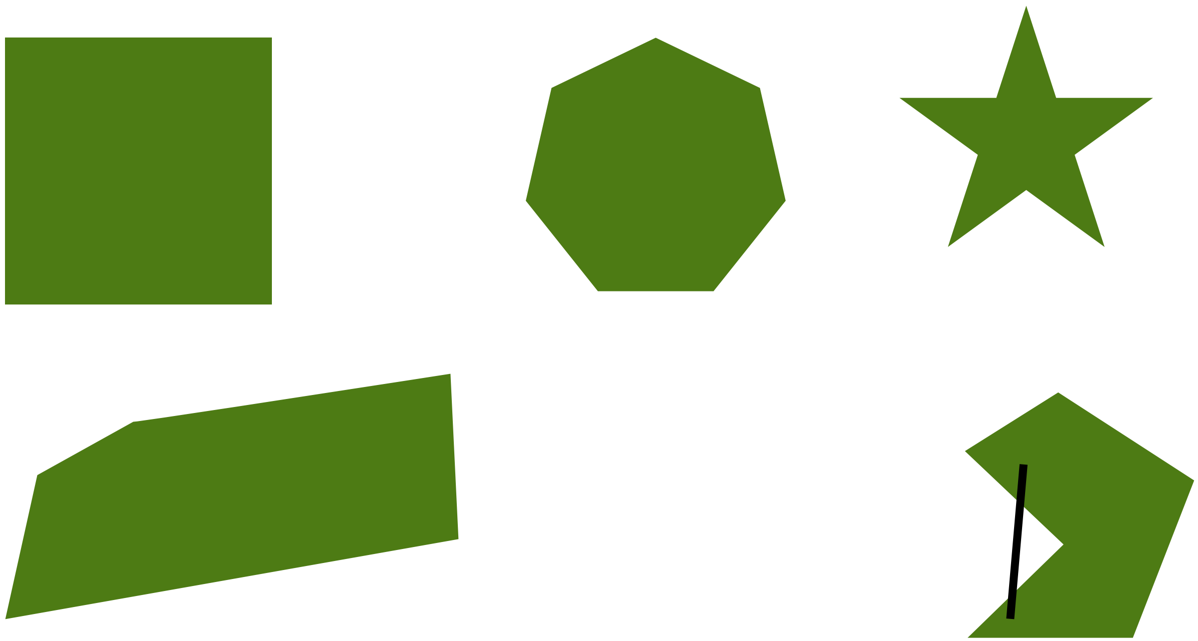
Convex Hulls

- What is a Convex Set?



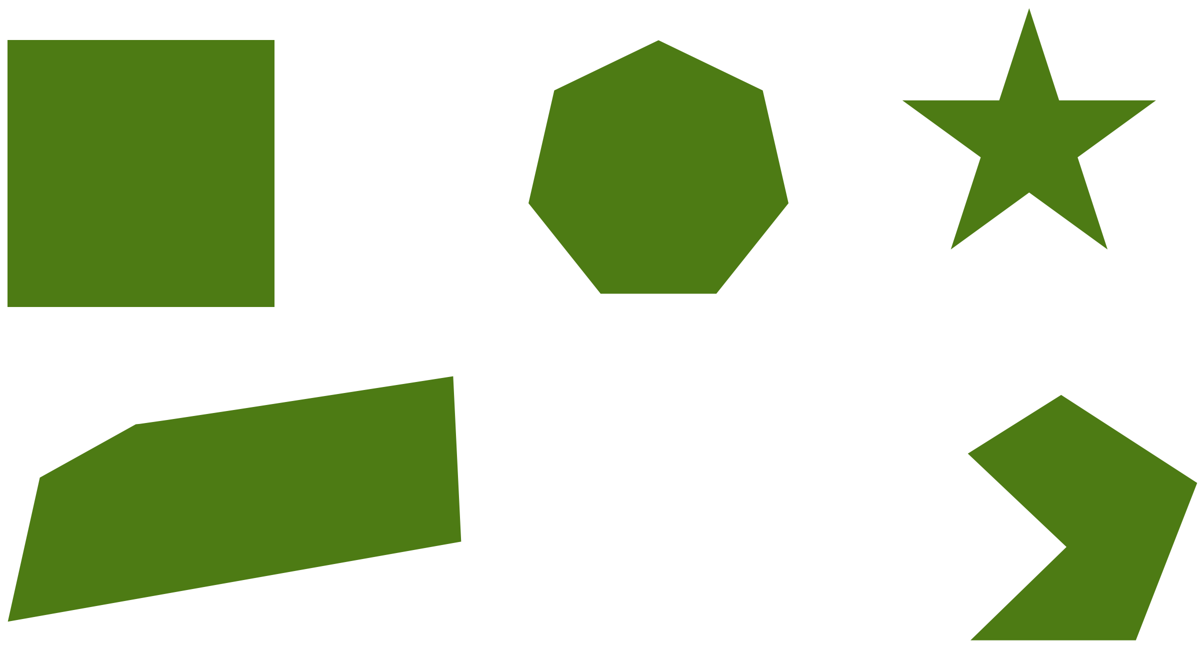
Convex Hulls

- What is a Convex Set?



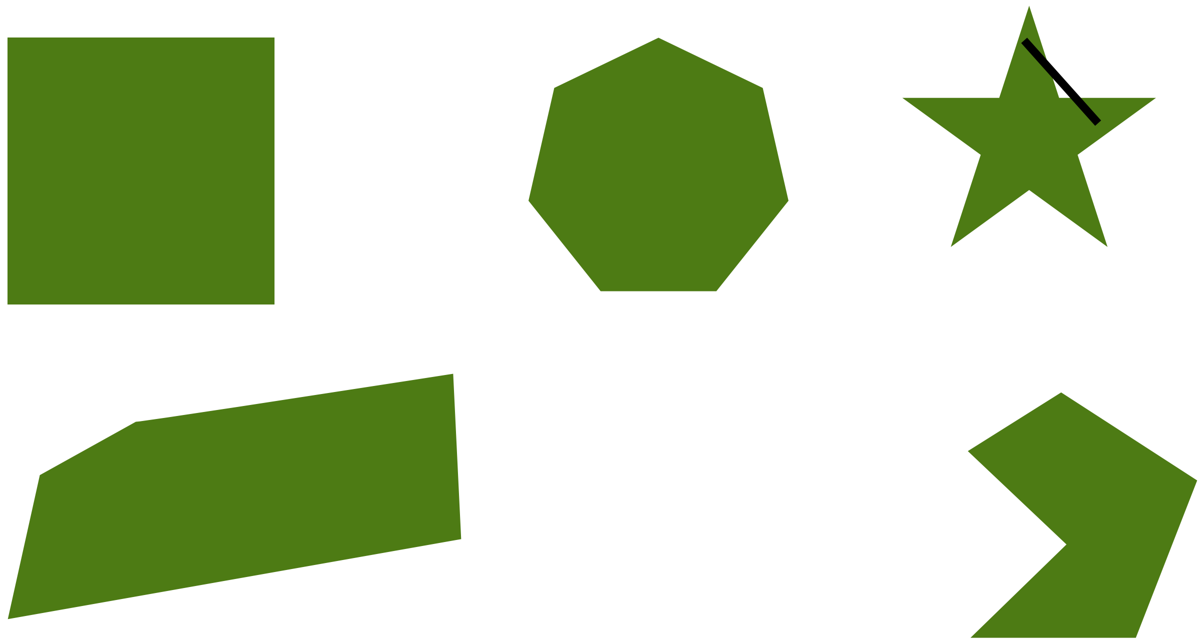
Convex Hulls

- What is a Convex Set?



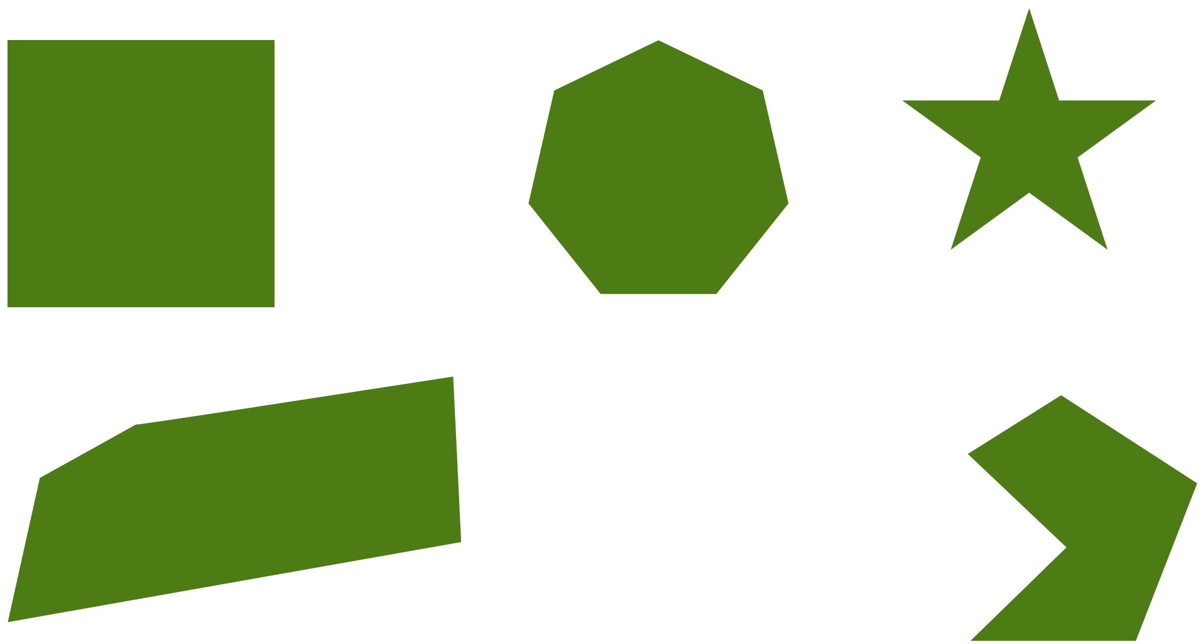
Convex Hulls

- What is a Convex Set?



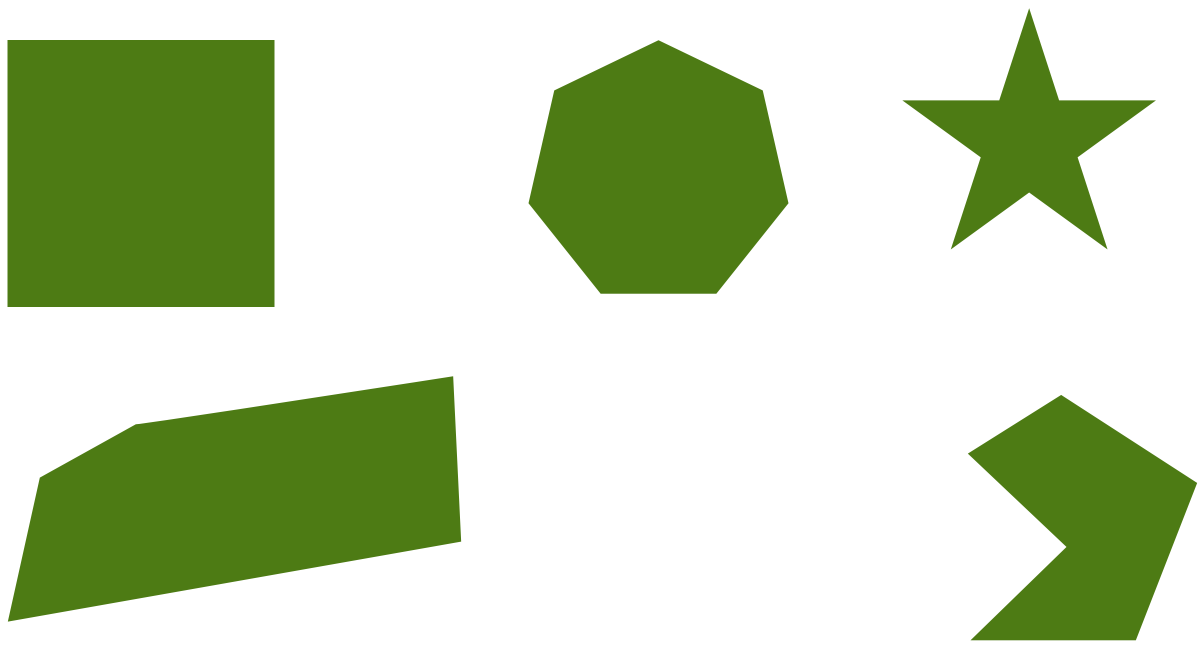
Convex Hulls

- What is a Convex Set?



