Deep Learning & Applied AI

Adversarial training

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SAPIENZA
UNIVERSITÀ DI ROMA

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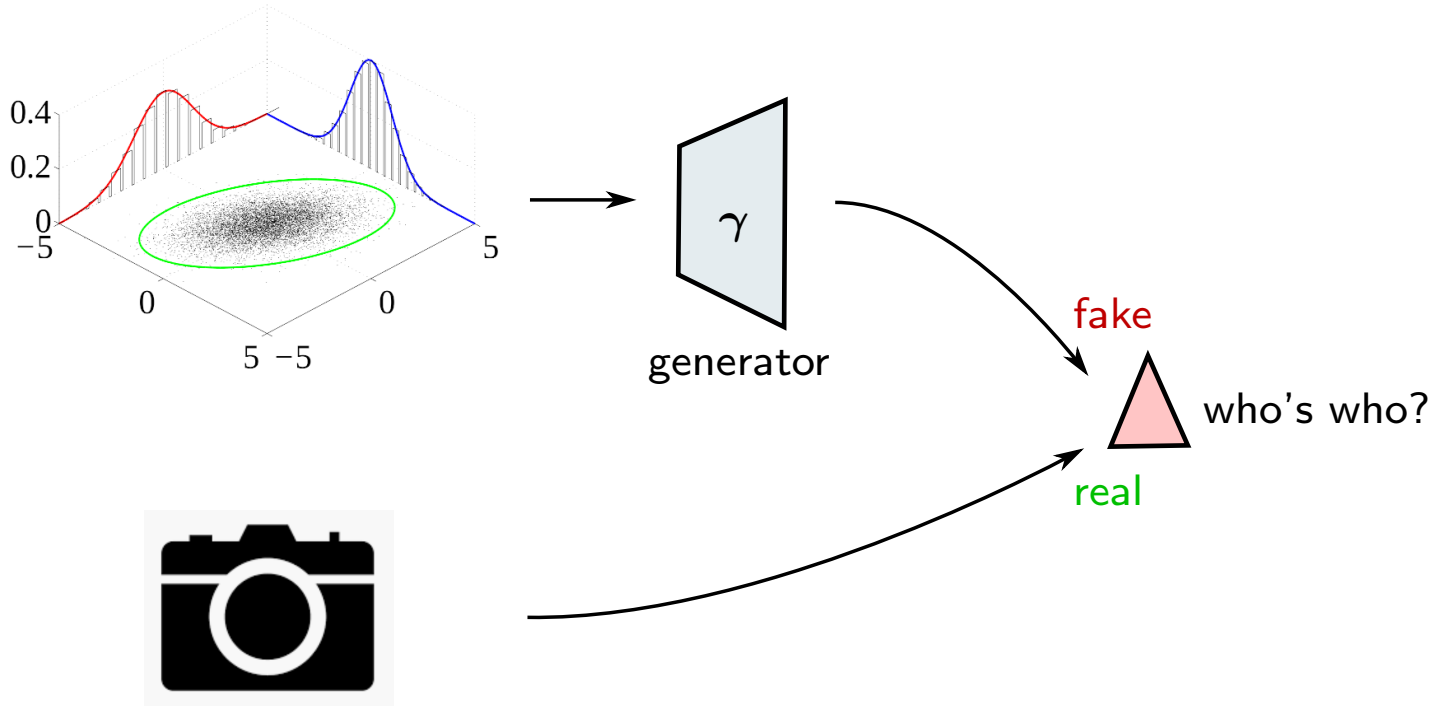
Generative adversarial networks (GAN)

Synthesize data points from a given generator (e.g. a VAE decoder), and sample real data from the actual data distribution:



Given instances of fake and real data, a generative model is "good" when you can not distinguish between the two.

