Reinforcement Learning

Introduction

Antonio Ricciardi, PhD

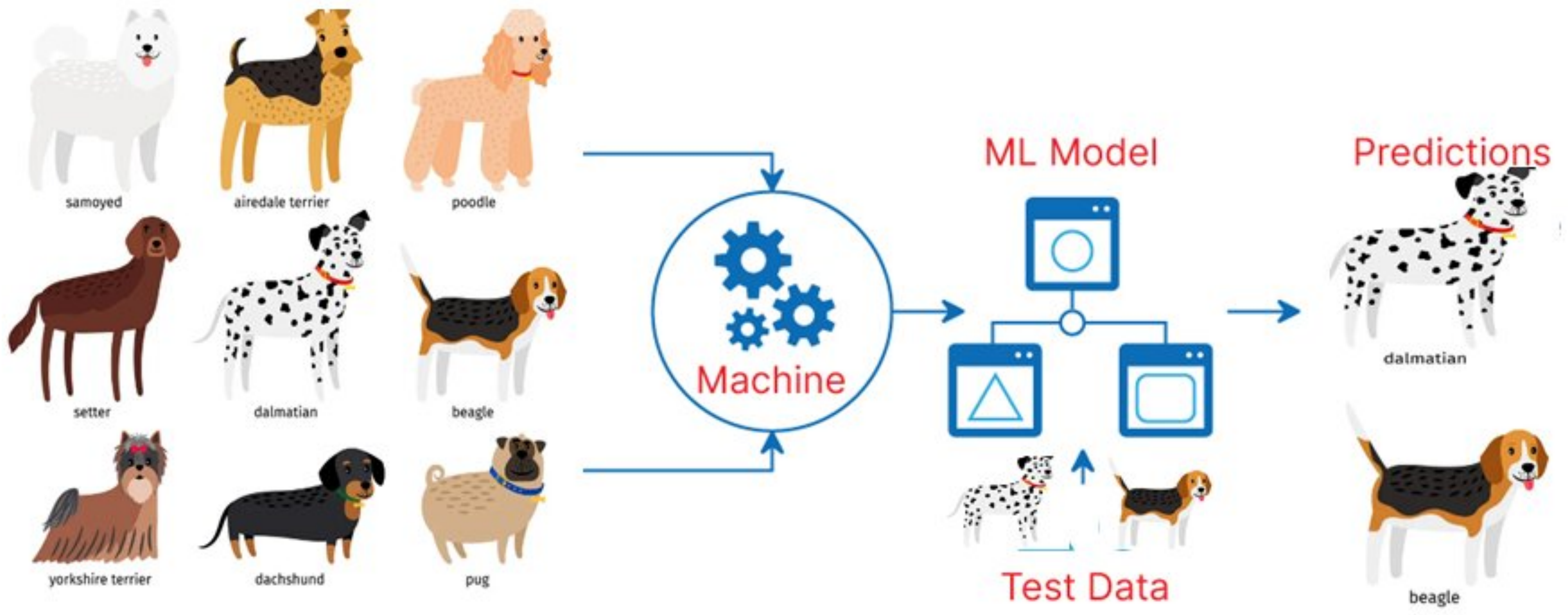
Machine Learning @ Mathematical Sciences for AI