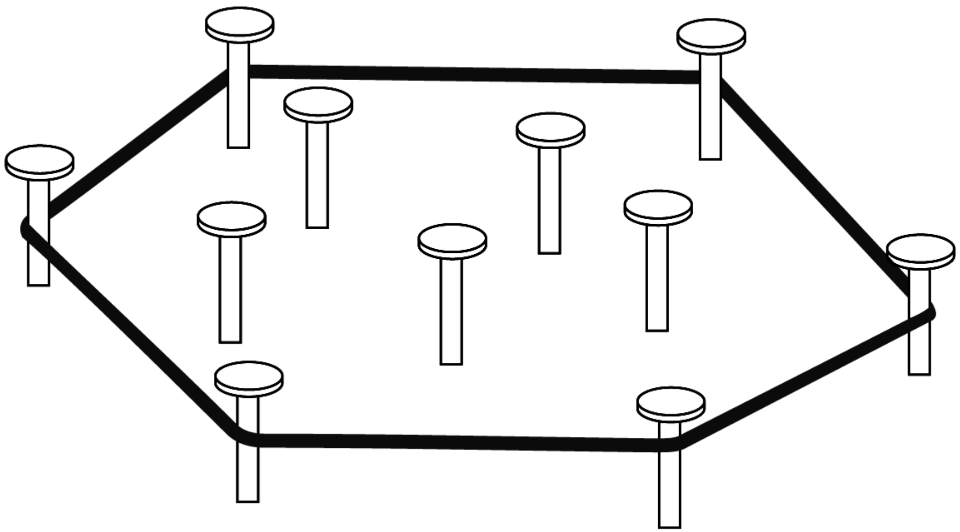
Convex Hulls

A set of points C is *convex* iff ∀ a, b ∈ C, all points on the line segment ab are entirely in C



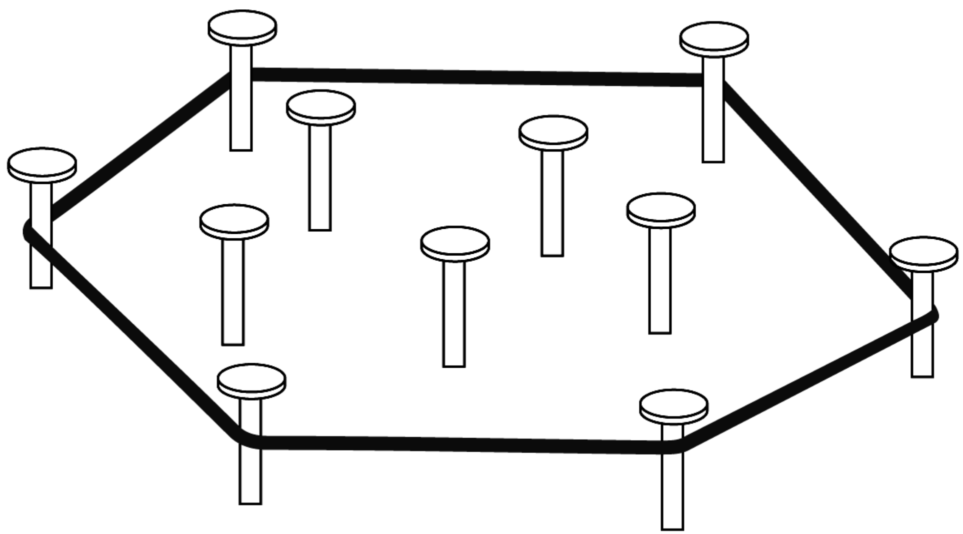
Portland State UNIVERSITY

Convex Hulls



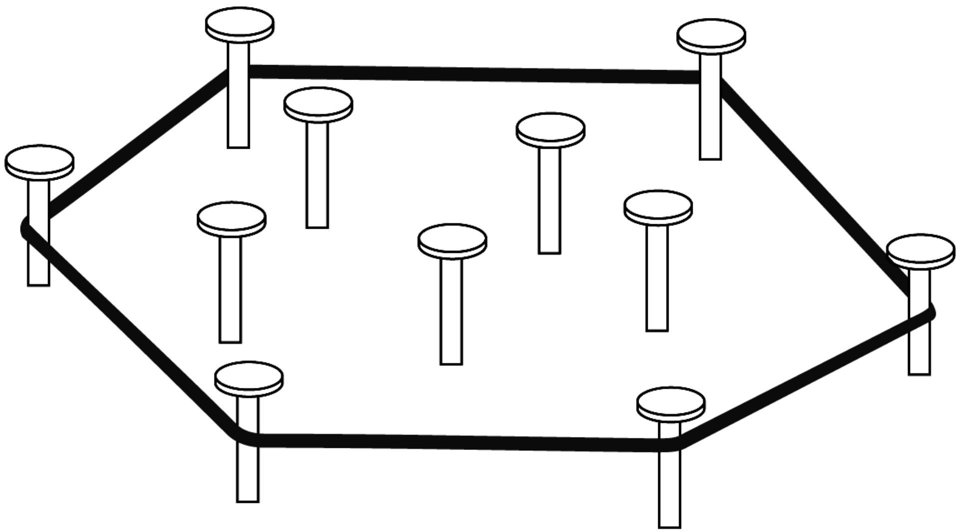
Convex Hulls

- Given an arbitrary set of points S, the convex hull of S is the smallest convex set that contain all the points in S.



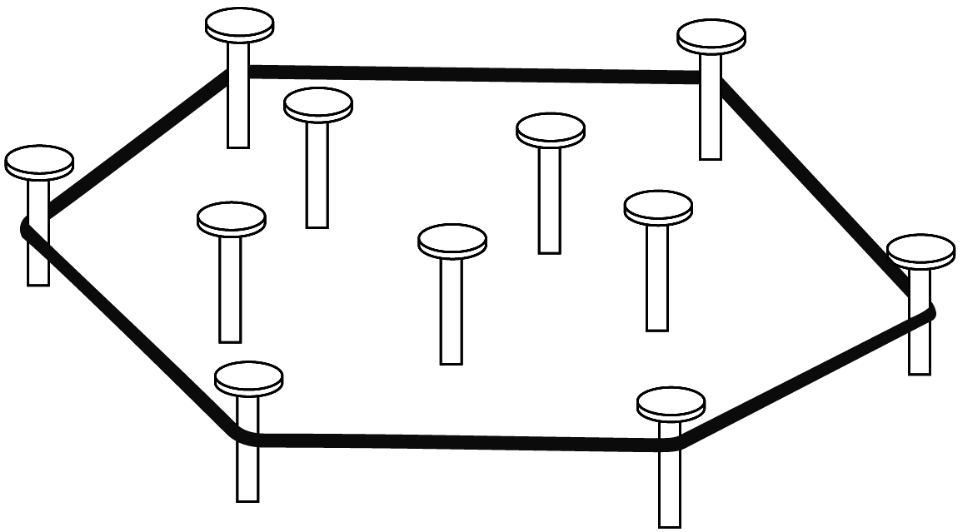
Convex Hulls

- Given an arbitrary set of points S, the convex hull of S is the smallest convex set that contain all the points in S.
    - Barricading sleeping tigers



Convex Hulls

- Given an arbitrary set of points S, the convex hull of S is the smallest convex set that contain all the points in S.
  - Barricading sleeping tigers
  - Rubber-band around nails

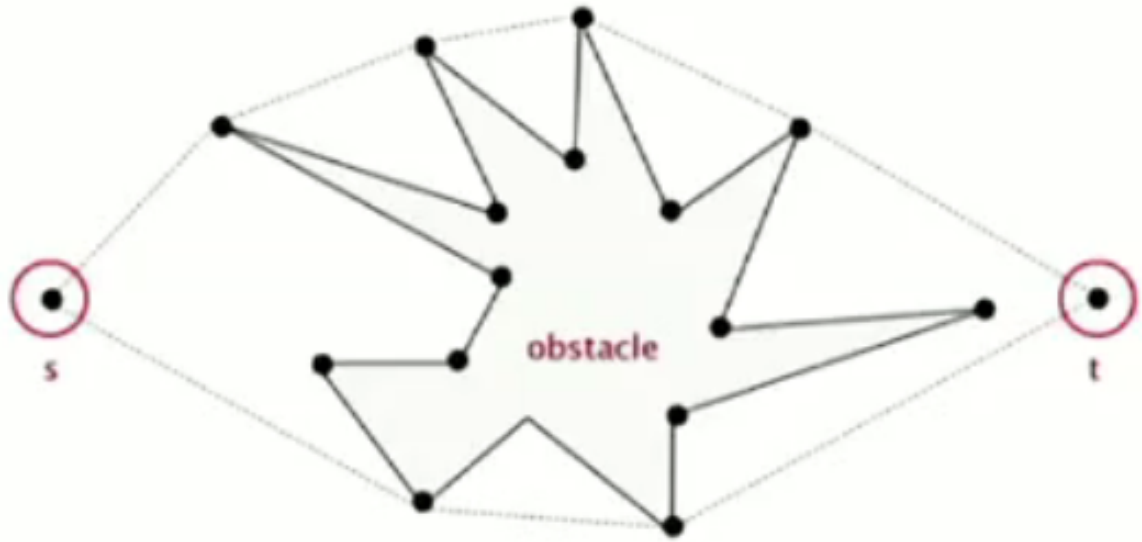


Applications of Convex Hull

- Collision-detection in video games

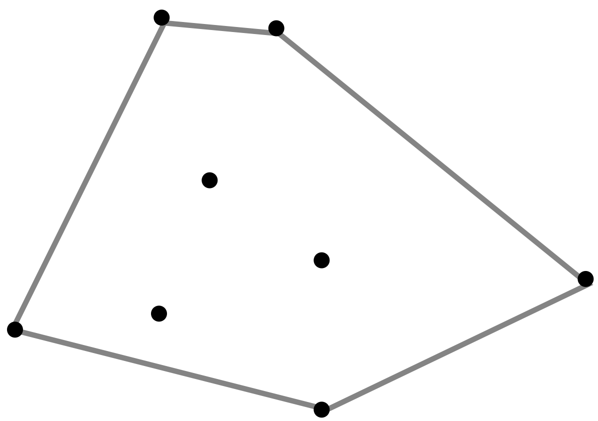
Applications of Convex Hull

- Collision-detection in video games
- Robot motion planning



Theorems about Convex Hulls

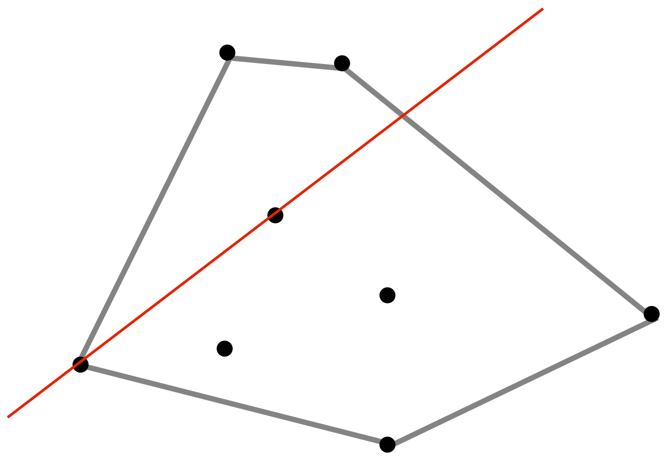
- The convex hull of a set S is a convex polygon all of whose vertices are at some of the points of S.
- A line segment ab is part of the boundary of the convex hull of S iff all the points of S lie *on the same side* of ab (or *on* ab)



Portland State UNIVERSITY

Theorems about Convex Hulls

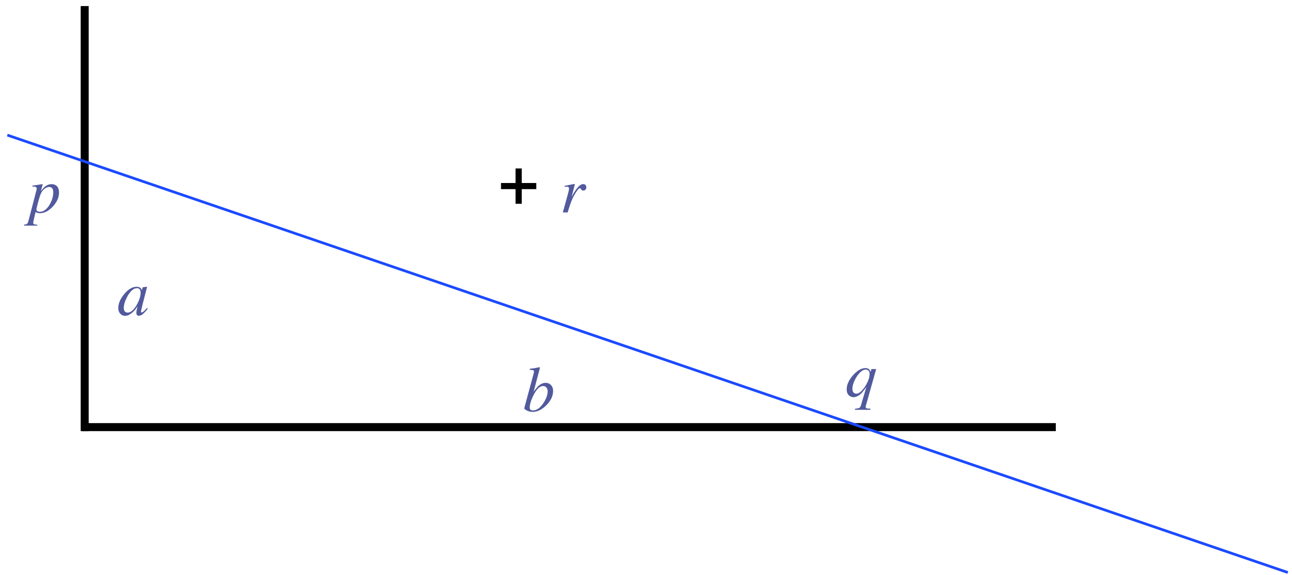
- The convex hull of a set S is a convex polygon all of whose vertices are at some of the points of S.
- A line segment ab is part of the boundary of the convex hull of S iff all the points of S lie *on the same side* of ab (or *on* ab)



Portland State UNIVERSITY

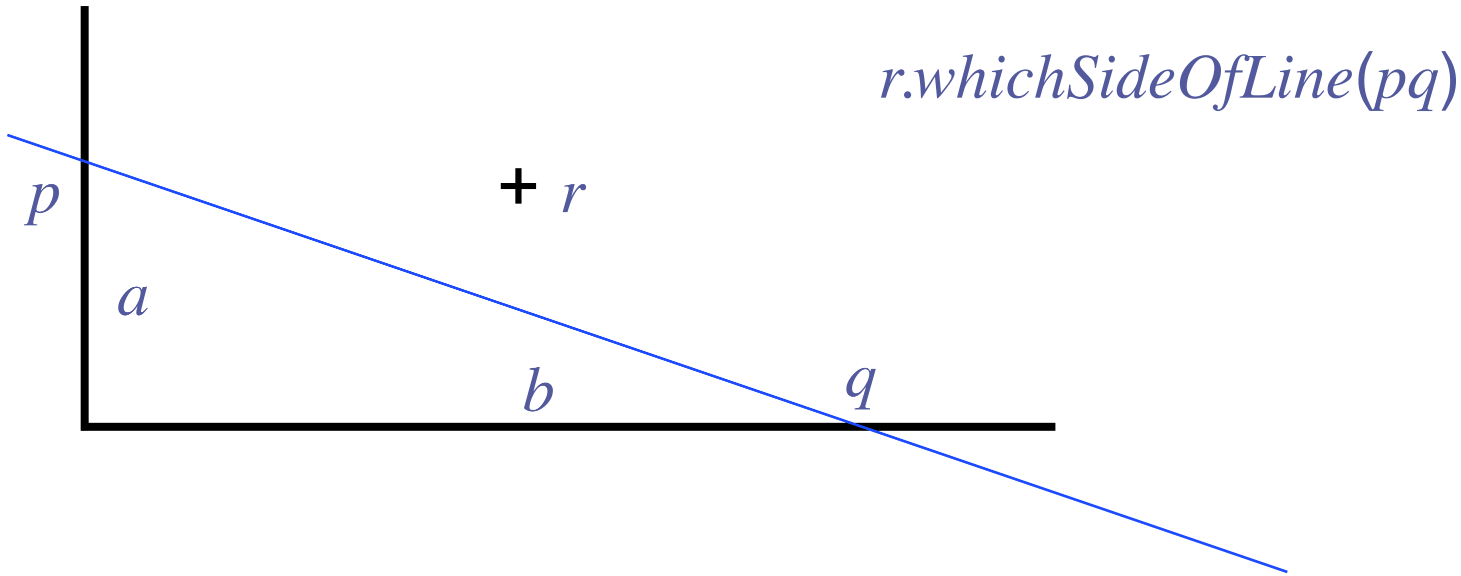
Brute-Force Algorithm for Convex Hull

- write it down!
  - Assume that you have a method for ascertaining if a point r is on a line pq, on the –ve side of line pq, or on the +ve side of pq



