Reinforcement Learning

Introduction

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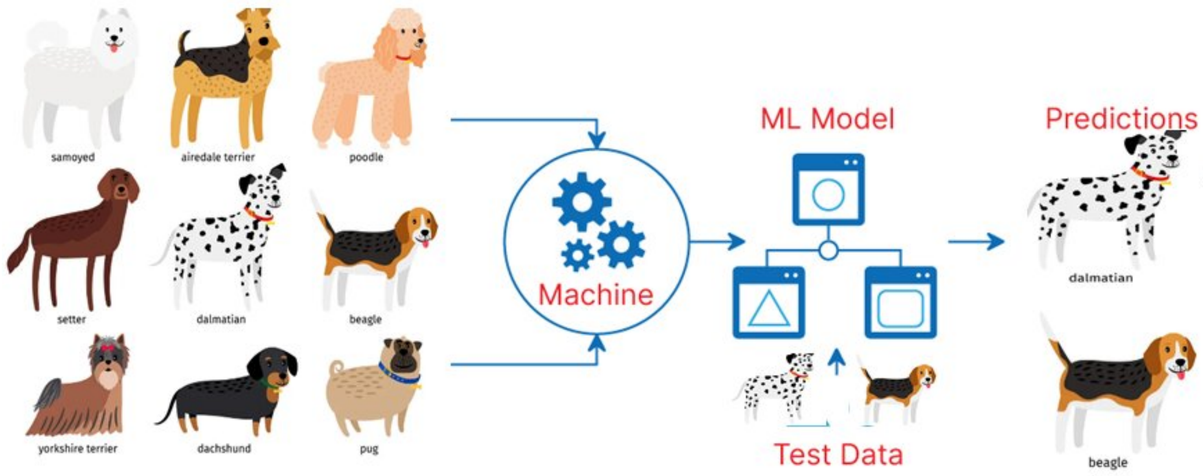
Machine Learning @ Mathematical Sciences for AI

Supervised Learning vs Reinforcement Learning

Supervised Learning

Learn using labeled data

Need pairs $\{x_i, y_i\}$



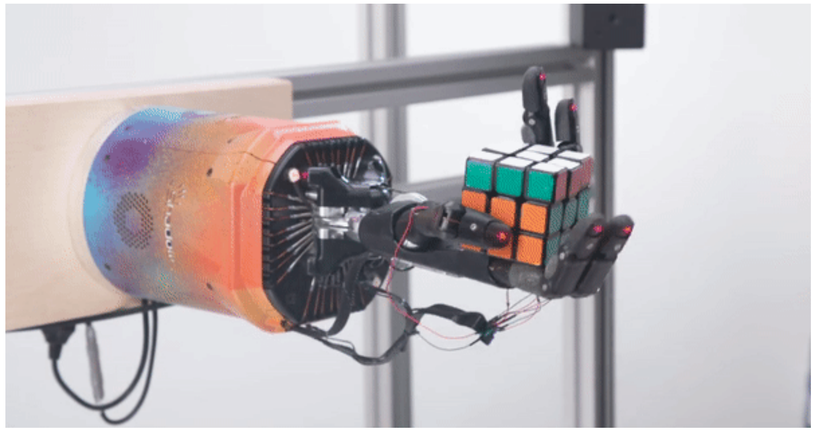
Reinforcement Learning

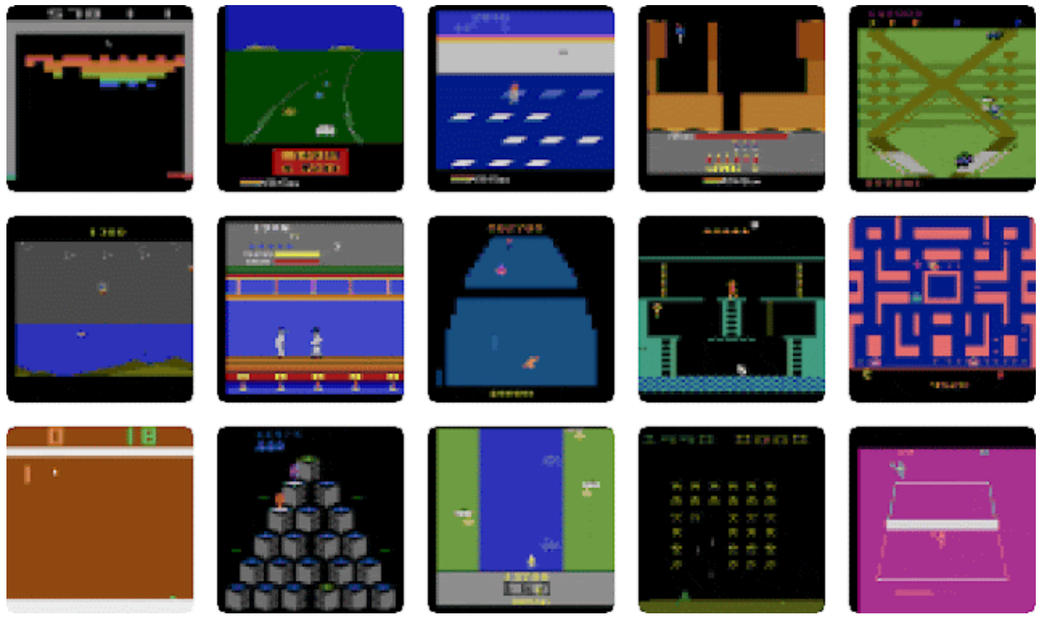
Framework for solving control tasks

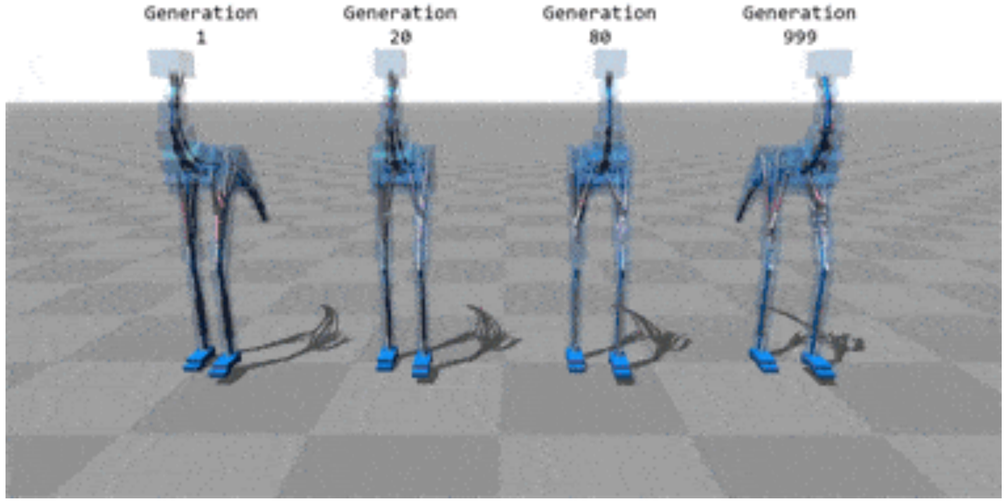
Learn from the environment through trial and error

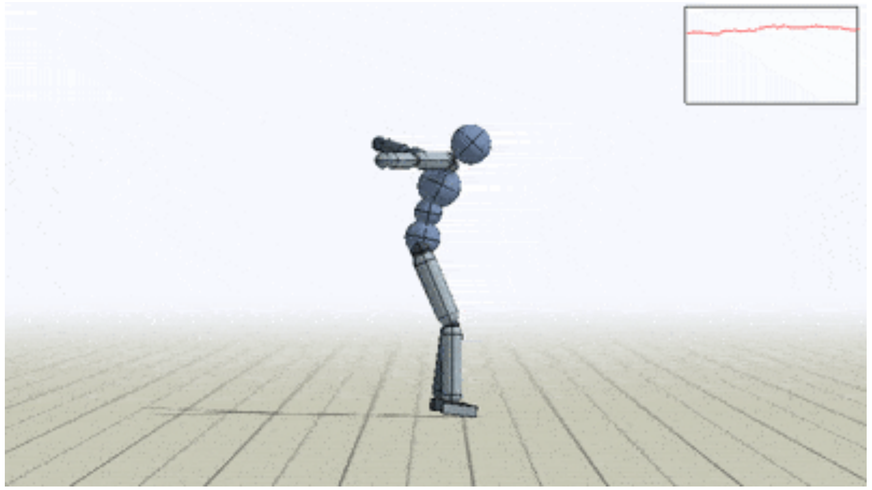
Receive rewards as feedback

Reinforcement Learning









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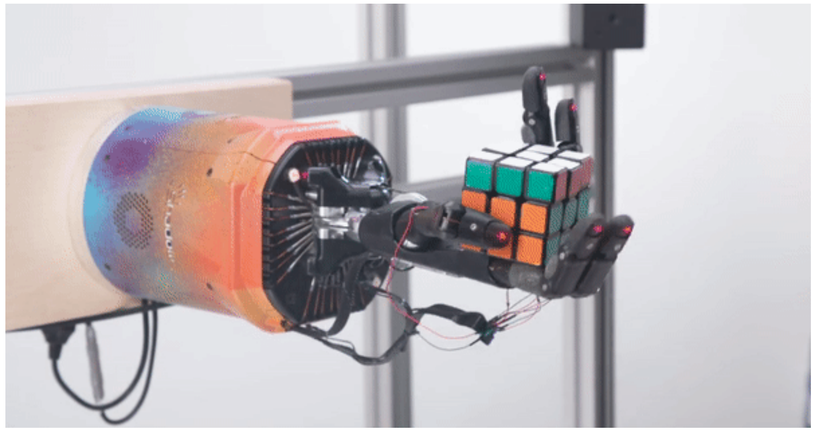

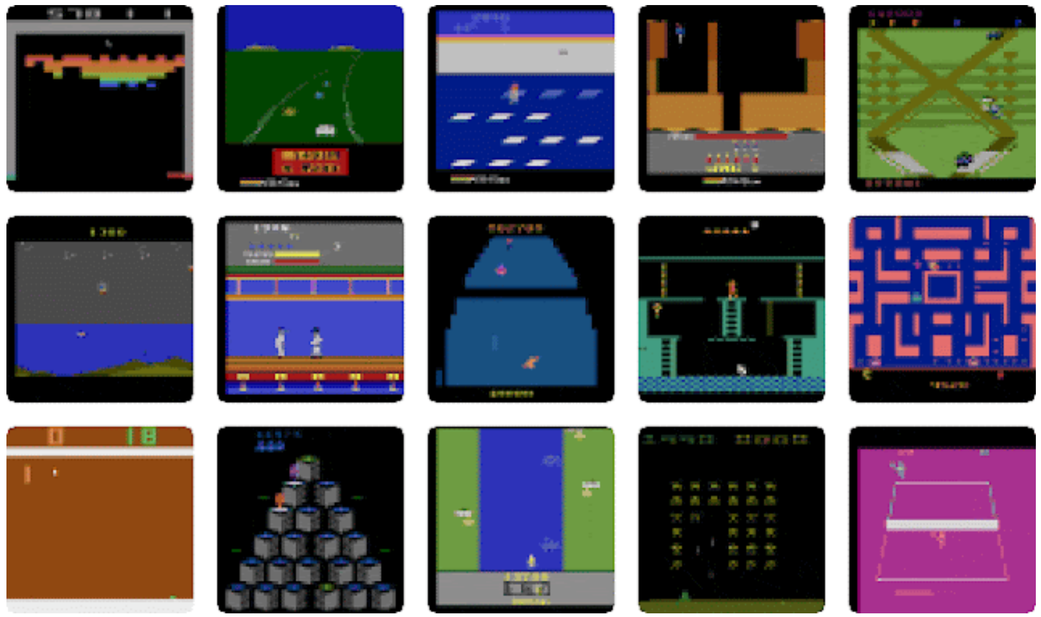

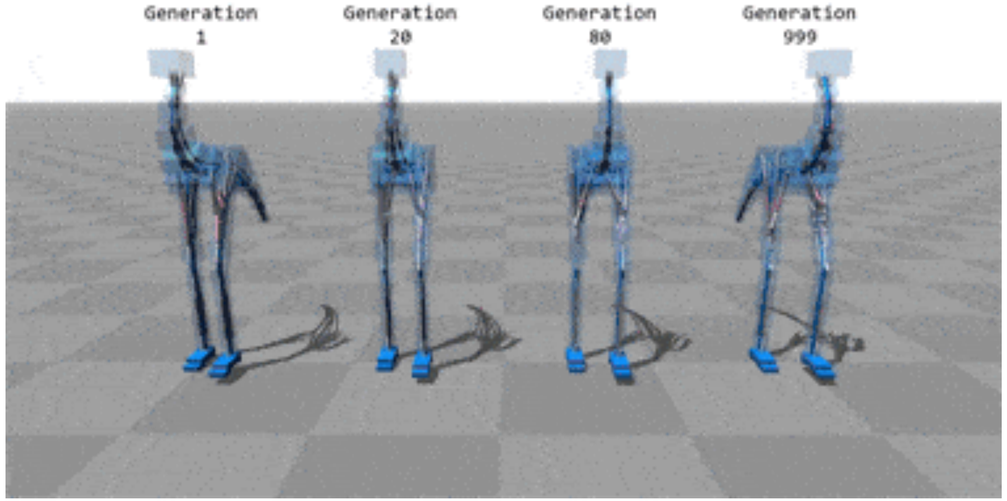

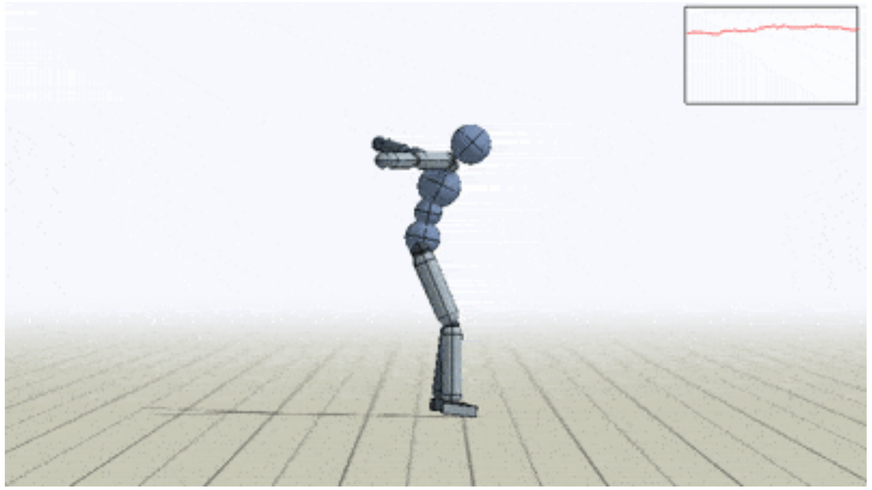

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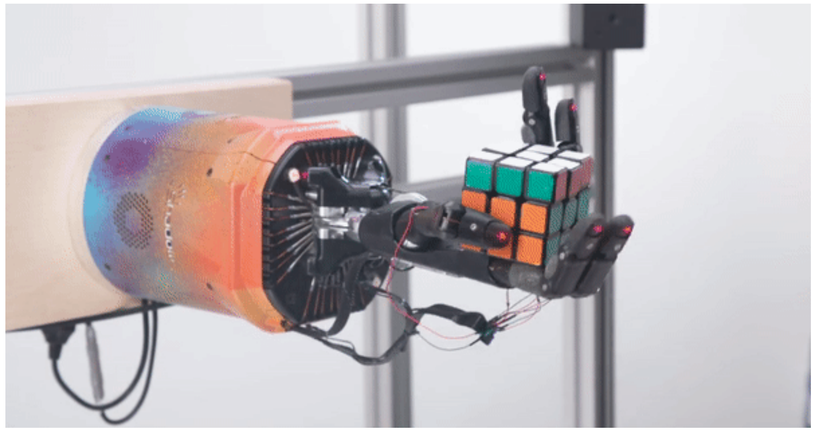