Goodfellow et al, "Generative adversarial networks", NIPS 2014

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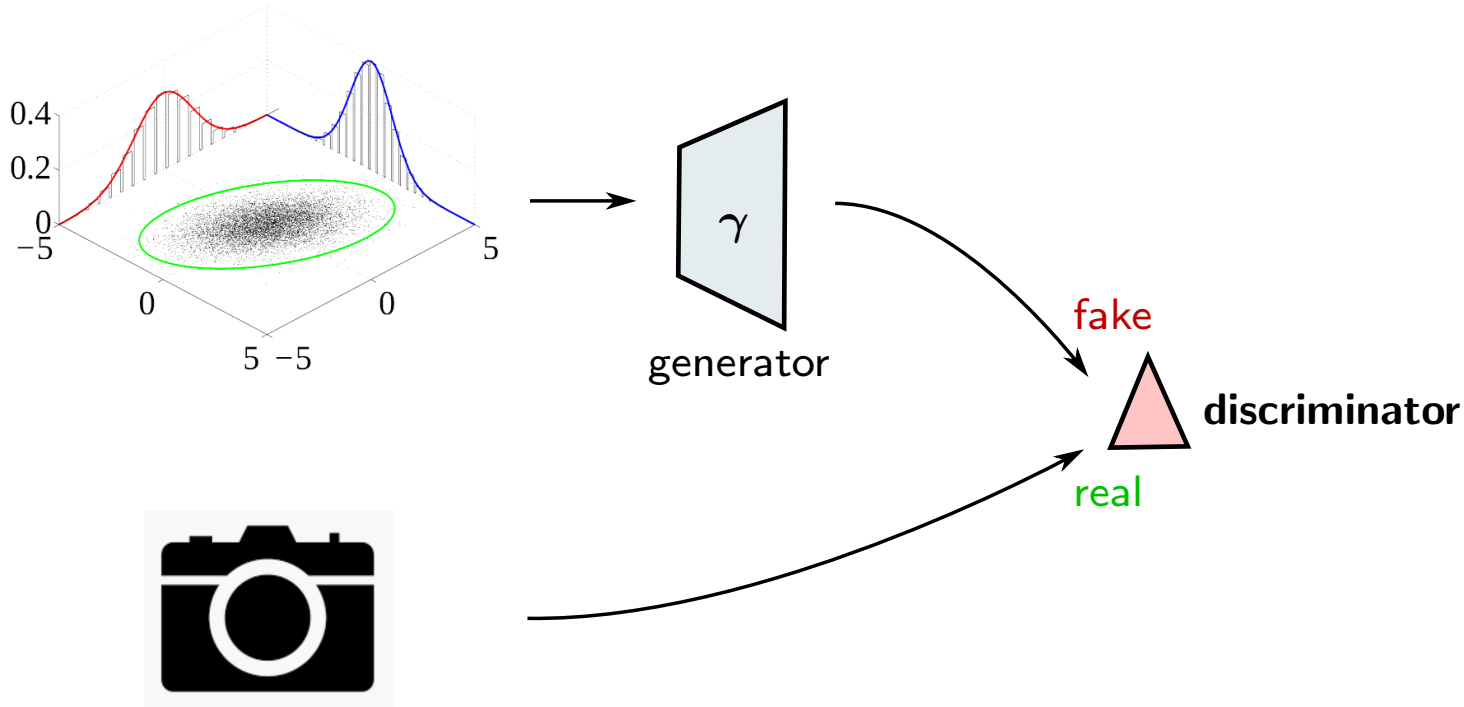

Let us train a discriminator Δ with parameters $\boldsymbol{\delta}$, whose output is the probability that the given data is real.

Goodfellow et al, "Generative adversarial networks", NIPS 2014

Generative adversarial networks (GAN)

