Algorithms

Stacks, queues, and linked lists

Emanuele Rodolà
rodola@di.uniroma1.it

SAPIENZA
UNIVERSITÀ DI ROMA

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Stack

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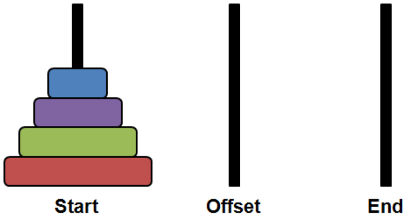

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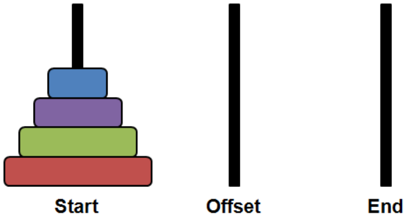

Insert (push) and remove (pop) operations must be efficient at the top.

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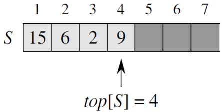

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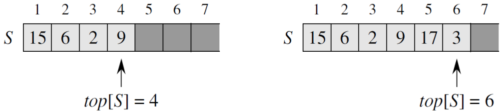

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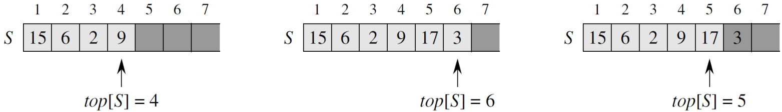

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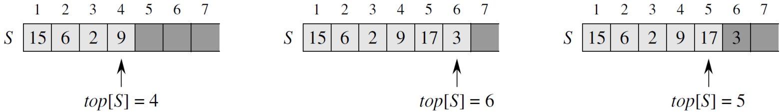

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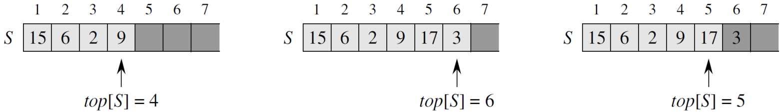

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Typical situation of a stack overflow: execution stack in recursive calls.

Stack operations

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if top[S] = 0
    then return TRUE
    else return FALSE
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PUSH(S, x)

$top[S] \leftarrow top[S] + 1$
$S[top[S]] \leftarrow x$

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↑ head ↑ tail

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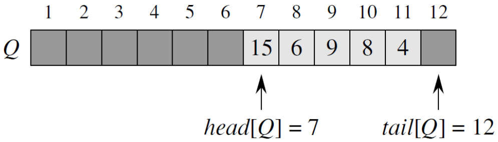
(In principle, we could avoid imposing a “length budget”.)



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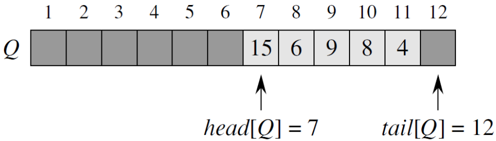

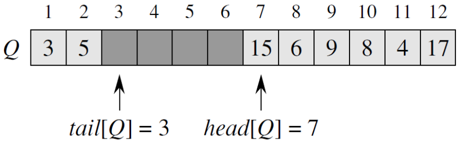

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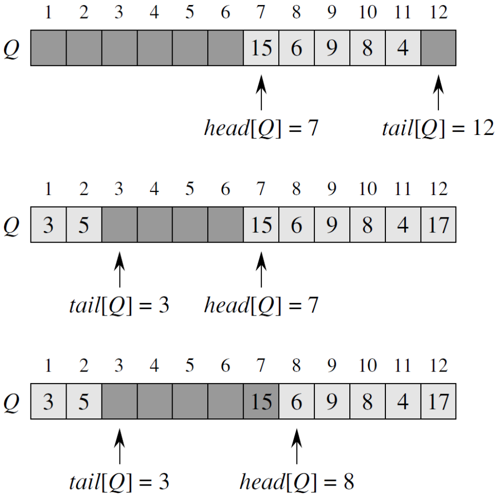