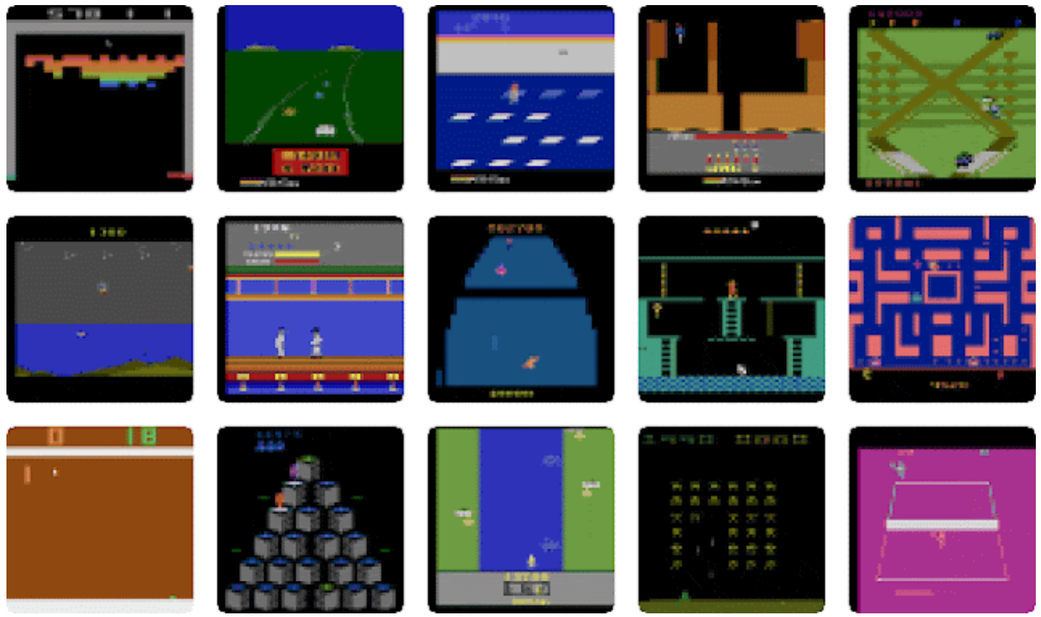

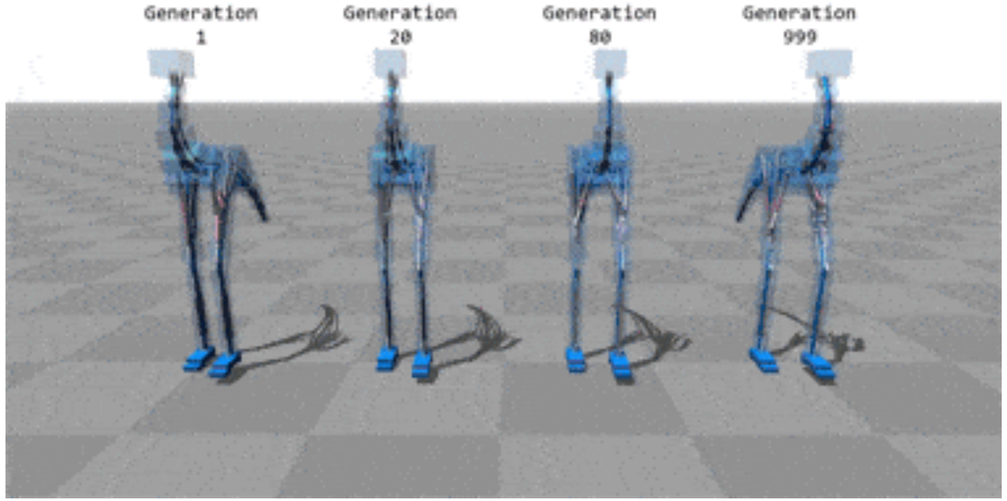

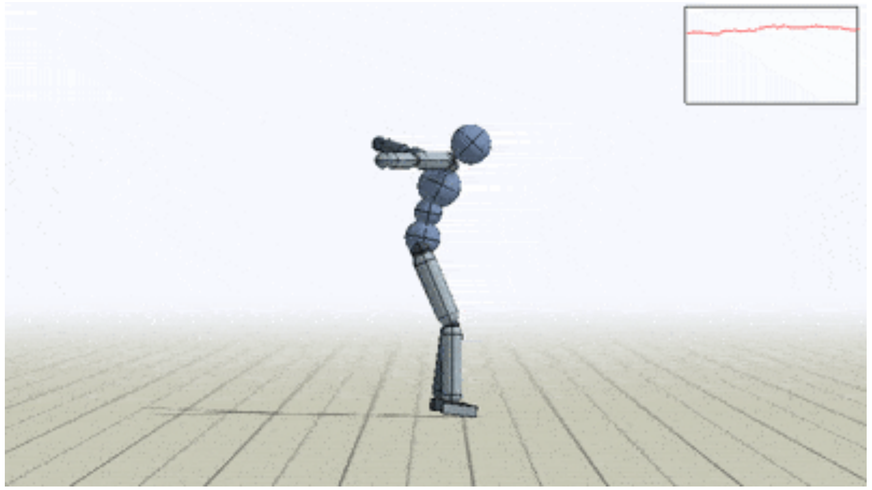

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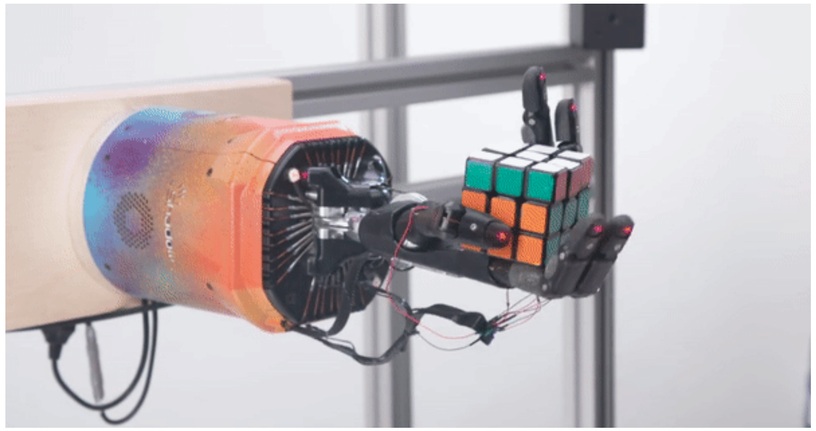

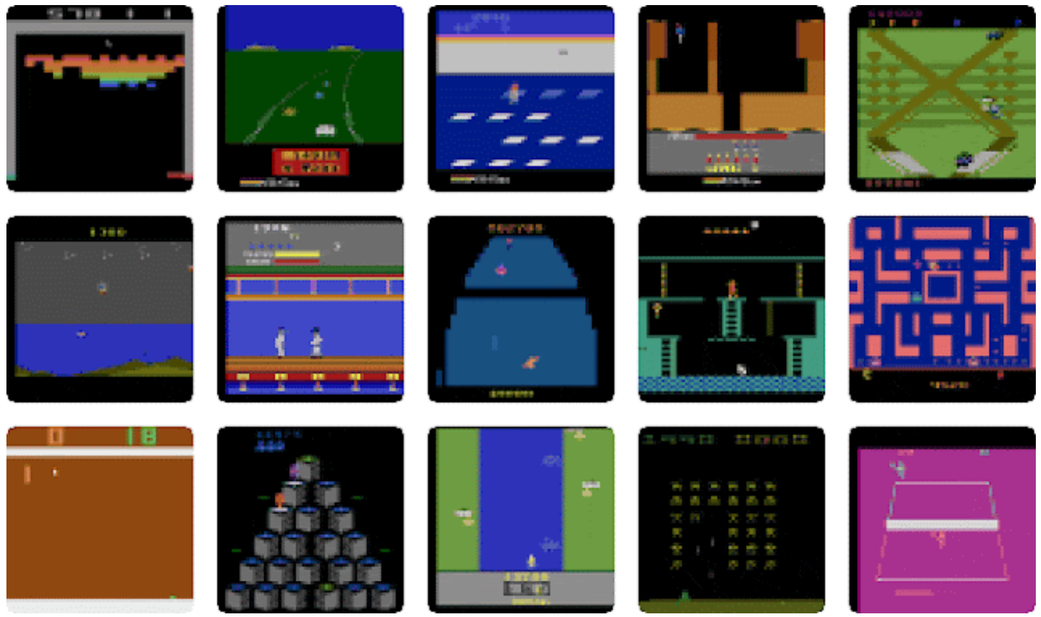

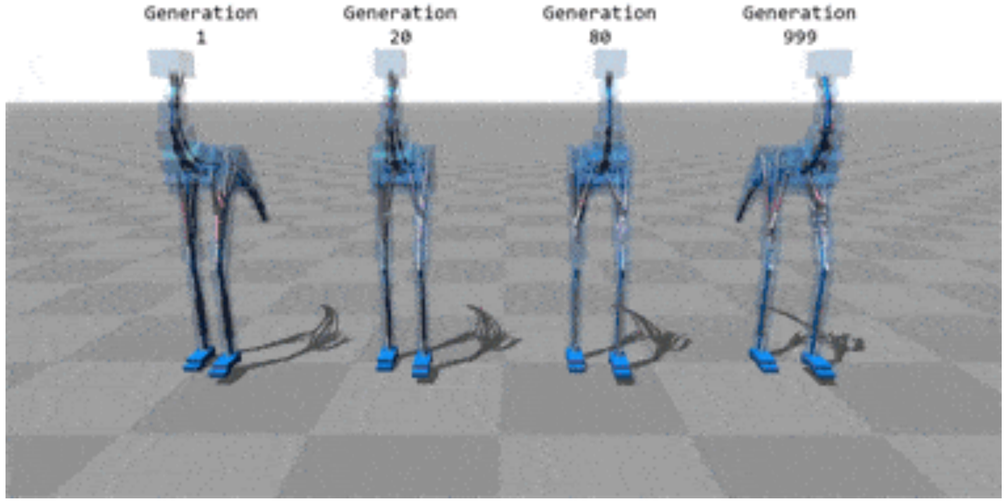

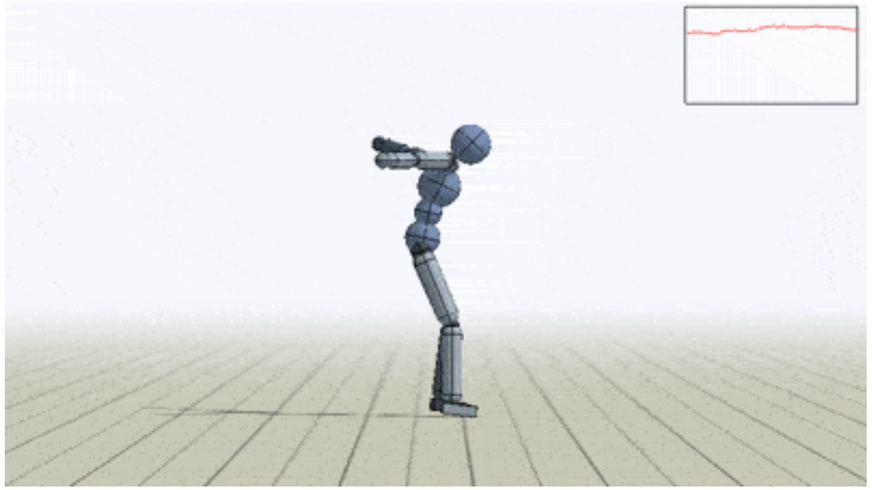

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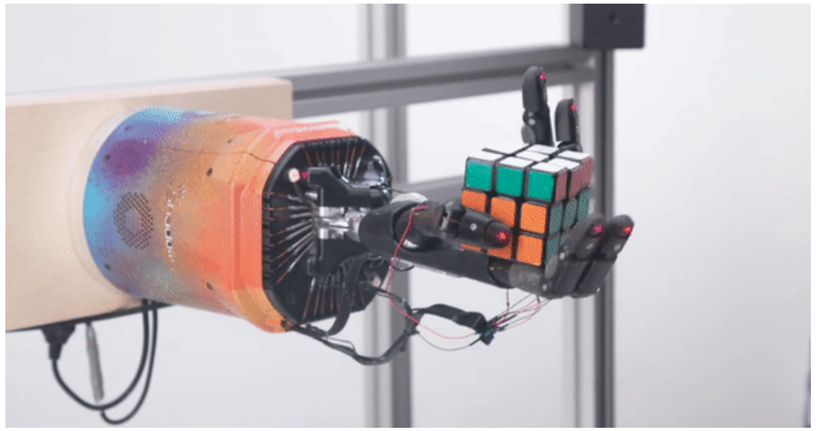

