Deep Learning & Applied AI

Adversarial training

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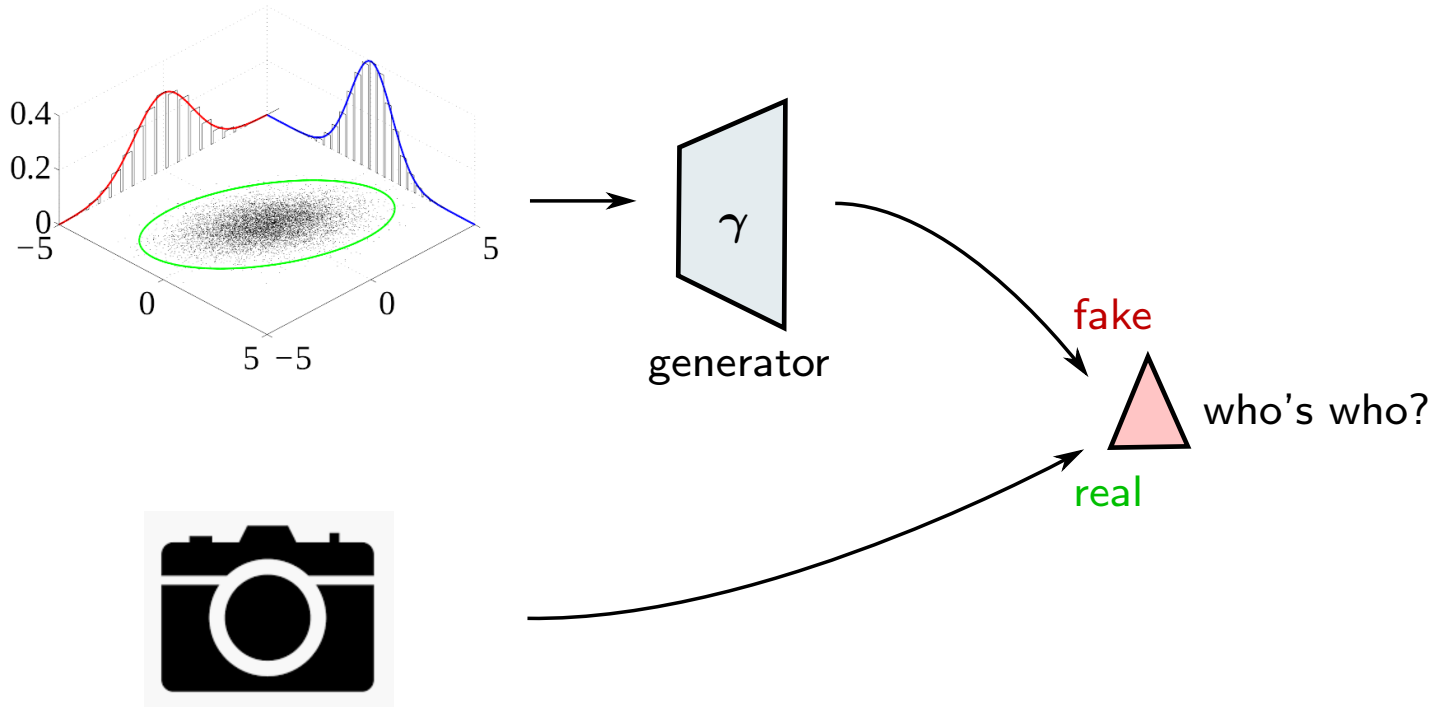
Generative adversarial networks (GAN)

Synthesize data points from a given generator (e.g. a VAE decoder), and sample real data from the actual data distribution:



Given instances of fake and real data, a generative model is "good" when you can not distinguish between the two.