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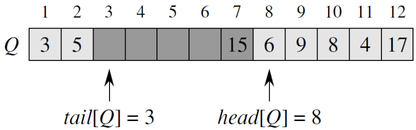

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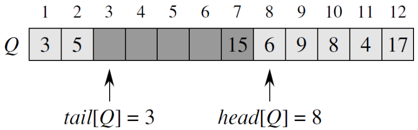

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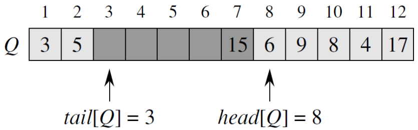

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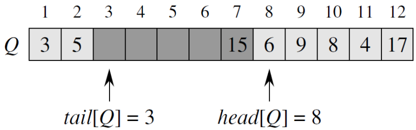

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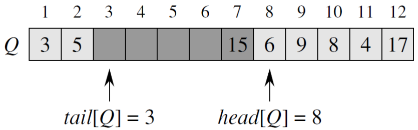

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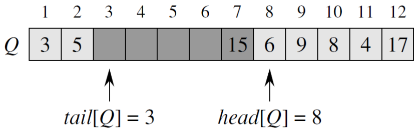

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- There is a circular shift at the boundaries of the container array. This way, we must not ensure that $head[Q] = 1$ always.
- $head[Q] = tail[Q] \Rightarrow$ empty queue.
- $head[Q] = tail[Q] + 1 \Rightarrow$ full queue, risk of overflow.

Queue operations

ENQUEUE(Q, x)

$Q[\mathit{tail}[Q]] \leftarrow x$
if $\mathit{tail}[Q] = \mathit{length}[Q]$
  then $\mathit{tail}[Q] \leftarrow 1$
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return x
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Each operation takes $O(1)$ time.

Linked list

An array is a sequence of objects arranged in linear order:

1	2	3	4	5	6	7	8	9	10
16	14	10	8	7	9	3	2	4	1

From a computational perspective, an array is a contiguous structure.

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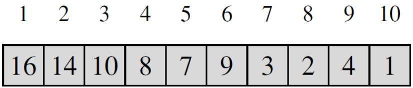

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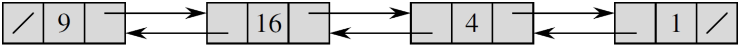

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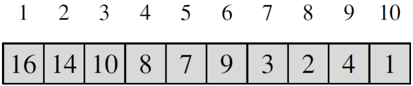

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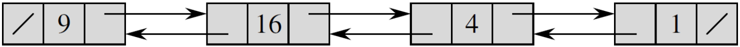
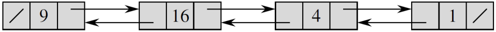


A linked list is not contiguous, as each element has its own context.

Linked list

(a)



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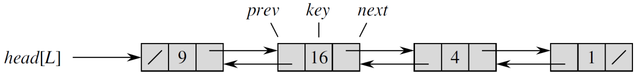

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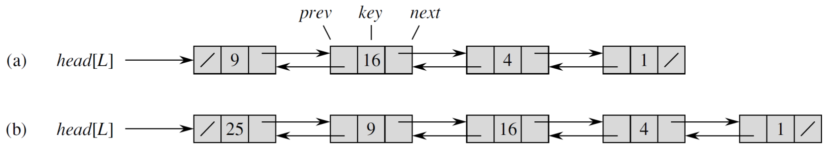

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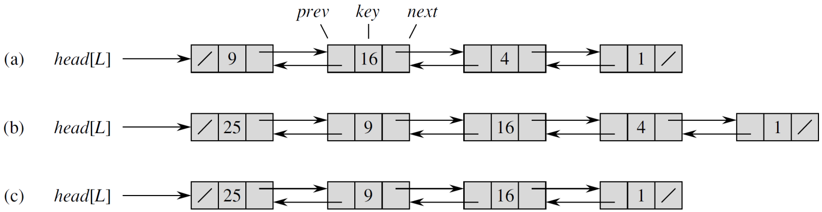