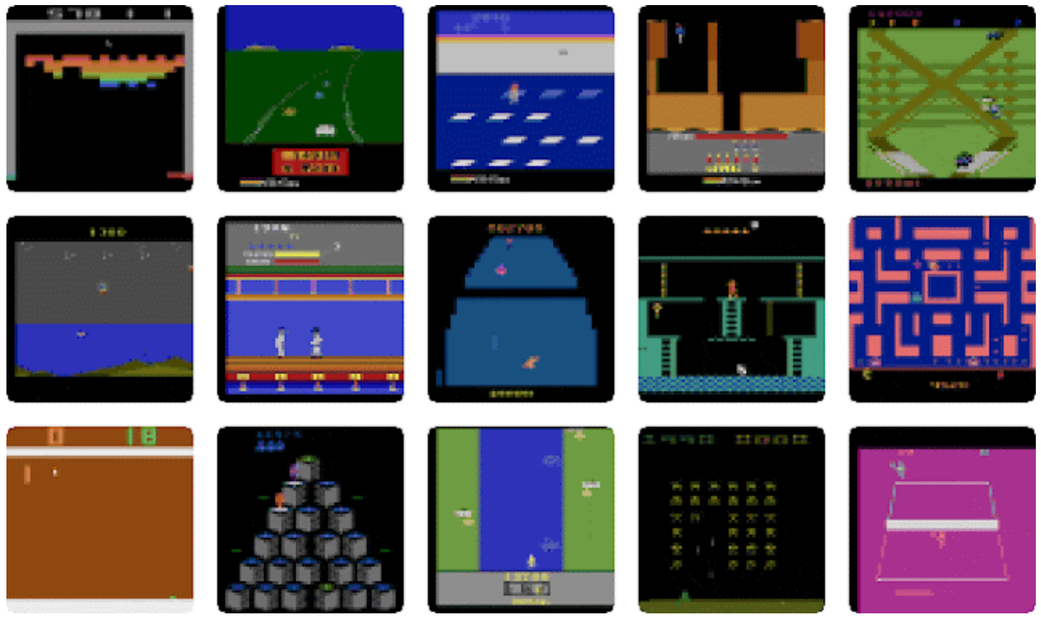

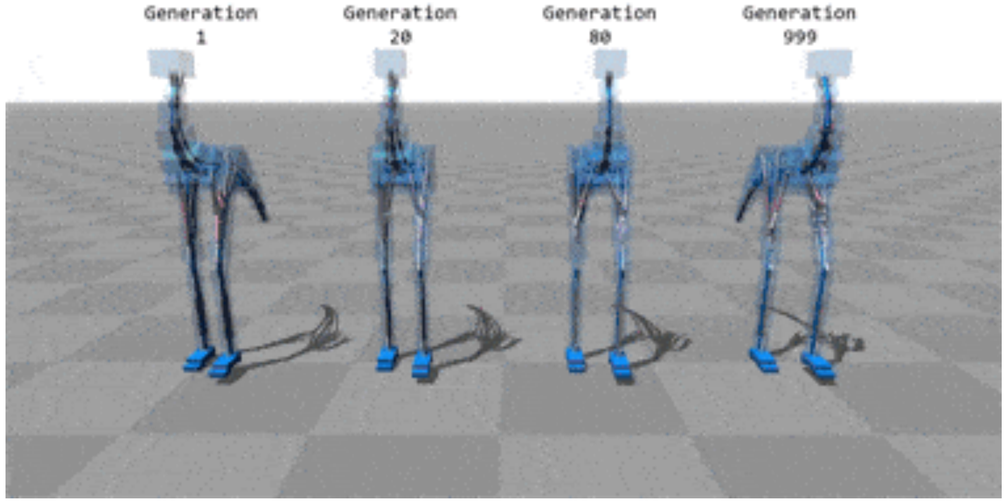

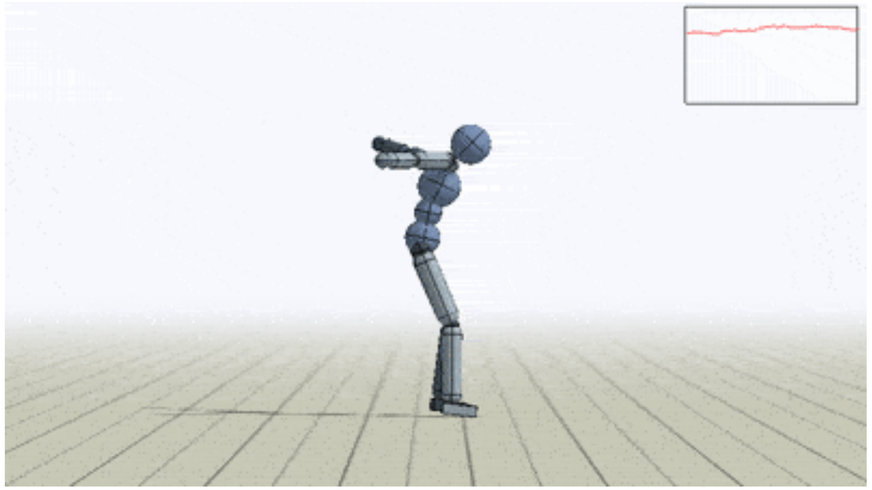

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Reinforcement Learning

https://www.youtube.com/watch?v=PxoG0A2QoFs

Reinforcement Learning

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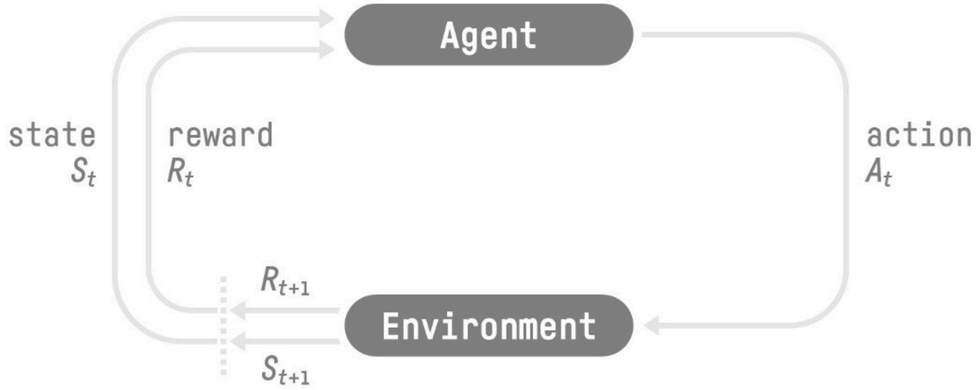
What you will learn

Basic concepts of RL

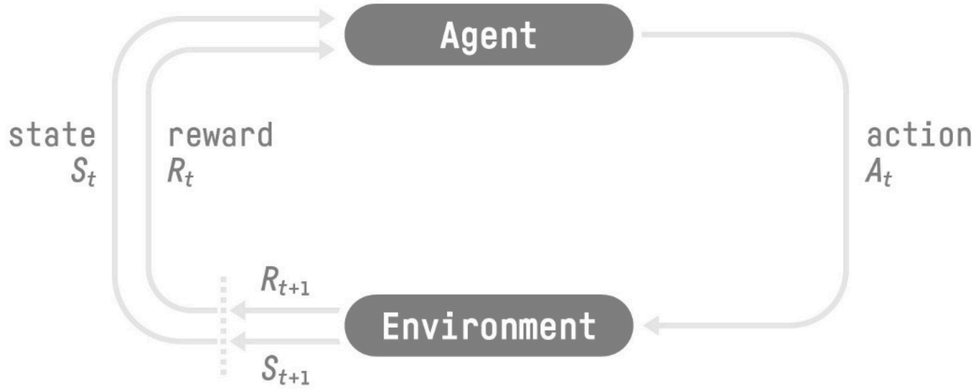
Q-Learning

Deep Q-Learning

Markov Decision Process (MDP) (finite)

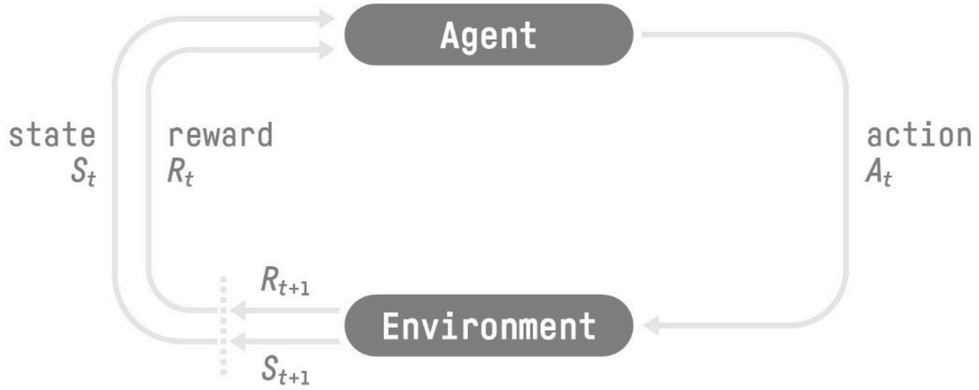


Markov Decision Process (MDP) (finite)



For t = 0, 1, 2, 3 …

Markov Decision Process (MDP) (finite)



For t = 0, 1, 2, 3 …

Trajectory: $S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$

Agent Inputs

Agent Inputs

How do agents get information from the environment?

Agent Inputs

How do agents get information from the environment?

States

Agent Inputs

How do agents get information from the environment?

States

Observations

Agent Inputs

How do agents get information from the environment?

States

Complete description of the state of the world
(no hidden information)

Observations

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Observations

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Complete description of the state of the world
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Observations

Partial description of the state of the world

Agent Inputs

How do agents get information from the environment?

States

Complete description of the state of the world
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Observations

Partial description of the state of the world

Agent Outputs

Agent Outputs

How do agents interact with the environment?

Agent Outputs

How do agents interact with the environment?

Action space

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

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Discrete

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

Discrete

Continuous

Agent Outputs

How do agents interact with the environment?

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Discrete

The number of possible actions is *finite*

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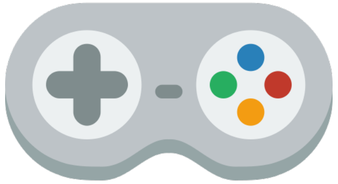

Discrete

The number of possible actions is *finite*

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Agent Outputs

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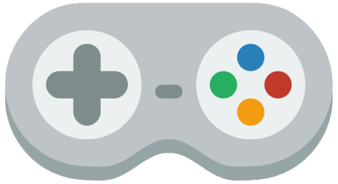

Continuous

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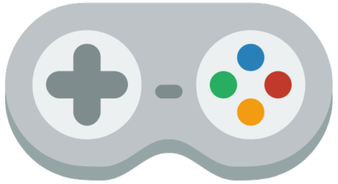

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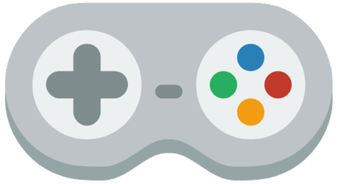

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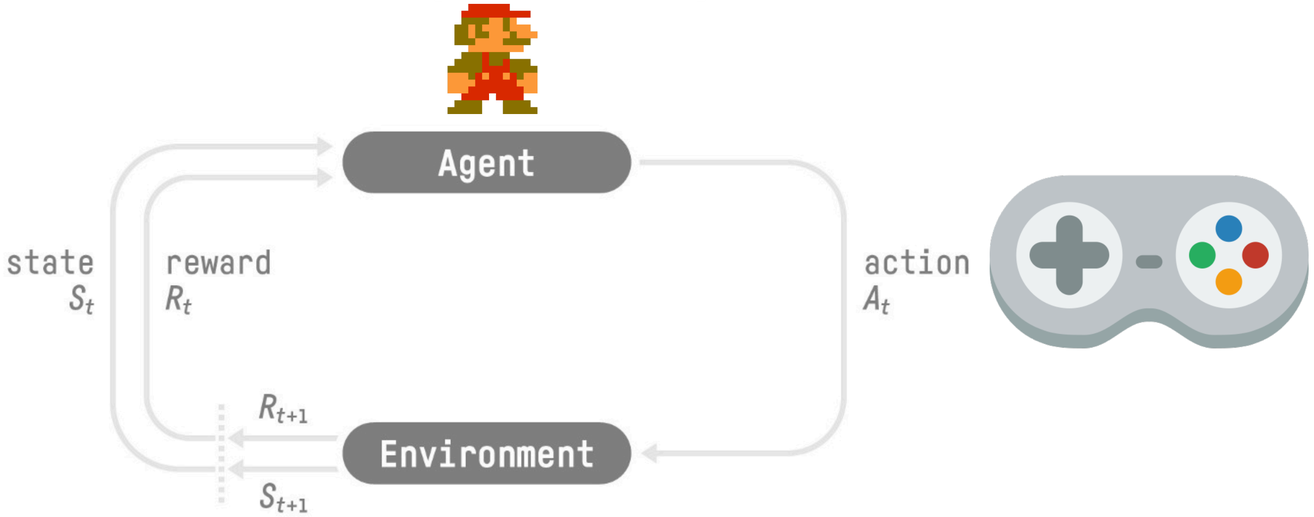
Markov Decision Process (MDP)