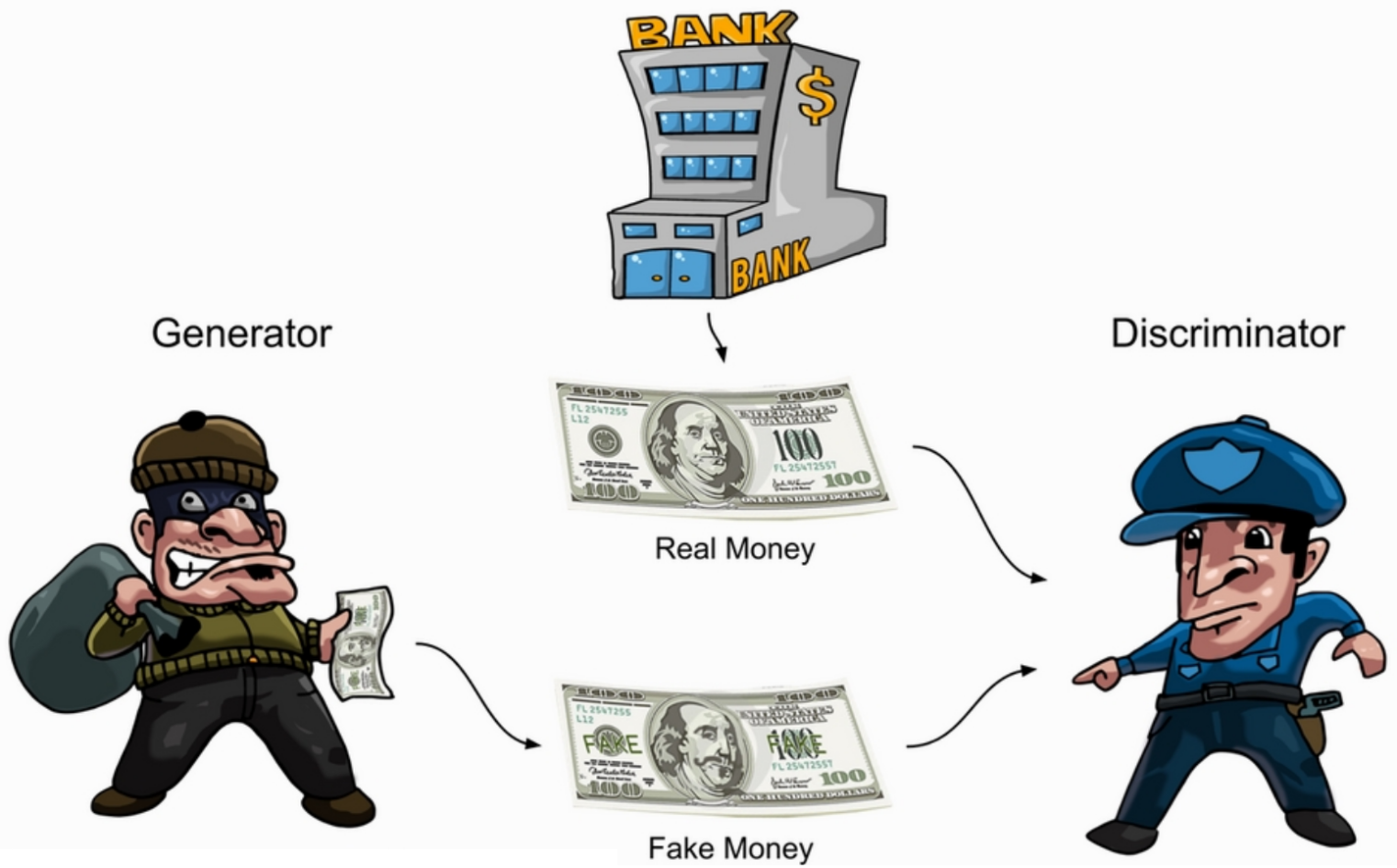

Figure: Rowel Atienza

Is this idea mathematically grounded?

Generative adversarial networks (GAN)

- $\mathbf{x}$: sample from the real distribution
- $\mathbf{x}' = D_{\boldsymbol{\gamma}}(\mathbf{z})$: generated sample, good if $p_g(\mathbf{x}') \approx p_{\text{data}}(\mathbf{x})$

A good discriminator should yield $\Delta_{\boldsymbol{\delta}}(\mathbf{x}) \approx 1$ on the real instances and $\Delta_{\boldsymbol{\delta}}(\mathbf{x}') \approx 0$ on the fake instances.

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Mathematically, we can use the following score:

$$\underbrace{\mathbb{E}_{\mathbf{x}} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x})}_{\text{real data}}$$

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We train a classifier to distinguish between generated and real data:

$$\max_{\boldsymbol{\delta}} \; \mathbb{E}_{\mathbf{x}} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{z}} \log(1 - \Delta_{\boldsymbol{\delta}}(D_{\boldsymbol{\gamma}}(\mathbf{z})))$$

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In contrast, the generator should make this value as small as possible:

$$\min_{\boldsymbol{\gamma}} \max_{\boldsymbol{\delta}} \; \mathbb{E}_{\mathbf{x}} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{z}} \log(1 - \Delta_{\boldsymbol{\delta}}(D_{\boldsymbol{\gamma}}(\mathbf{z})))$$

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The generator competes against the adversarial discriminator and tries to minimize its success rate.

Goodfellow et al, "Generative adversarial networks", NIPS 2014

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Assume $\mathbf{x} \sim p_g$ for the generated data, and $\mathbf{x} \sim p_{\text{data}}$ for the real data.

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Assume $\mathbf{x} \sim p_g$ for the generated data, and $\mathbf{x} \sim p_{\text{data}}$ for the real data.

Let us try to maximize the discriminator score given a generator G:

$$J(G) = \max_{\boldsymbol{\delta}} \; \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{x} \sim p_g} \log(1 - \Delta_{\boldsymbol{\delta}}(\mathbf{x}))$$

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$$= \max_{\boldsymbol{\delta}} \int [\log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) p_{\text{data}}(\mathbf{x}) + \log(1 - \Delta_{\boldsymbol{\delta}}(\mathbf{x})) p_g(\mathbf{x})] \, d\mathbf{x}$$

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For any given $\mathbf{x}$, we want to maximize $\Delta_{\boldsymbol{\delta}}(\mathbf{x}) = a$; let's rename for simplicity $p_{\text{data}}(\mathbf{x}) \equiv p$ and $p_g(\mathbf{x}) \equiv q$, we get to:

$$\max_a \; p \log a + q \log(1 - a)$$

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This is maximized when the derivative w.r.t. a is zero:

$$\frac{p}{a} - \frac{q}{1 - a} = 0$$

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We thus have a closed-form solution for the optimal discriminator:

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We thus have a closed-form solution for the optimal discriminator:

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Let us define the distribution $\rho = \frac{1}{2}p_{\text{data}} + \frac{1}{2}p_g$. We get:

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$$\begin{aligned} J(G) &= \mathbb{E}_{\mathbf{x}\sim p_{\text{data}}} \log \frac{p_{\text{data}}(\mathbf{x})}{\rho(\mathbf{x})} + \mathbb{E}_{\mathbf{x}\sim p_g} \log \frac{p_g(\mathbf{x})}{\rho(\mathbf{x})} + \text{const.} \\ &= KL(p_{\text{data}} \| \rho) + KL(p_g \| \rho) + \text{const.} \end{aligned}$$

Goodfellow et al, "Generative adversarial networks", NIPS 2014

Generative adversarial networks (GAN)

Therefore, the optimal GAN generator is found by minimizing:

$$J(G) = KL(p_{\text{data}} \| \rho) + KL(p_g \| \rho) + \text{const.}$$

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$$\min_{p_g} \underbrace{KL(p_{\text{data}} \| \rho) + KL(p_g \| \rho)}_{\substack{2 \times \text{Jensen-Shannon divergence} \\ \text{between } p_{\text{data}} \text{ and } p_g}}$$

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Therefore, the optimal GAN generator is found by minimizing:

$$\min_{p_g} \underbrace{KL(p_{\text{data}} \| \rho) + KL(p_g \| \rho)}_{\approx JS(p_{\text{data}} \| p_g)}$$

Property: $p_{\text{data}} = p_g \;\Leftrightarrow\; JS(p_{\text{data}} \| p_g) = 0$

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Within the GAN paradigm, the globally optimal generator has a data distribution exactly equal to the real distribution of the data.

