Algorithms
Prof. Rodolà
2^{nd} Semester 2019/2020

Bioinformatics
Sapienza University of Rome

Midterm self-evaluation

Tue 21 Apr 2020

Name: _____

Department: _____

Matr. number: _____

There is a total of **10** questions.

The maximum possible score is **32**.

Note: This test is not graded and will not concur to the final exam.

- The solutions will be discussed in the next online lecture.

  If you need more time, just avoid watching the recorded lecture of Thu 23 April.

- To send me your solutions: you can print these sheets and scan them, or use any other medium that is easier for you.

Question 1 (1 point)**.**

Explain why the following statement is meaningless: "The running time of algorithm A is at least $O(n^2)$".

Question 2 (2 points)**.**

Is $2^{n+1} = O(2^n)$? Why yes/no?

Is $2^{2n} = O(2^n)$? Why yes/no?

Question 3 (4 points)**.**

Using the substitution method, show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

Question 4 (6 points)**.**

Consider the recursion tree for $T(n) = 3T(\lfloor n/2 \rfloor) + n$.

- What is the subproblem size for a node at depth i?
- How many levels are in the tree?
- How many leaves are in the tree?
- What is the total cost over all nodes at depth i (excluding the leaf level)?
- Using your answers to the previous points, provide an asymptotic upper bound for the recursion.

Question 5 (3 points)**.**

Is an array that is in sorted order a min-heap?

Is the array with values $\langle 23, 17, 14, 6, 13, 10, 1, 5, 7, 12 \rangle$ a max-heap?

Question 6 (3 points)**.**

Illustrate the operation of MAX-HEAP-INSERT$(A, 10)$ on the heap $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$.

Question 7 (3 points)**.**

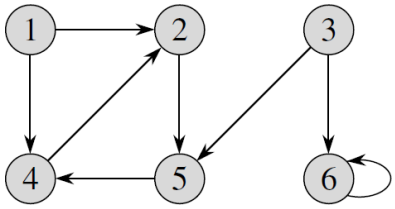
Illustrate the result of each operation in the sequence ENQUEUE$(Q, 4)$, ENQUEUE$(Q, 1)$, ENQUEUE$(Q, 3)$, DEQUEUE(Q), ENQUEUE$(Q, 8)$, and DEQUEUE(Q) on an initially empty queue Q stored in array $Q[1..6]$.

Question 8 (2 points)**.**

Write an $O(n)$-time recursive procedure that, given an n-node binary tree, prints out the key of each node in the tree.

Question 9 (2 points)**.**

For the directed graph in the figure, specify for each node the d and π values that result from running breadth-first search using vertex 3 as the source.



Question 10 (6 points)**.**

Run Dijkstra's algorithm on the directed graph below.

- Do it twice, with two different sources: first s, and then z.
- For each iteration of the while loop, show the d and π values for each node.