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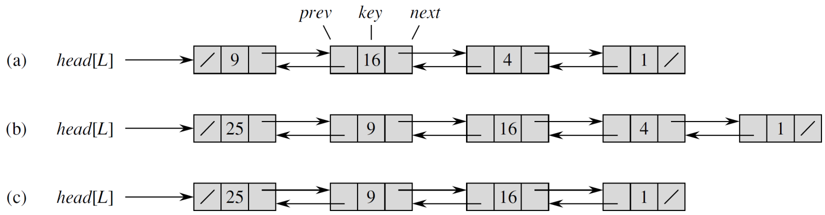

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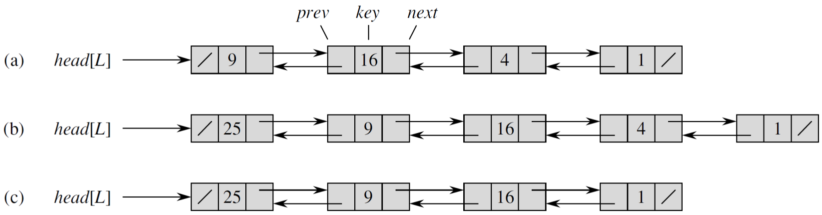

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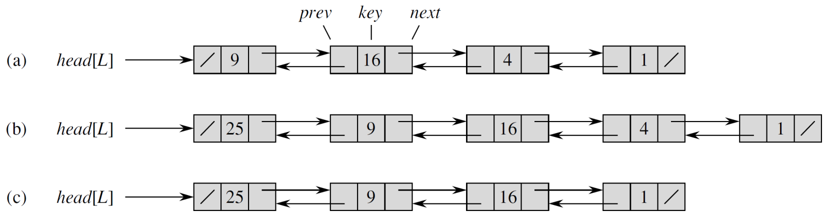

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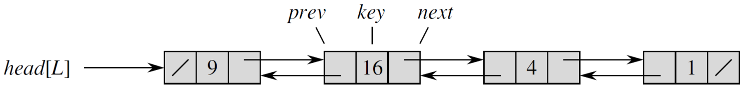
- A circular list has the tail pointing to the head.

Linked list: Search

Look for element with key k, return a pointer to it.

LIST-SEARCH(L, k)

$x \leftarrow head[L]$
while $x \neq$ NIL and $key[x] \neq k$
    do $x \leftarrow next[x]$
return x



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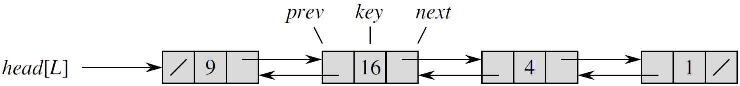


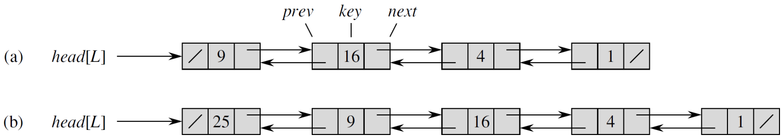
If the list has n objects, complexity is upper bounded as $O(n)$.

Linked list: Insert

Insert an element x at the front of the list.

LIST-INSERT(L, x)

```
next[x] ← head[L]
if head[L] ≠ NIL
    then prev[head[L]] ← x
head[L] ← x
prev[x] ← NIL
```



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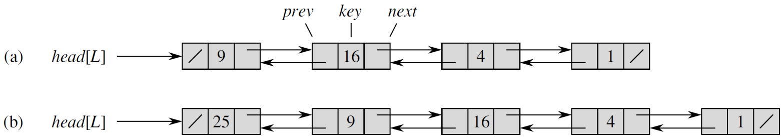


Complexity is $O(1)$.