Goodfellow et al, "Generative adversarial networks", NIPS 2014

Adversarial training

Training the generator on data produced by an adversary is an example of a more general concept called adversarial training.

The generated data samples used for training are adversarial examples.

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Adversarial examples can be used maliciously.

"speed limit 50mph"

Adversarial attacks

Szegedy et al, “Intriguing properties of neural networks”, 2013

Adversarial attacks

"truck"

"ostrich"

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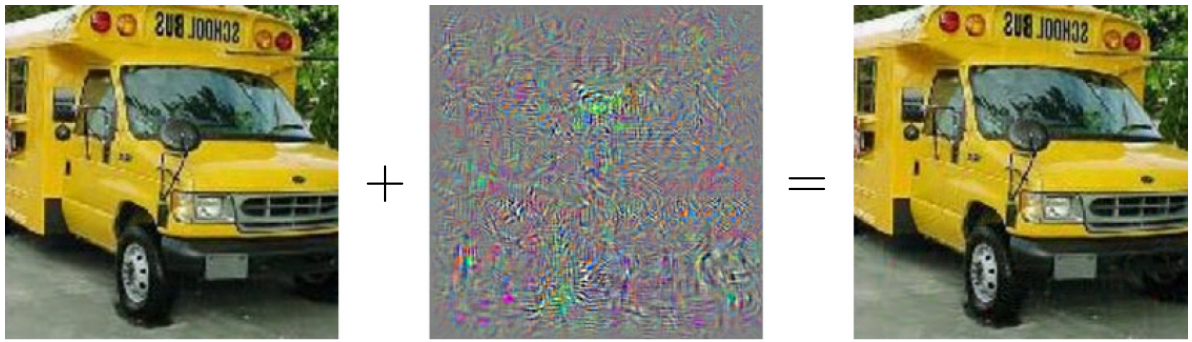

"truck" perturbation "ostrich"

The perturbation can be explicitly optimized for.

Adversarial attacks can cause a system to take unwanted actions.

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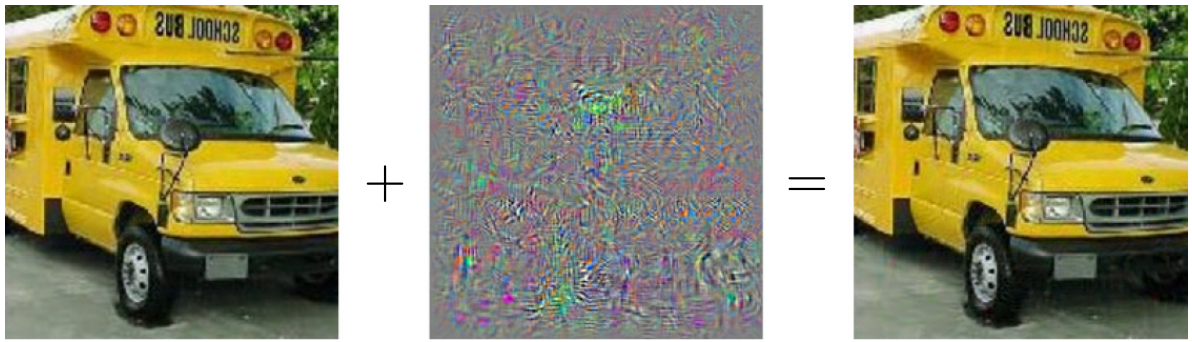
Adversarial attacks



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How to construct undetectable adversarial examples?

Szegedy et al, "Intriguing properties of neural networks", 2013

Types of attack

Distinction based on the amount of information available to the attacker:

- Black-box attack:

  Can only query the target model.

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  Can only query the target model.
- Gray-box attack:
  Access to partial information (only the features, architecture, etc.).
- White-box attack:
  Complete access to the network (architecture, parameters, etc.).
  You are not allowed to touch the network weights.

Targeted attacks

Given an input sample $\mathbf{x}$, a classifier C, and a target class t, consider:

$$\min_{\mathbf{x}' \in [0,1]^n} \|\mathbf{x} - \mathbf{x}'\|_2^2$$
$$\text{s.t. } C(\mathbf{x}') = t$$

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Relax the difficult constraint to a penalty term:

$$\min_{\mathbf{x}' \in [0,1]^n} \|\mathbf{x} - \mathbf{x}'\|_2^2 + c\, L(\mathbf{x}', t)$$

where L is the cross-entropy loss.