Goodfellow et al, "Generative adversarial networks", NIPS 2014

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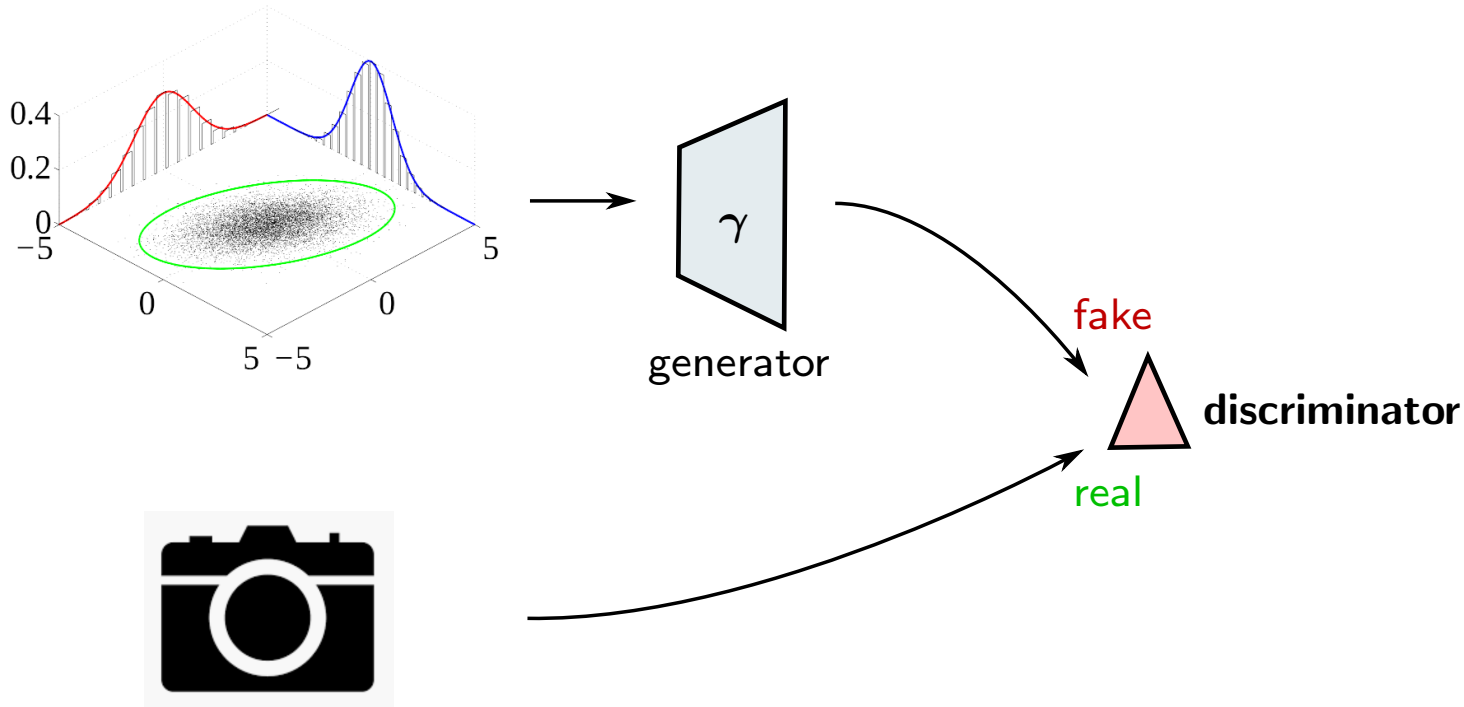

Let us train a discriminator Δ with parameters $\boldsymbol{\delta}$, whose output is the probability that the given data is real.

Goodfellow et al, "Generative adversarial networks", NIPS 2014

Generative adversarial networks (GAN)

- $\mathbf{x}$: sample from the real distribution
- $\mathbf{x}' = D_{\boldsymbol{\gamma}}(\mathbf{z})$: generated sample, good if $p_g(\mathbf{x}') \approx p_d(\mathbf{x})$

A good discriminator should yield $\Delta_{\boldsymbol{\delta}}(\mathbf{x}) \approx 1$ on the real instances and $\Delta_{\boldsymbol{\delta}}(\mathbf{x}') \approx 0$ on the fake instances.

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In contrast, the generator should make this value as small as possible:

$$\min_{\boldsymbol{\gamma}} \max_{\boldsymbol{\delta}} \; \mathbb{E}_{\mathbf{x}} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{z}} \log(1 - \Delta_{\boldsymbol{\delta}}(D_{\boldsymbol{\gamma}}(\mathbf{z})))$$

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The generator competes against the adversarial discriminator and tries to minimize its success rate.

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Let us try to maximize the discriminator score given a generator G:

$$J(G) = \max_{\boldsymbol{\delta}} \; \mathbb{E}_{\mathbf{x} \sim p_d} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{x} \sim p_g} \log(1 - \Delta_{\boldsymbol{\delta}}(\mathbf{x}))$$

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$$= \max_{\boldsymbol{\delta}} \; \int [\log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) p_d(\mathbf{x}) + \log(1 - \Delta_{\boldsymbol{\delta}}(\mathbf{x})) p_g(\mathbf{x})] \, d\mathbf{x}$$

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For any given $\mathbf{x}$, we want to maximize $\Delta_{\boldsymbol{\delta}}(\mathbf{x}) = a$; let's rename for simplicity $p_d(\mathbf{x}) \equiv p$ and $p_g(\mathbf{x}) \equiv q$, we get to:

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This is maximized when the derivative w.r.t. a is zero:

$$\frac{p}{a} - \frac{q}{1 - a} = 0$$

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$$J(G) \propto \frac{1}{2}\mathbb{E}_{\mathbf{x}\sim p_d} \log \frac{p_d(\mathbf{x})}{\rho(\mathbf{x})} + \frac{1}{2}\mathbb{E}_{\mathbf{x}\sim p_g} \log \frac{p_g(\mathbf{x})}{\rho(\mathbf{x})} + \text{const.}$$

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Therefore, the optimal GAN generator is found by minimizing:

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$$\min_{p_g} \underbrace{\frac{1}{2}KL(p_d\|\rho) + \frac{1}{2}KL(p_g\|\rho)}_{\text{Jensen-Shannon divergence between } p_d \text{ and } p_g}$$

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Therefore, the optimal GAN generator is found by minimizing:

$$\min_{p_g} \underbrace{\frac{1}{2}KL(p_d\|\rho) + \frac{1}{2}KL(p_g\|\rho)}_{JS(p_d\|p_g)}$$

Property: $p_d = p_g \;\Leftrightarrow\; JS(p_d\|p_g) = 0$

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Within the GAN paradigm, the globally optimal generator has a data distribution exactly equal to the real distribution of the data.

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Training the generator on data produced by an adversary is an example of a more general concept called adversarial training.

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Adversarial examples can be used maliciously.

"speed limit 50mph"

Adversarial attacks

Szegedy et al, “Intriguing properties of neural networks”, 2013

Adversarial attacks

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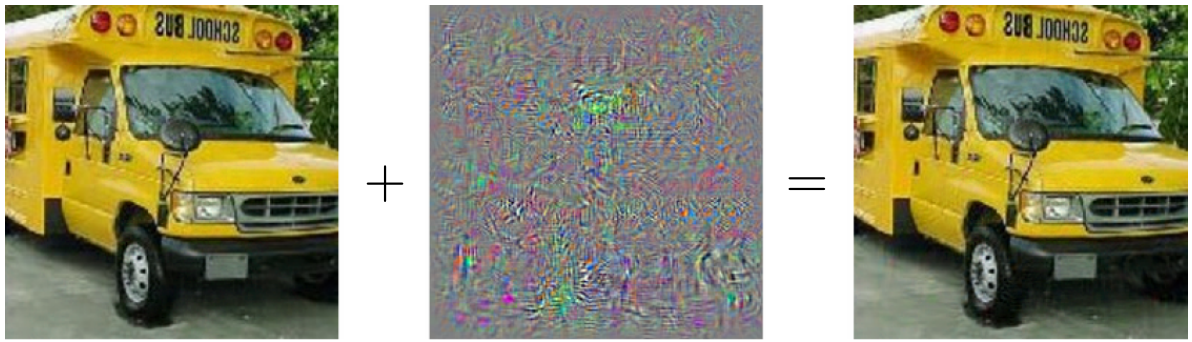

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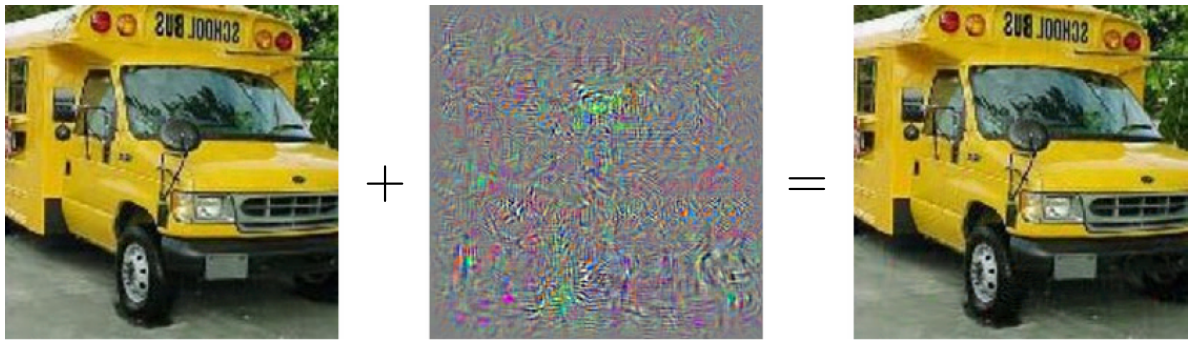

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How to construct undetectable adversarial examples?

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An attack is undetectable if it can not be perceived as such.