Supervised Learning vs Reinforcement Learning

Supervised Learning

Learn using labeled data

Need pairs $\{x_i, y_i\}$



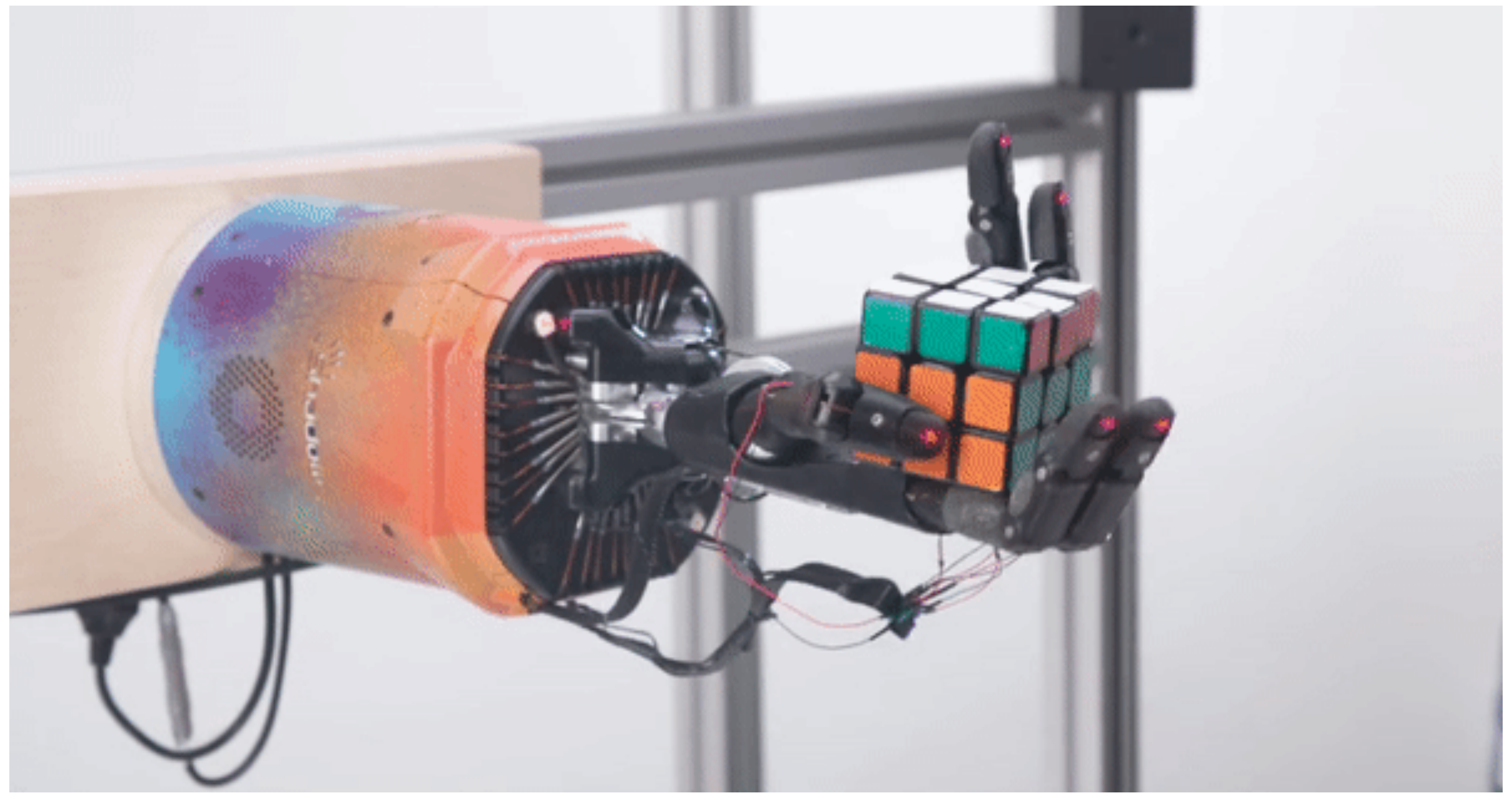
Reinforcement Learning

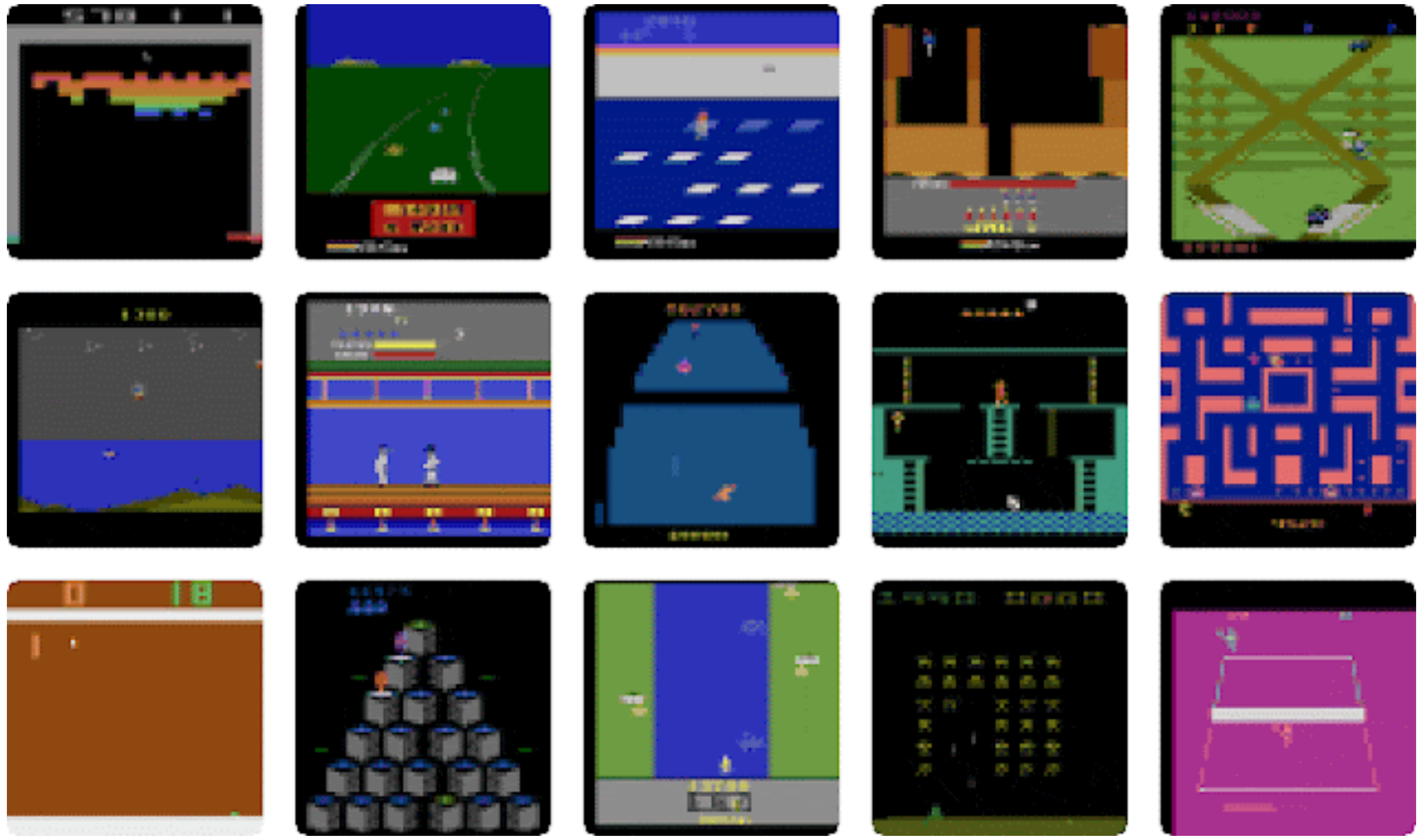
Framework for solving control tasks

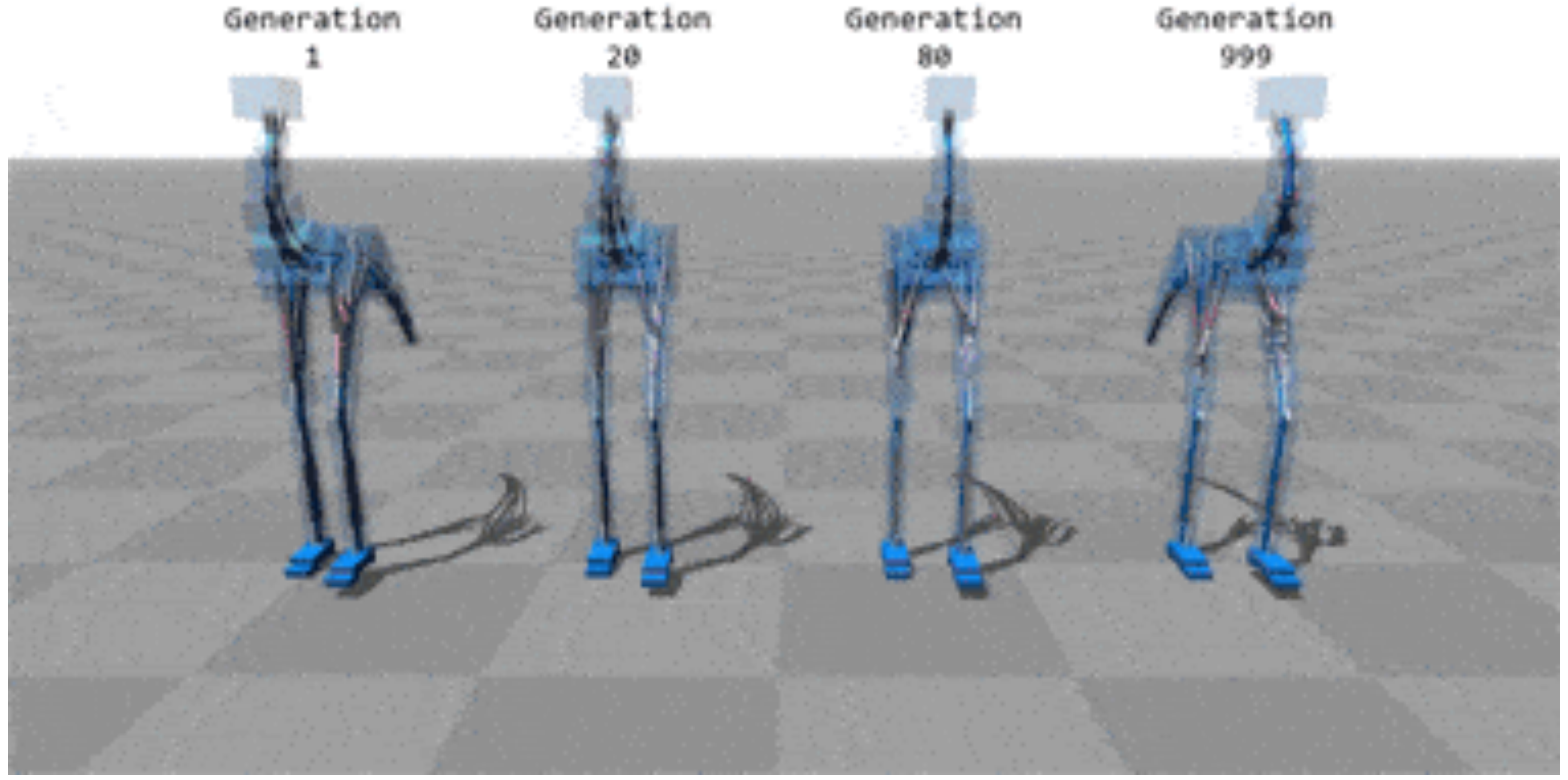
Learn from the environment through trial and error

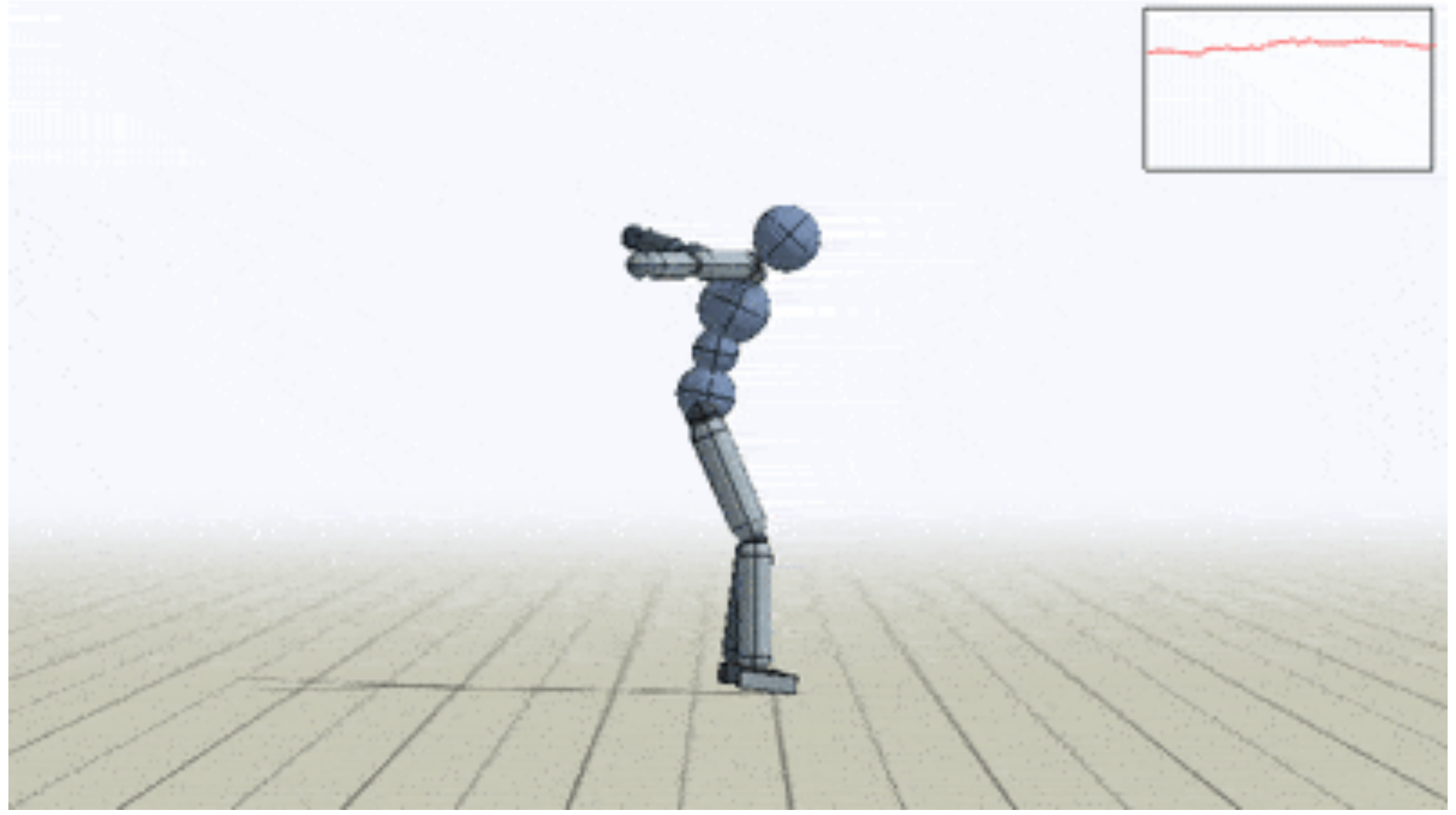
Receive rewards as feedback

Reinforcement Learning









Akkaya, Ilge, et al. "Solving rubik's cube with a robot hand." arXiv preprint arXiv:1910.07113 (2019). (https://openai.com/research/solving-rubiks-cube)

Mnih, V., Kavukcuoglu, K., Silver, D. et al. Human-level control through deep reinforcement learning. Nature **518**, 529–533 (2015).

Geijtenbeek, Thomas, Michiel Van De Panne, and A. Frank Van Der Stappen. "Flexible muscle-based locomotion for bipedal creatures." ACM Transactions on Graphics (TOG) 32.6 (2013): 1-11.

Peng, Xue Bin, et al. "Deepmimic: Example-guided deep reinforcement learning of physics-based character skills." ACM Transactions On Graphics (TOG) 37.4 (2018): 1-14.