For t = 0, 1, 2, 3 …

Paradigm

Paradigm

s_0

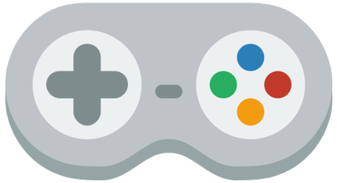
Paradigm

s_0

Policy π

Paradigm

s_0

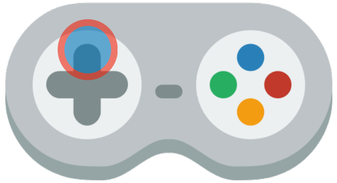


Policy π

Paradigm

S_0

A_0

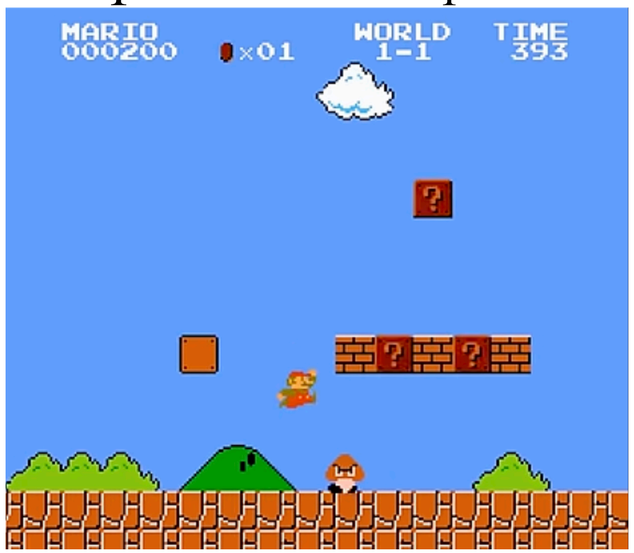


Policy π

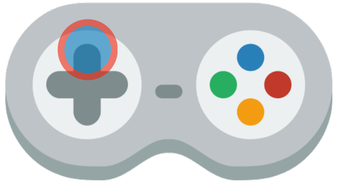
Paradigm

S_0

S_1 $r_1 = 0$



A_0



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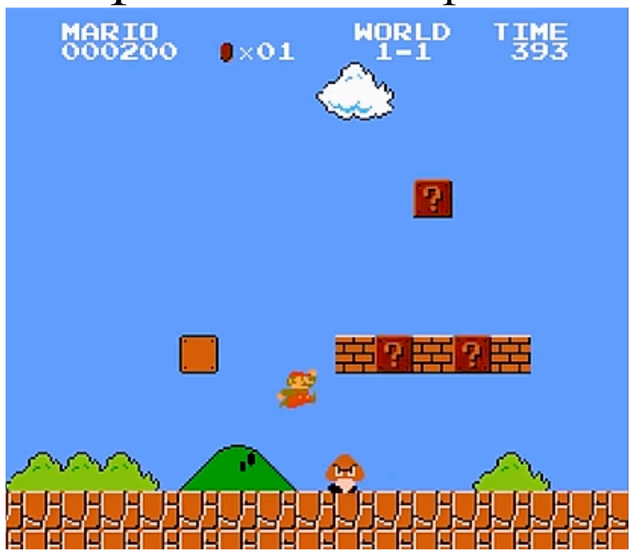
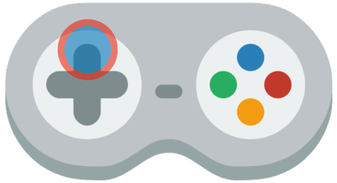
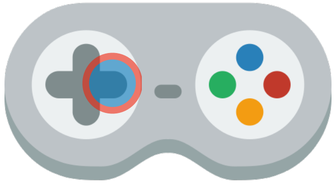


A_0



A_1

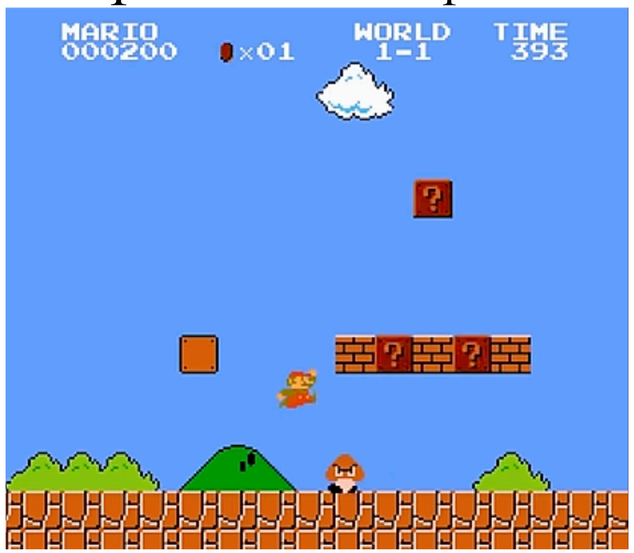


Policy π

Paradigm

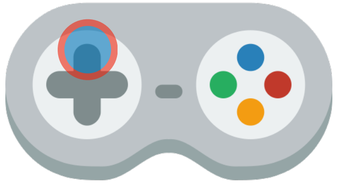
S_0

S_1 $r_1 = 0$



S_2 $r_2 = 100$

A_0



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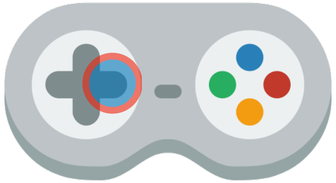
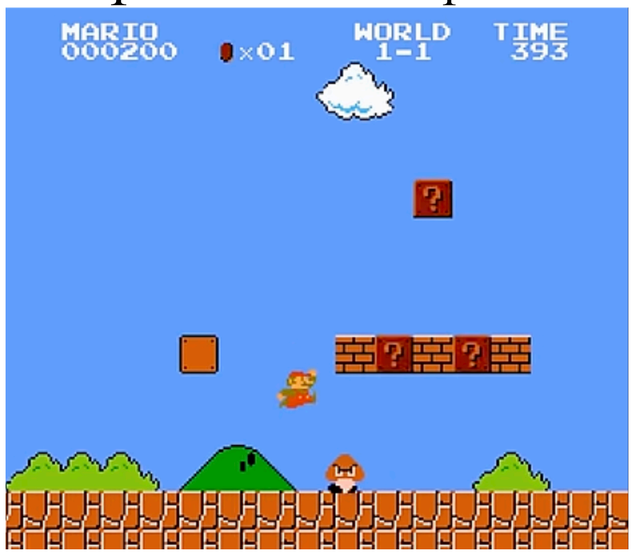


Policy π

Paradigm

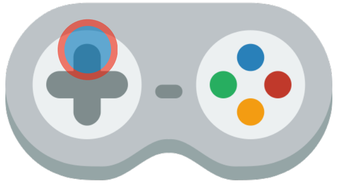
S_0

S_1 $r_1 = 0$

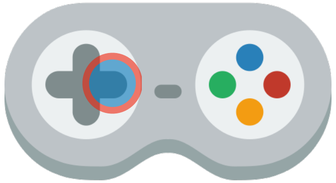


S_2 $r_2 = 100$

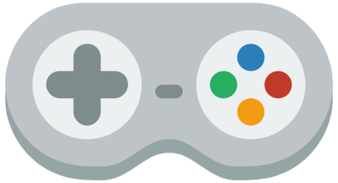
A_0



A_1



A_2



Policy π

Goal

Goal

Complete tasks successfully

Goal

Complete tasks successfully

Use rewards

Goal

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Rewards communicate an agent *what* we want to achieve (NOT *how* to do it)

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Maximize total amount of reward received

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$$R_{t+1} + R_{t+2} + R_{t+3}, \ldots + R_T$$

Goal

Complete tasks successfully

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Goal

Complete tasks successfully

Use rewards

Rewards communicate an agent *what* we want to achieve (NOT *how* to do it)

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$$G_t = R_{t+1} + R_{t+2} + R_{t+3}, \ldots + R_T$$

Episodic tasks

Continuing tasks

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T is the final step. RL breaks into *episodes*.

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$T = \infty$, return could be infinite

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$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \quad = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Policies and Value functions

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RL involves evaluating value functions

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Functions of states or state-action pairs

Policies and Value functions

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Functions of states or state-action pairs

- How good is it to be in a certain state (or to take an action in a certain state)

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Policies π decide actions, maps states to actions

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At time t, $\pi(a \mid s)$ is the probability that $A_t = a$ given $S_t = s$

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RL is about changing the policy as a result of its experience

Policies and Value functions

Policies and Value functions

Value function of state s under policy π

Expected return when starting in state s and following policy π

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State-value function for policy π

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$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \,|\, S_t = s] = \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,|\, S_t = s\Big], \quad \text{for all } s \in \mathcal{S}$$

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Action-value function for policy π

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Action-value function for policy π

Expected return when starting in state s and taking action a, according to policy π

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Action-value function for policy π

Expected return when starting in state s and taking action a, according to policy π

$$q_\pi(s, a) \doteq \mathbb{E}_\pi[G_t \,|\, S_t = s, A_t = a] = \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,|\, S_t = s, A_t = a\Big], \quad \text{for all } s \in \mathcal{S}$$

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v_π and q_π can be estimated from experience

Q-Learning

Q-Learning

One of the first major *online* RL algorithms

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Bellman Equation for Q-Learning

Q-Learning

One of the first major *online* RL algorithms

Bellman Equation for Q-Learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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Q-Learning converges to the optimal value function, as long as the agent continues to explore and samples all areas at the state-action space

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Q-Learning target policy is Greedy wrt its current values (max operator)

Its behaviour policy can be anything that continues to visit all state actions pairs during learning (e.g. $\varepsilon -$ greedy)

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
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What is $Q(S, A)$?

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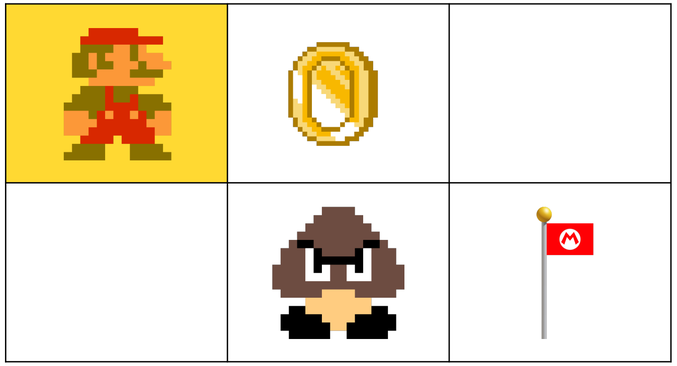
Bellman equation for state-action values

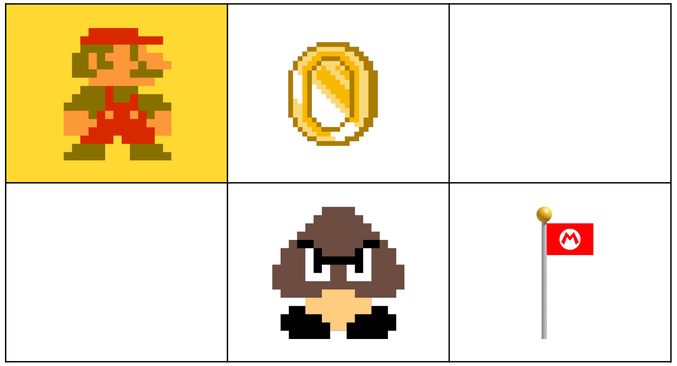
What is $Q(S, A)$?

Encode Q-values in a table, each cell corresponds to a state-action pair value.

Q-Learning example

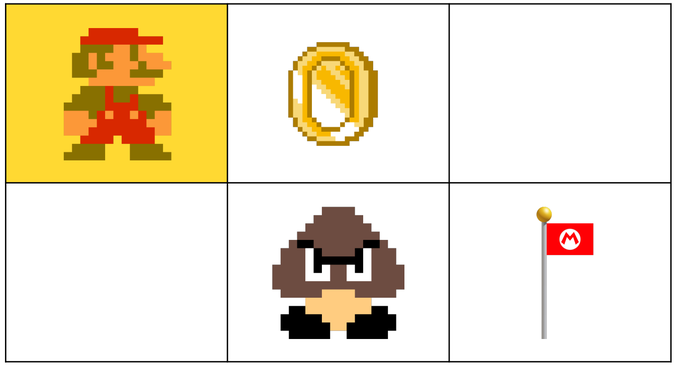


Q-Learning example



Policy π

Q-Learning example



Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

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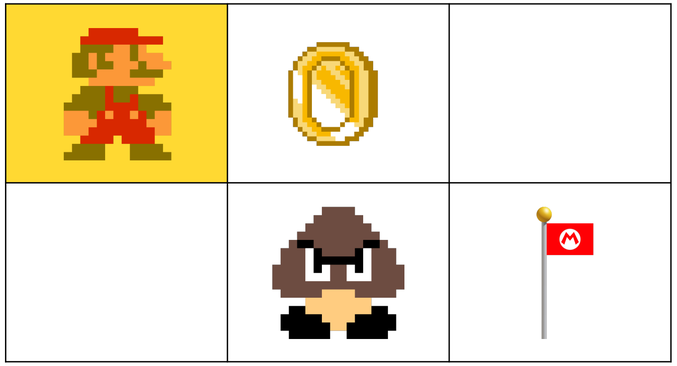
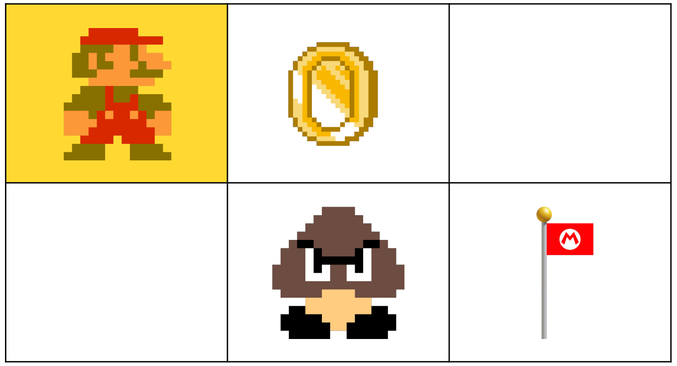


Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

+1

Q-Learning example



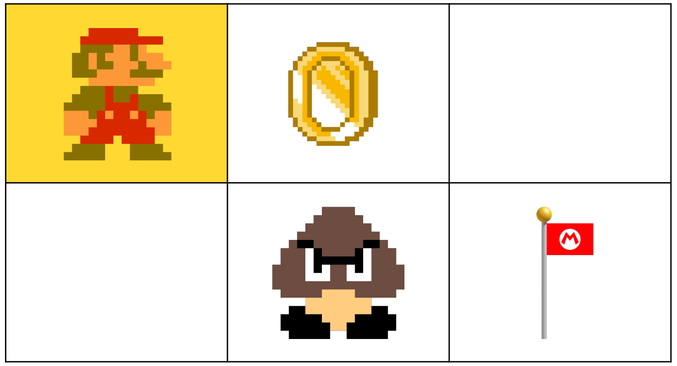
Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

+1

-10

Q-Learning example



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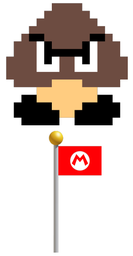
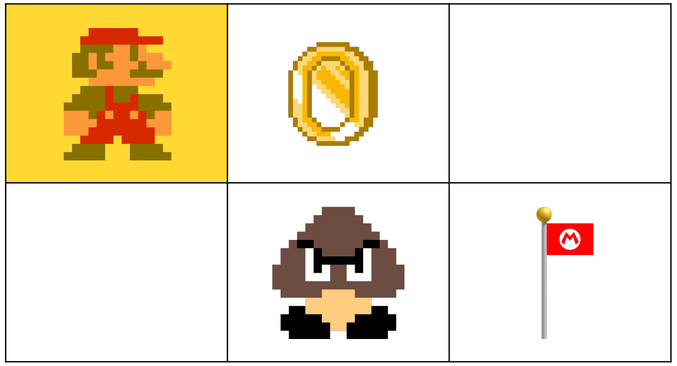


-10

+10
(terminal)

Q-Learning example



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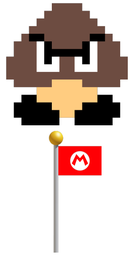

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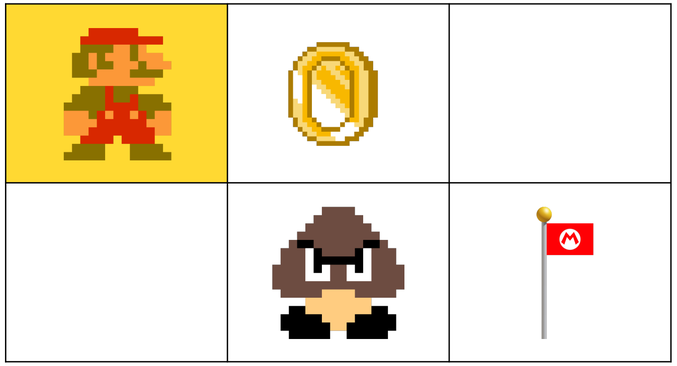

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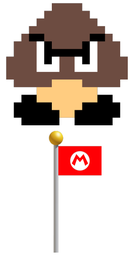

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$$\alpha = 0.1$$

$$\gamma = 0.99$$

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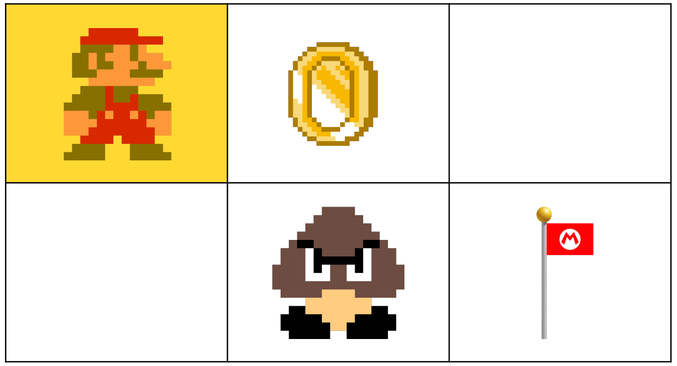

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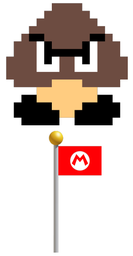

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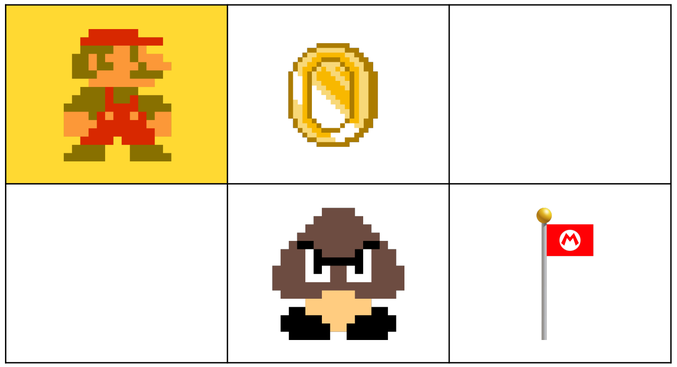
How to represent the Q-Table?

Q-Learning example



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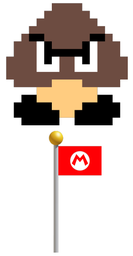

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	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$

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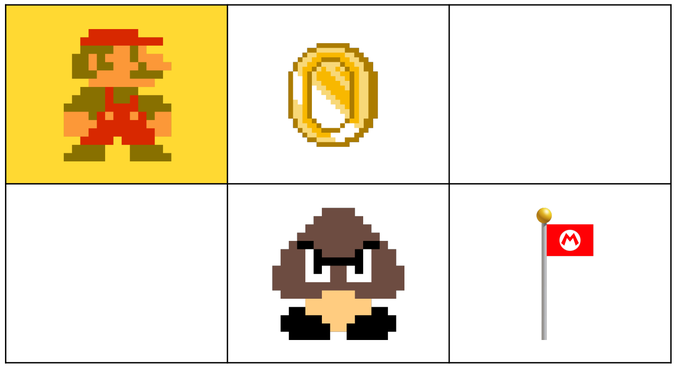

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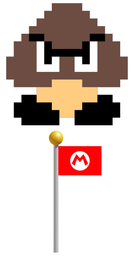

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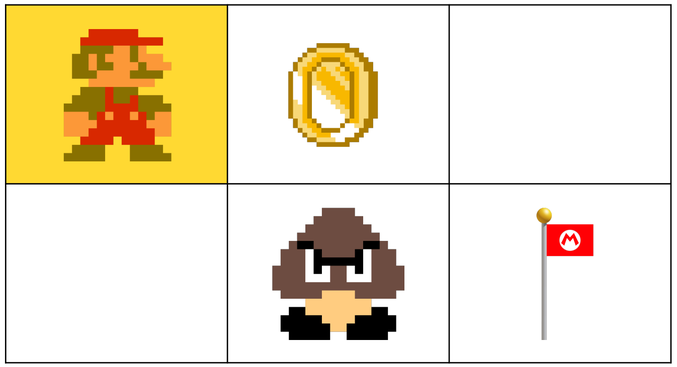
How to represent the Q-Table?

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$

How to move?

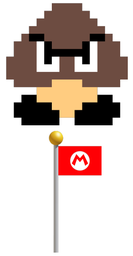
Q-Learning example



Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

+1



-10

+10 (terminal)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

$$\alpha = 0.1$$

$$\gamma = 0.99$$

How to represent the Q-Table?

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$

How to move?

$$\pi = \text{argmax}_A Q(S, A)$$

Q-Learning example

Q-Learning example

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Q-Learning example

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

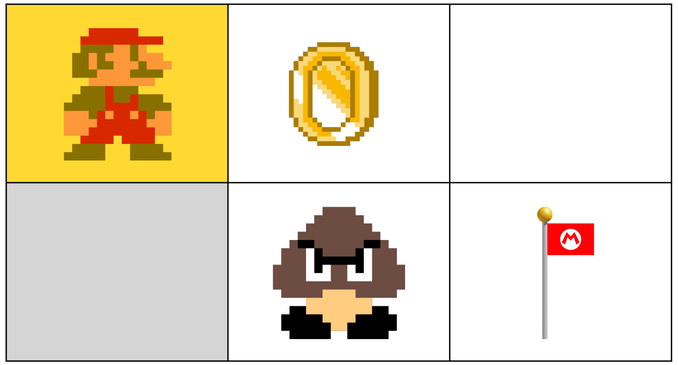
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning example

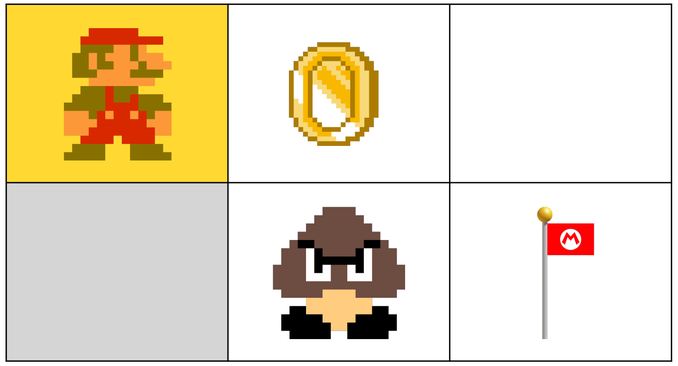
	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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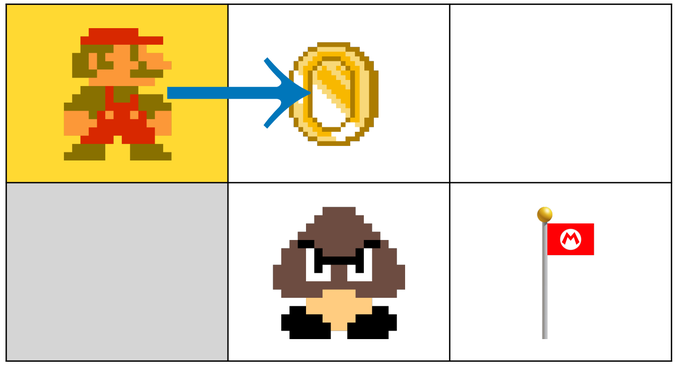
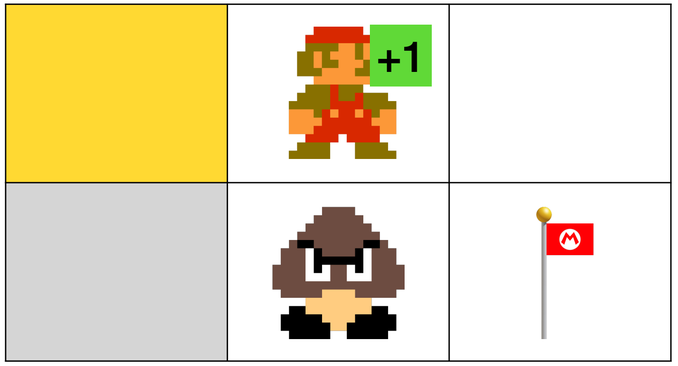


Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

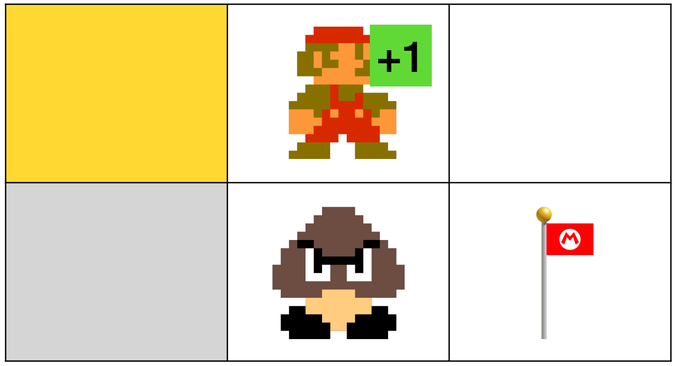
Take action A, observe R, S'



Q-Learning example

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'

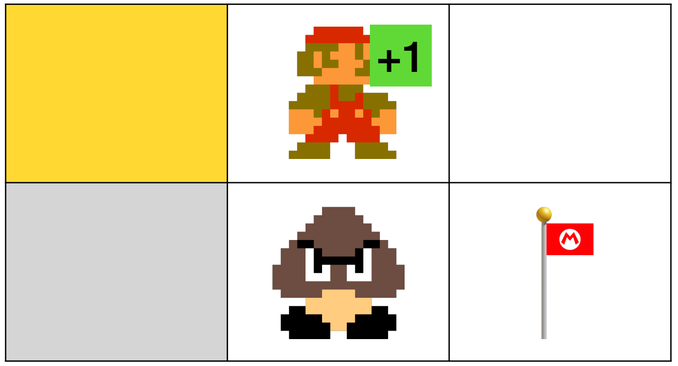


Update $Q(S, A)$

Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

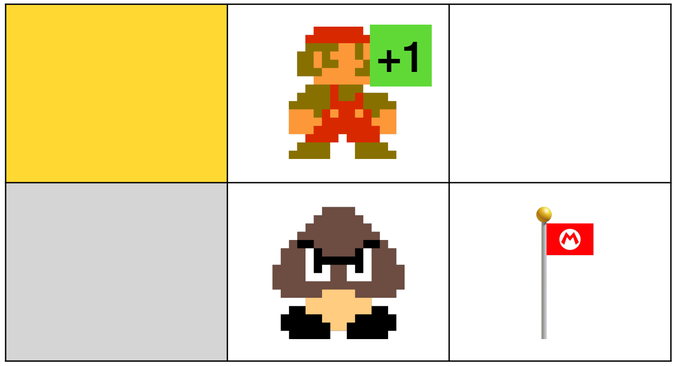
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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	←	→	↑	↓
	0	0	0	0
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	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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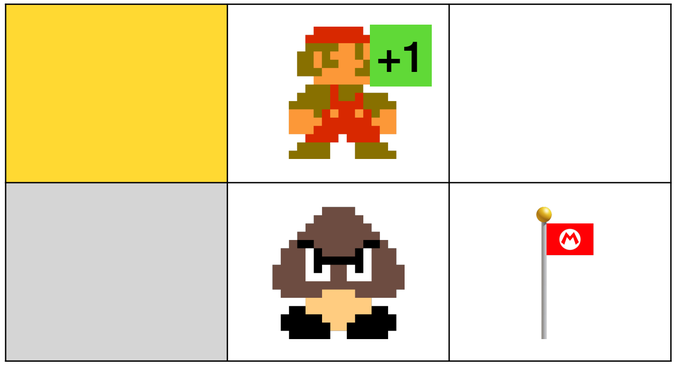

