Algorithms

Stacks, queues, and linked lists

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2nd semester a.y. 2020/2021 · March 25, 2021

Stack

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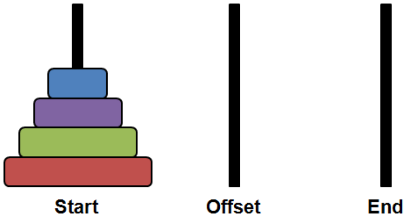

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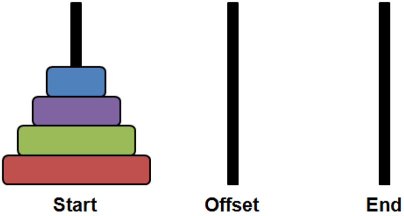

Insert (push) and remove (pop) operations must be efficient at the top.

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Further, we use a pointer $top[S]$ to the most recently inserted element.

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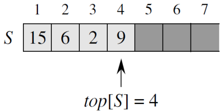

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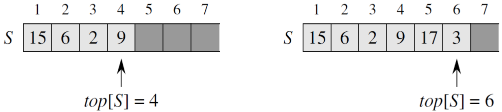

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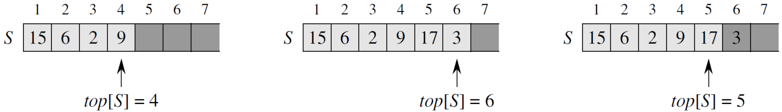

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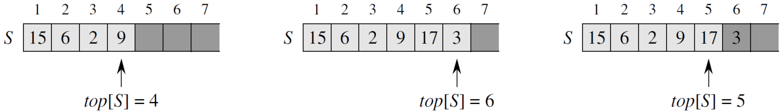

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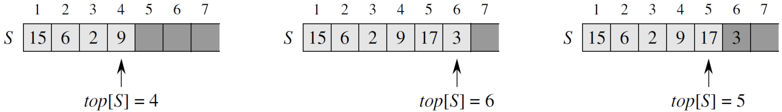

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Typical situation of a stack overflow: execution stack in recursive calls.

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if top[S] = 0
    then return TRUE
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$top[S] \leftarrow top[S] + 1$
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↑ head ↑ tail

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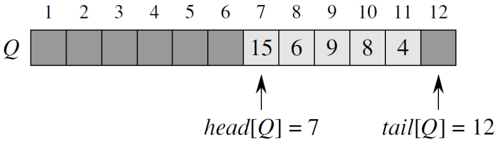
(In principle, we could avoid imposing a "length budget".)



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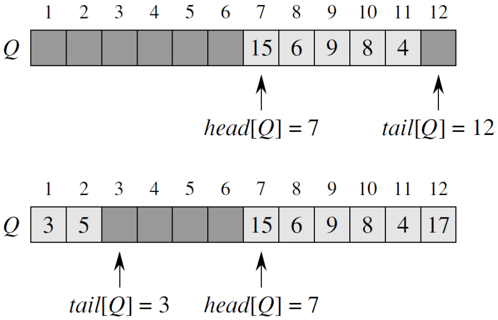

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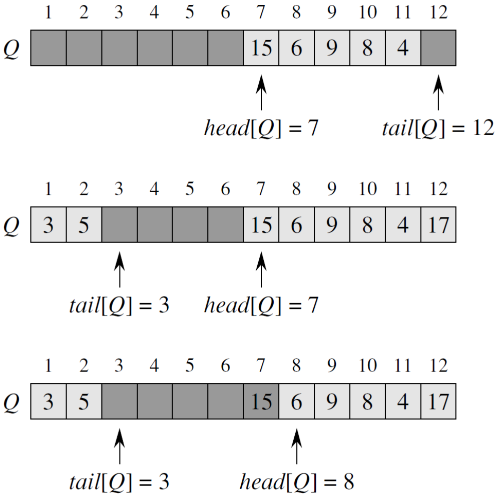

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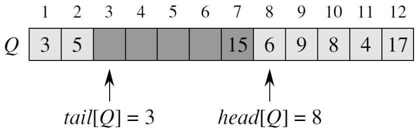

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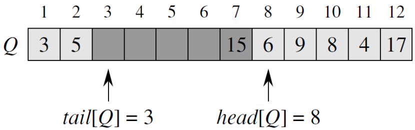

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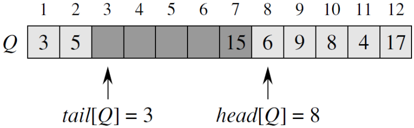

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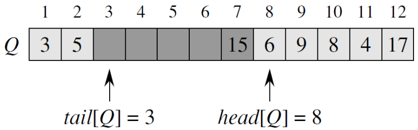

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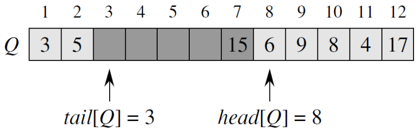

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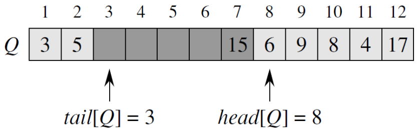

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- $head[Q]=tail[Q]\ \Rightarrow$ empty queue.
- $head[Q]=tail[Q]+1\ \Rightarrow$ full queue, risk of overflow.