Reinforcement Learning

https://www.youtube.com/watch?v=PxoG0A2QoFs

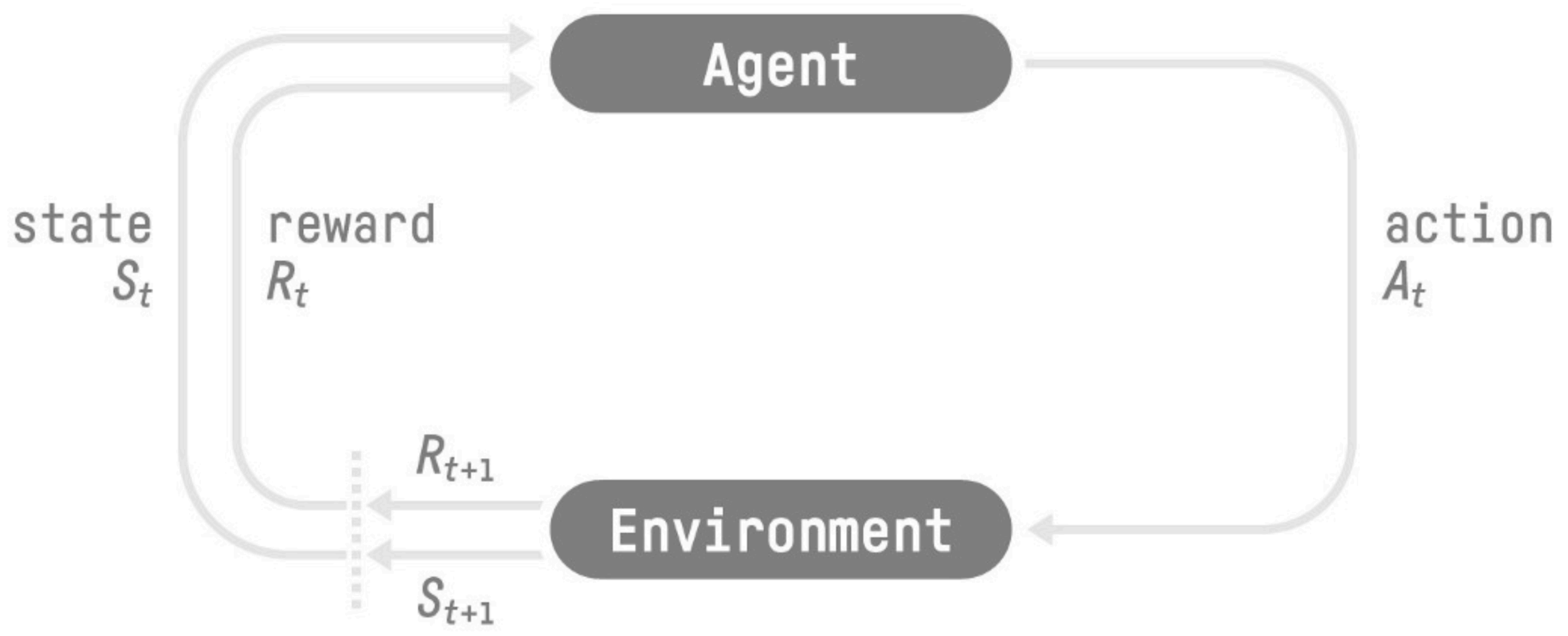
What you will learn

Basic concepts of RL

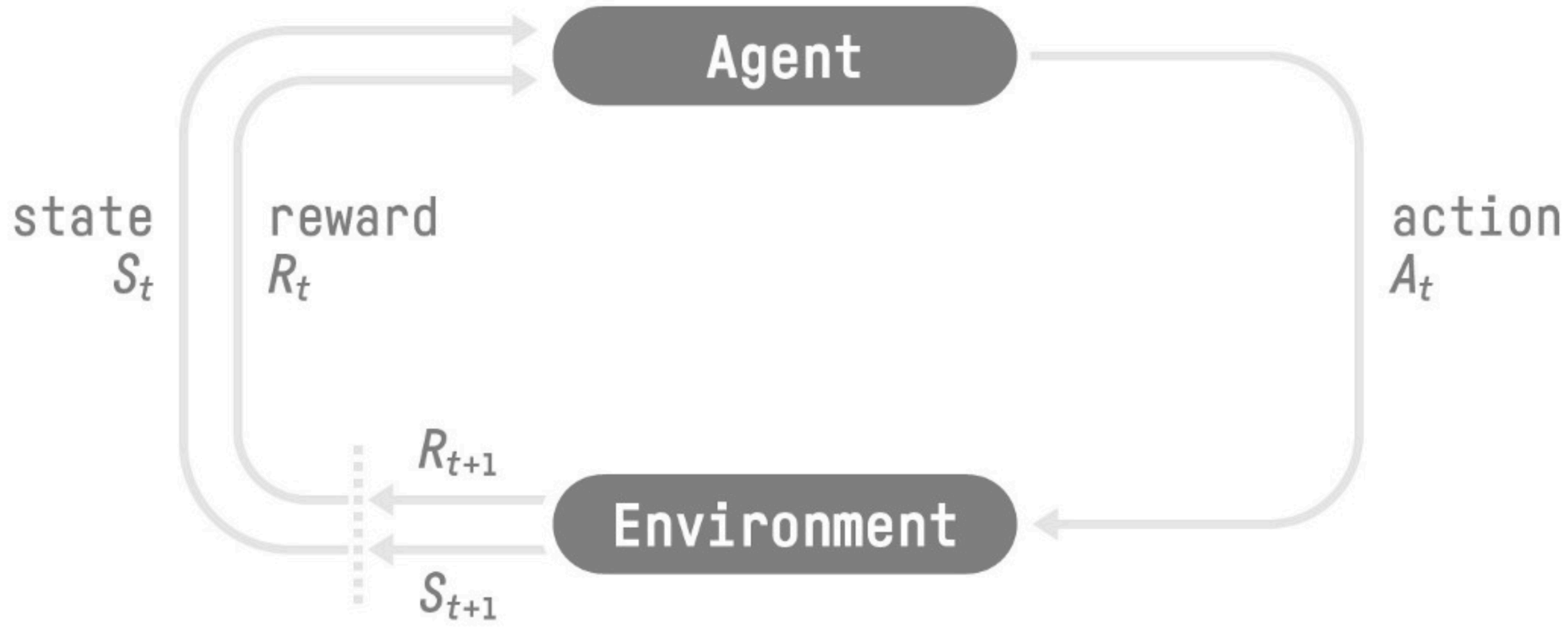
Q-Learning

Deep Q-Learning

Markov Decision Process (MDP) (finite)



Markov Decision Process (MDP) (finite)



For t = 0, 1, 2, 3 …

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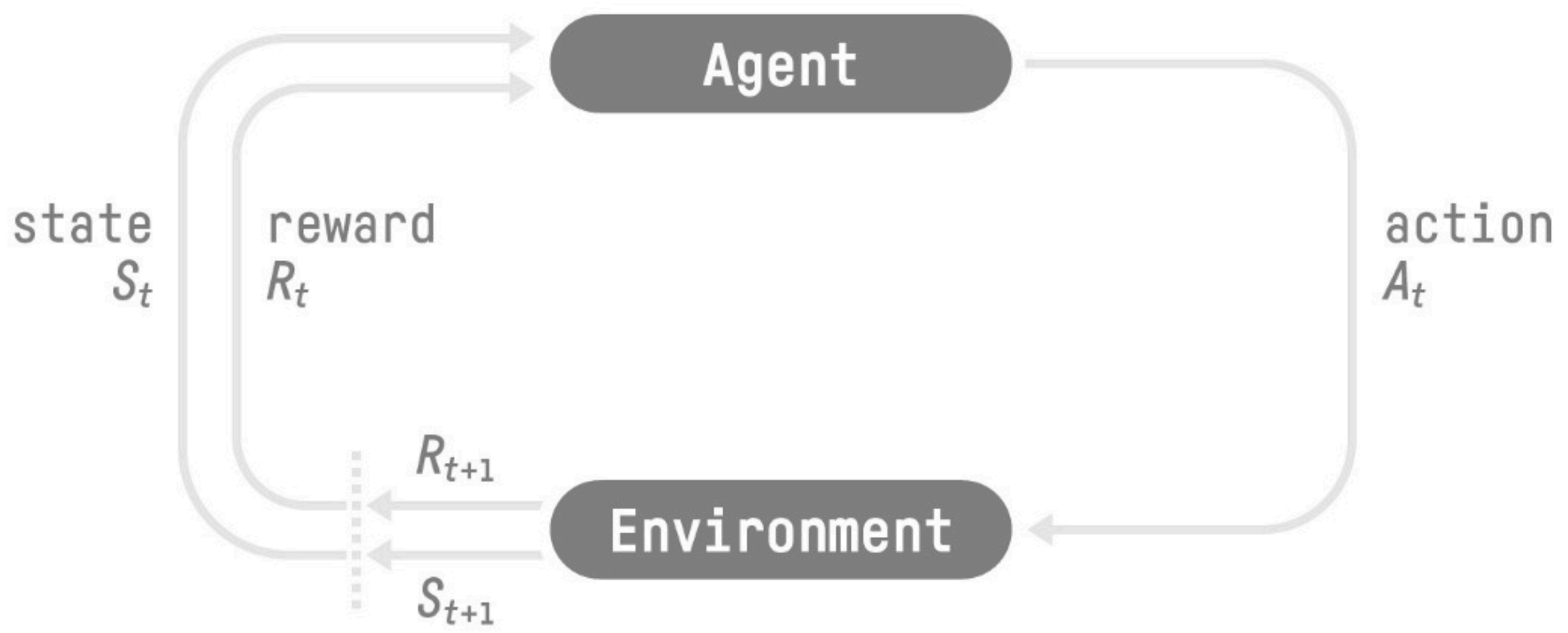

For t = 0, 1, 2, 3 …

Trajectory: $S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$

Agent Inputs

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How do agents get information from the environment?

Agent Inputs

How do agents get information from the environment?

States

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Observations

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Complete description of the state of the world
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Action space Set of all possible actions in an environment.

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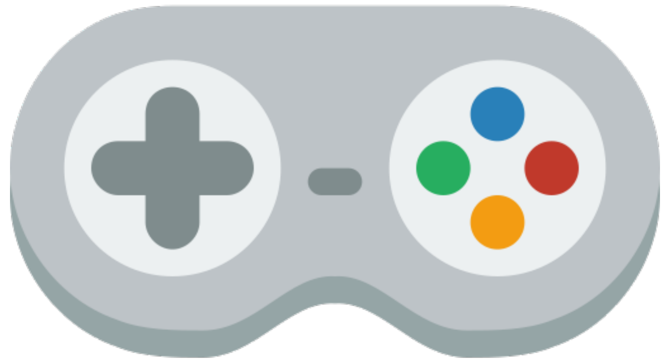

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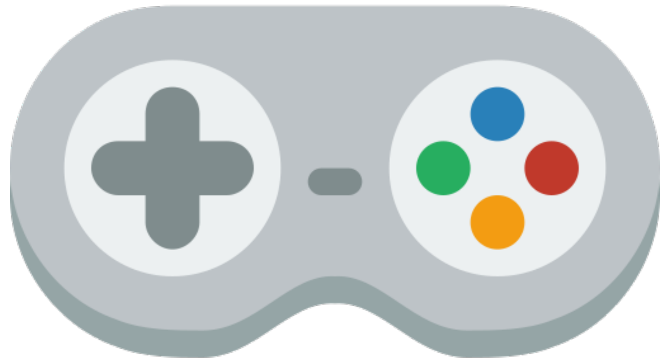

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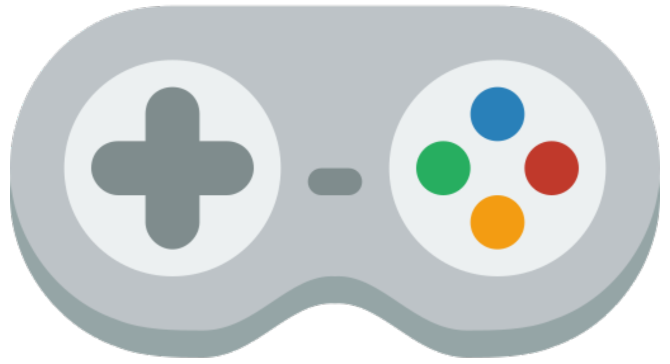

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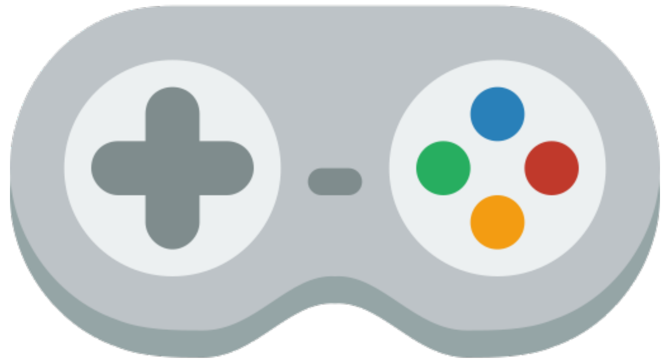

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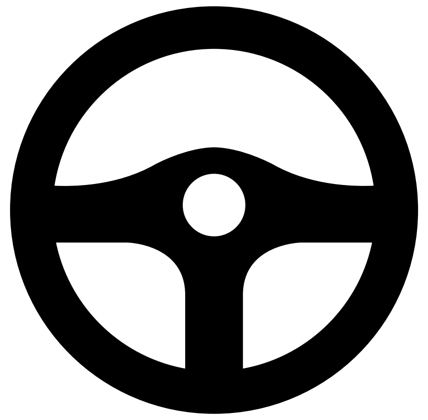