Linked list: Delete

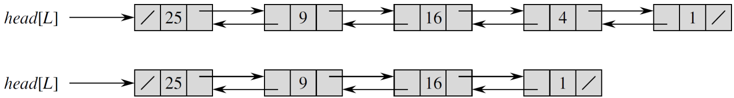
Remove an element x from any location of the list.

LIST-DELETE(L, x)

if $prev[x] \neq$ NIL
  then $next[prev[x]] \leftarrow next[x]$
  else $head[L] \leftarrow next[x]$
if $next[x] \neq$ NIL
  then $prev[next[x]] \leftarrow prev[x]$

(b)

(c)



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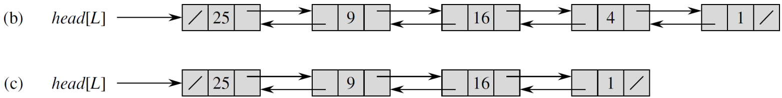

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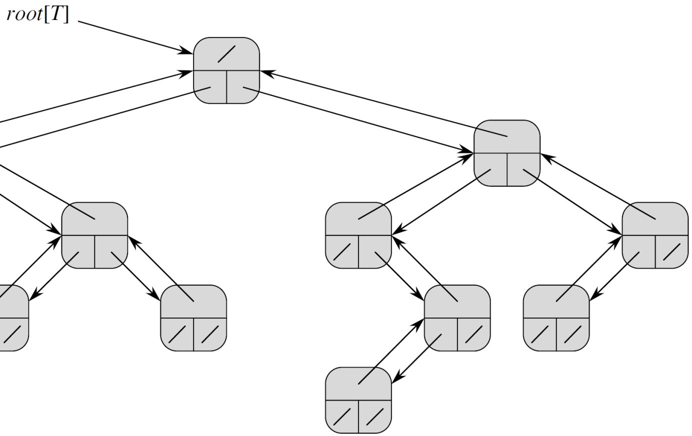
Trees

Linked lists can be used to represent general trees.

For example, consider a binary tree:



Trees

If the tree has > 2 children per node, use the following trick at each node:

- Store a pointer to the leftmost child.
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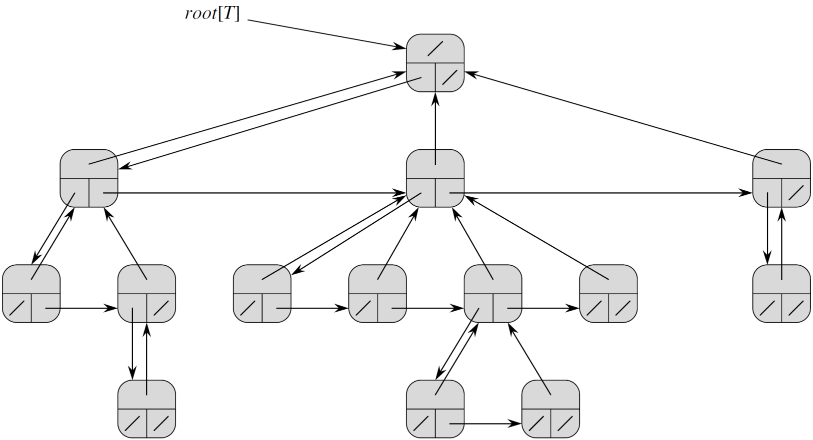

Exercises

Write Python code implementing all the data structures, together with the associated operations, that we have seen till now.

In particular:

- The stack.
- The queue.
- Using an array as the container, implement a deque (**d**ouble-**e**nded queue), which is similar to the queue, but allows insertion and deletion at both ends. The four operations should each take $O(1)$ time.
- The linked list, the doubly linked list, and the circular linked list.
- The binary tree, using a (doubly) linked list.
- The tree with > 2 children.
  Also write iterative or recursive code for traversing the tree.

Suggested reading

Chapters 10 Introduction, 10.1, 10.2 (skip the "Sentinels" paragraph), and 10.4 of:

"Introduction to Algorithms – 2nd Ed.", Cormen et al.