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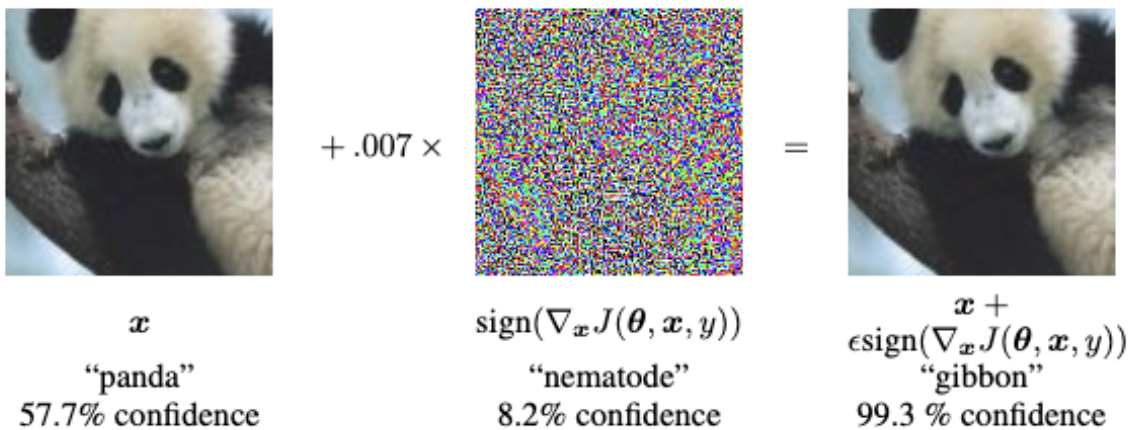

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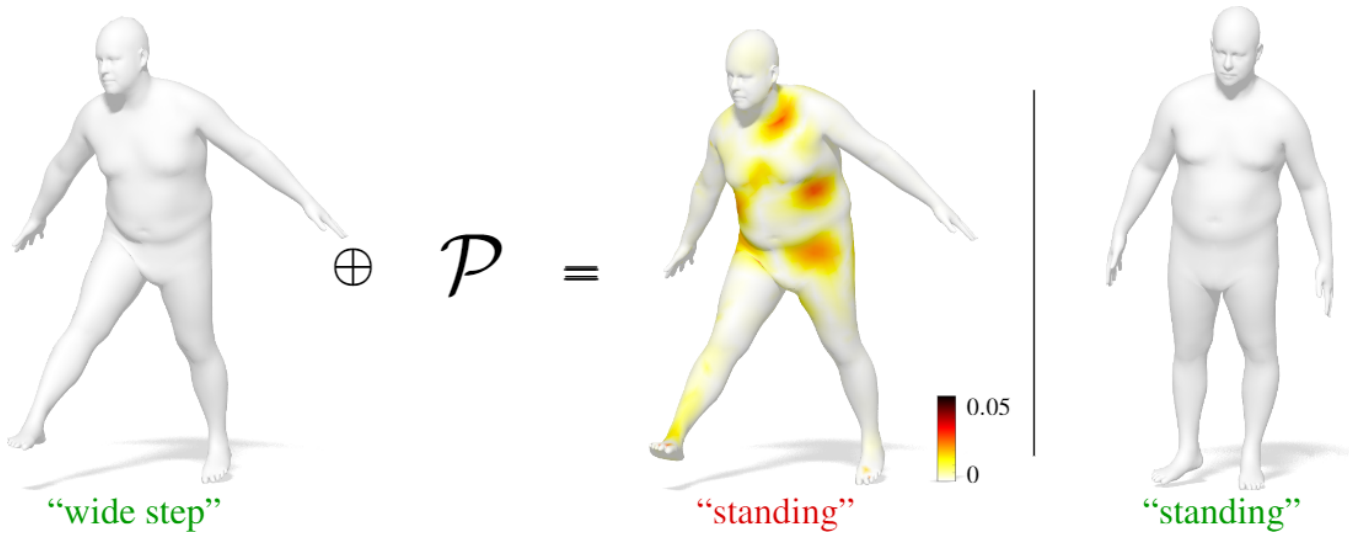

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It is possible to train a substitute model with black-box access to the target; by attacking the substitute model (now with white-box access), we can transfer these attacks to the black-box target.

Papernot et al, "Transferability in machine learning: from phenomena to black-box attacks using adversarial samples", 2016

Targeted attacks

Given an input sample $\mathbf{x}$, a classifier C, and a target class t, consider:

$$\min_{\mathbf{x}' \in [0,1]^n} \|\mathbf{x} - \mathbf{x}'\|_2^2$$
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$c > 0$ is chosen as the one giving an example of minimum distance, can be found via line search algorithms.