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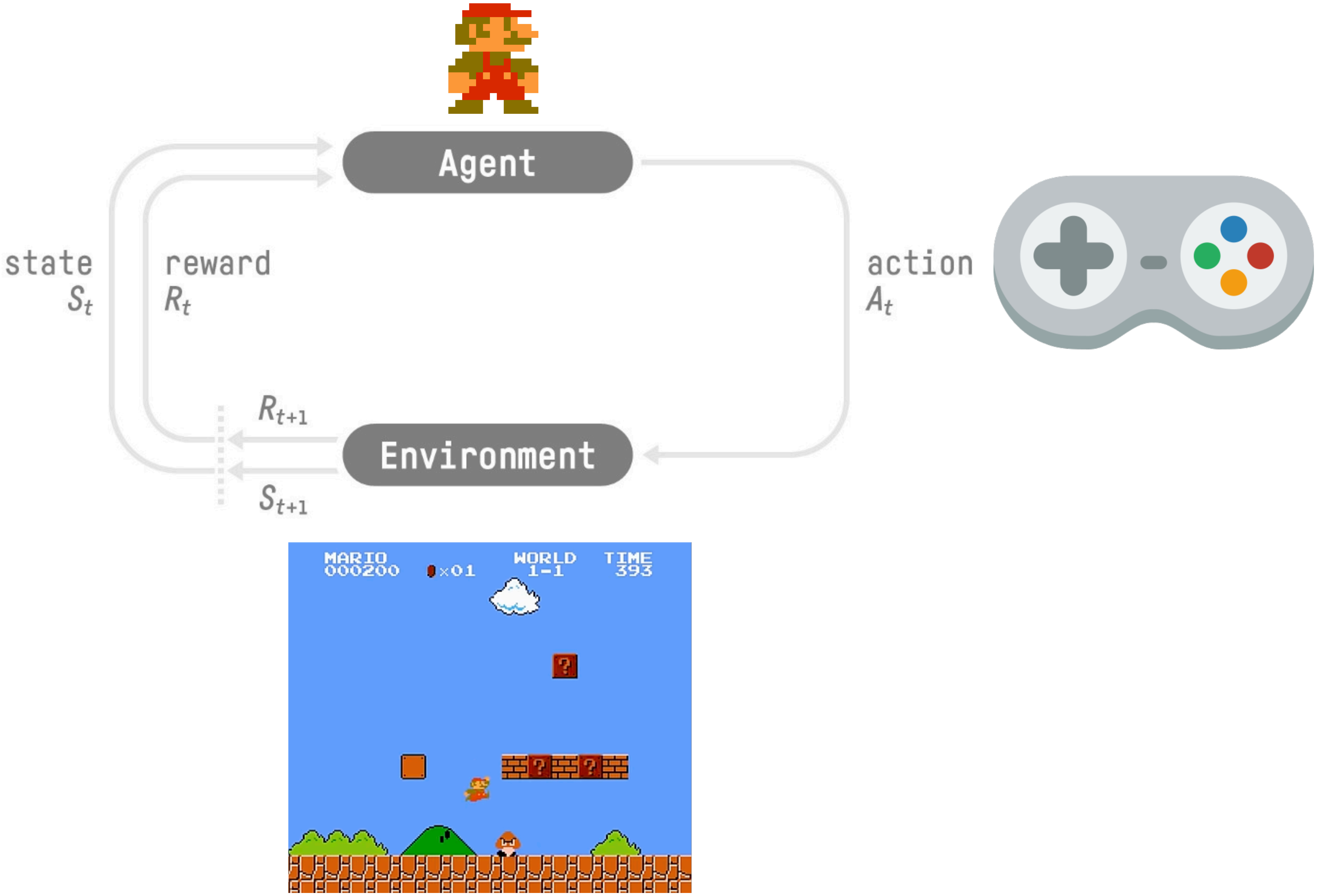


For t = 0, 1, 2, 3 …

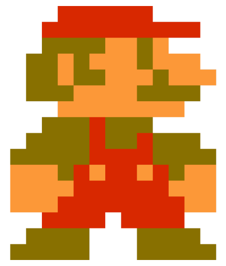
Paradigm

Paradigm

S_0

Paradigm

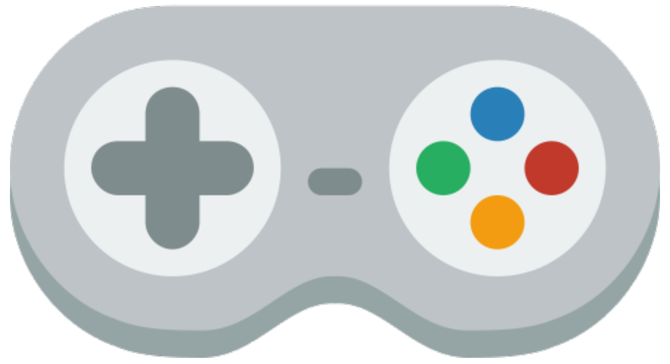
S_0

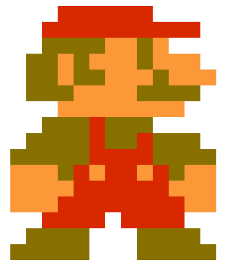


Policy π

Paradigm

S_0





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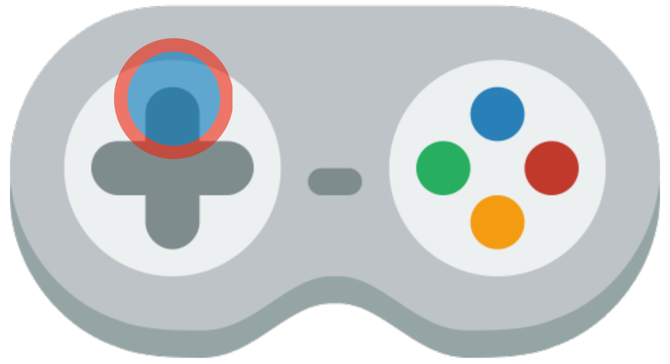
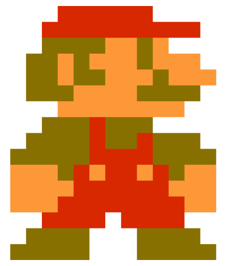
A_0 

 Policy π

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S_1 $r_1 = 0$

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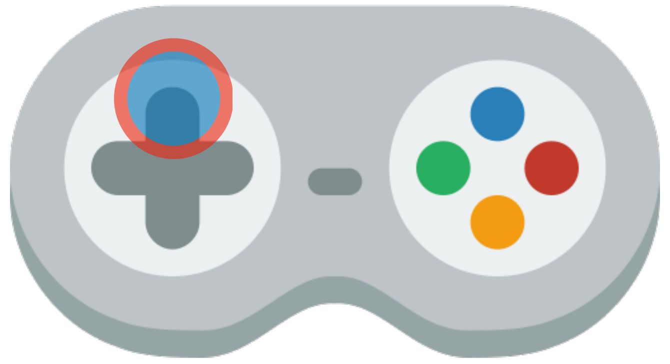

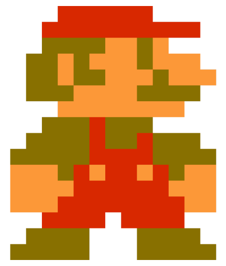

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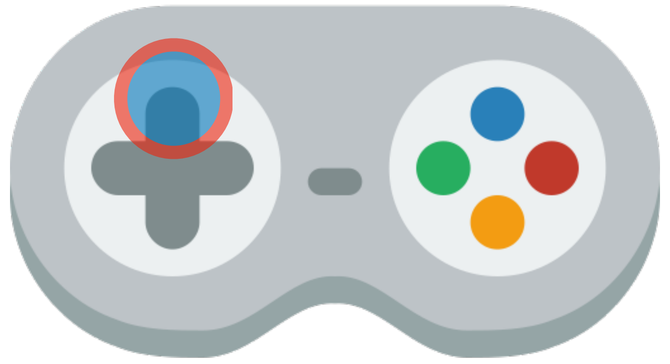
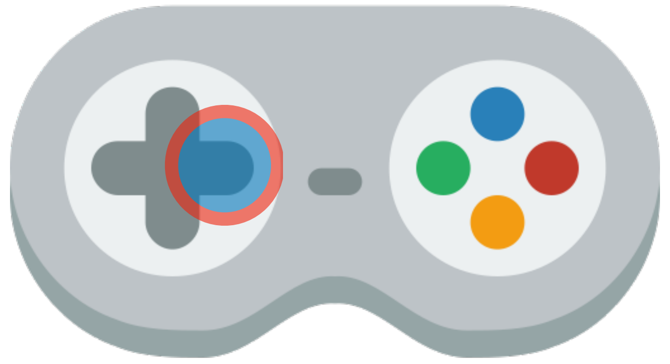
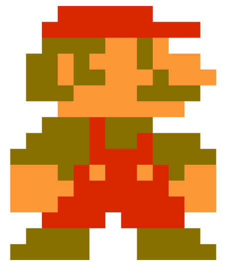
A_0 

A_1 

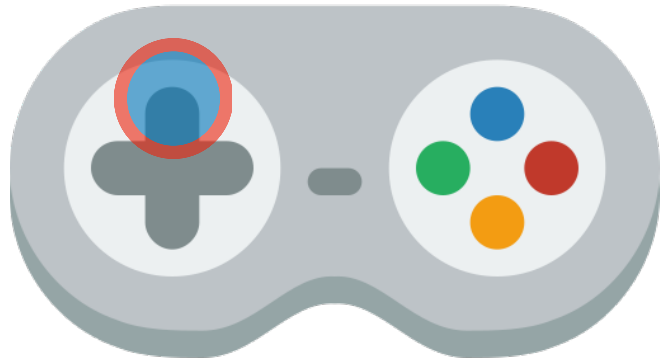
 Policy π

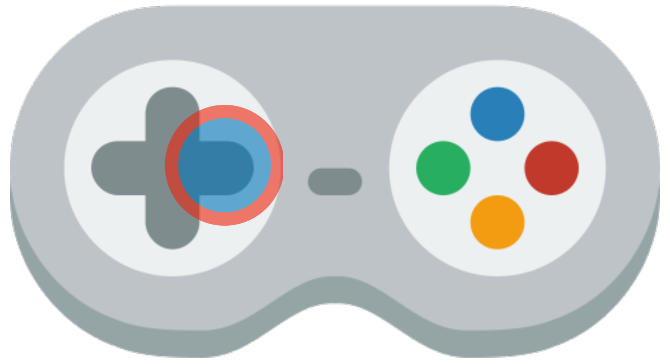
Paradigm

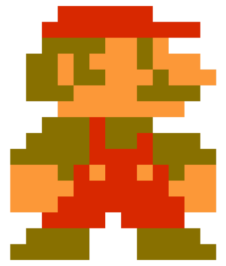
S_0

S_1 $r_1 = 0$

S_2 $r_2 = 100$

A_0 

A_1 

 Policy π

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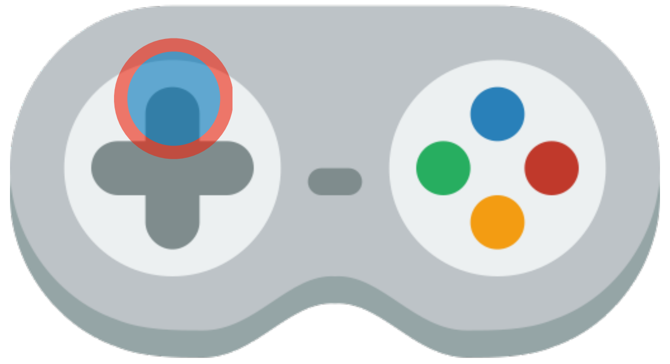

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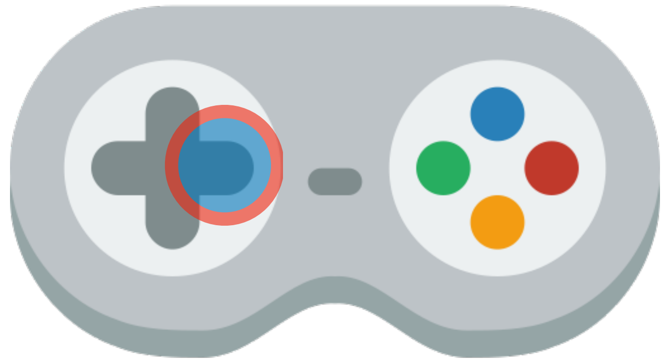
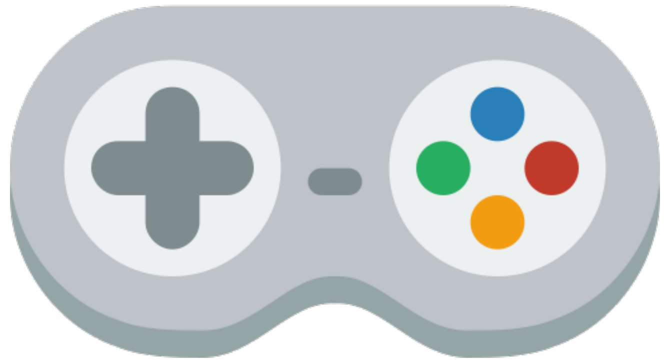
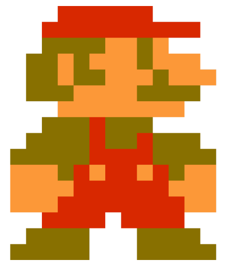