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$c > 0$ is a trade-off parameter that is chosen as small as possible; it can be found via line search.

Szegedy et al, "Intriguing properties of neural networks", 2013

Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \; d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta})$$
$$\text{s.t. } C(\mathbf{x} + \boldsymbol{\delta}) = t$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d is the data term (e.g. a distance) that depends on the specific task.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

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A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \; d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta})$$
$$\text{s.t. } f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d is the data term (e.g. a distance) that depends on the specific task.

f is such that $C(\mathbf{x} + \boldsymbol{\delta}) = t$ if and only if $f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta}) + c\, f(\mathbf{x} + \boldsymbol{\delta})$$

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Possible instance of the problem above:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \|\boldsymbol{\delta}\|_p + c\, \left(\max\{F(\mathbf{x} + \boldsymbol{\delta})_i \ : \ i \neq t\} - F(\mathbf{x} + \boldsymbol{\delta})_t\right)^+$$

where $F : \mathbf{x} \mapsto [0,1]^k$ is the full neural network yielding a probability distribution over all k classes for a given input $\mathbf{x}$, and $(a)^+ = \max(a, 0)$.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

Untargeted attacks

If there is no specific target toward which to misclassify, consider the closed-form expression for a given input $\mathbf{x}$ with ground-truth label ℓ_{gt}:

$$\mathbf{x}' = \mathbf{x} + \alpha \underbrace{\operatorname{sign}\left(\nabla L(\mathbf{x}, \ell_{\text{gt}})\right)}_{\text{perturbation}}$$

which adds a perturbation maximizing the cost.

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$$\mathbf{x}'_{(i)} = \operatorname{clip}_{\epsilon}\left(\mathbf{x}'_{(i-1)} + \alpha \operatorname{sign}\left(\nabla L(\mathbf{x}'_{(i-1)}, \ell_{\text{gt}})\right)\right)$$

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The clip operation projects back into an ϵ-neighborhood from $\mathbf{x}$.

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This method is designed to be fast, since 1 iteration ≈ 1 backprop step.

Kurakin et al, "Adversarial examples in the physical world", 2016

Example: Targeted attack for adversarial training

Using adversarial examples as training data improves the robustness of the attacked learning model:

original

target

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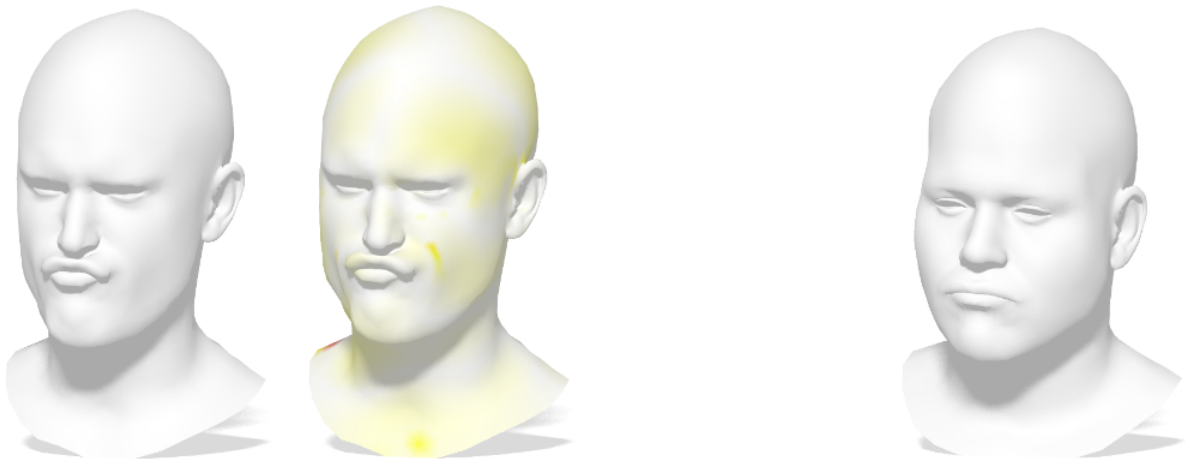

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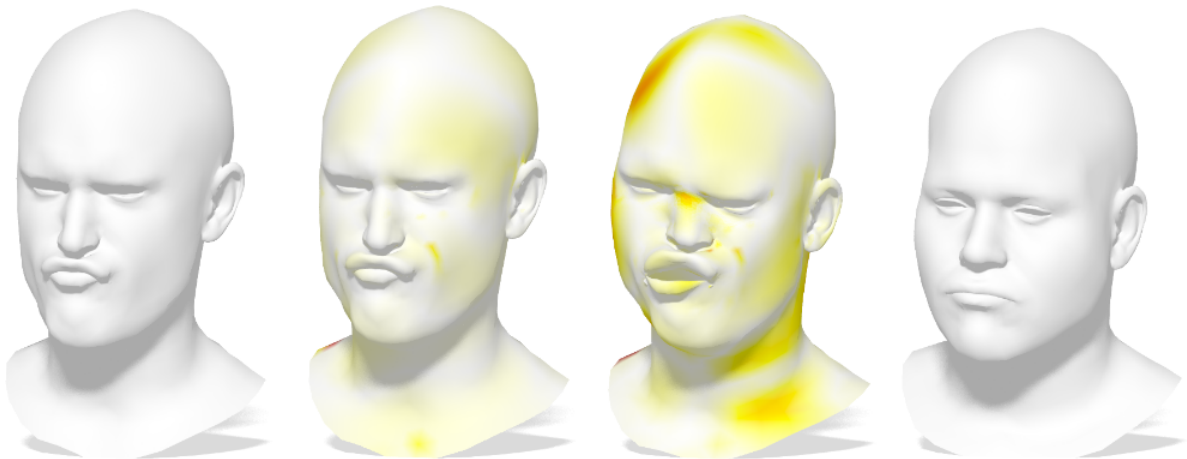