Queue operations

ENQUEUE(Q, x)

$Q[tail[Q]] \leftarrow x$
if $tail[Q] = length[Q]$
  then $tail[Q] \leftarrow 1$
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Each operation takes $O(1)$ time.

Linked list

An array is a sequence of objects arranged in linear order:

1	2	3	4	5	6	7	8	9	10
16	14	10	8	7	9	3	2	4	1

From a computational perspective, an array is a contiguous structure.

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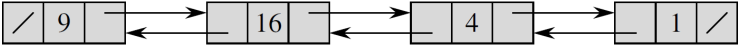

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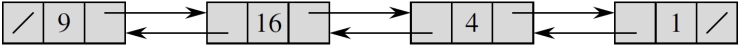
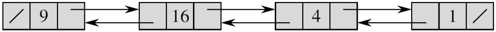


A linked list is not contiguous, as each element has its own context.

Linked list

(a)



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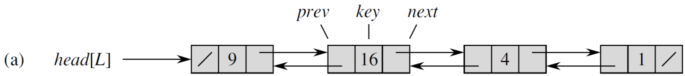

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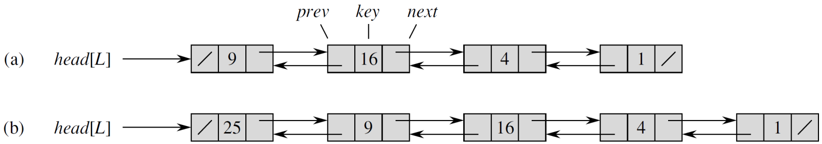

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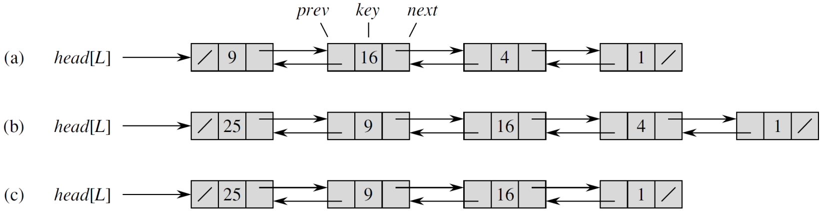

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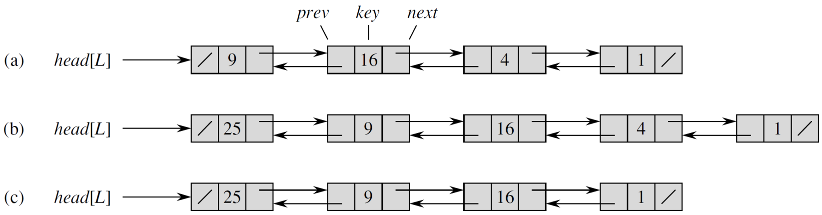

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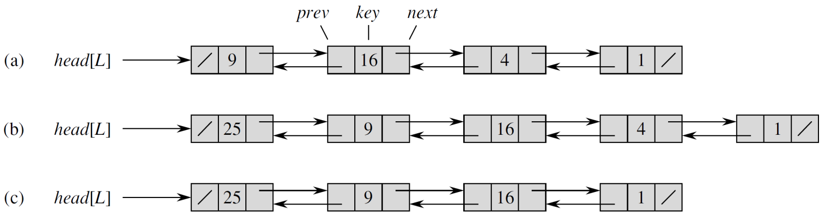

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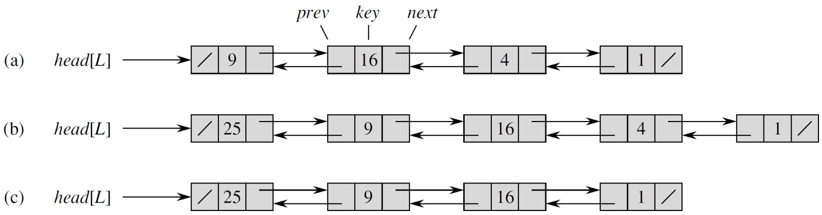

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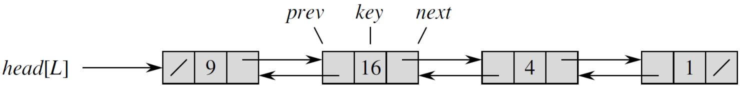
- A circular list has the tail pointing to the head.

Linked list: Search

Look for element with key k, return a pointer to it.

LIST-SEARCH(L, k)

$x \leftarrow head[L]$
while $x \neq$ NIL and $key[x] \neq k$
    do $x \leftarrow next[x]$
return x



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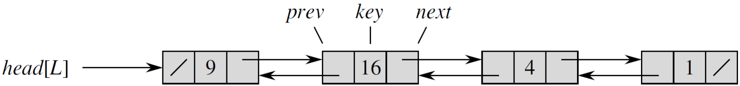

If the list has n objects, complexity is upper bounded as $O(n)$.