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Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \; d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta})$$
$$\text{s.t. } C(\mathbf{x} + \boldsymbol{\delta}) = t$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d is some distance function that depends on the specific task.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \; d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta})$$
$$\text{s.t. } f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d is some distance function that depends on the specific task.

f is such that $C(\mathbf{x} + \boldsymbol{\delta}) = t$ if and only if $f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$.

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$$\min_{\boldsymbol{\delta} \in [0,1]^n} d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta}) + c\, f(\mathbf{x} + \boldsymbol{\delta})$$

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Possible instance of the problem above:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \|\boldsymbol{\delta}\|_p + c\, \left(\max\{F(\mathbf{x} + \boldsymbol{\delta})_i \ : \ i \neq t\} - F(\mathbf{x} + \boldsymbol{\delta})_t\right)^+$$

where $F : \mathbf{x} \mapsto [0,1]^k$ is the full neural network yielding a probability distribution over all k classes for a given input $\mathbf{x}$, and $(a)^+ = \max(a, 0)$.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

Untargeted attacks

If there is no specific target toward which to misclassify, consider the closed-form expression for a given input $\mathbf{x}$ with ground-truth label ℓ_{gt}:

$$\mathbf{x}' = \mathbf{x} + \epsilon \underbrace{\text{sign}\left(\nabla L(\mathbf{x}, \ell_{\text{gt}})\right)}_{\text{perturbation}}$$

which adds a perturbation maximizing the cost.

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$$\mathbf{x}'_{(i)} = \operatorname{clip}_{\epsilon}\left(\mathbf{x}'_{(i-1)} + \alpha \operatorname{sign}\left(\nabla L(\mathbf{x}'_{(i-1)}, \ell_{\text{gt}})\right)\right)$$

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The clip operation projects back into an ϵ-neighborhood from $\mathbf{x}$.

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This method is designed to be fast, since 1 iteration ≈ 1 backprop step.

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Example: Targeted attack for adversarial training

Using adversarial examples as training data improves the robustness of the attacked learning model:

original

target

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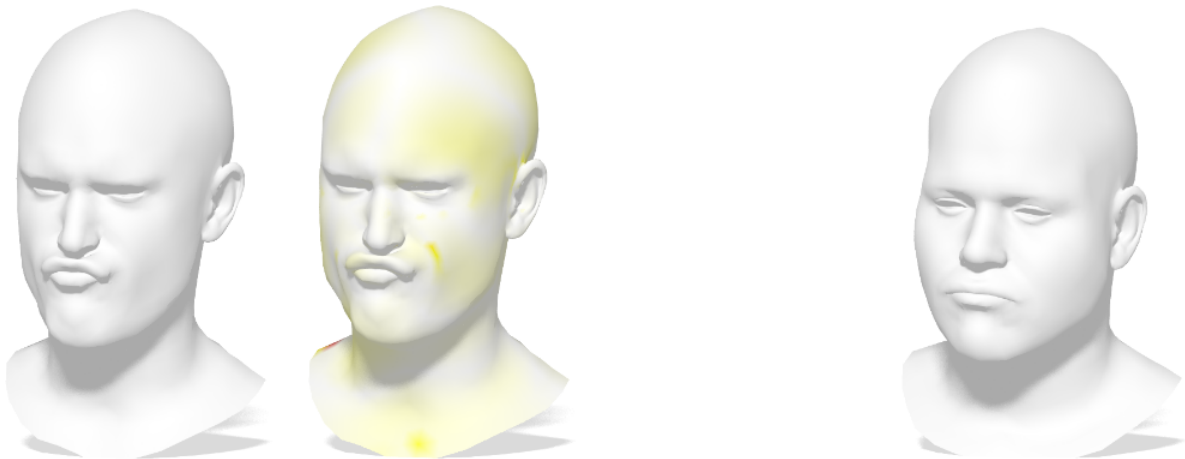
Using adversarial examples as training data improves the robustness of the attacked learning model:



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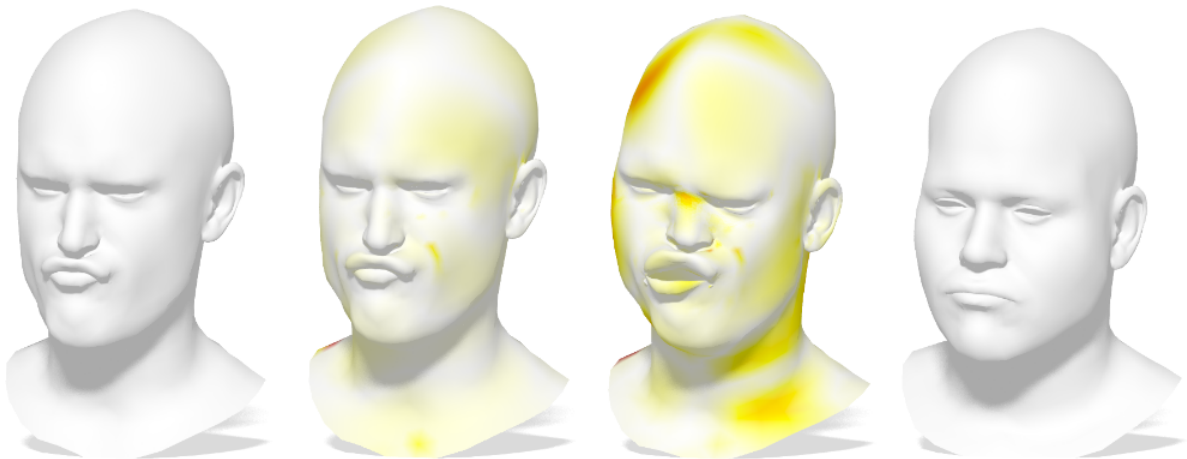

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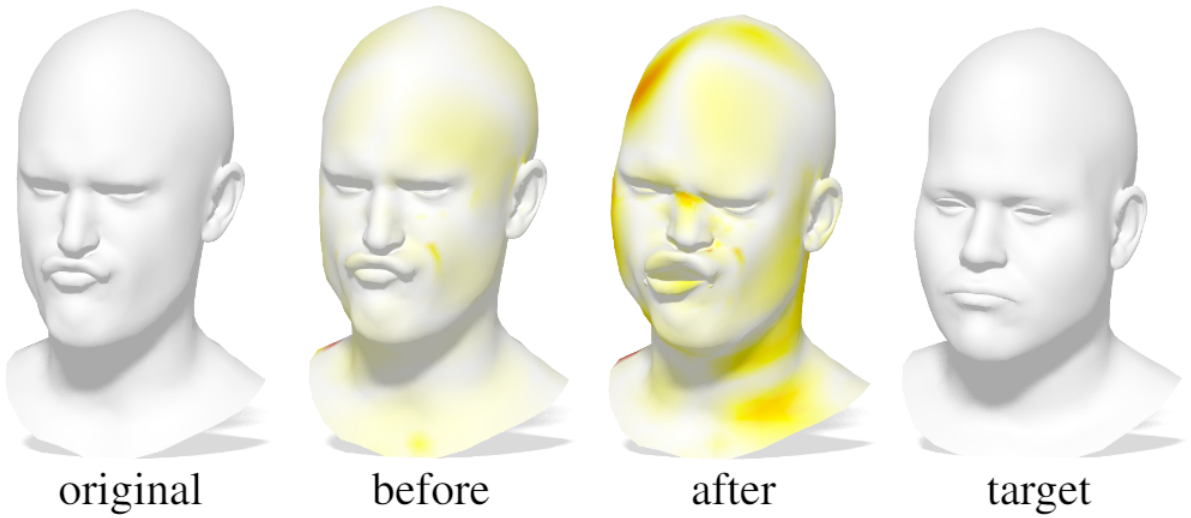

But however we improve robustness, classifiers are always vulnerable!

Vulnerability

There exist small adversarial perturbations for arbitrary classifiers.

There is a maximal achievable robustness.

Fawzi et al, "Robustness of classifiers: from adversarial to random noise", 2016; Fawzi et al, "Adversarial vulnerability for any classifier", 2018