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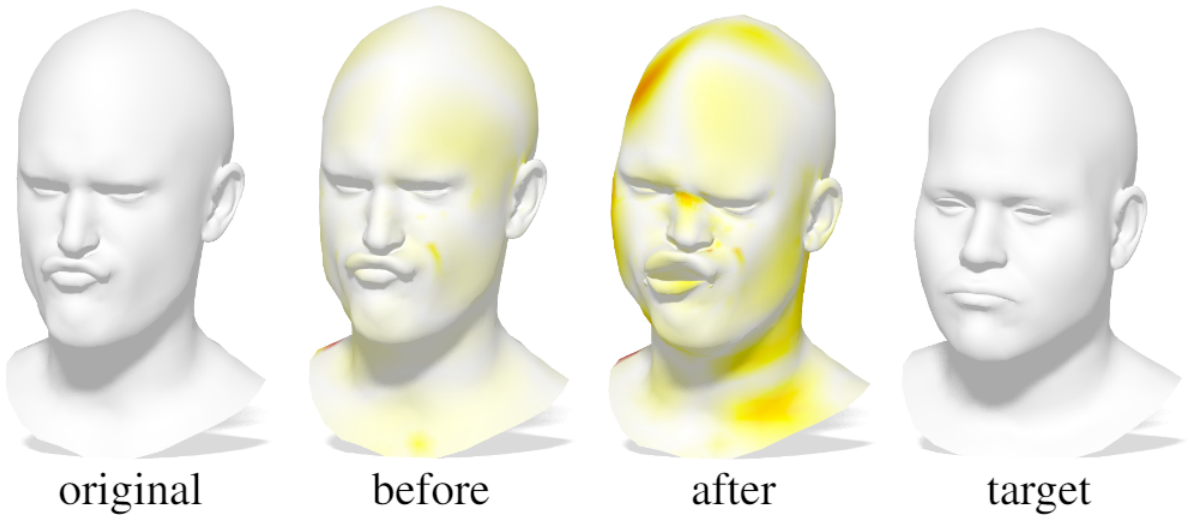
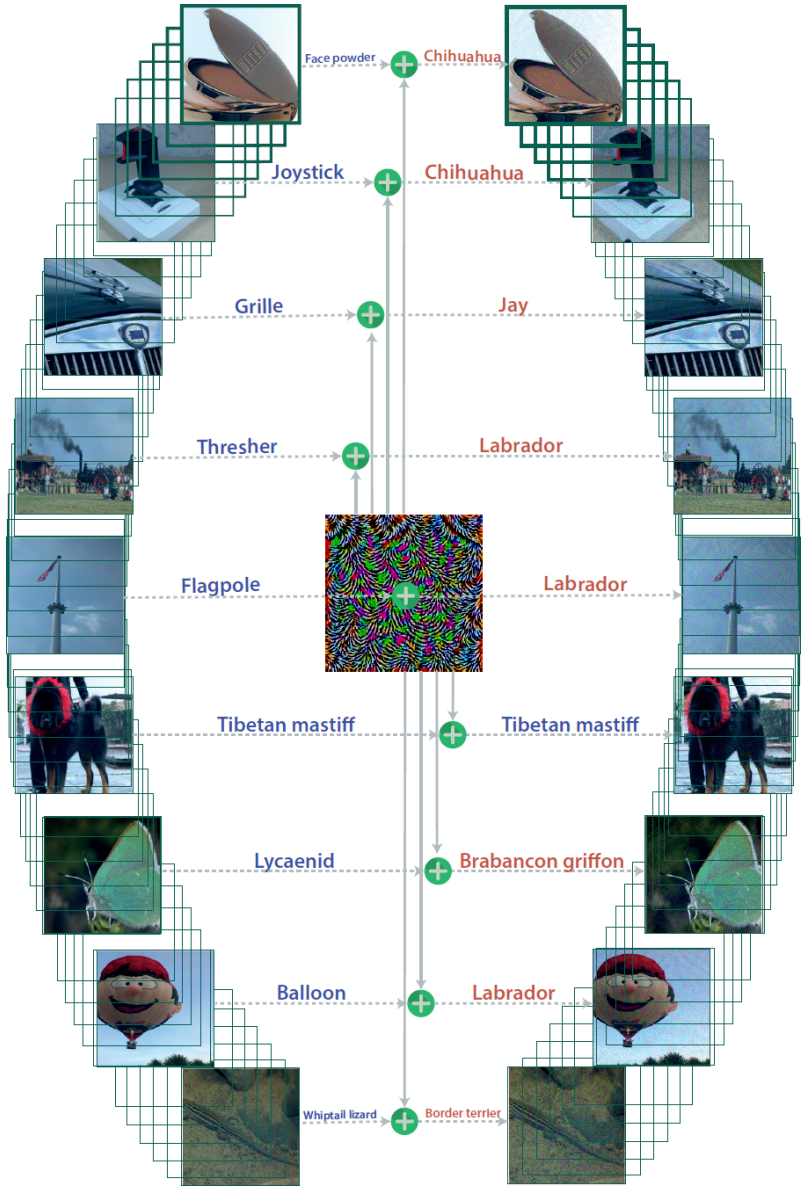