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Q(initial, →) = 0 + 0.1*[1 + 0.99*0 - 0] = 0.1

Q-Learning example

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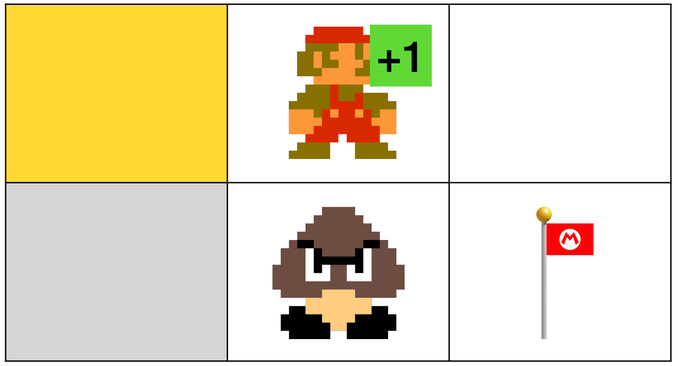
	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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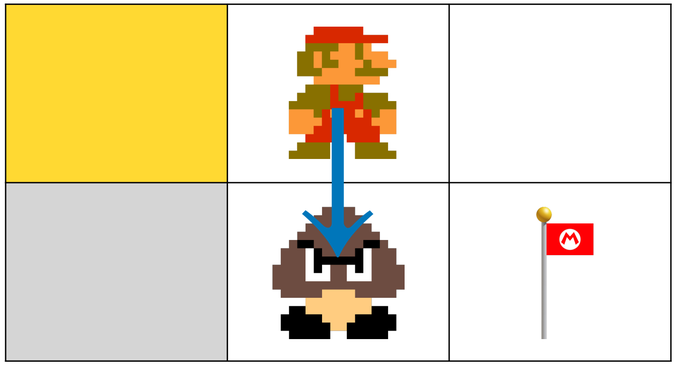
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



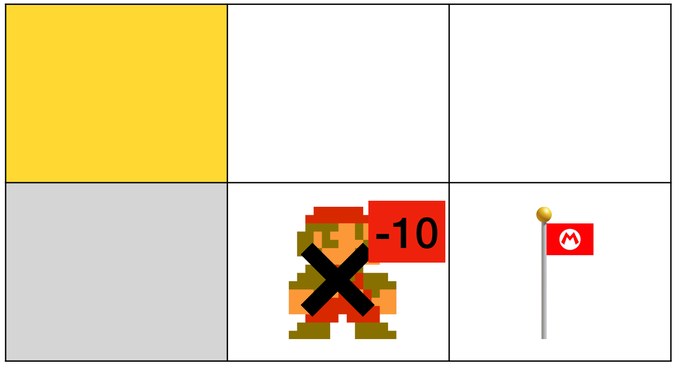
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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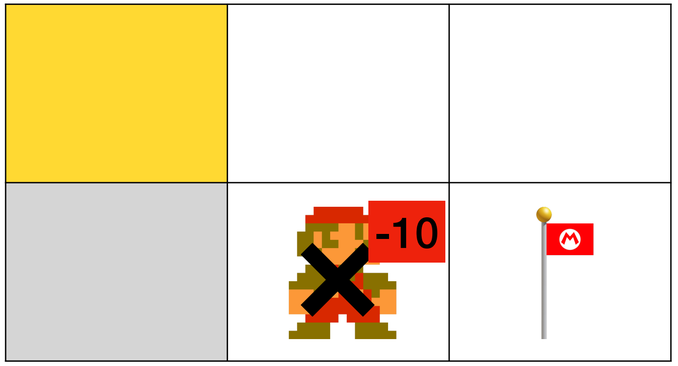


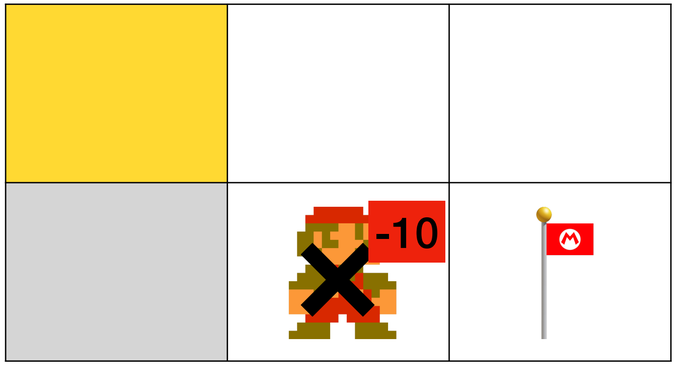
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q(state_2, ↓) = 0 + 0.1*[-10 + 0.99*0 - 0] = -1

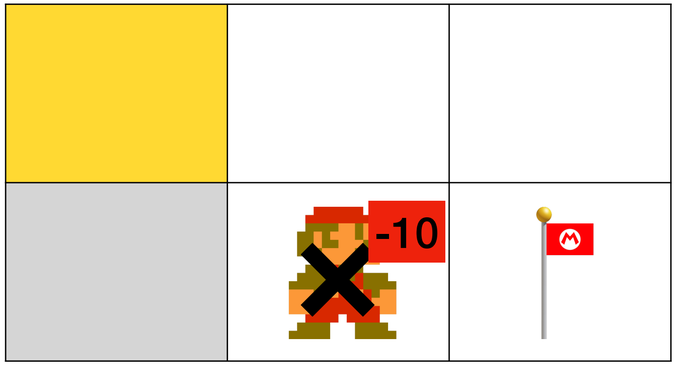
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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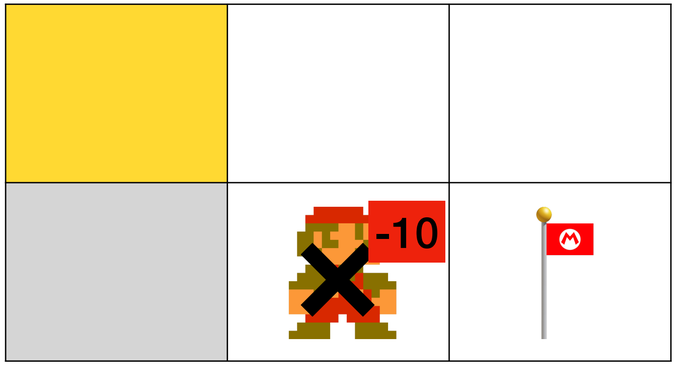
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Terminal state, Episode resets

Q-Learning example

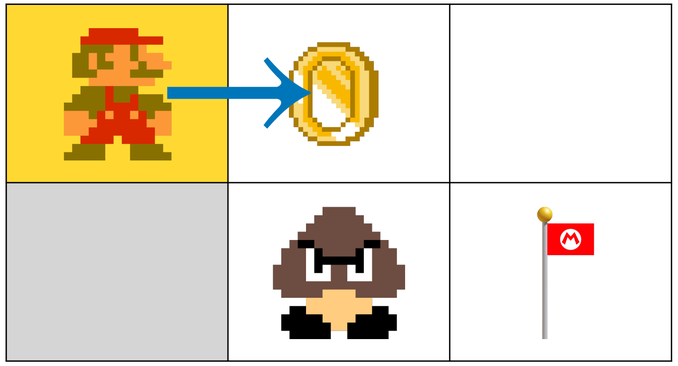
$\alpha = 0.1$

$\gamma = 0.99$

New episode

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



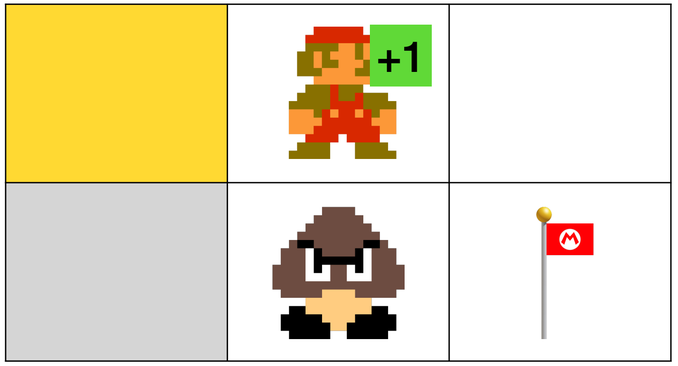
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0.1 + 0.1*[1 + 0.99*0 - 0] = 0.2

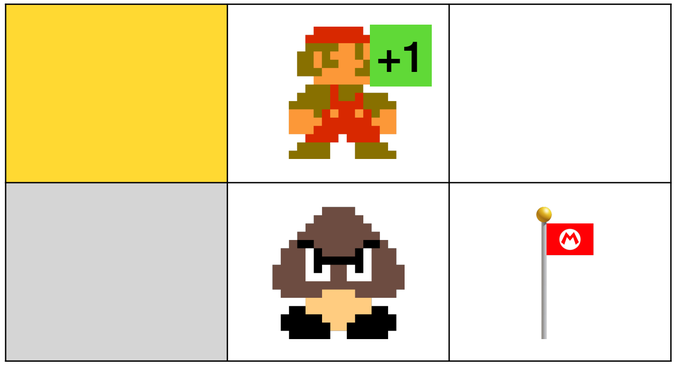
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q-Learning example

$\alpha = 0.1$

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	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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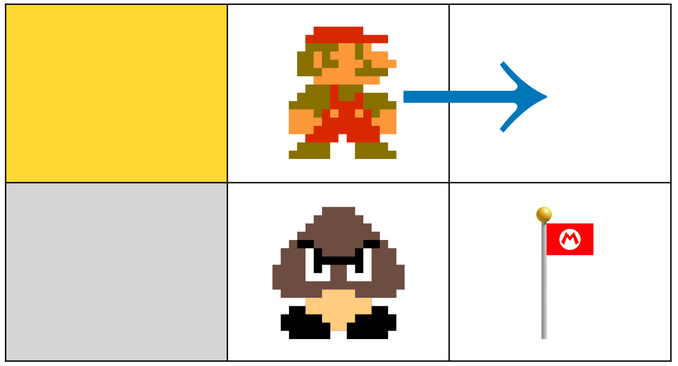


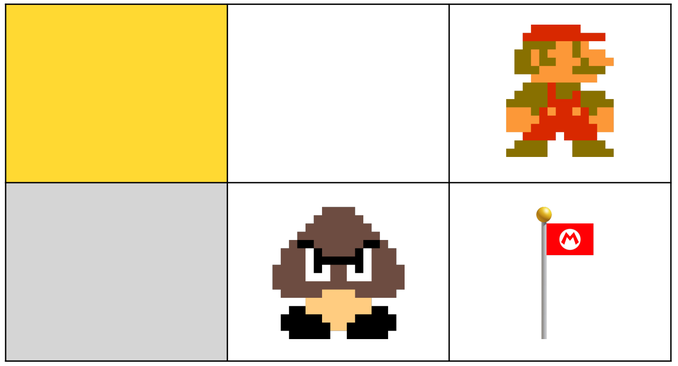
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Q-Learning example

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	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
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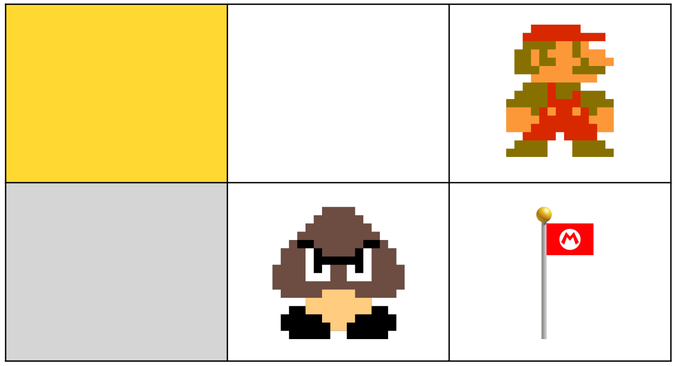

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Q-Learning example

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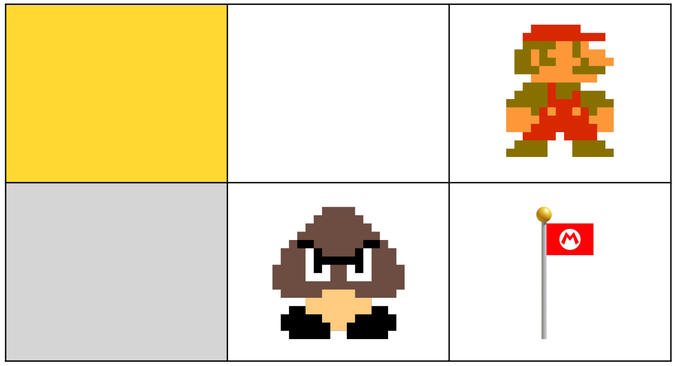
	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q-Learning example

$\alpha = 0.1$

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	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
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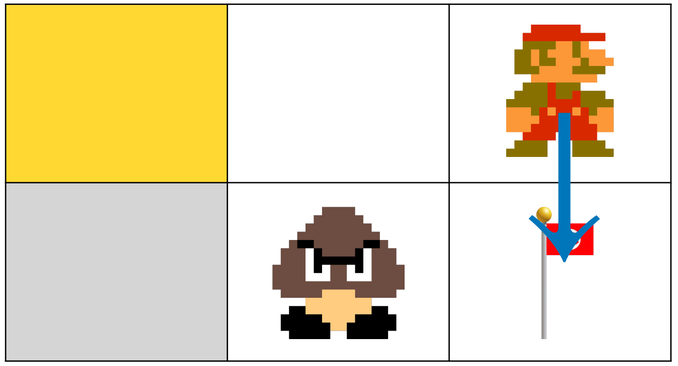