Brute-Force Algorithm for Convex Hull

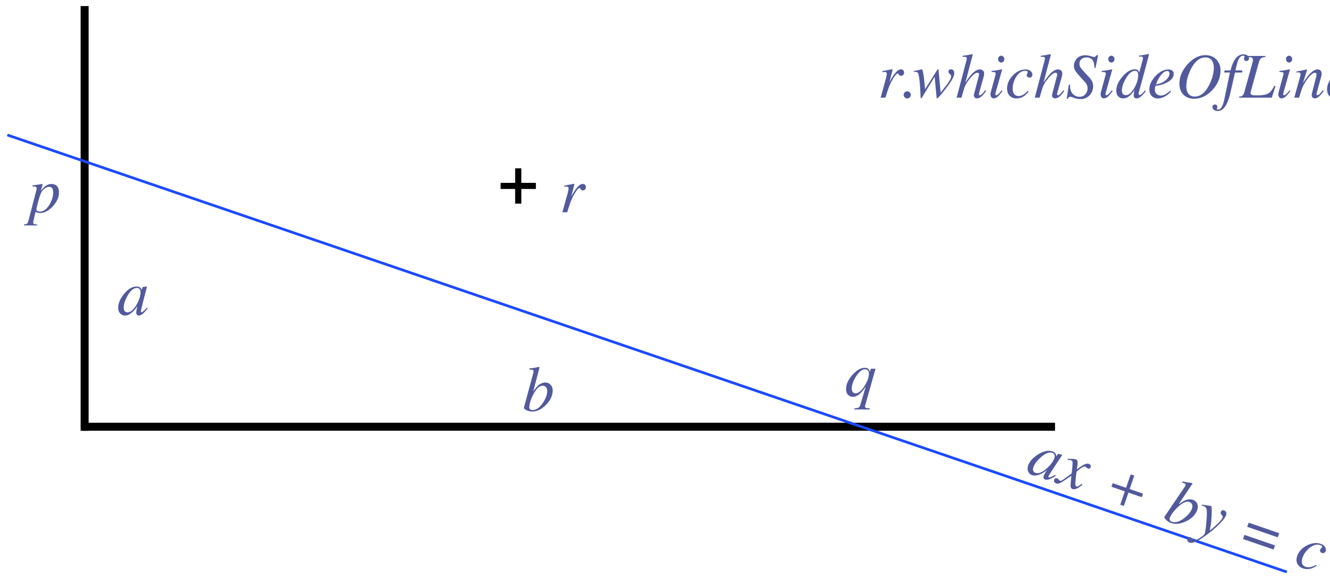
- write it down!
  - Assume that you have a method for ascertaining if a point r is on a line pq, on the –ve side of line pq, or on the +ve side of pq



Brute-Force Algorithm for Convex Hull

- write it down!
  - Assume that you have a method for ascertaining if a point r is on a line pq, on the –ve side of line pq, or on the +ve side of pq

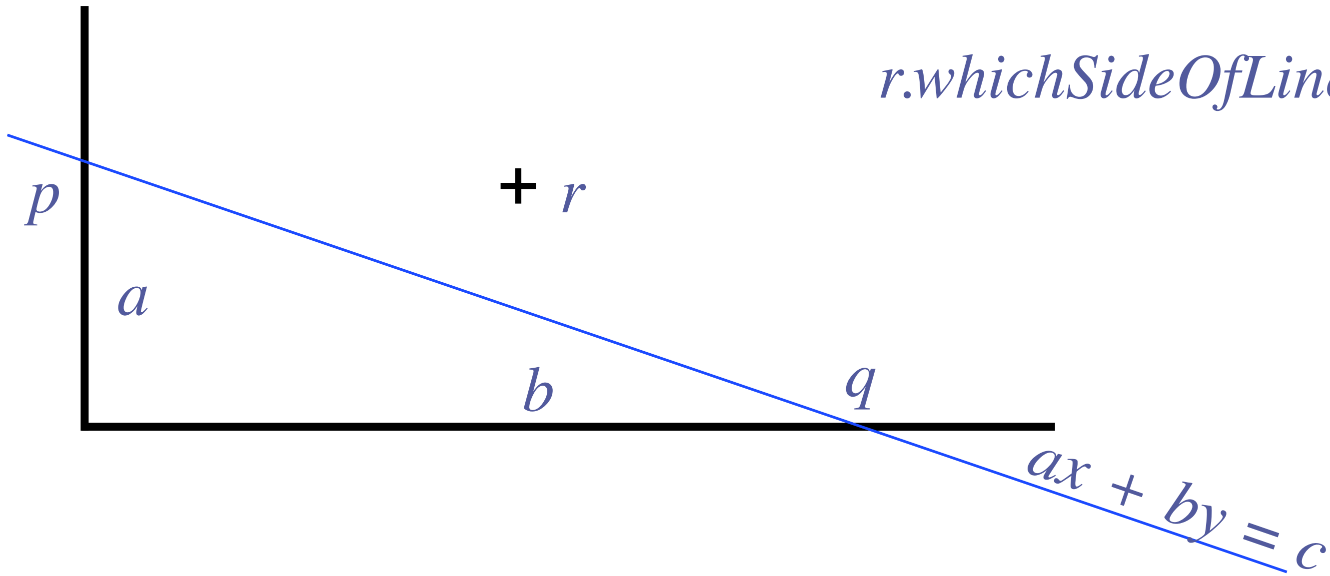
$r.whichSideOfLine(pq)$



Brute-Force Algorithm for Convex Hull

- write it down!
  - Assume that you have a method for ascertaining if a point r is on a line pq, on the –ve side of line pq, or on the +ve side of pq

$$r.whichSideOfLine(pq)$$



$$c = p_x q_y - q_x p_y$$

Brute-Force Algorithm for Convex Hull

```
edgeSet ← {}
P: for p in S do:
   Q: for q in S, q ≠ p do:
      goodSide ← 0
      R: for r in S, r≠p ∧ r≠q do:
         side ← r.whichSideOfLine(pq)
         if  goodSide = – side then exit Q.
         if  goodSide = 0 then goodSide ← side
      edgeSet ← edgeSet ∪ {pq}
```

Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time *only* on *very small* instances
- In some cases, there are *much* better alternatives!
    - Euler circuits
    - shortest paths
    - minimum spanning tree
    - assignment problem
- However, in many cases, exhaustive search (or a variation) is the only known way to find an exact solution

Searching in Graphs

Exhaustively search a graph, by traversing the edges, visiting every node once

Two approaches:

- Depth-first search and
- Breadth-first search

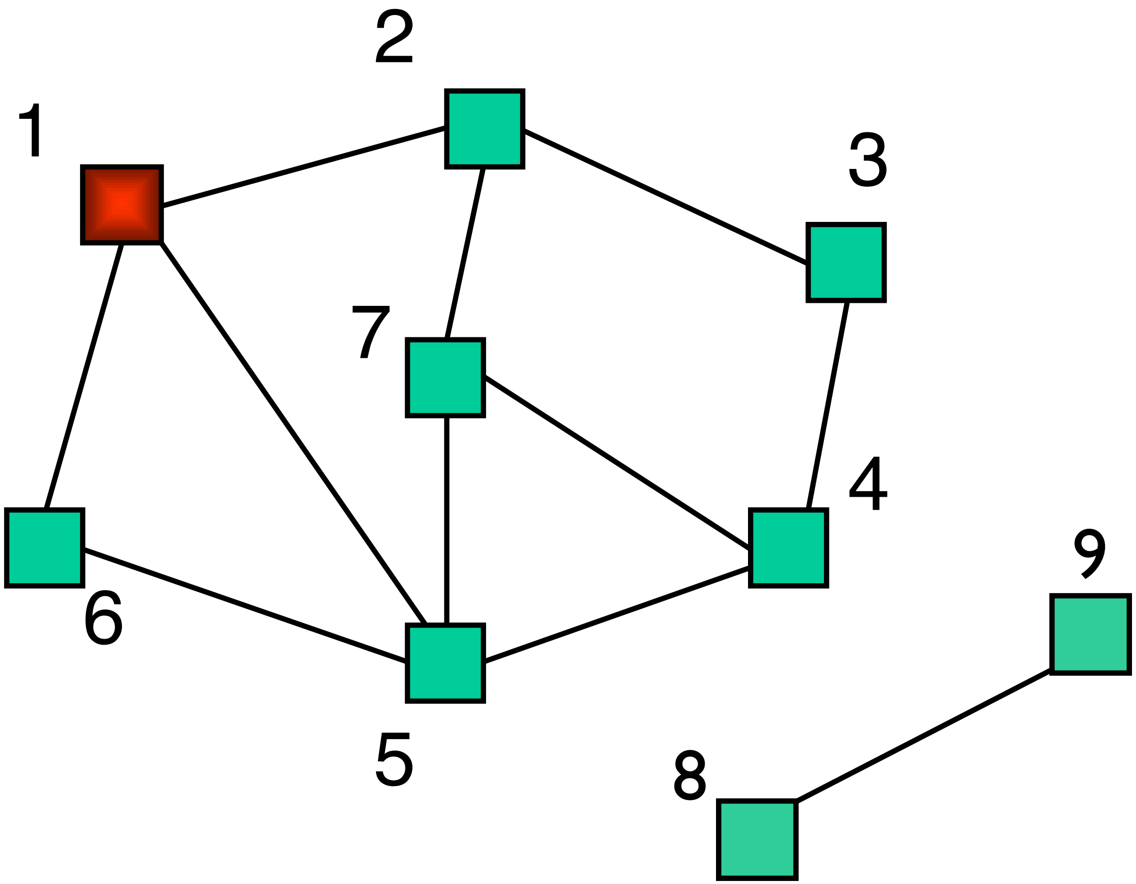
ALGORITHM $DFS(G)$

//Implements a depth-first search traversal of a given graph
//Input: Graph $G = \langle V, E \rangle$
//Output: Graph G with its vertices marked with consecutive integers
// in the order they are first encountered by the DFS traversal
mark each vertex in V with 0 as a mark of being "unvisited"
$count \leftarrow 0$
for each vertex v in V **do**
    if v is marked with 0
        $dfs(v)$

$dfs(v)$
//visits recursively all the unvisited vertices connected to vertex v
//by a path and numbers them in the order they are encountered
//via global variable $count$
$count \leftarrow count + 1$; mark v with $count$
for each vertex w in V adjacent to v **do**
    if w is marked with 0
        $dfs(w)$