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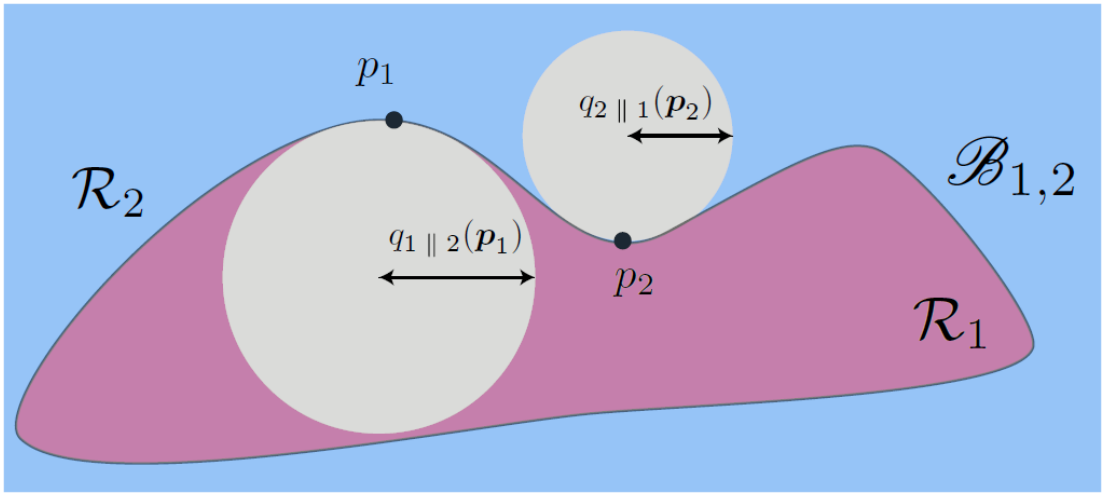
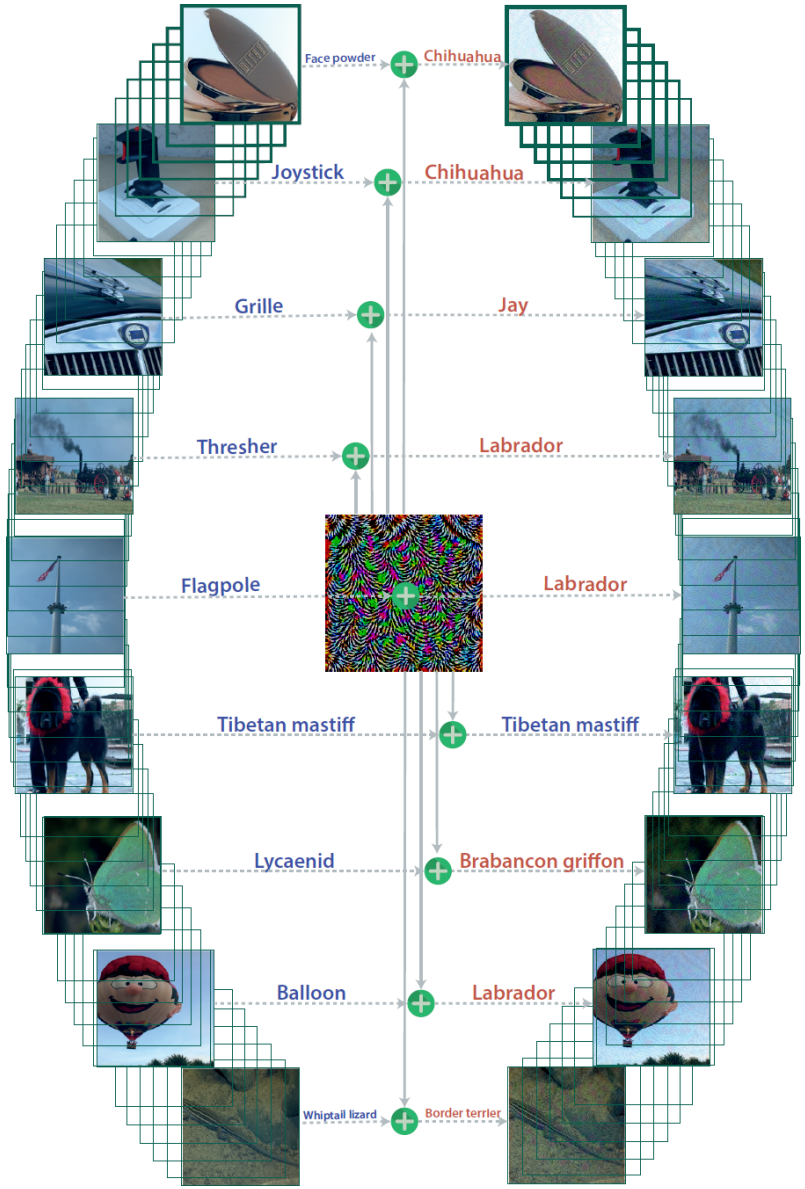


Key observation: Data points often lie very close to the decision boundary of the classifier.

Robustness can be quantified w.r.t. the curvature of the boundary.

Fawzi et al, "Robustness of classifiers: from adversarial to random noise", 2016; Fawzi et al, "Adversarial vulnerability for any classifier", 2018