Q-Learning example

$\alpha = 0.1$

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	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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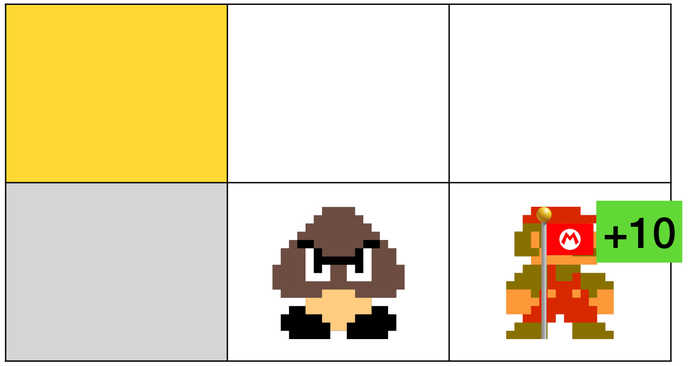

Q-Learning example

$\alpha = 0.1$

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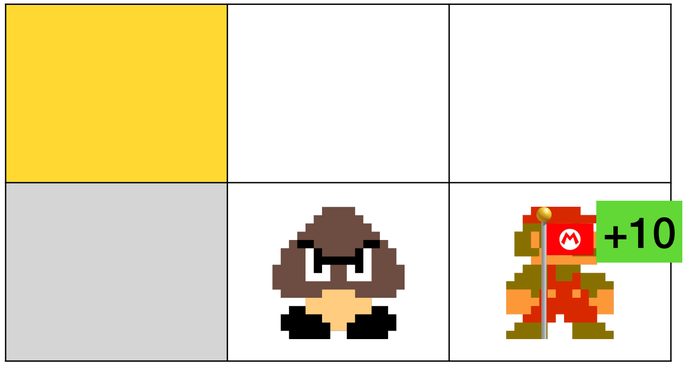
	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q(, ↓) = 0 + 0.1*[10 + 0.99*0 - 0] = 1

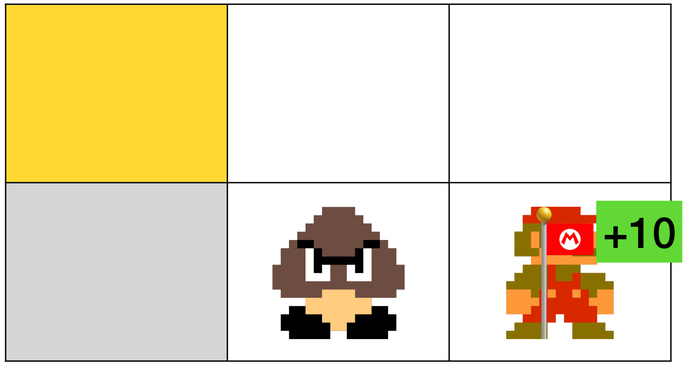
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	1
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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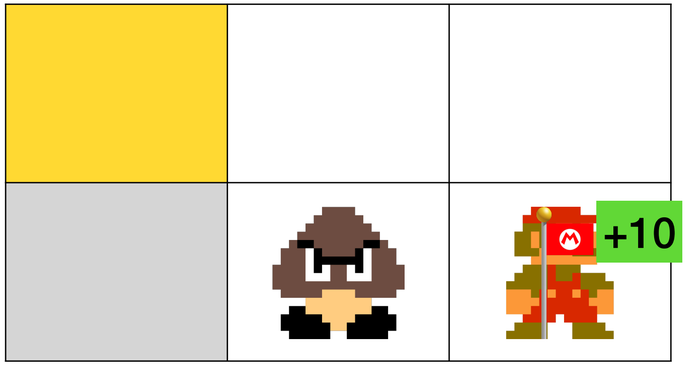

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Episode won! Terminal state

Why ε-greedy?

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Exploration-exploitation tradeoff

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Take greedy actions with probability $1 - \varepsilon$

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Exploit what the agent already knows

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$0 \leq \varepsilon \leq 1$

Why ε-greedy?

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

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-1		0				0
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If we take only greedy actions…

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-1		0				0
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We end up with a suboptimal solution

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
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Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

G = 10

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

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$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

If we explore…

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

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If we explore…

We might discover more valuable states

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
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That lead to higher returns

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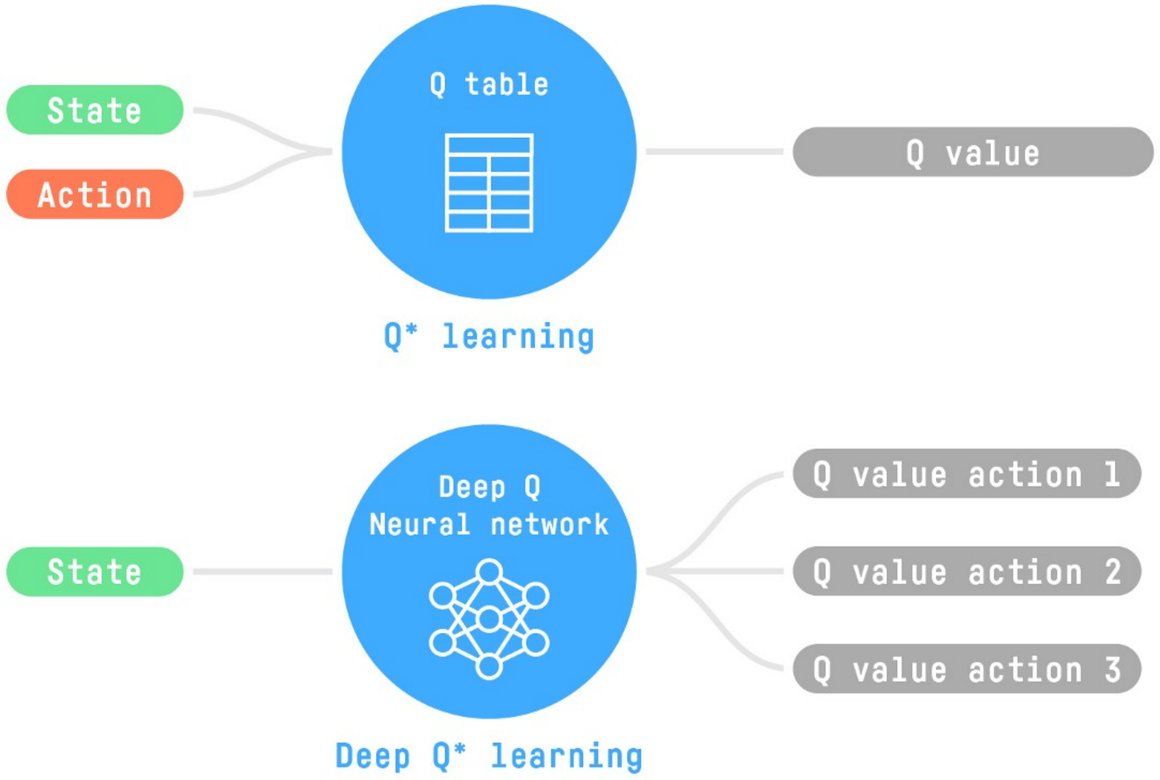
Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

G = 22

Deep Q-Learning


State
Action
Q table
Q value
Q* learning
State
Deep Q
Neural network
Q value action 1
Q value action 2
Q value action 3
Deep Q* learning

Deep Q-Learning

Deep Q-Learning

Q-Learning works well for small state spaces

Deep Q-Learning

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Atari games, however, have observation space of shape (210, 160, 3), values in [0, 255]

Deep Q-Learning

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This gives us $256^{210 \times 160 \times 3} = 256^{100800}$ possible observations

Deep Q-Learning

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Approximate Q-values using a neural network

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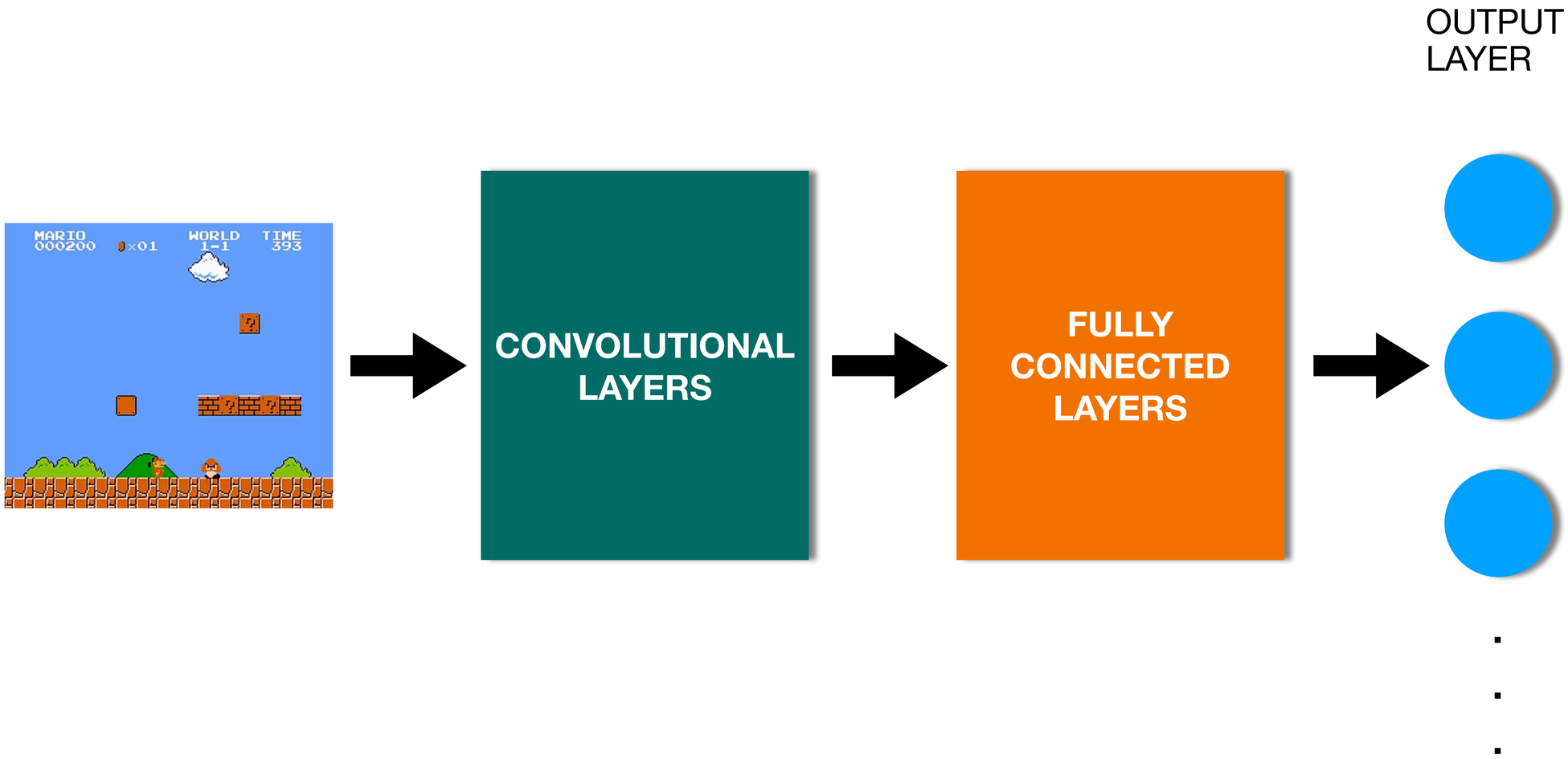
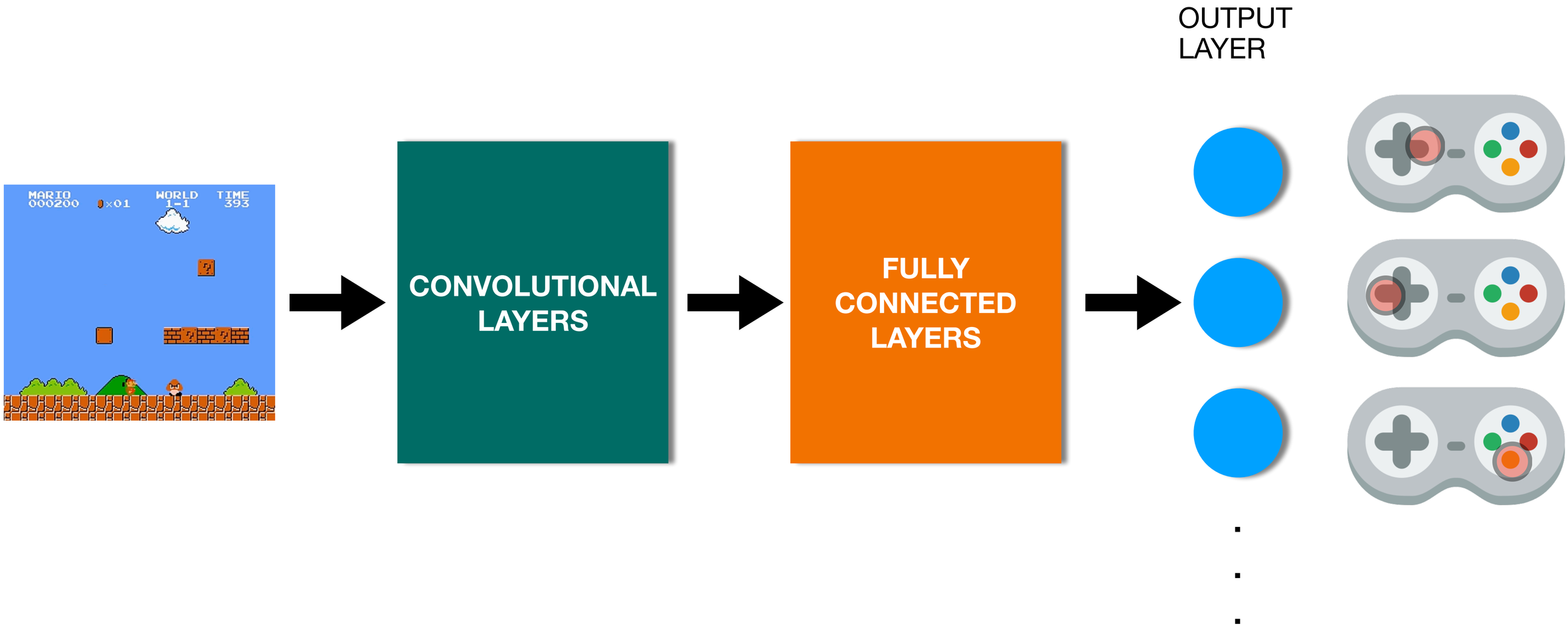
Approximate Q-values using a neural network

