Algorithms
Prof. Rodolà
2^{nd} Semester 2020/2021

Bioinformatics
Sapienza University of Rome

Midterm self-evaluation

Thu 08 Apr 2021

Name: ______________________________

Matr. number: ______________

There is a total of **10** questions.

The maximum possible score is **32**.

Note: This test is <u>not graded</u> and will not concur to the final exam.

Question 1 (1 point).

Explain why the following statement is meaningless: "The running time of algorithm A is at least $O(n^2)$".

Question 2 (2 points).

Is $2^{n+1} = O(2^n)$? Why yes/no?

Is $2^{2n} = O(2^n)$? Why yes/no?

Question 3 (4 points).

Using the substitution method, show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

Question 4 (6 points).

Consider the recursion tree for $T(n) = 3T(\lfloor n/2 \rfloor) + n$.

- What is the subproblem size for a node at depth i?
- How many levels are in the tree?
- How many leaves are in the tree?
- What is the total cost over all nodes at depth i (excluding the leaf level)?
- Using your answers to the previous points, provide an asymptotic upper bound for the recursion.

Question 5 (3 points).

Is an array that is in sorted order a min-heap?

Is the array with values $[23, 17, 14, 6, 13, 10, 1, 5, 7, 12]$ a max-heap?

Question 6 (3 points).

Illustrate the operation of MAX-HEAP-INSERT$(A, 10)$ on the heap $A = [15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1]$.

Question 7 (3 points).

Illustrate the result of each operation in the sequence ENQUEUE$(Q, 4)$, ENQUEUE$(Q, 1)$, ENQUEUE$(Q, 3)$, DEQUEUE(Q), ENQUEUE$(Q, 8)$, and DEQUEUE(Q) on an initially empty queue Q stored in array $Q[1..6]$.

Question 8 (2 points).

Write an $O(n)$-time recursive procedure that, given an n-node binary tree, prints out the key of each node in the tree.

Question 9 (2 points).

For the directed graph in the figure, specify for each node the d and π values that result from running breadth-first search using vertex 3 as the source.