But however we improve robustness, classifiers are always vulnerable!

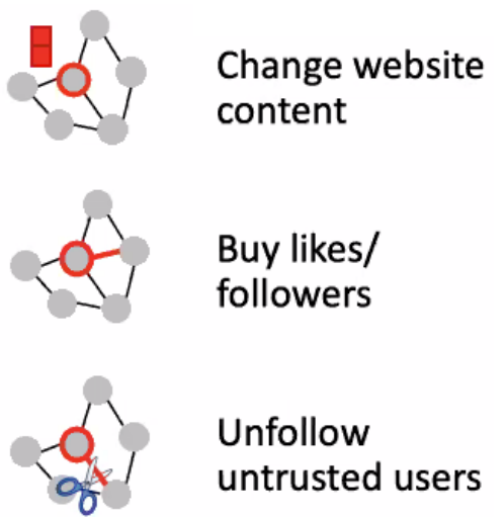
Universal perturbations

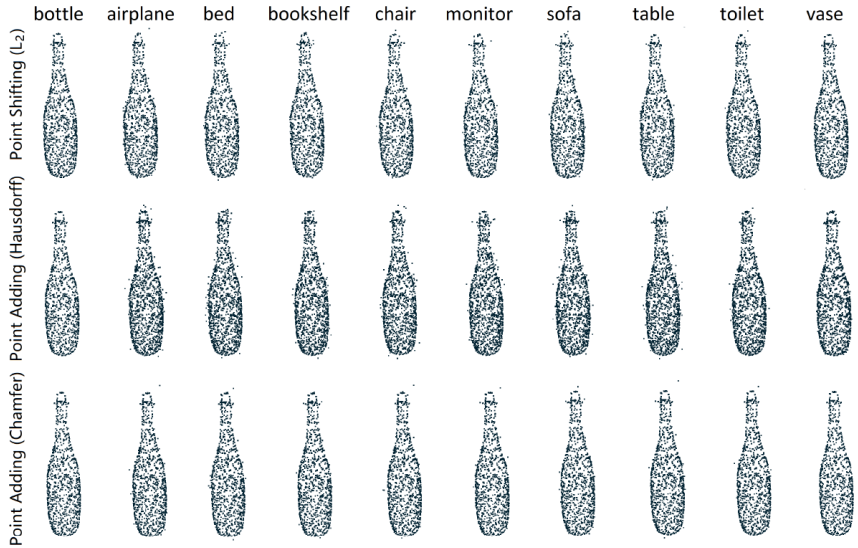


Moosavi-Dezfooli et al, "Universal adversarial perturbations", 2017

Non-Euclidean domains

Adversarial training can also be conducted on geometric domains.





Zügner et al, "Adversarial attacks on neural networks for graph data", 2018; Xiang et al, "Generating 3D Adversarial Point Clouds", 2018

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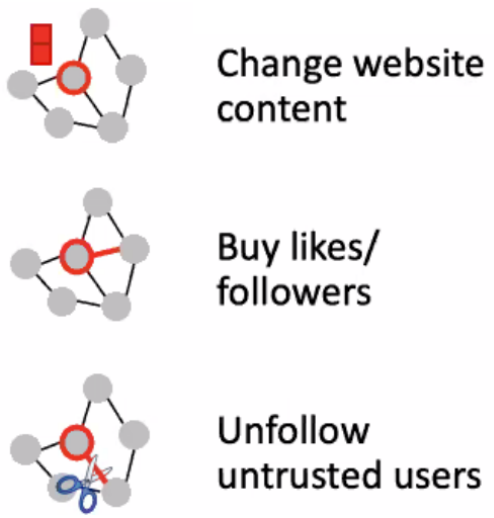