Parametrized Q-function $Q_\theta(s, a)$

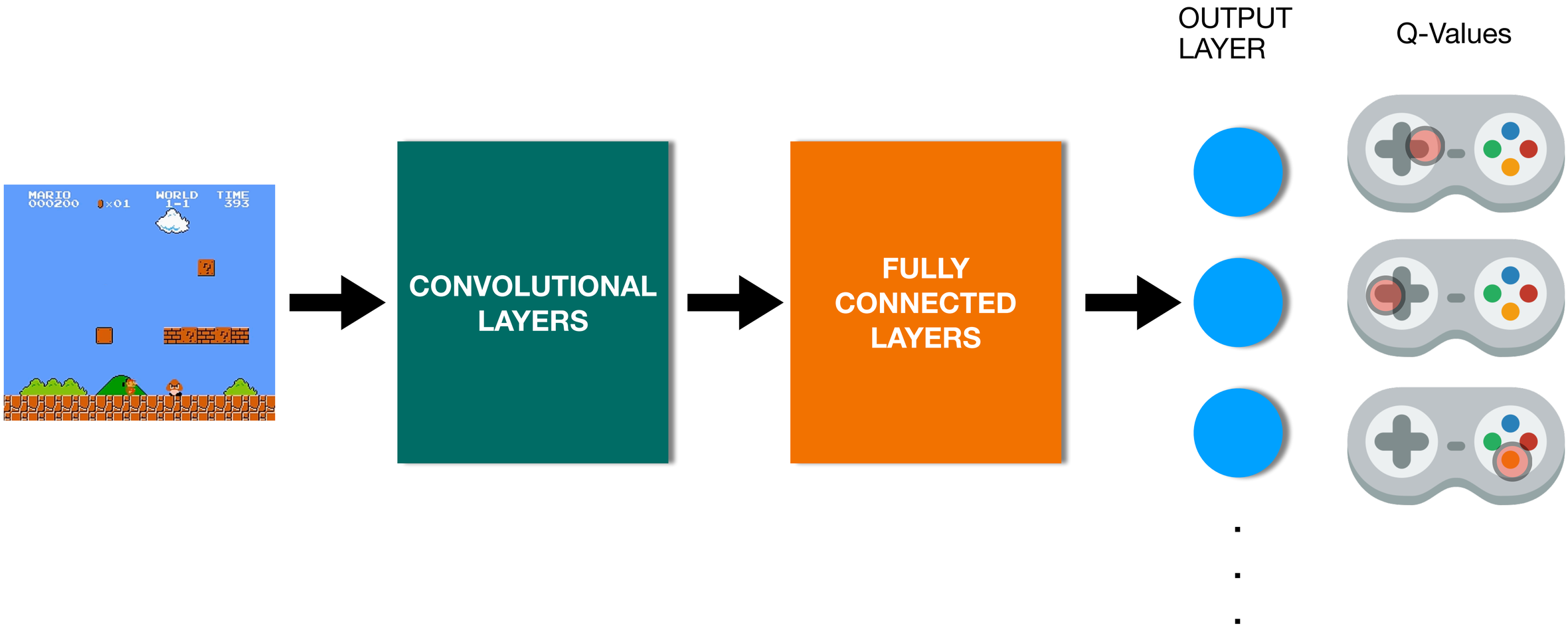
Deep Q-Learning



Deep Q-Learning



Deep Q-Learning



Deep Q-Learning algorithm

For episode $= 1, M$ **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1$,T **do**

With probability ε select a random action a_t

otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

Set $y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$

Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

End For

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Deep Q-Learning algorithm

ϕ represents the NN

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Sampling (interaction with the environment)

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Sampling (interaction with the environment)

Training

Deep Q-Learning algorithm

Tabular Q-Learning

Deep Q-Learning

Deep Q-Learning algorithm

Tabular Q-Learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Deep Q-Learning

Deep Q-Learning algorithm

Tabular Q-Learning

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Deep Q-Learning

$$y_j = \; r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

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Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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Deep Q-Learning algorithm

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Deep Q-Learning

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TD-target

TD-error

Deep Q-Learning

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}}(\underbrace{\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma\, max_{a'}\, Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}}}_{\text{TD-target}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}})$$

TD-error

Deep Q-Learning

Q-Target

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$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

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TD-target

TD-error

Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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TD-target

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Deep Q-Learning

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Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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TD-error

Deep Q-Learning

Q-Target

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Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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TD-error

Deep Q-Learning

Q-Target

$$y_j = \underbrace{\underbrace{r_j}_{\text{Reward}} + \underbrace{\gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)}_{\text{Discounted Estimate (optimal Q-value of next state)}}}_{\text{TD-target}}$$

Q-Loss

$$\underbrace{\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2}_{\text{TD-error}}$$

To the notebook!

Deep Q-Learning algorithm

For episode $= 1, M$ **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1$,T **do**

With probability ε select a random action a_t

otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

Set $y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$

Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

End For

End For

Sample-Based RL Methods

Only learn from experience - lots of trial-error interactions

No need for a model of the world - only data collected while interacting

Temporal Difference Learning

Based on value functions and dynamic programming ideas

Monte Carlo methods for prediction and control

Often used for estimation that relies on repeated random sampling

Monte Carlo methods

Monte Carlo for prediction and control

Often used for estimation that relies on repeated random sampling

Monte Carlo in RL

Estimate values directly from experience (sequence of S, A, R)

Learning from experience

Agent can accurately estimate a value function without prior knowledge of environment dynamics

No need to know transition probability p

Monte Carlo for policy evaluation

Value functions represent expected returns

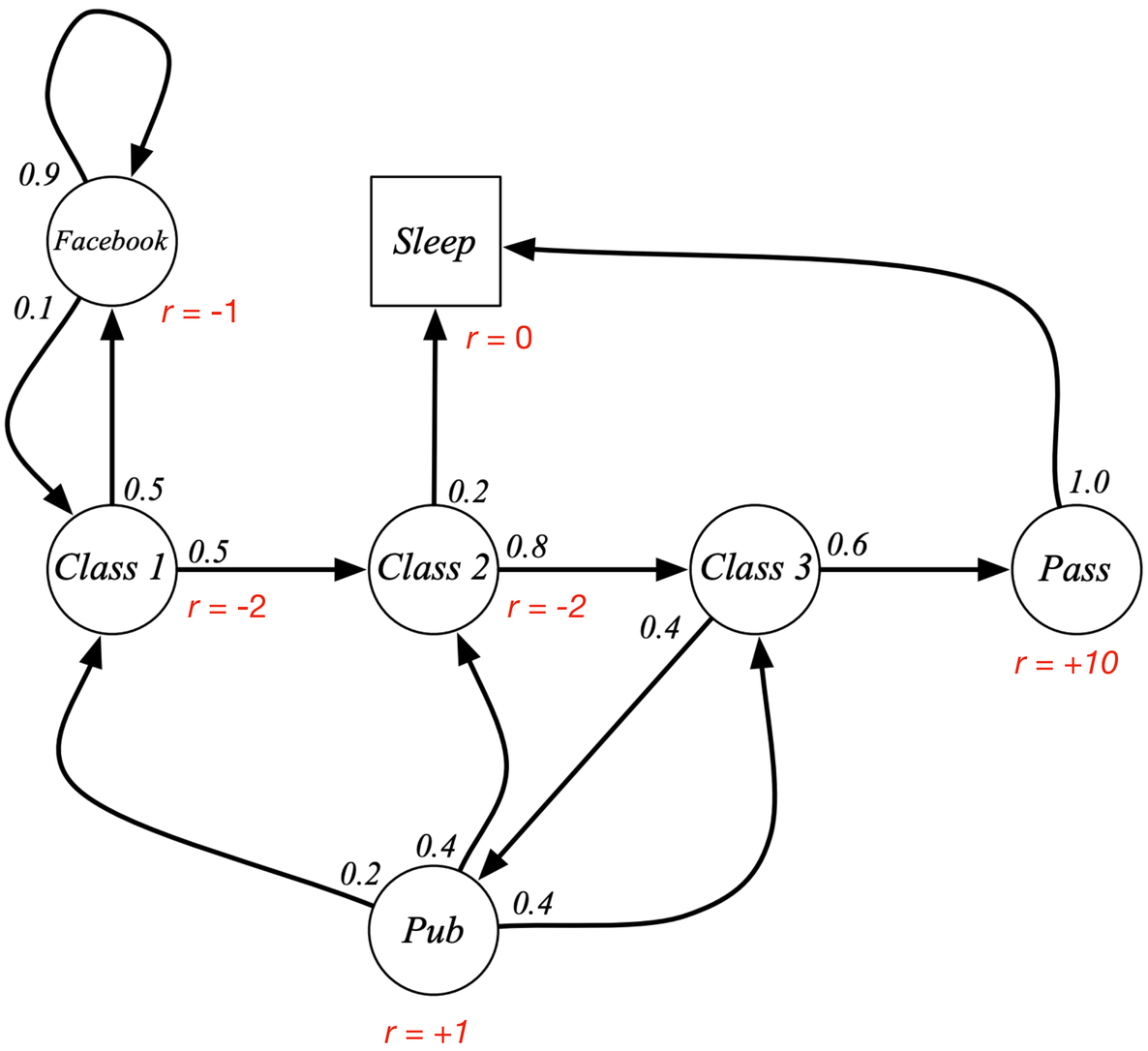
Observe multiple returns from the same state

Then average those to estimate te expected return from that state

---

As the number of samples increases, the expected terutn gets closer to real avg

Example


0.9
Facebook
0.1
r = -1
Sleep
r = 0
0.5
0.2
1.0
Class 1
0.5
r = -2
Class 2
0.8
r = -2
Class 3
0.6
Pass
r = +10
0.4
0.4
0.2
Pub
0.4
r = +1

Temporal-Difference Learning

$$V(S_t) = V(S_t) + \alpha\left[V(S_{t+1} - V(S_t)\right]$$

Finite Markov Decision Process (MDP)

Bellman Equation

Bellman Equation

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Off-policy:

Bellman Equation

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

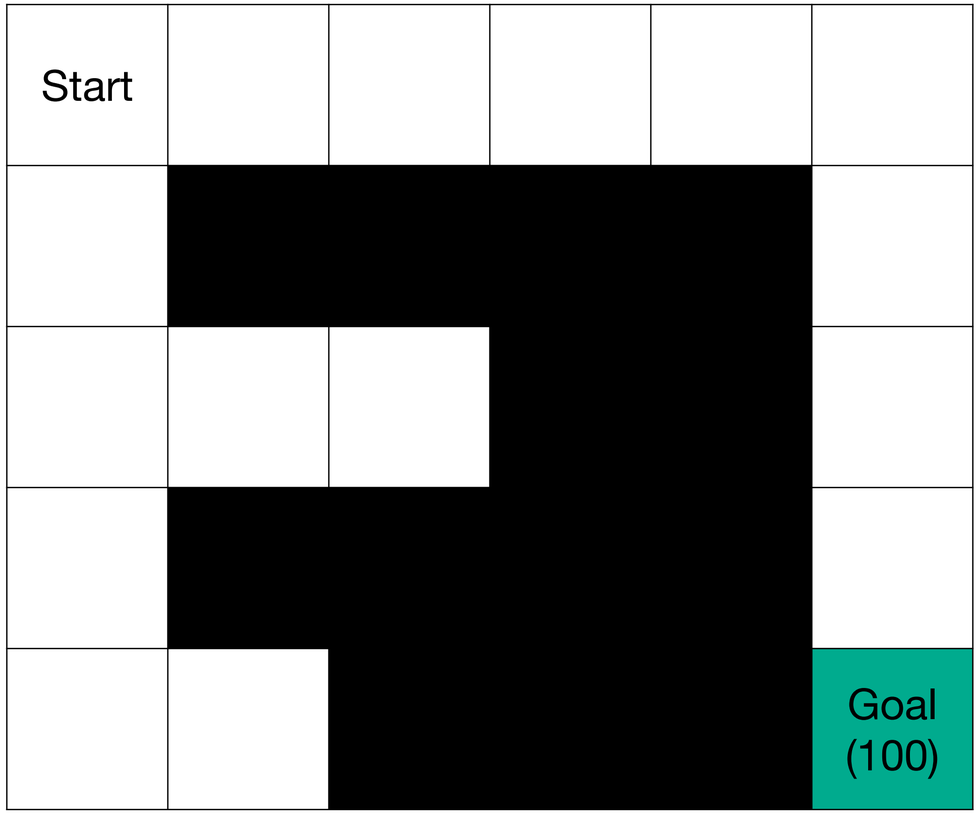
Q-Learning algorithm

Algorithm parameters: step size $\alpha \in (0,1]$, small $\varepsilon > 0$
Initialize $Q(s,a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$
  until S is terminal

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$



Updating the Q Table

Da controllare

Quando MDP non è a finite states.
Come si definisce nel caso di una NN (che ha immagini in input)?
Assumiamo la stessa cosa?