A_2





Policy π

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$$G_t = R_{t+1} + R_{t+2} + R_{t+3}, \ldots + R_T$$

Episodic tasks

Continuing tasks

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$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \quad = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Policies and Value functions

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RL involves evaluating value functions

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Functions of states or state-action pairs

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RL is about changing the policy as a result of its experience

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Bellman Equation for Q-Learning

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Bellman Equation for Q-Learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\ max_{a'}\ Q(S_{t+1}, a') - Q(S_t, A_t))$$

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Its behaviour policy can be anything that continues to visit all state actions pairs during learning (e.g. $\varepsilon -$ greedy)

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
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Bellman equation for state-action values

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Bellman equation for state-action values

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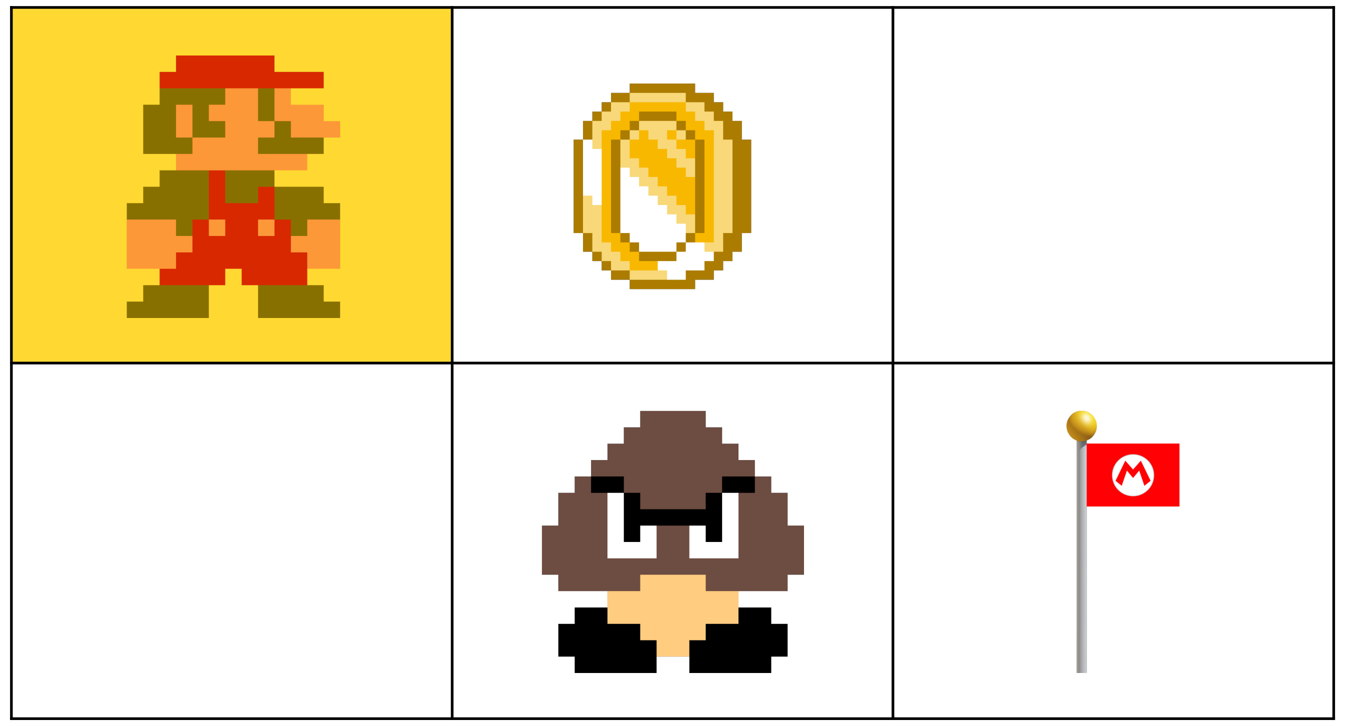
Bellman equation for state-action values

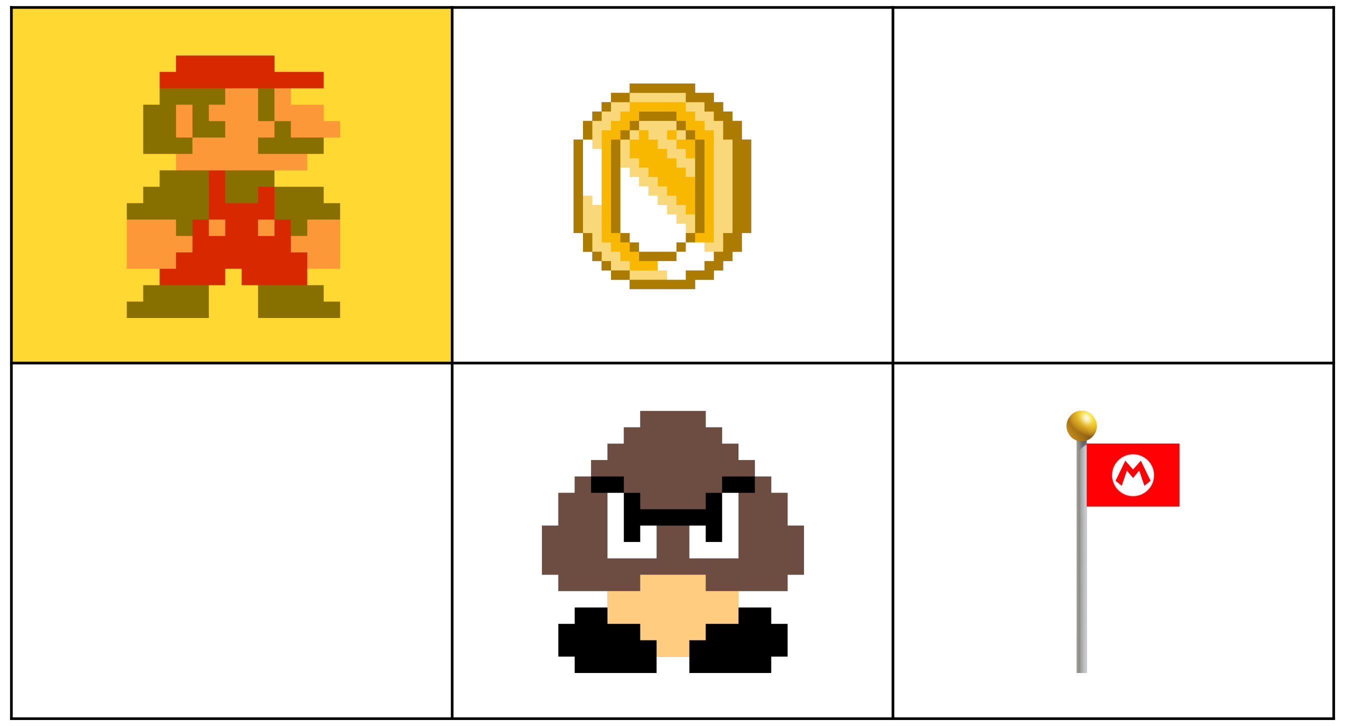
What is $Q(S, A)$?

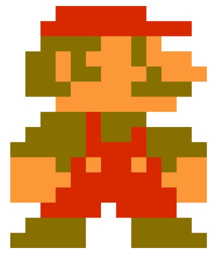
Encode Q-values in a table, each cell corresponds to a state-action pair value.