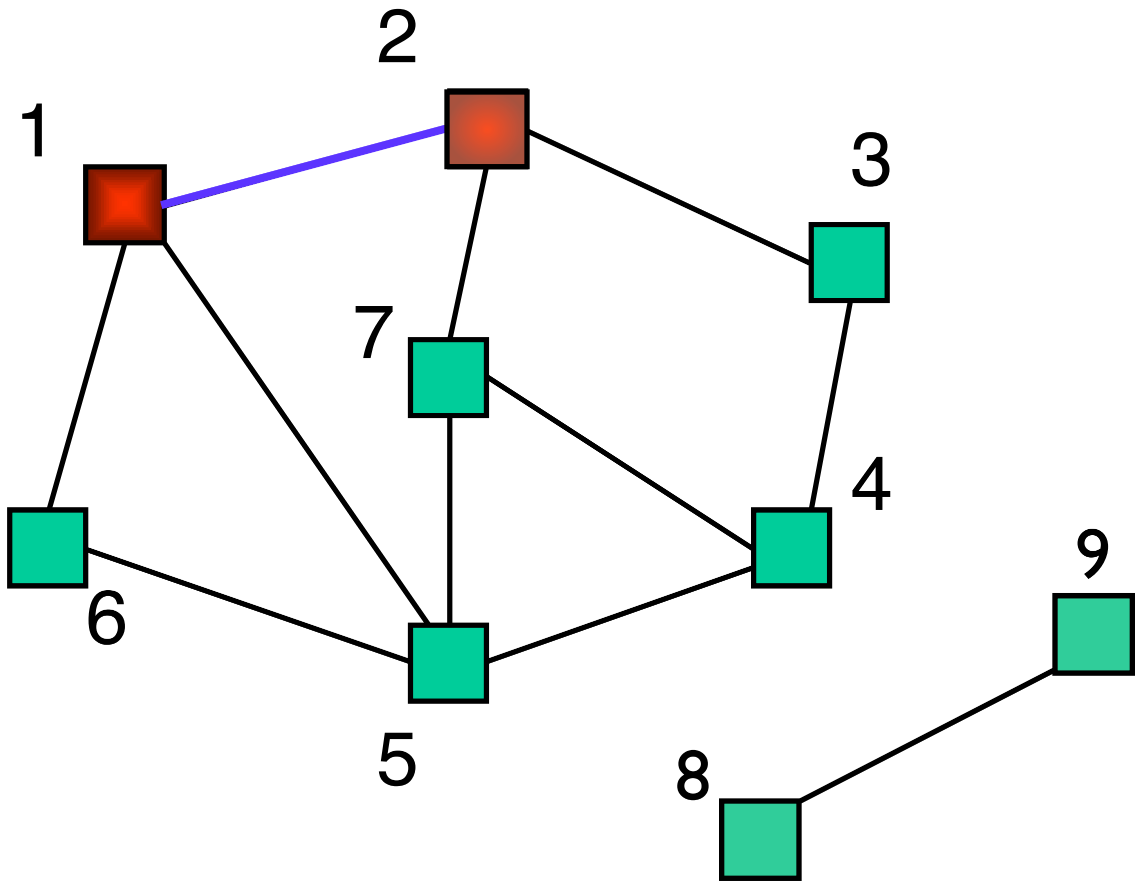
Example

$dfs(1)$



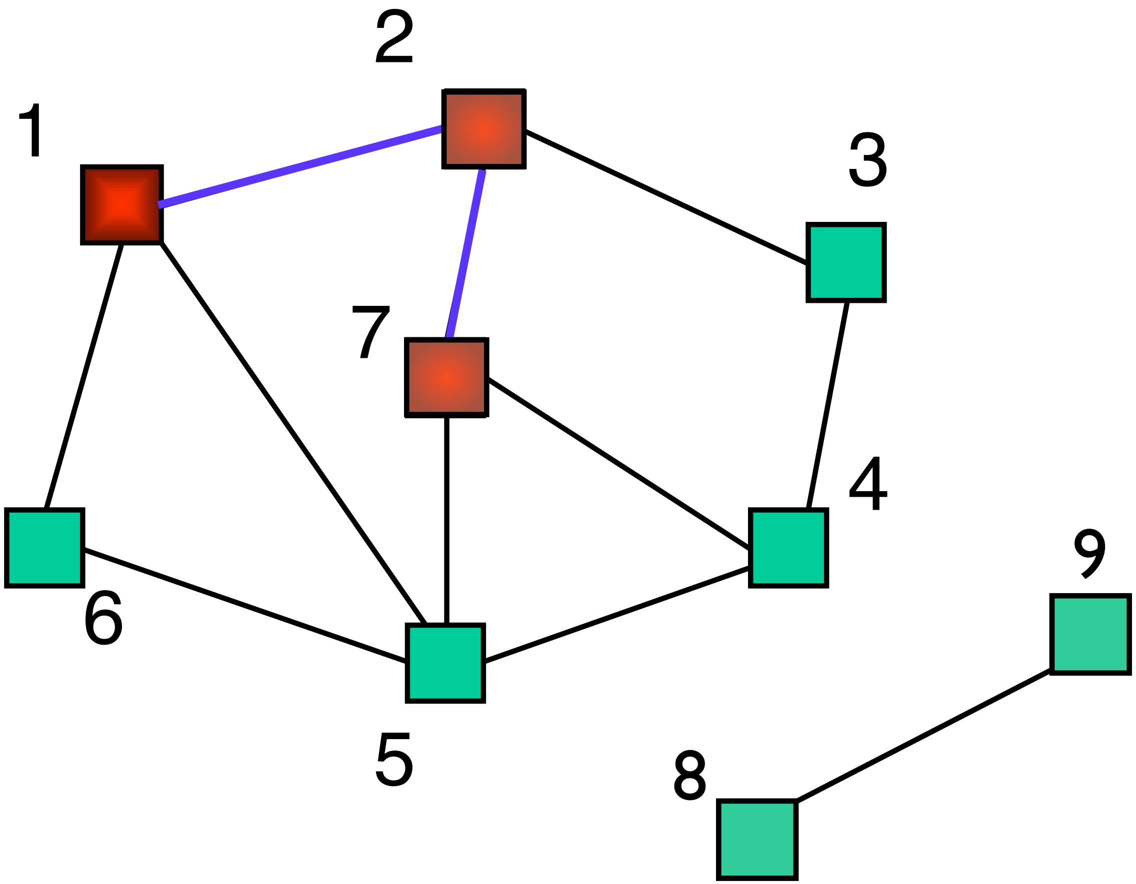
Example

$dfs(1)$
$dfs(2)$



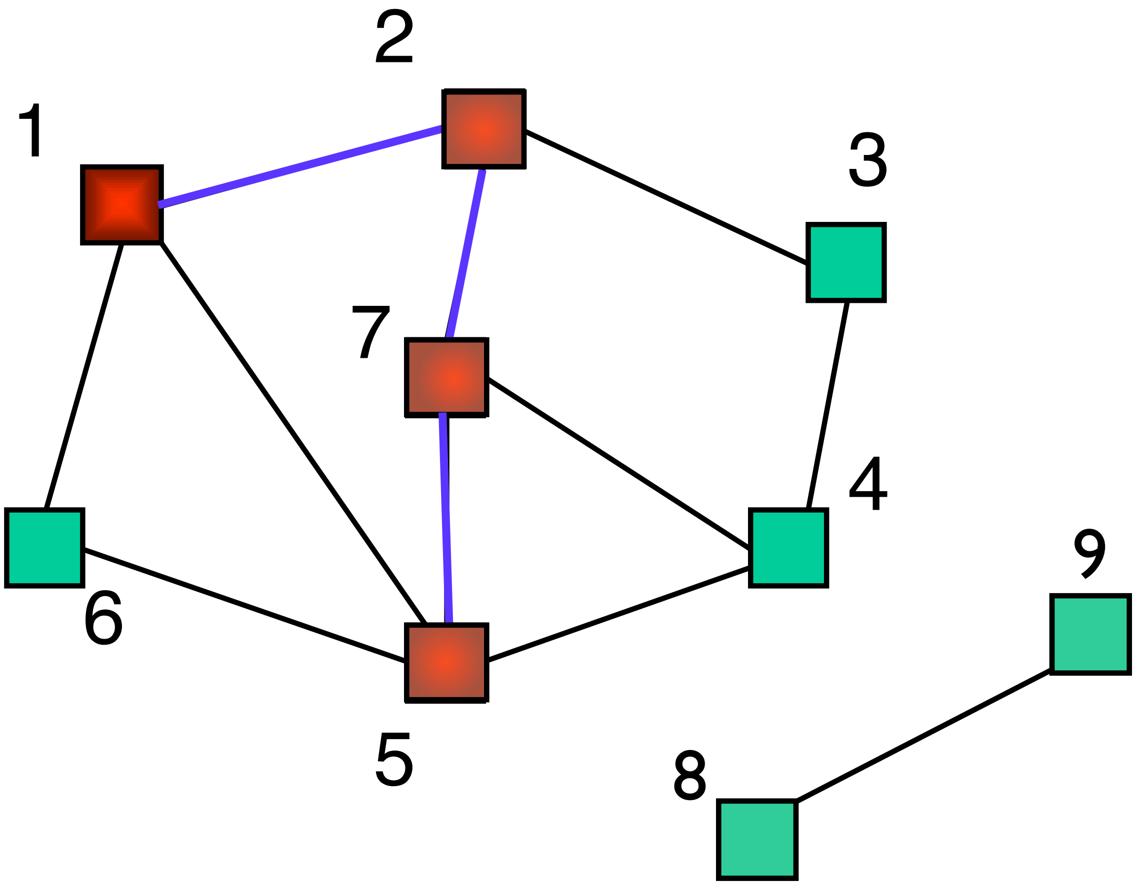
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$



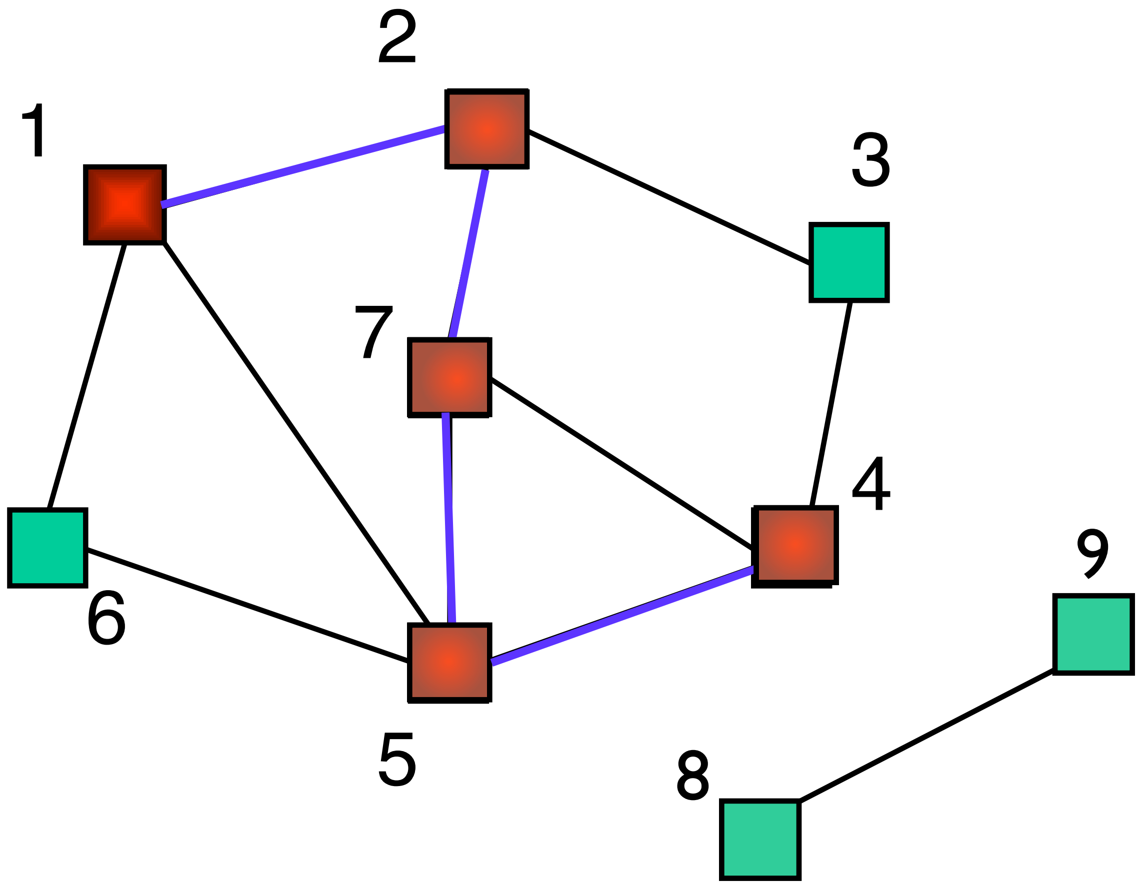
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$
$dfs(5)$



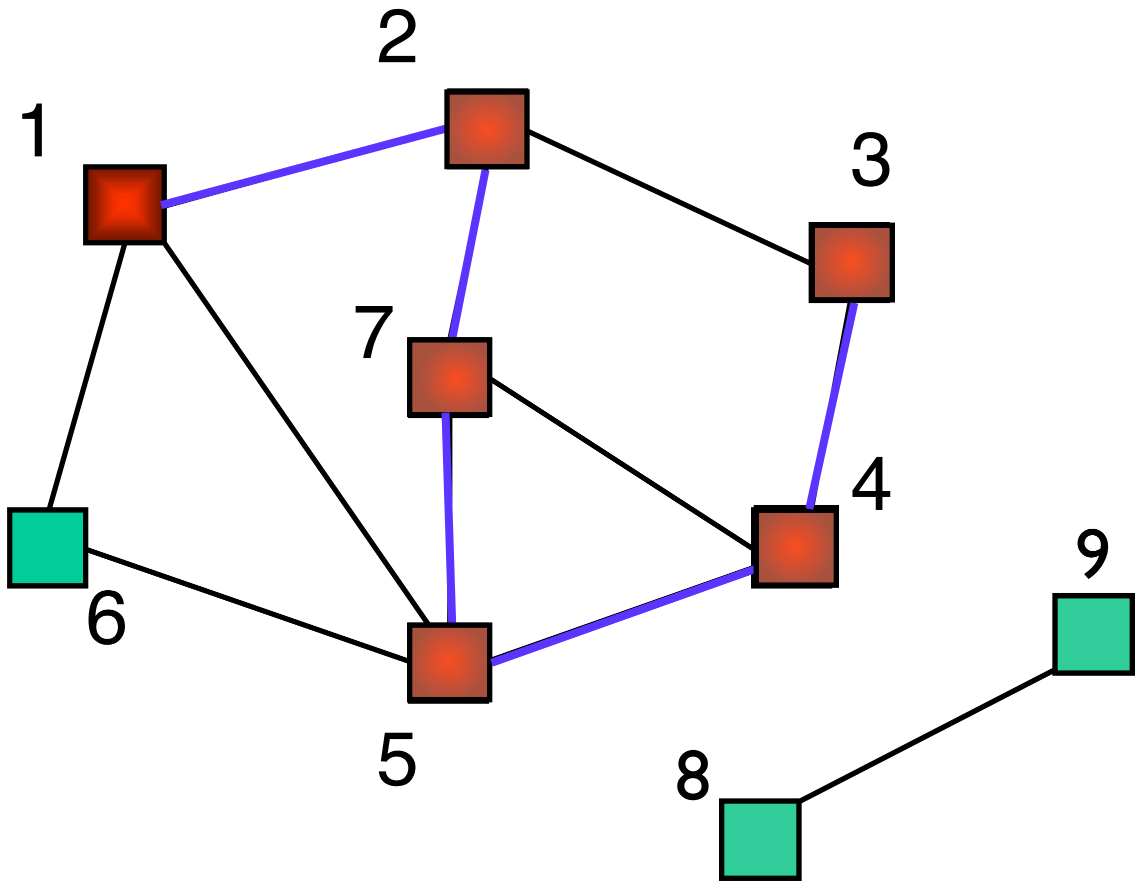
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$
$dfs(5)$
$dfs(4)$



