Distances between probability distributions

P,Q,R probability distributions over E, with densities piq, [[pmf piq, r]

ACE. P(A) = [Ap(x)dx [= [p(x)]] (PO)OEB. Q: d(PO, PO) = ???

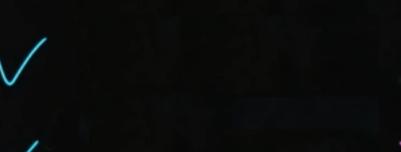
Total Variation Distance (TV)

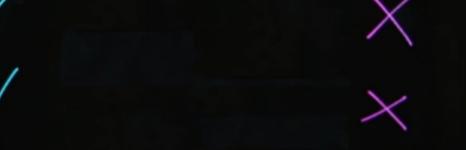
 $TV(P_{1}Q) = \sup_{A \in E} |P(A) - Q(A)|$ $= \frac{1}{2} \int_{E} |p(x) - q(x)| dx$ $\left[= \frac{1}{2} \sum_{x \in E} |p(x) - q(x)| \right]$

Kullback-Leibler Divergence KL(P,Q) = KL(PIIQ) $\int_{E}^{P(x)} \log \frac{P(x)}{q(x)} dx, \quad q(x) = 0$ $= \left\{ \left\{ \sum_{x \in E} \rho(x) \left\{ \log \frac{P(x)}{q(x)} - \mu - \frac{1}{q(x)} \right\} \right\} \right\}$ $+\infty$, =0,p(x)+0

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KL







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(1)
$$P_n = Poi(\frac{1}{n})$$
, $Q = S_0$. Show: $TV(P_n(Q)) \xrightarrow{n \to \infty} = S_0(Q)$

$$[Poi(\lambda) \text{ has pmf } P_{\lambda}(k) - \frac{\lambda^k}{k!} e^{-\lambda}, k = 0, 1, 2, ...; q(k) = \begin{cases} 1, k = 0 \\ 0, \text{ otherwise} \end{cases}$$

$$Proof: TV(P_n, Q) = \frac{1}{2} \sum_{k=0}^{\infty} |P_{\lambda}(k) - q(k)| = \frac{1}{2} \sum_{k=0}^{\infty} |\frac{1}{n} e^{-\lambda n} - S_0(k)|$$

$$= \frac{1}{2} |\frac{1}{(n)} e^{-\lambda n} - 1| + \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{(n)} e^{-\lambda n} \xrightarrow{n \to \infty} 0 \quad \text{What about}$$

$$= \frac{1}{2} |P_n(\xi_1, ..., \xi_n)|$$

$$= |P_n(\xi_1$$

Distances between probability distributions (2) P=Bin(n,p), Q=Bin(n,q), p,q ∈ CO,1), f(p,k)=(k)p(1-p)-k $KL(PRQ) = \sum_{k=0}^{\infty} f(q_1 k) \cdot \log \frac{f(q_1 k)}{f(q_1 k)} = \sum_{k=0}^{\infty} f(q_1 k) \cdot \log \left[\frac{(n)}{(n)} \frac{p^k (1-q)^{n-k}}{(n)} \frac{p^k (1-q)^{n-k}}{(n)} \right]$ $= \sum_{k=0}^{n} f(\rho_1 k) \left[log \left(\frac{\rho}{2} \right) + log \left(\frac{1-\rho}{1-q} \right)^{n-k} \right]$ X~Bin(n,p) E[X]= n.p = = = f(p, &) [k log(f) + (n-k) log(1-p)-9-30: KL(P,Q)->00 9=0,pe(0,D: KL(P,Q)=00 = log(=) np + log(1-p) (n-np) d-21: 6-0:6-21 535

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$$\begin{array}{l}
\exists P = \mathcal{N}(a, 1), \quad Q = \mathcal{N}(b, 1), \quad \alpha_1 b \in \mathbb{R}; \quad f_{a_1 o}(x) = \frac{1}{(2\pi o^2)^2} e^{-\frac{1}{2}o}(x - a)^2 \\
\hline
\text{KL}(P|Q) = \int_{\mathbb{R}^2} f_{a_1}(x) \log \left[\frac{f_{a_1}(x)}{f_{b_1}(x)} \right] dx = \int_{\mathbb{R}^2} f_{a_1}(x) \log \left[\frac{1}{\sqrt{2}} e^{-\frac{1}{2}}(x - a)^2 - \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 + \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 + \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 + \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^2 + \frac{1}{2}(x - a)^2 - \frac{1}{2}(x - a)^$$