Q-Learning example



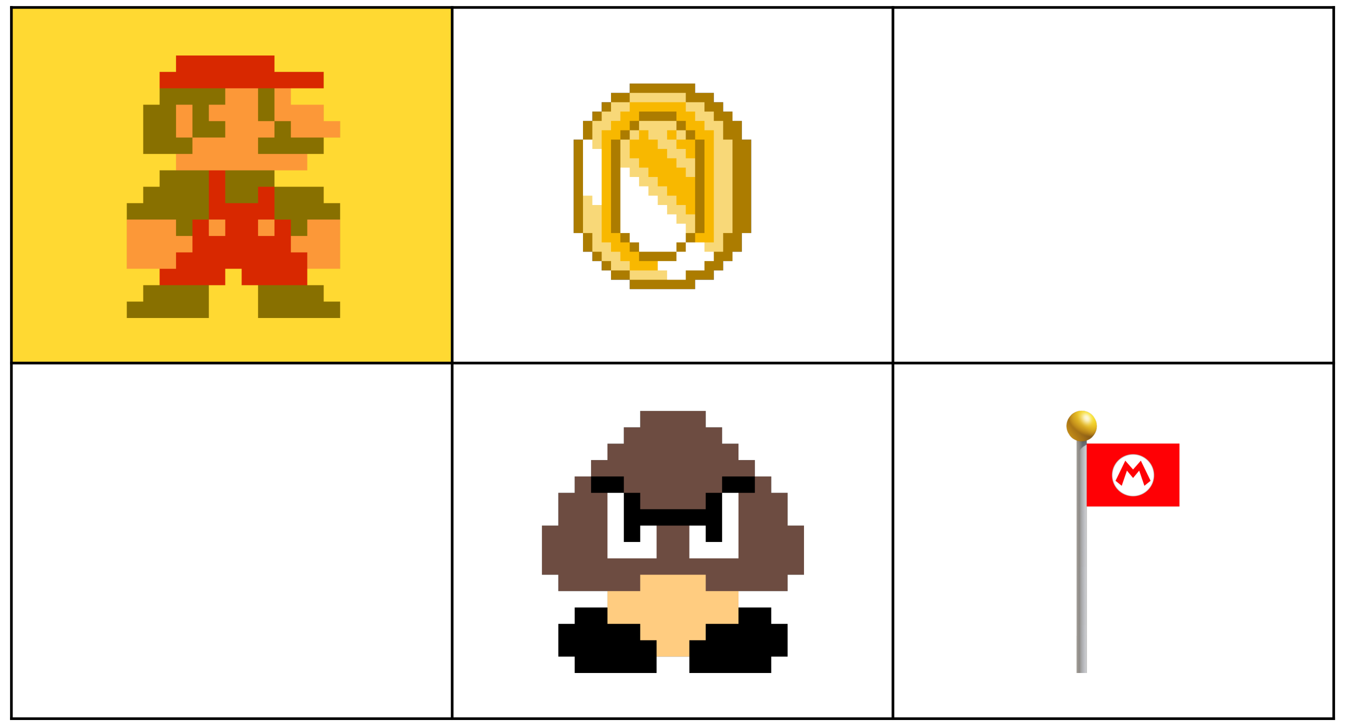
Q-Learning example

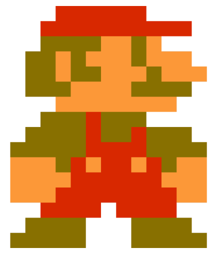




Policy π

Q-Learning example

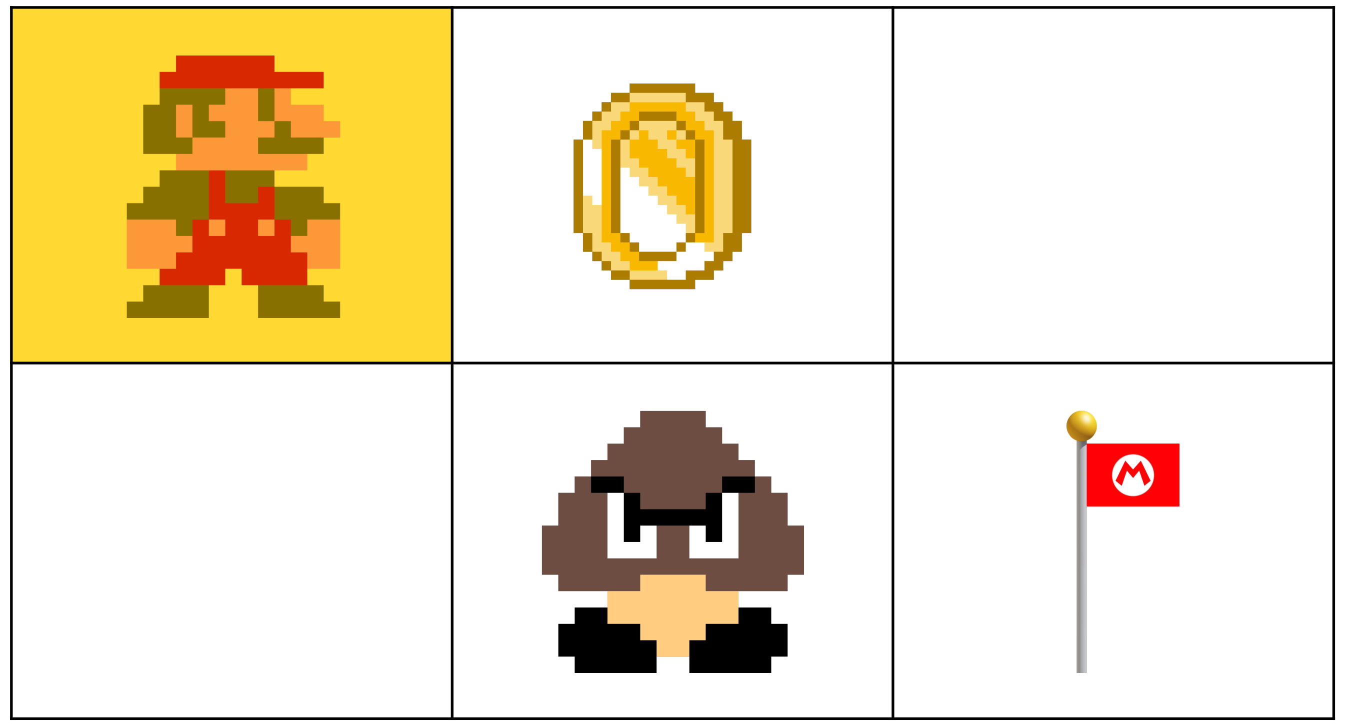


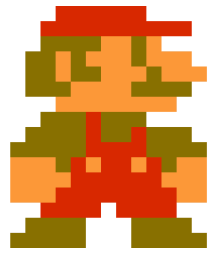


Policy π

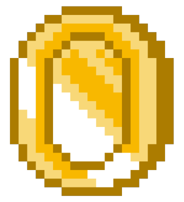
Actions $\uparrow \downarrow \leftarrow \rightarrow$

Q-Learning example

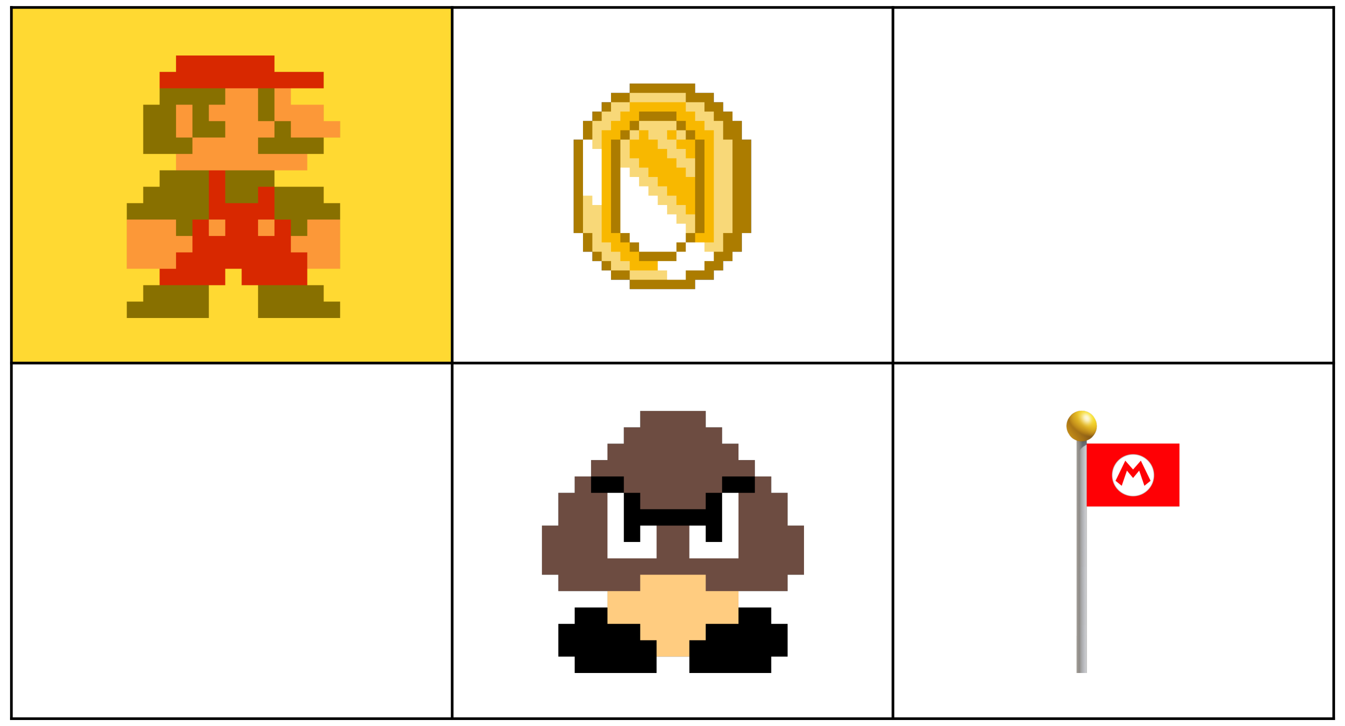


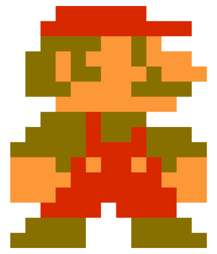
 Policy π

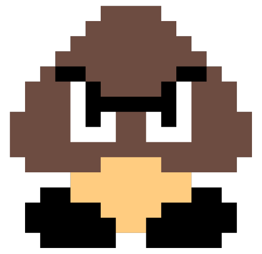
Actions $\uparrow \downarrow \leftarrow \rightarrow$