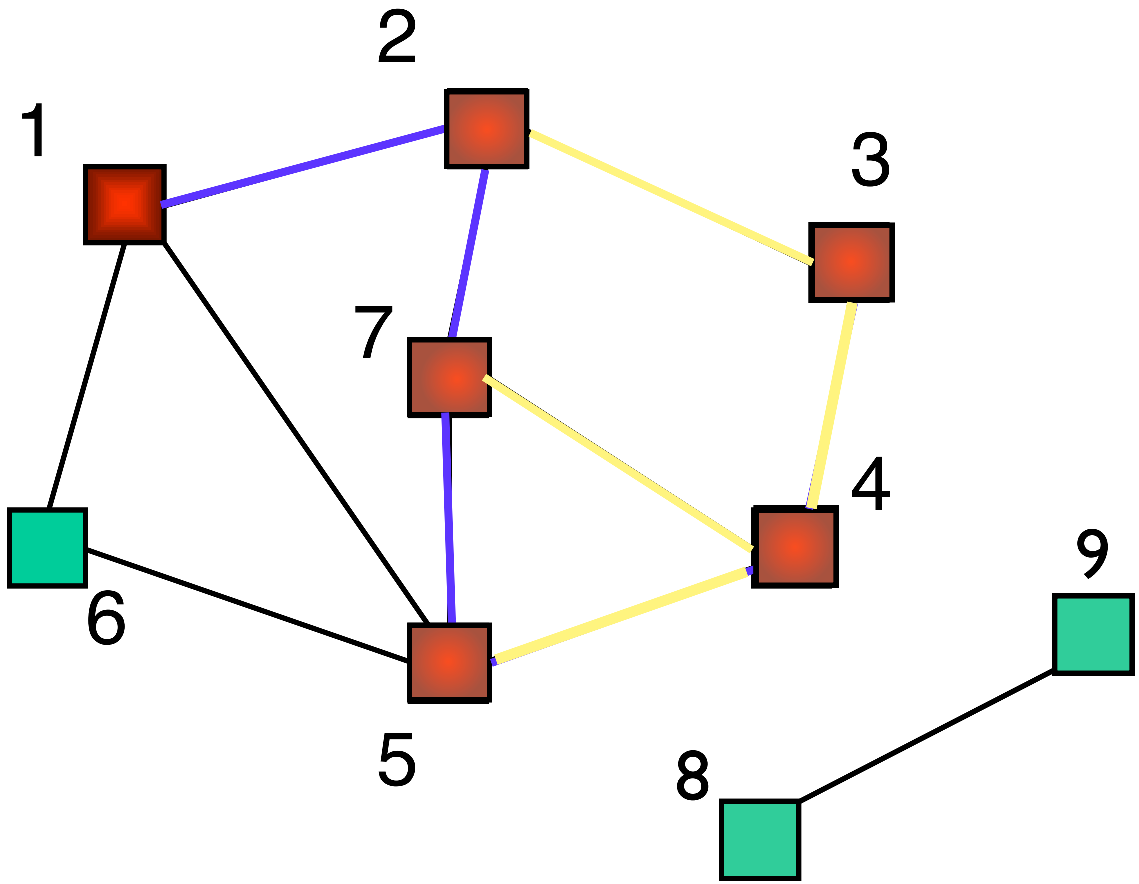
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$
$dfs(5)$
$dfs(4)$
$dfs(3)$



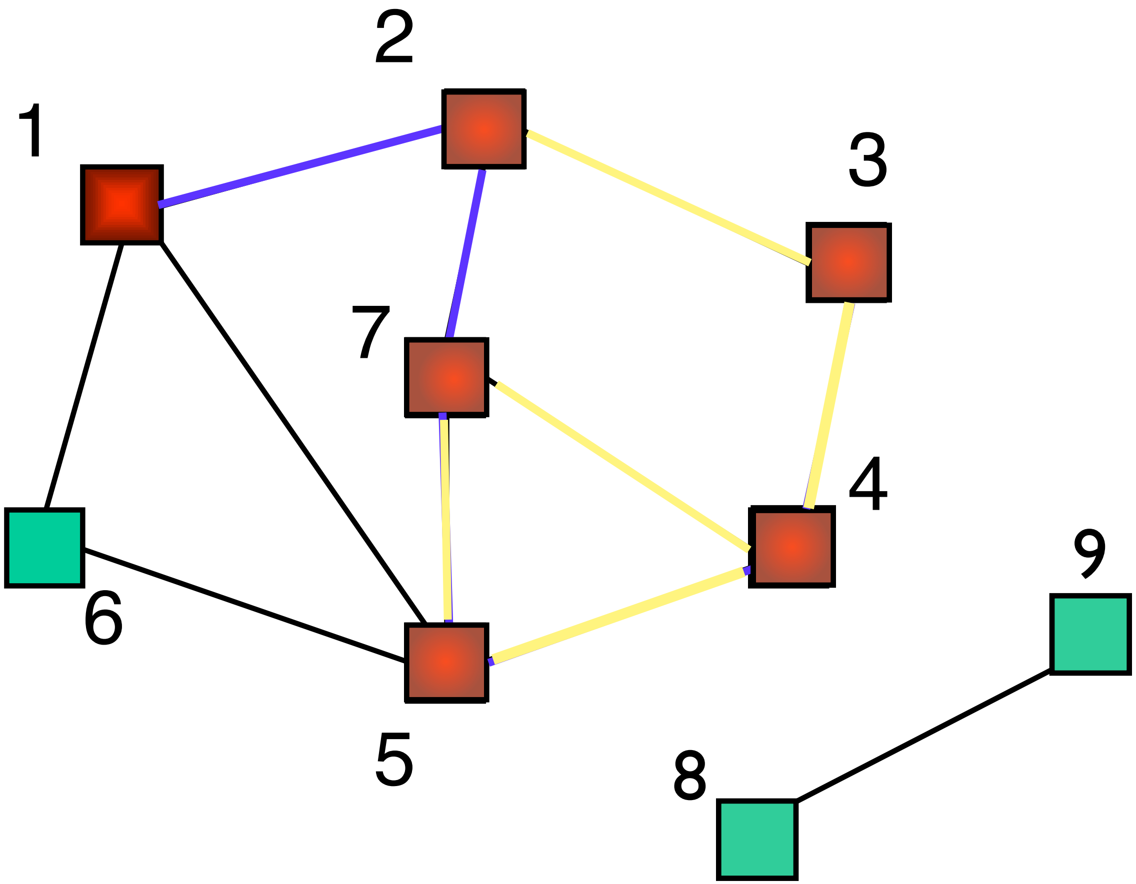
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$
$dfs(5)$
$dfs(4)$



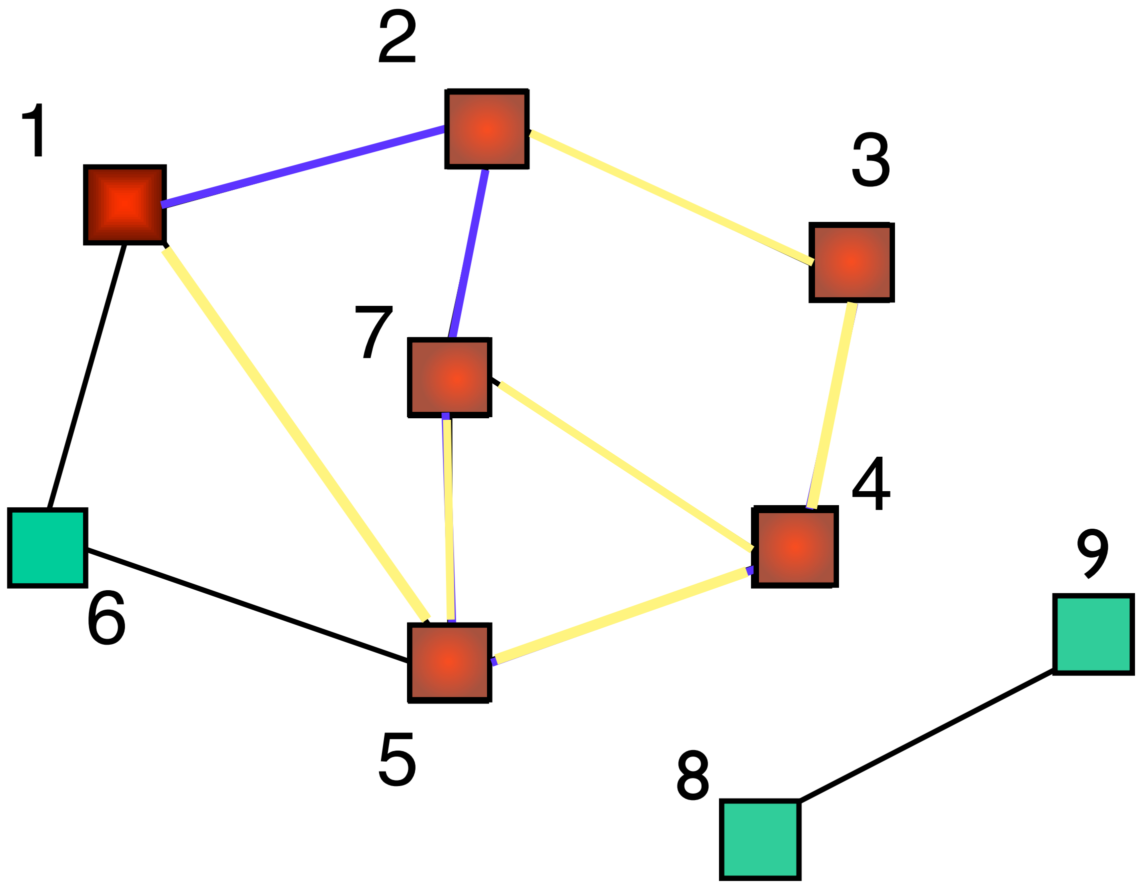
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$
$dfs(5)$



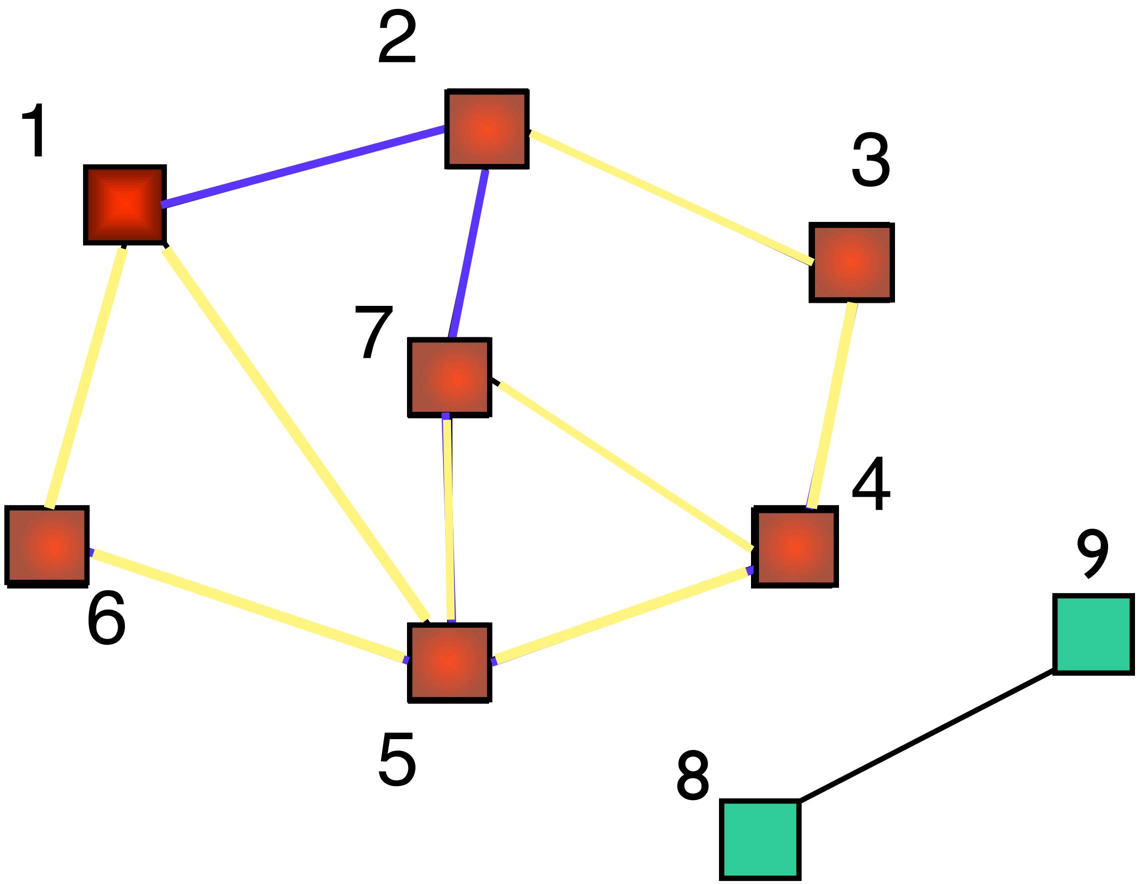
Example

$dfs(1)$
$dfs(2)$
$dfs(7)$
$dfs(5)$



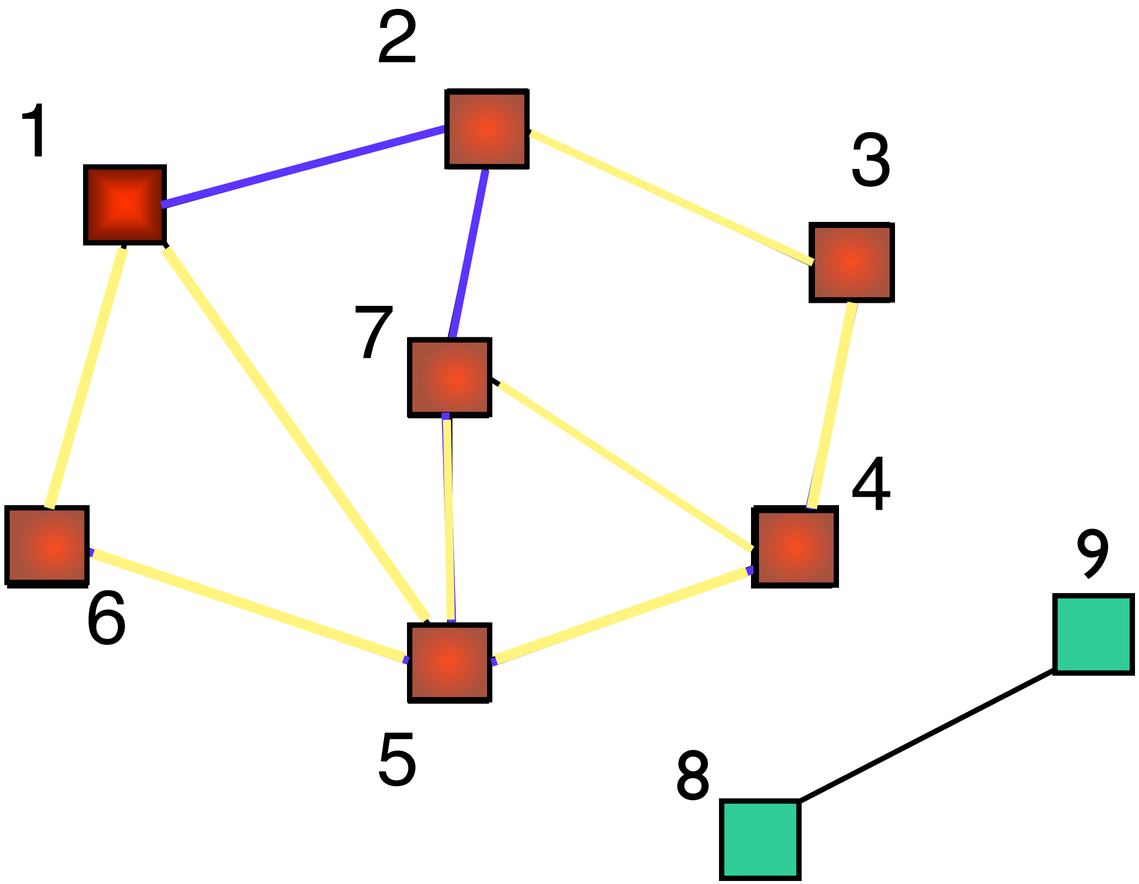
Example

dfs(1)
dfs(2)
dfs(7)
dfs(5)
dfs(6)