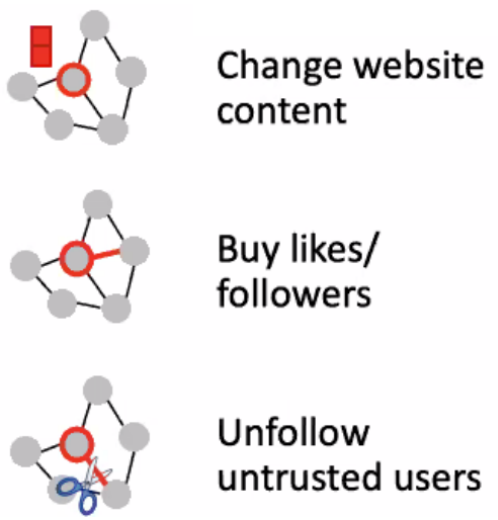
Universal perturbations

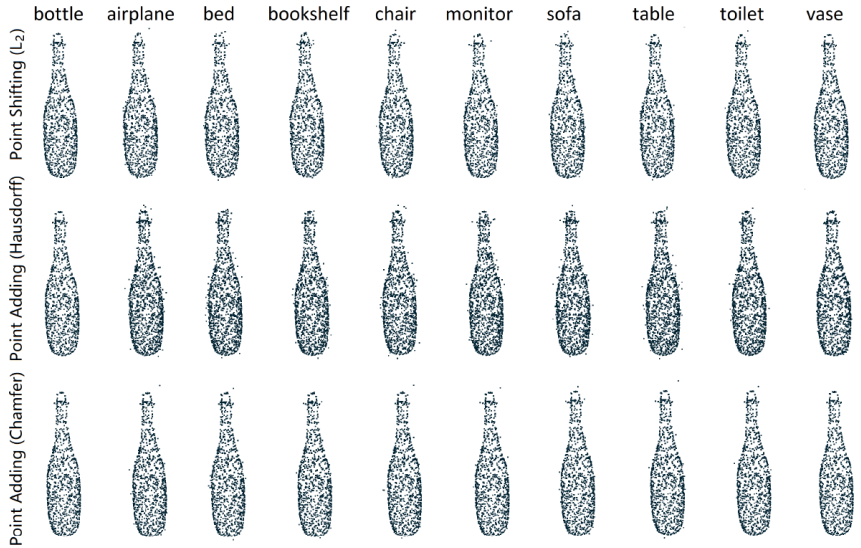


Moosavi-Dezfooli et al, "Universal adversarial perturbations", 2017

Non-Euclidean domains

Adversarial training can also be conducted on geometric domains.





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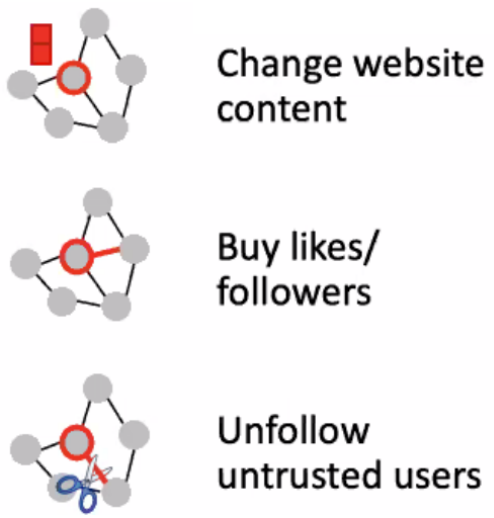

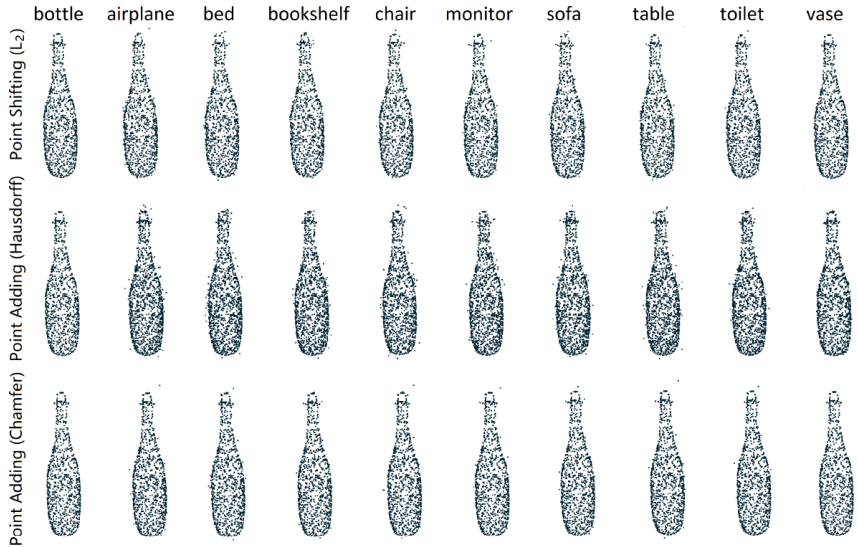

- The notion of perceptible is different than with images.
- Can alter the domain (e.g. the graph connections) rather than just the features (e.g. the values stored at the nodes).

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