Linked list: Insert

Insert an element x at the front of the list.

LIST-INSERT(L, x)

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next[x] ← head[L]
if head[L] ≠ NIL
    then prev[head[L]] ← x
head[L] ← x
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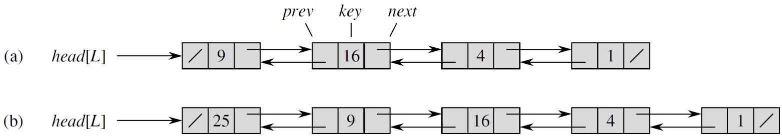

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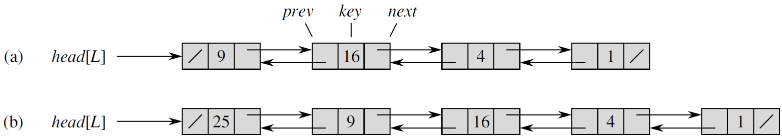

Complexity is $O(1)$.

Linked list: Delete

Remove an element x from any location of the list.

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LIST-DELETE(L, x)
if prev[x] ≠ NIL
   then next[prev[x]] ← next[x]
   else head[L] ← next[x]
if next[x] ≠ NIL
   then prev[next[x]] ← prev[x]
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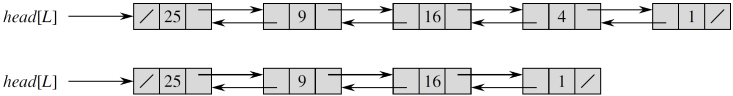
(b)

(c)



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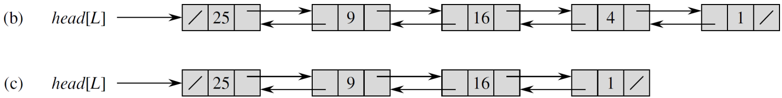

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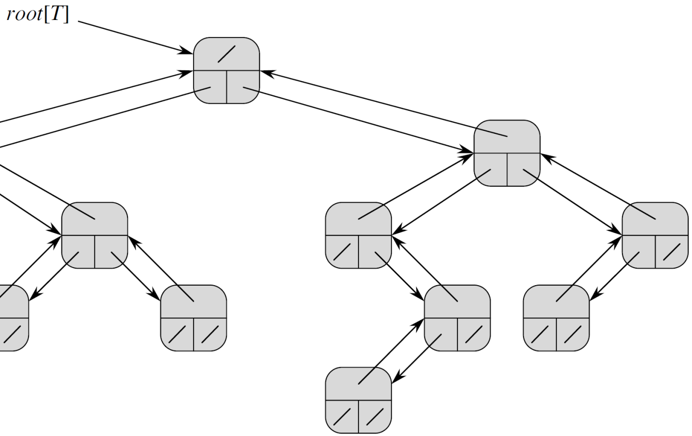
Trees

Linked lists can be used to represent general trees.

For example, consider a binary tree:



Trees

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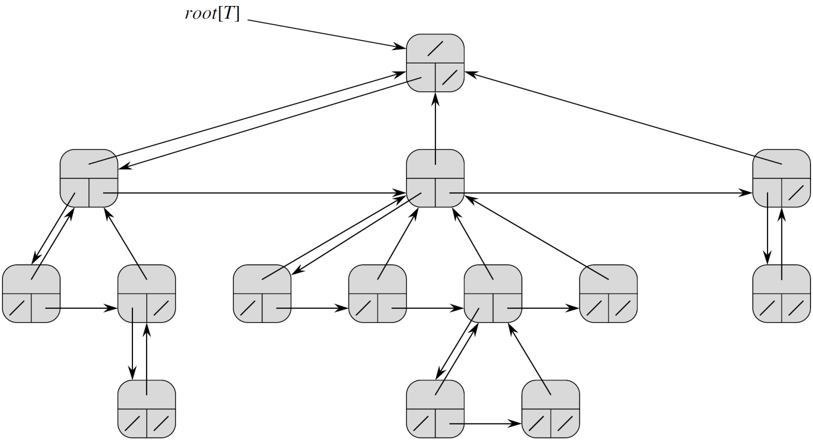

Exercises

- Write Python code implementing a tree with > 2 children, together with iterative or recursive code for traversing the tree.
- Using an array as the container, implement a deque (**d**ouble-**e**nded queue), which is similar to the queue, but allows insertion and deletion at both ends, so that each of these operations has $\Theta(1)$ complexity.

Send your solutions to rodola@di.uniroma1.it

Suggested reading

Chapters 10 Introduction, 10.1, 10.2 (skip the "Sentinels" paragraph), and 10.4 of:

"Introduction to Algorithms – 2nd Ed.", Cormen et al.