 +1

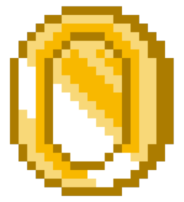
Q-Learning example



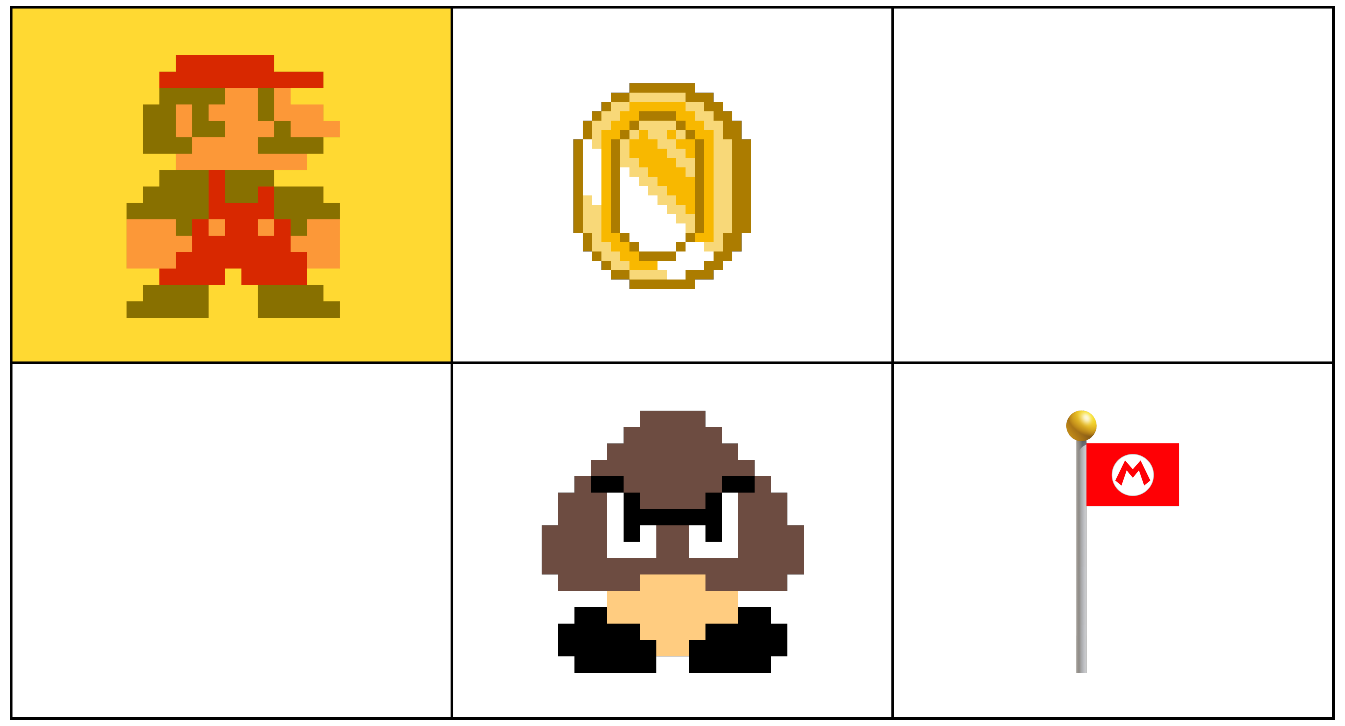
 Policy π

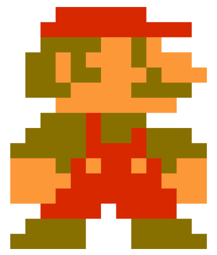
 -10

Actions $\uparrow \downarrow \leftarrow \rightarrow$

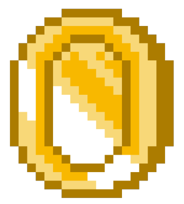
 +1

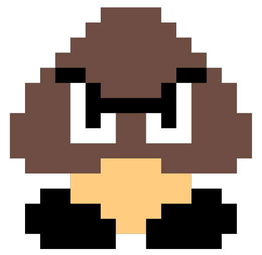
Q-Learning example



 Policy π

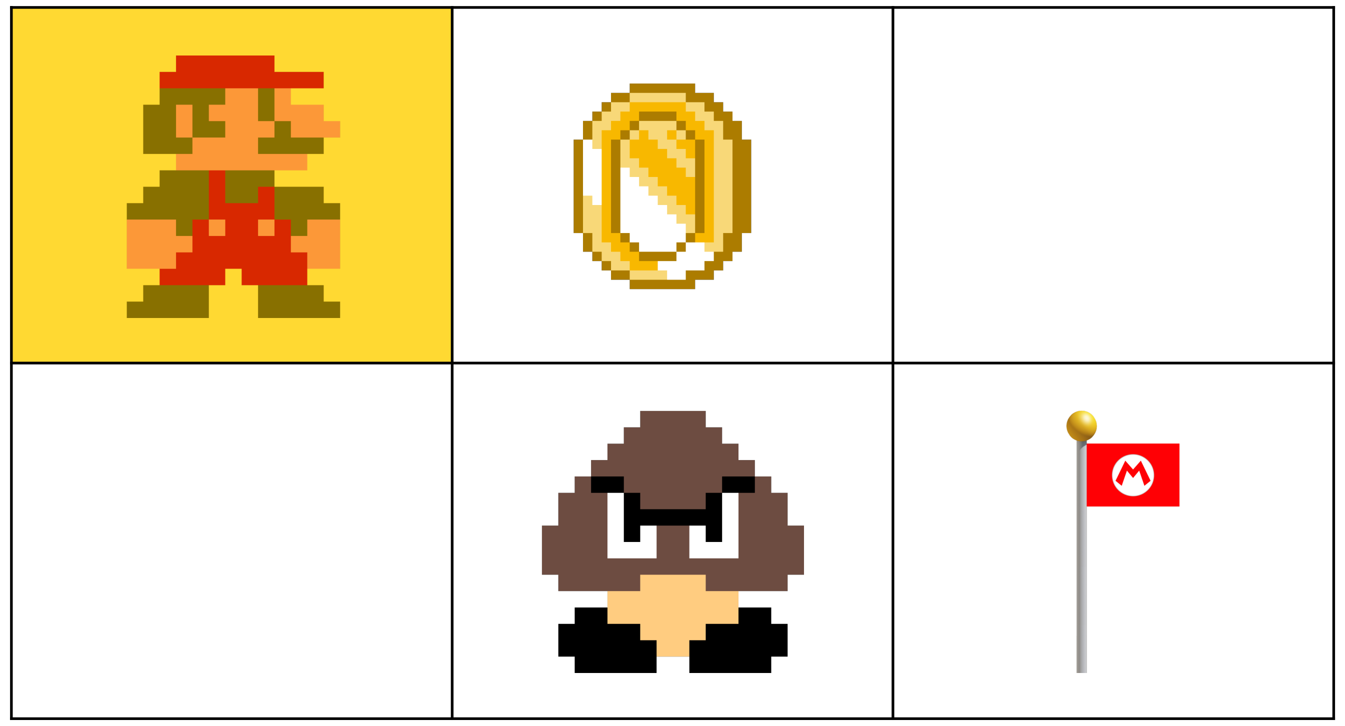
Actions $\uparrow \downarrow \leftarrow \rightarrow$

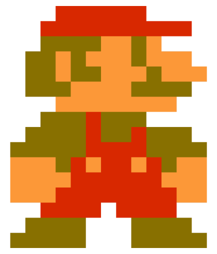
 +1

 -10

 +10 (terminal)

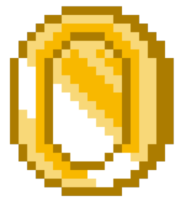
Q-Learning example



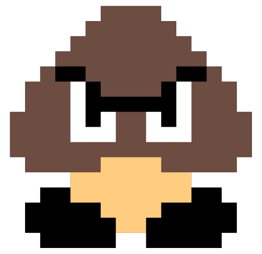


Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$



+1

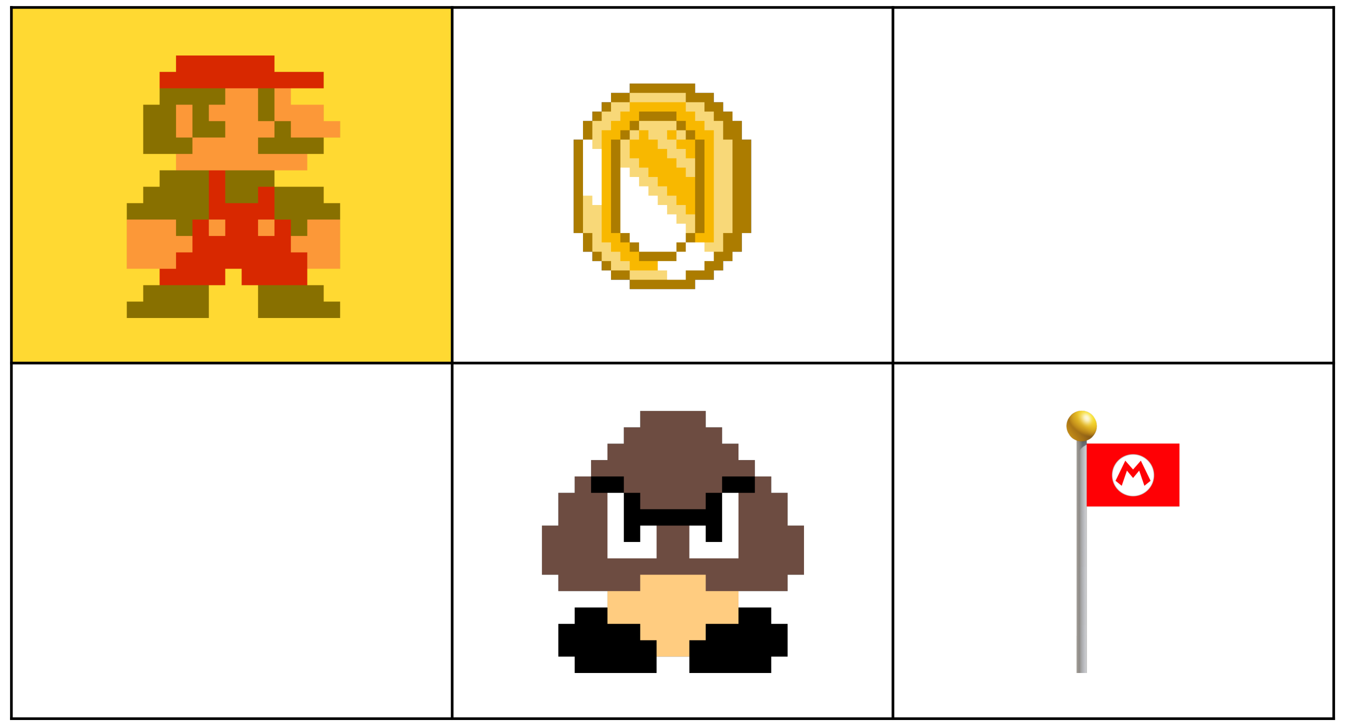


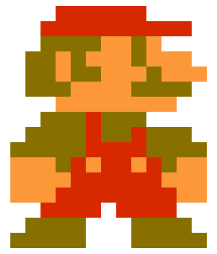
-10

+10
(terminal)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \ max_{a'} \ Q(S_{t+1}, a') - Q(S_t, A_t))$$

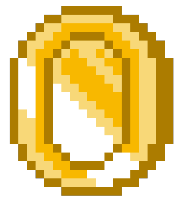
Q-Learning example



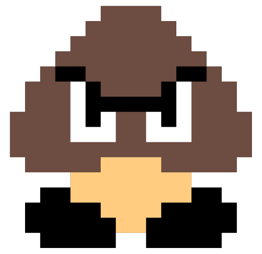


Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$



+1



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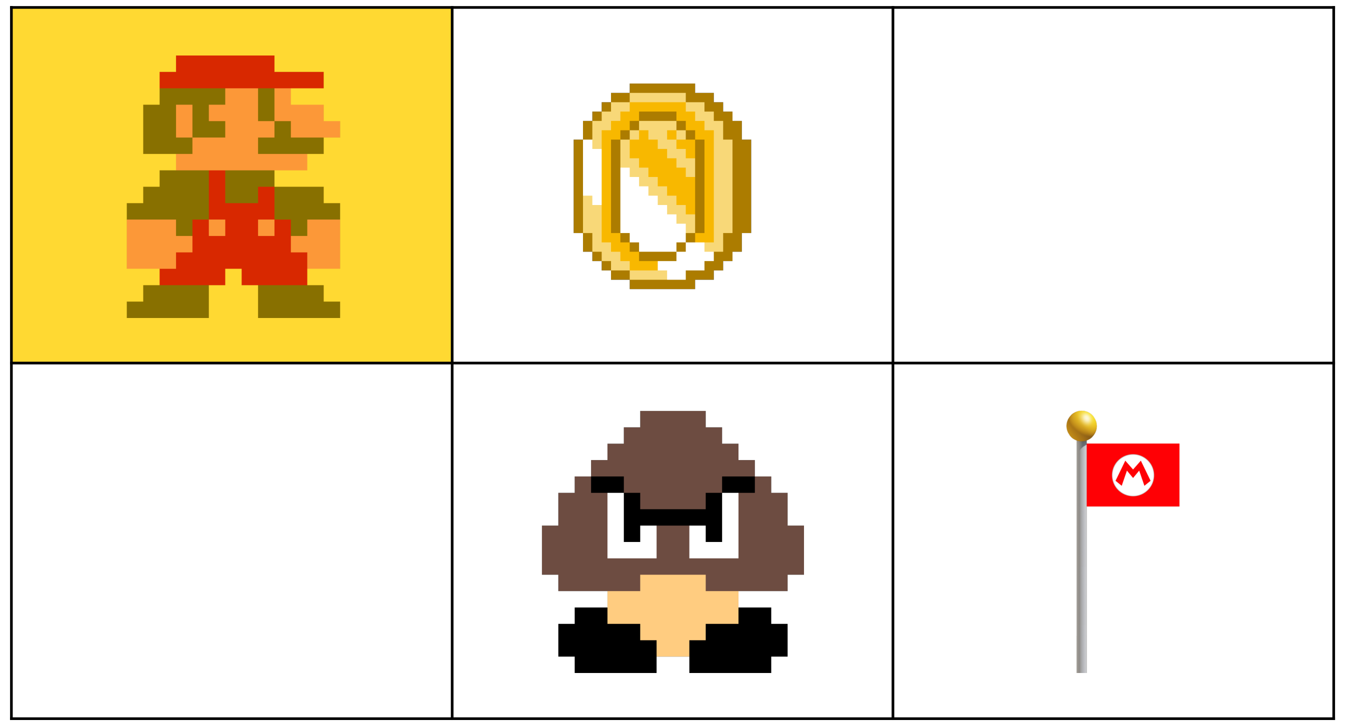
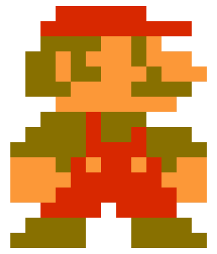
$$\alpha = 0.1$$

$$\gamma = 0.99$$

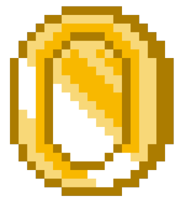
Q-Learning example



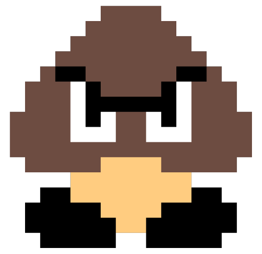


Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$



+1



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+10
(terminal)