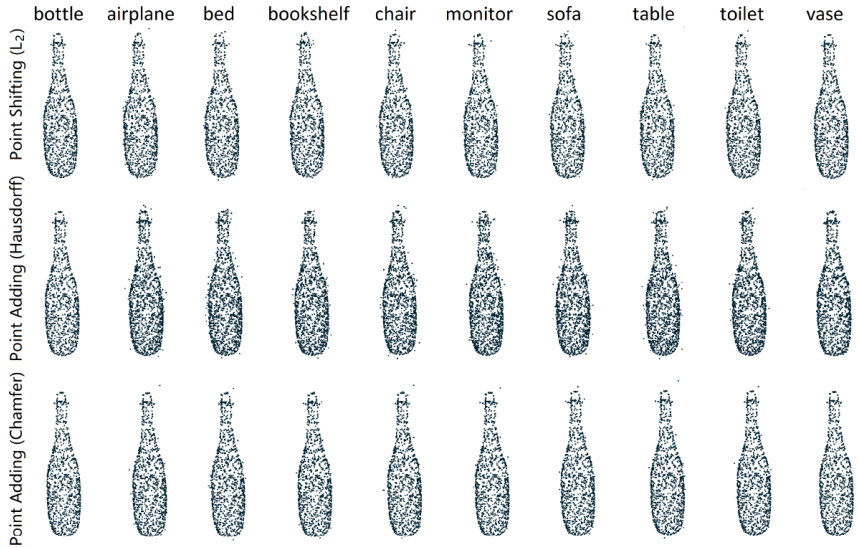

- The notion of perceptible is different than with images.
- Can alter the domain (e.g. the graph connections) rather than just the features (e.g. the values stored at the nodes).

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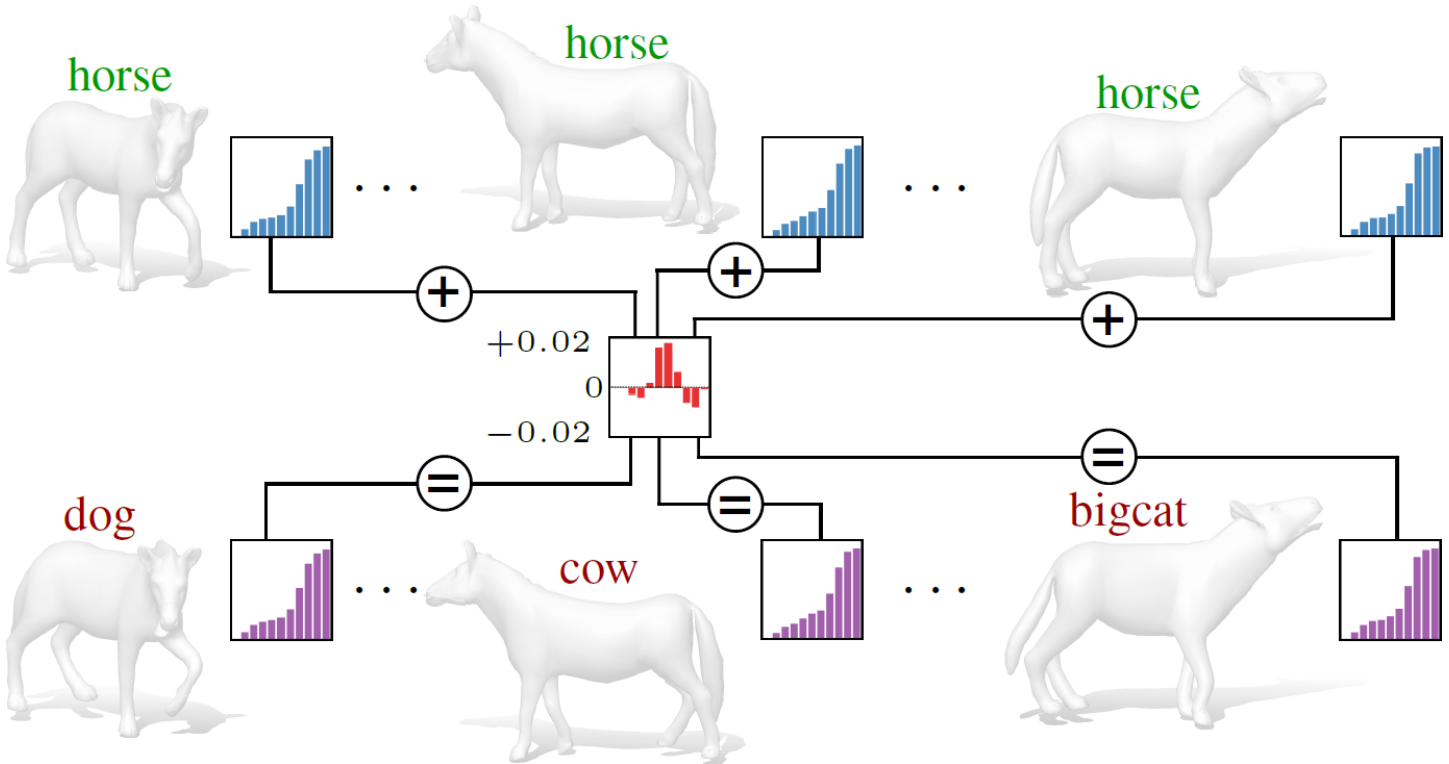
Universal perturbations on 3D data



Rampini, Pestarini, Cosmo, Melzi, Rodolà, "Universal Spectral Adversarial Attacks for Deformable Shapes", 2021