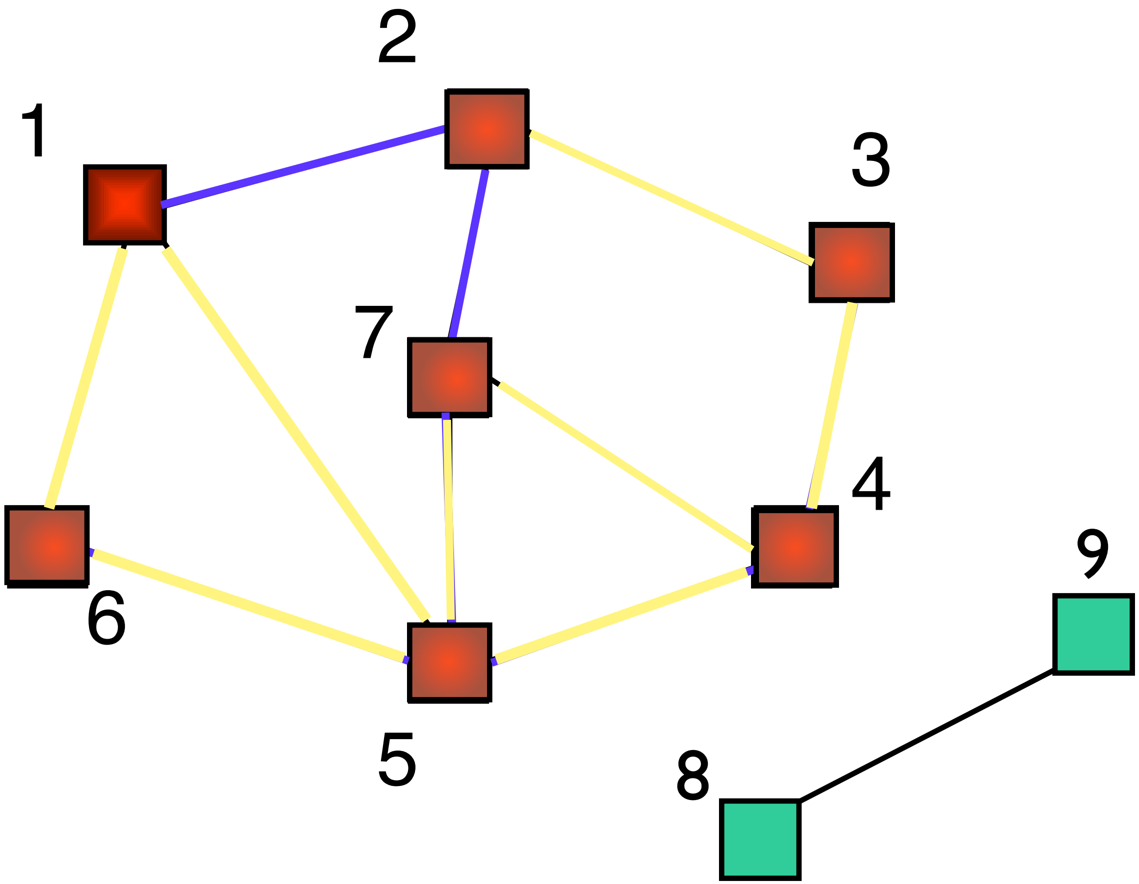
Example

dfs(1)
dfs(2)
dfs(7)
dfs(5)



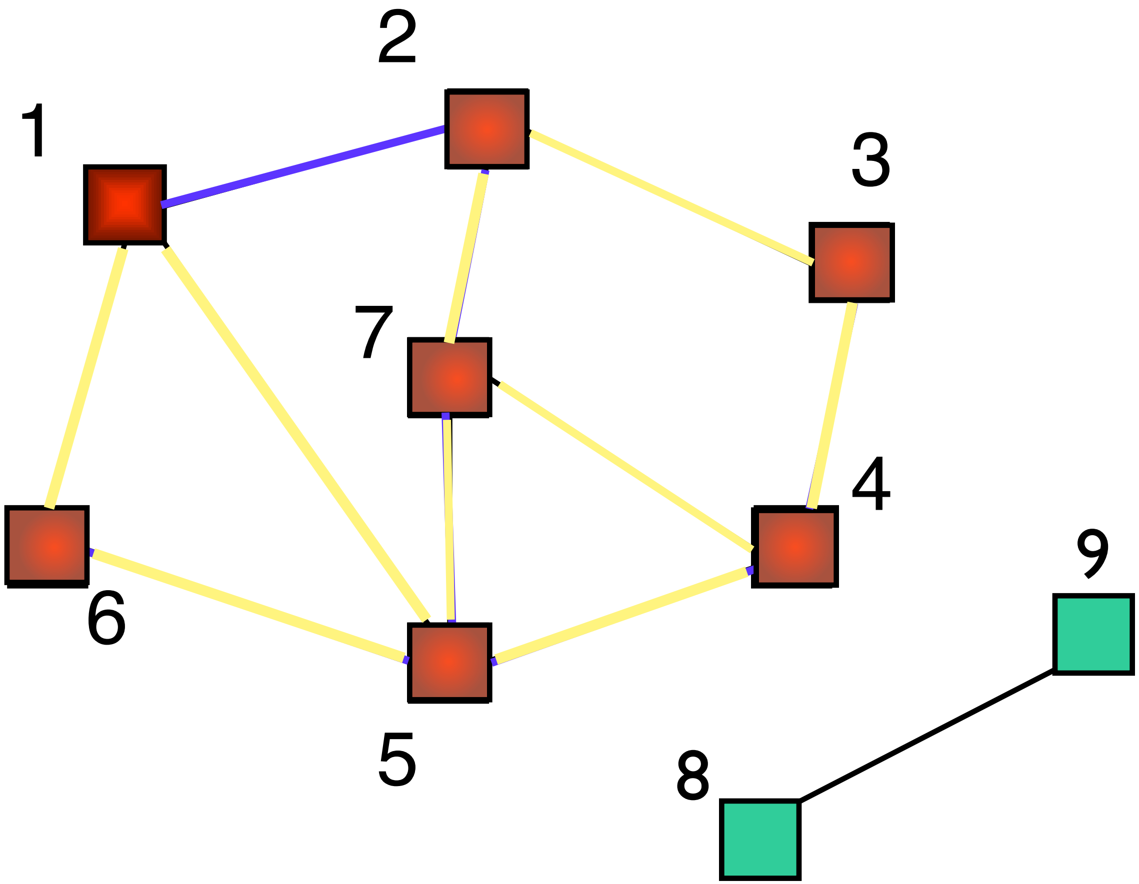
Example

dfs(1)
dfs(2)
dfs(7)



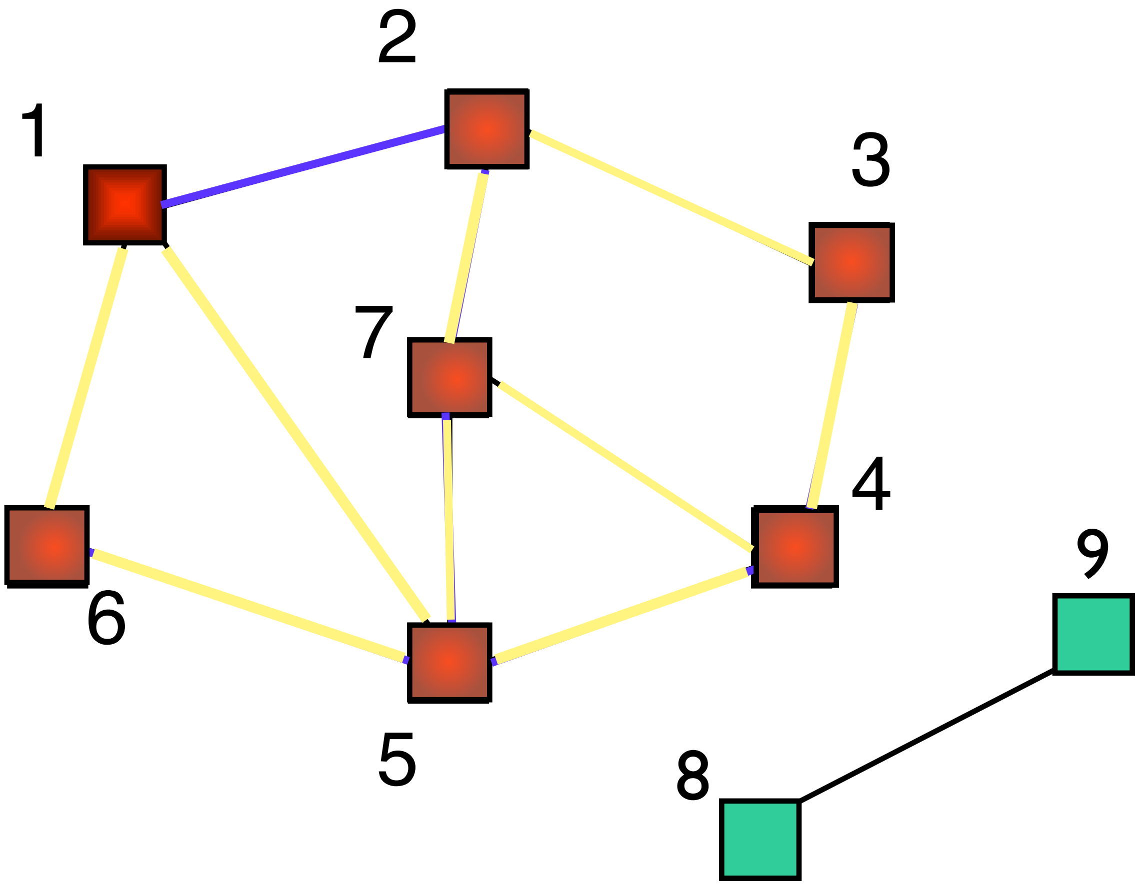
Example

dfs(1)
dfs(2)
dfs(7)



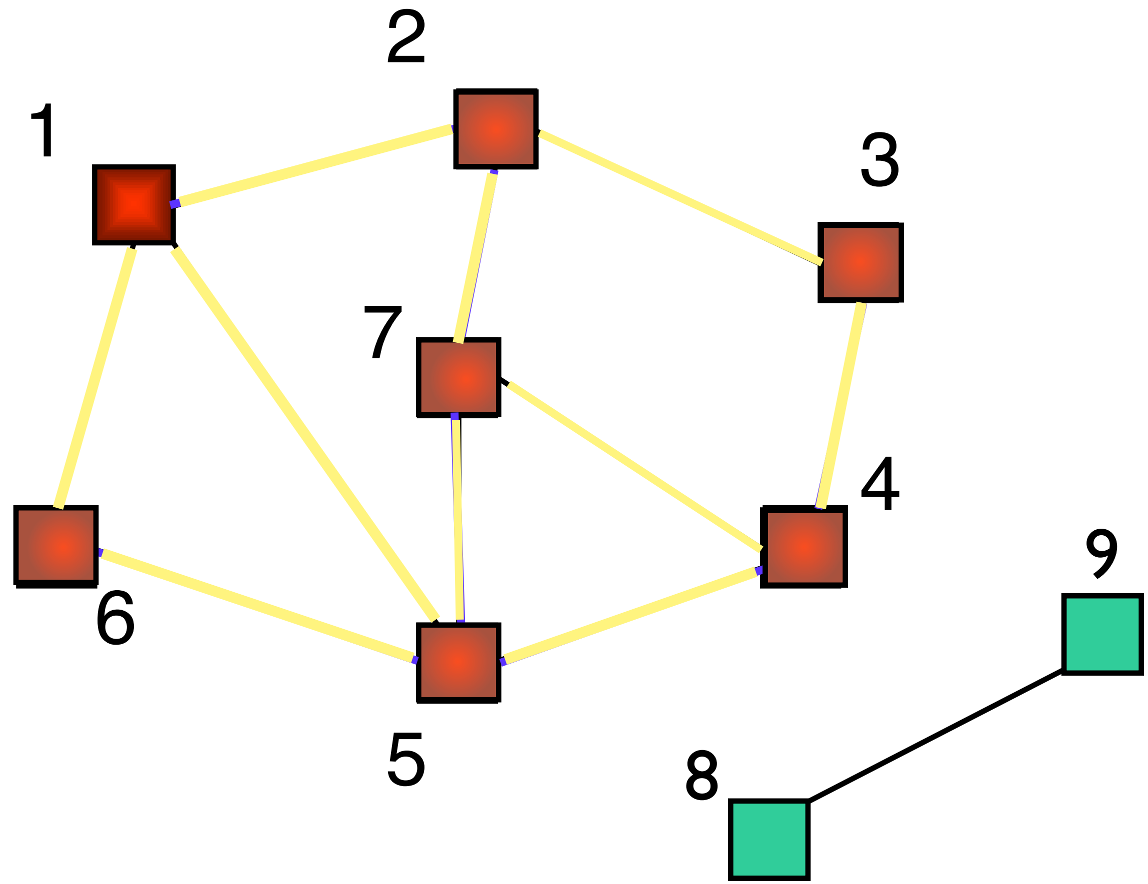
Example

$dfs(1)$
$dfs(2)$



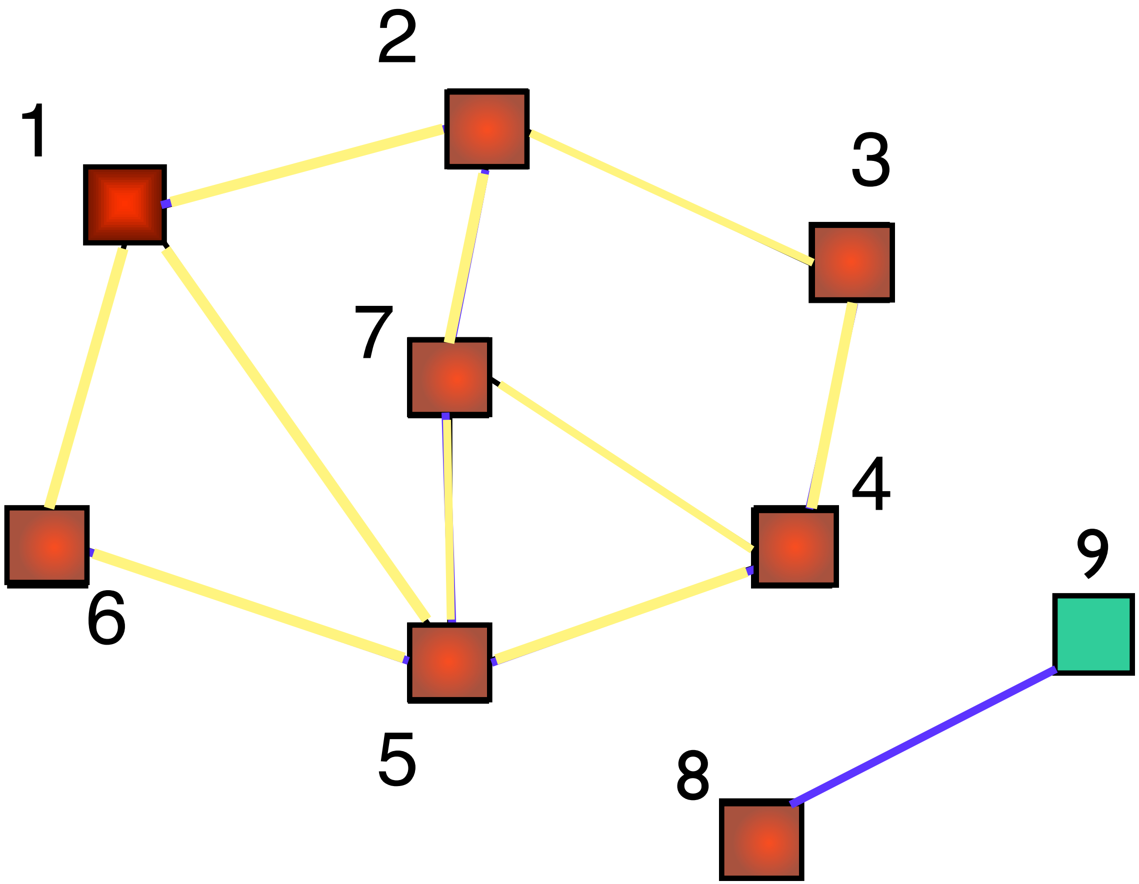
Example

dfs(1)



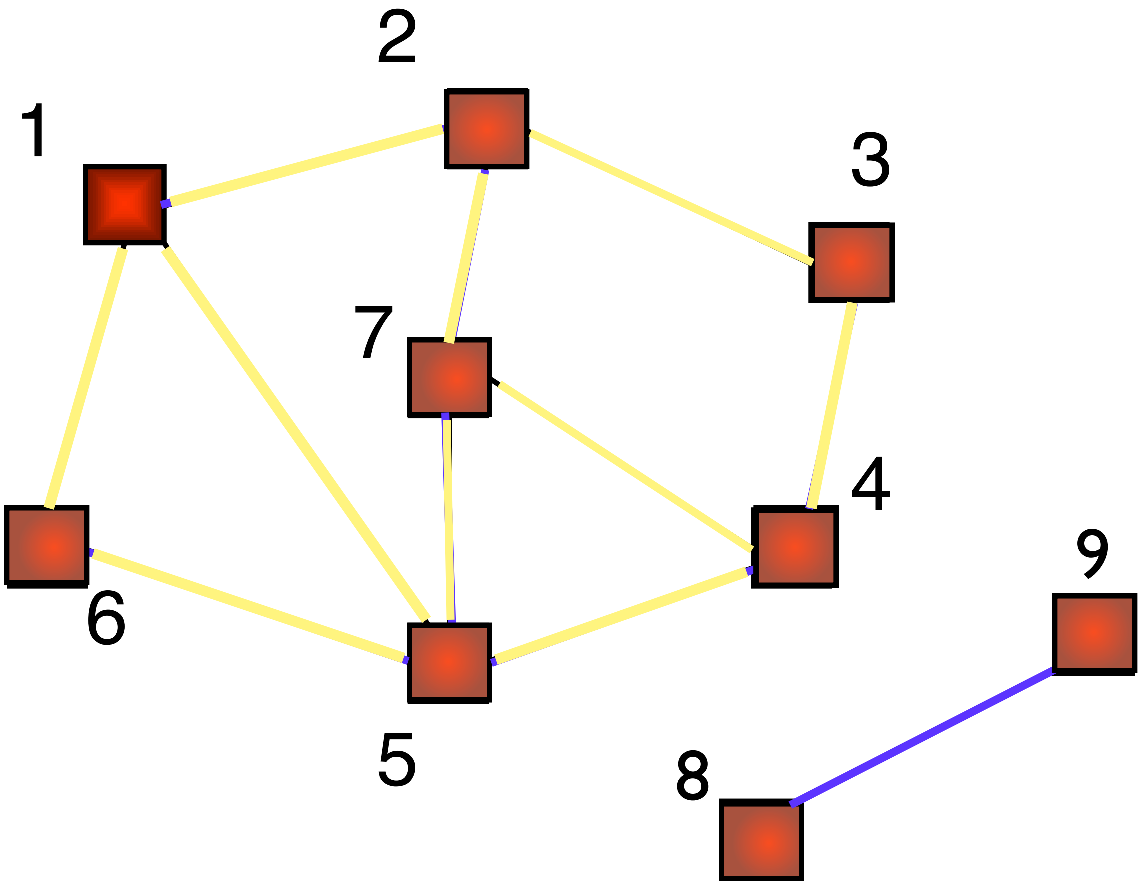
Example

dfs(8)



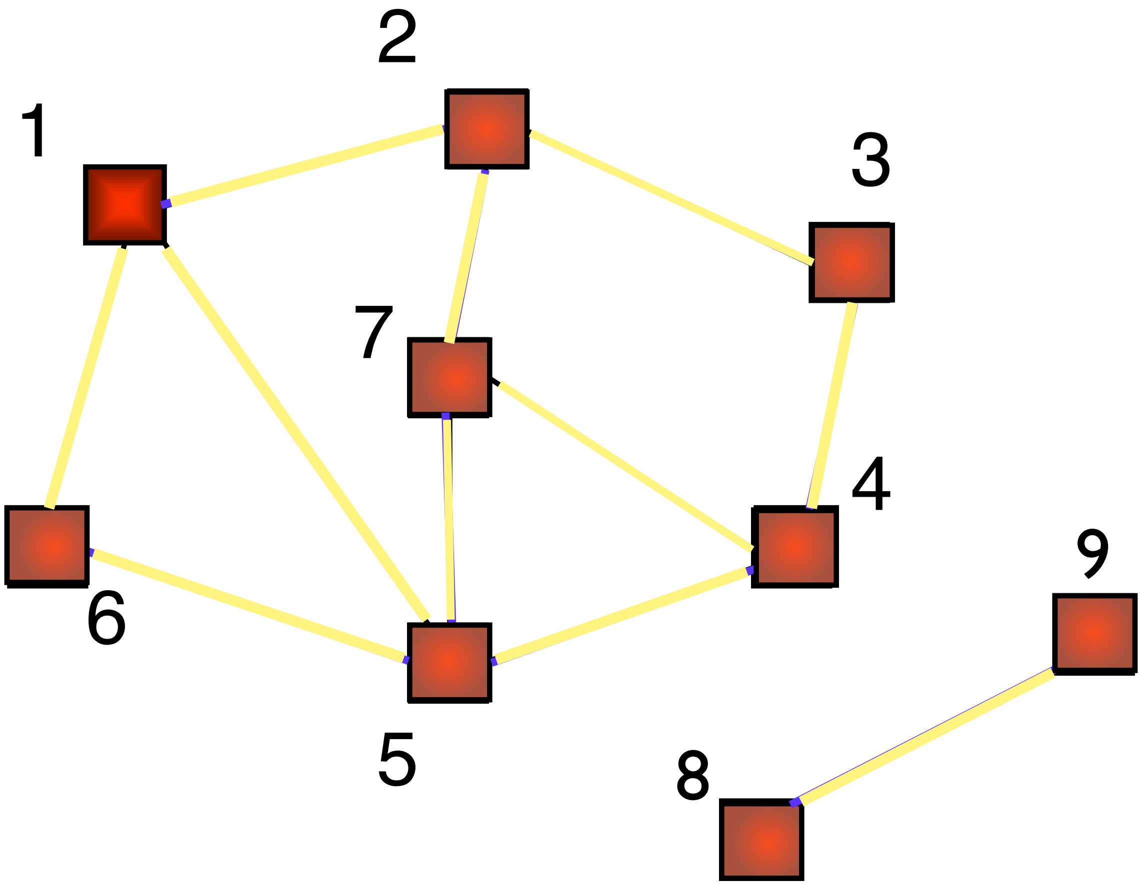
Example

dfs(8)
dfs(9)



Example

$dfs(8)$
$dfs(9)$

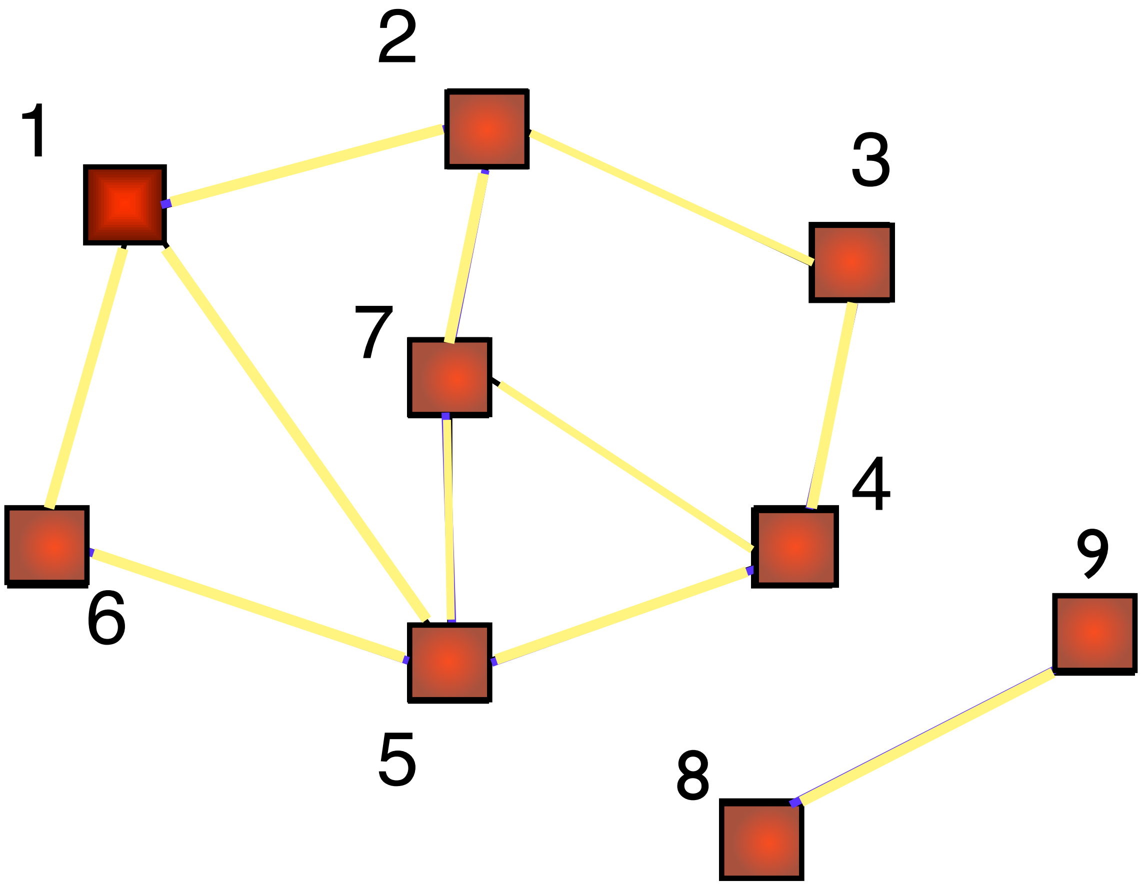


Portland State University

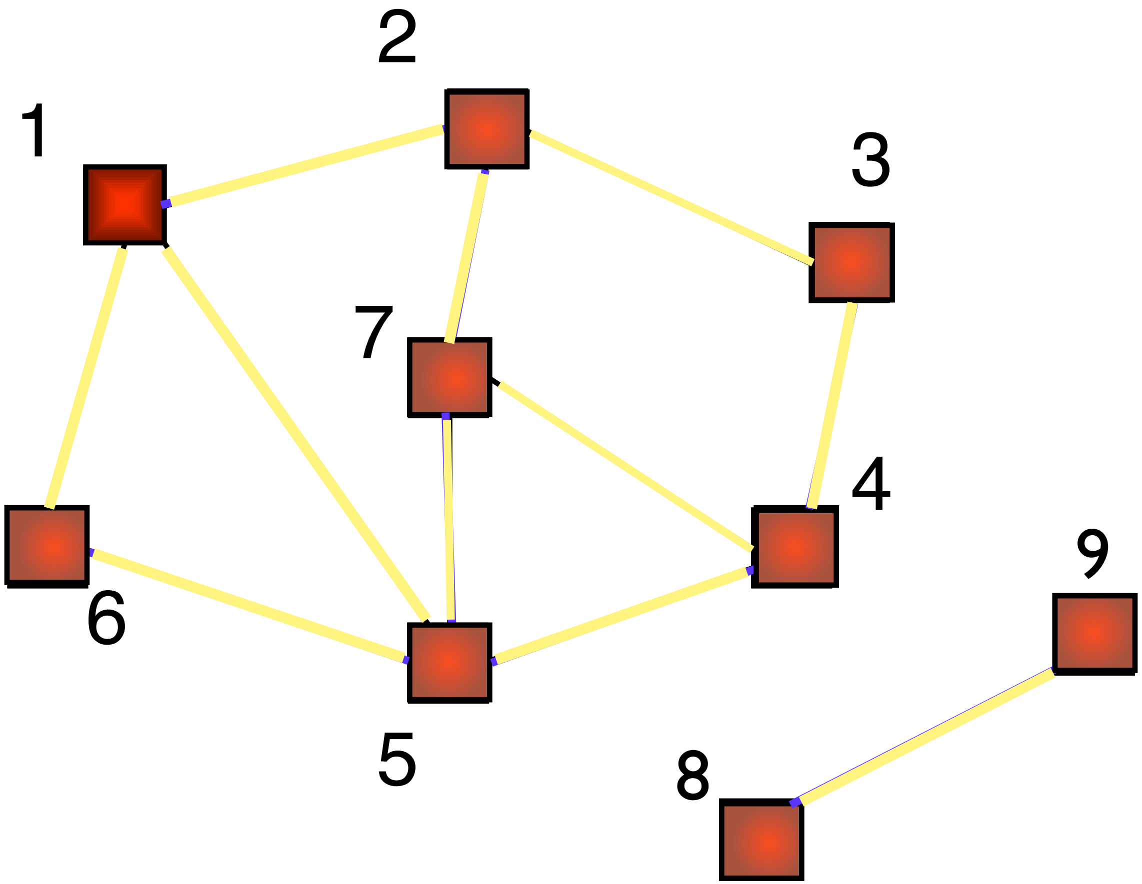
Thursday, 15 October 2015

Example

$dfs(8)$



Example



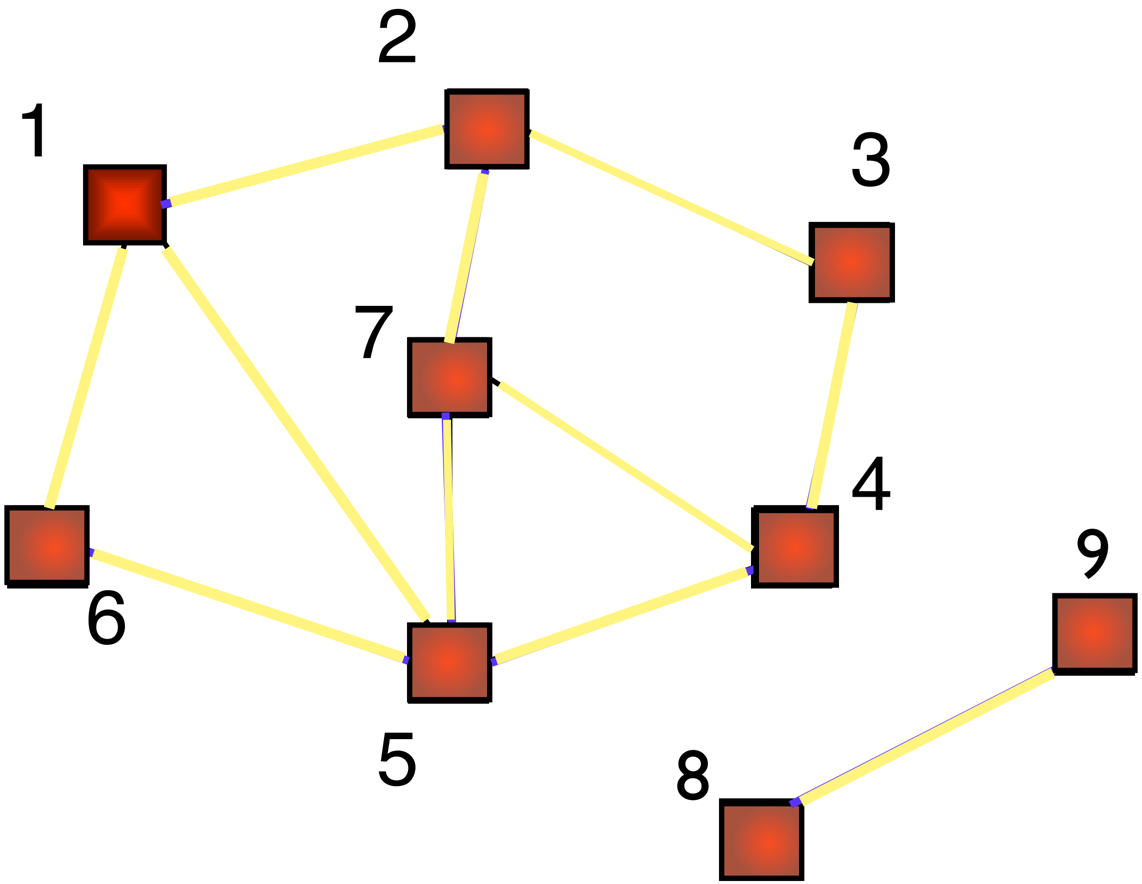
Complexity?

- What's the basic operation?
  - finding all the Vertices in the graph?
  - making a mark?
  - checking a mark?
  - finding all the neighbors of a node?
- Cost depends on the data structure used to represent the graph

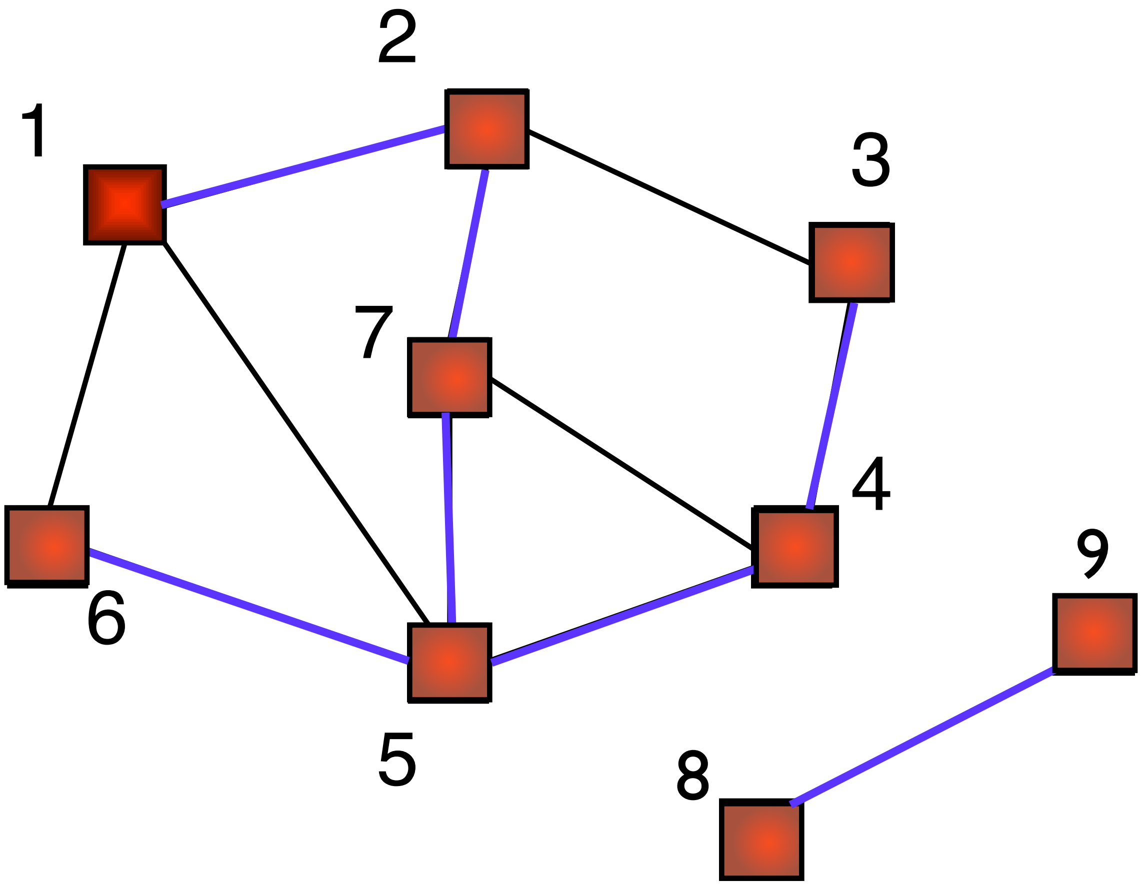
Two choices of data structure:

- Adjacency Matrix: $\Theta(|V|^2)$
- Adjacency List: $\Theta(|V| + |E|)$

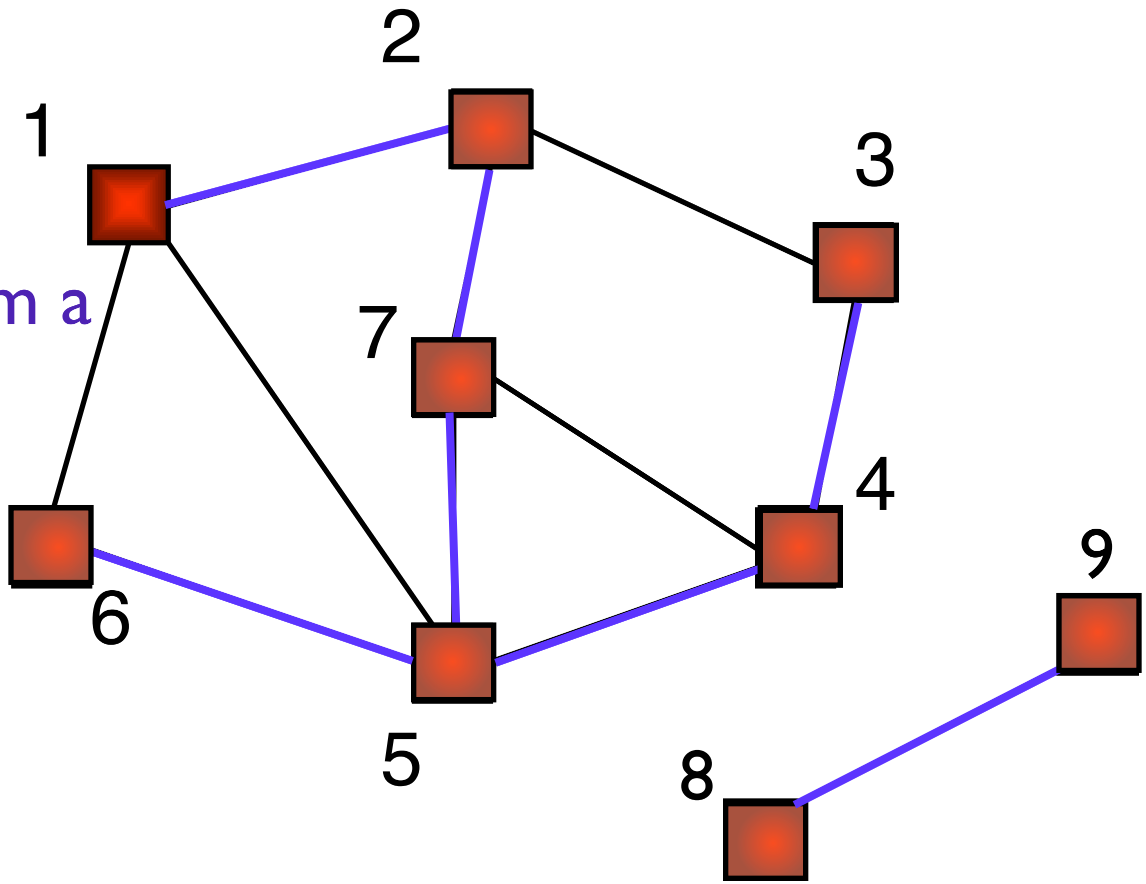
One Last look at the Example



One Last look at the Example

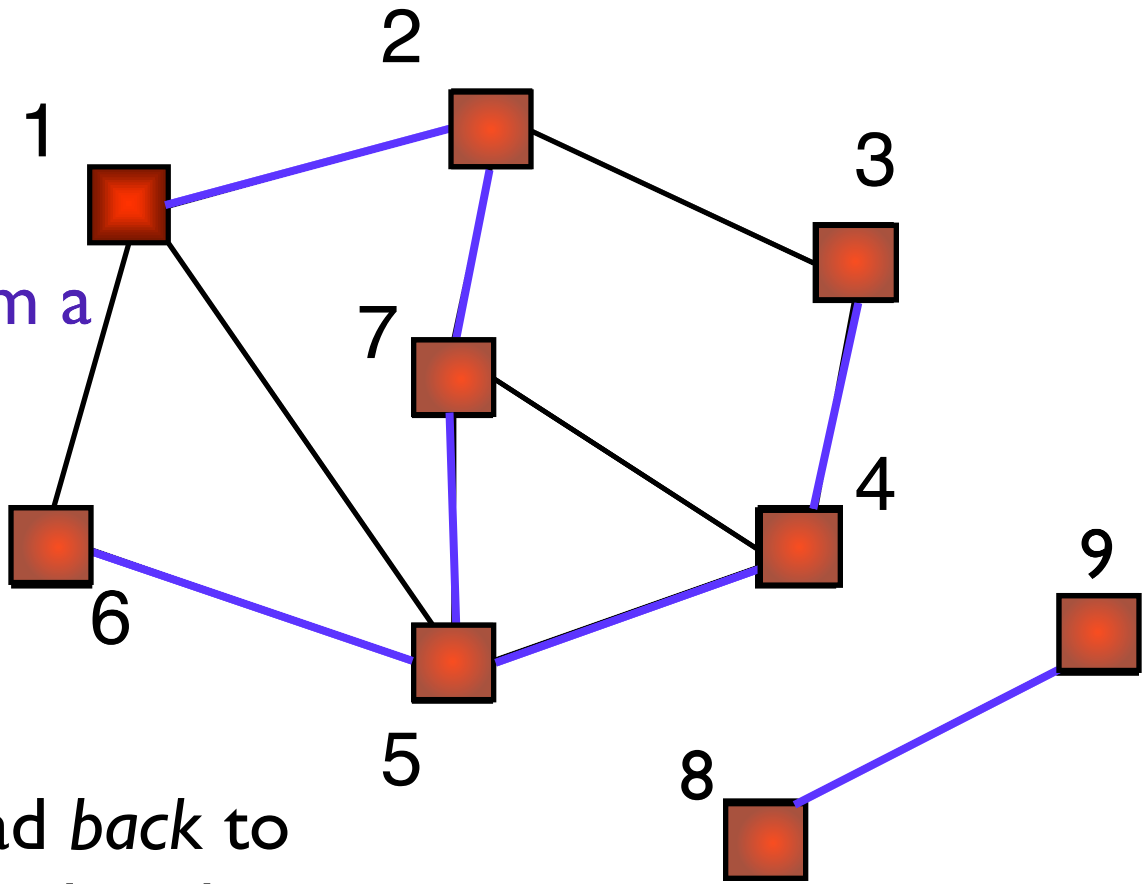


One Last look at the Example



Blue edges form a spanning three

One Last look at the Example



Blue edges form a spanning three

Black edges lead *back* to an already-visited node

Applications

- Checking for connectivity
  - How?
- Checking for Cycles
  - How?