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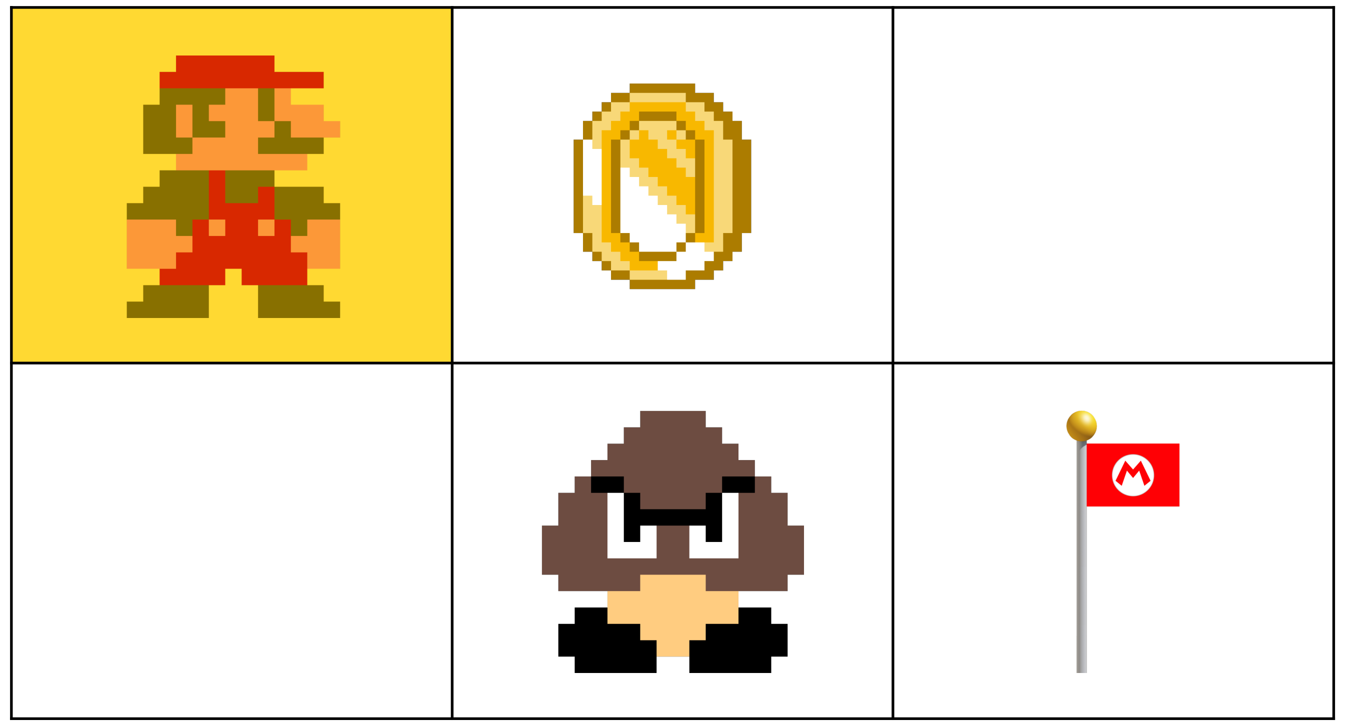
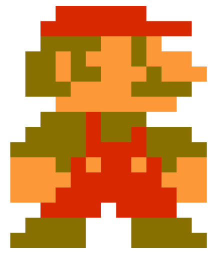
How to represent the Q-Table?

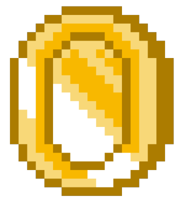
Q-Learning example



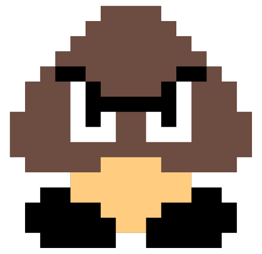


Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$



+1



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+10
(terminal)

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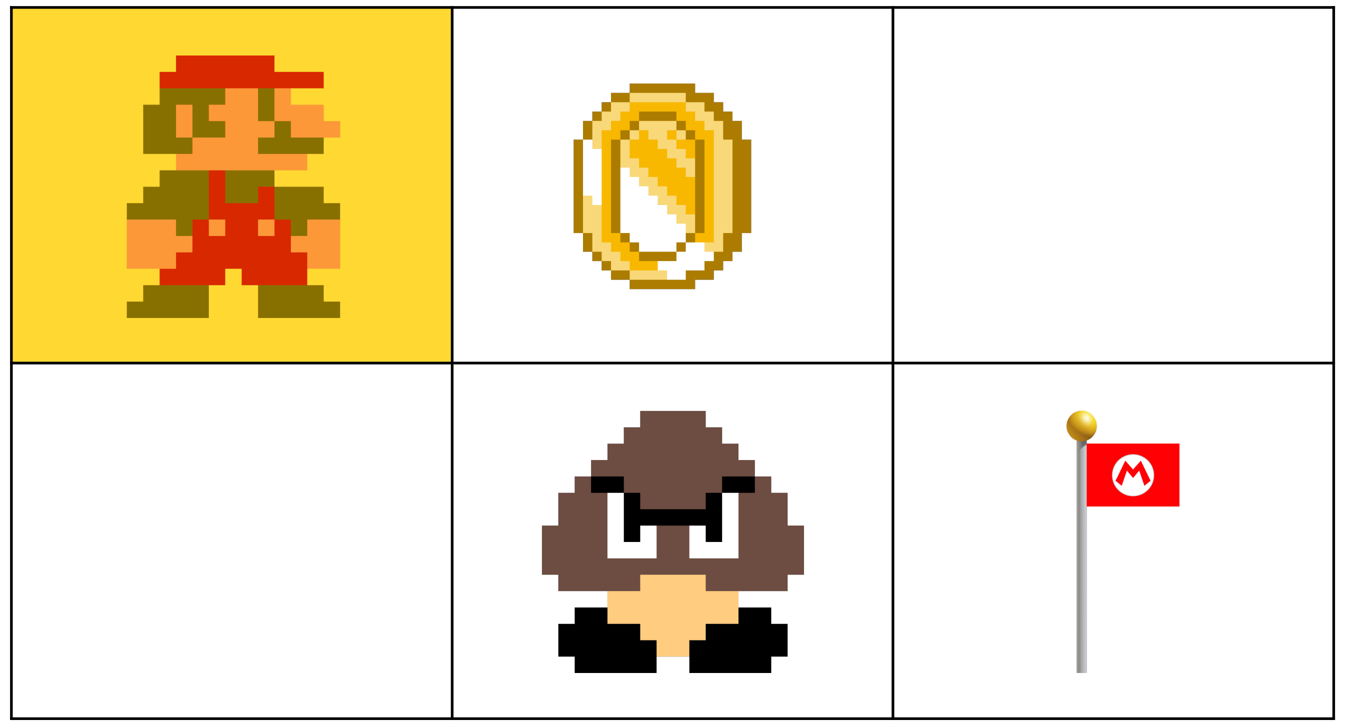
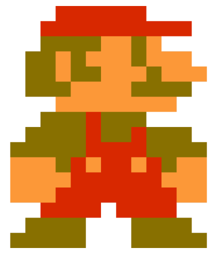
How to represent the Q-Table?

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$

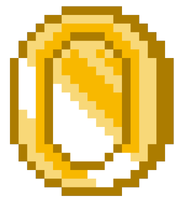
Q-Learning example



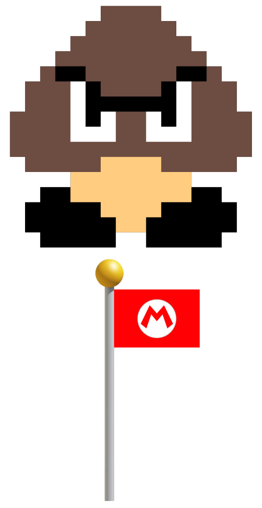


Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$



+1



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+10
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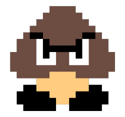
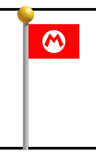
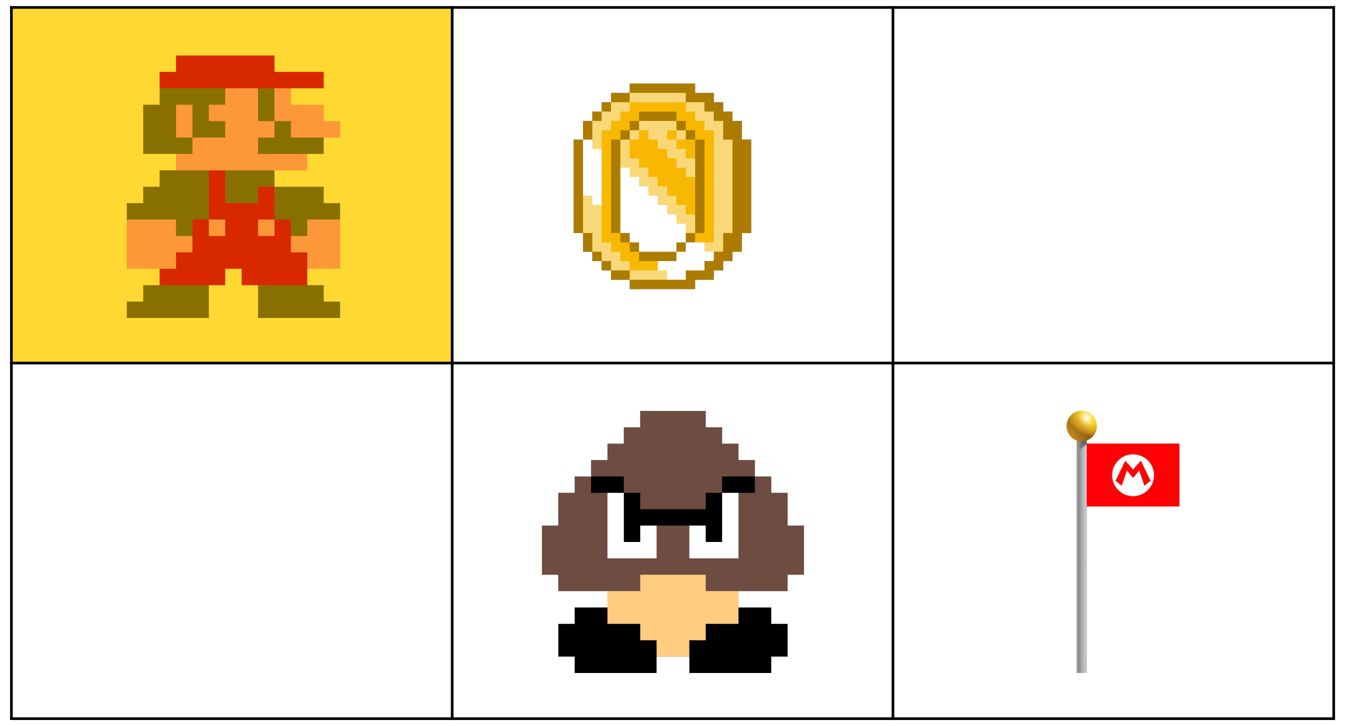
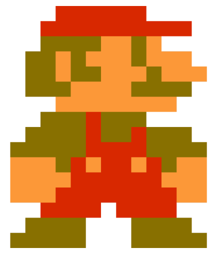
How to represent the Q-Table?

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
				
				
				

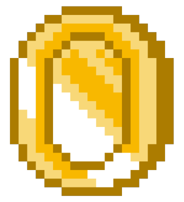
How to move?

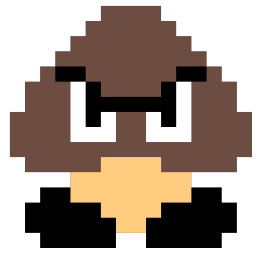
Q-Learning example



 Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

 +1

 -10

 +10 (terminal)

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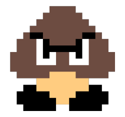
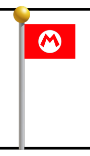
How to represent the Q-Table?

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
				
				
				

How to move?

$\pi = \text{argmax}_A Q(S, A)$

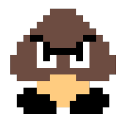
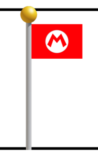
Q-Learning example

Q-Learning example

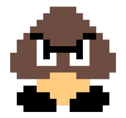
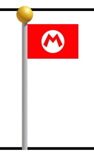
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Q-Learning example

Initialize $Q(s,a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

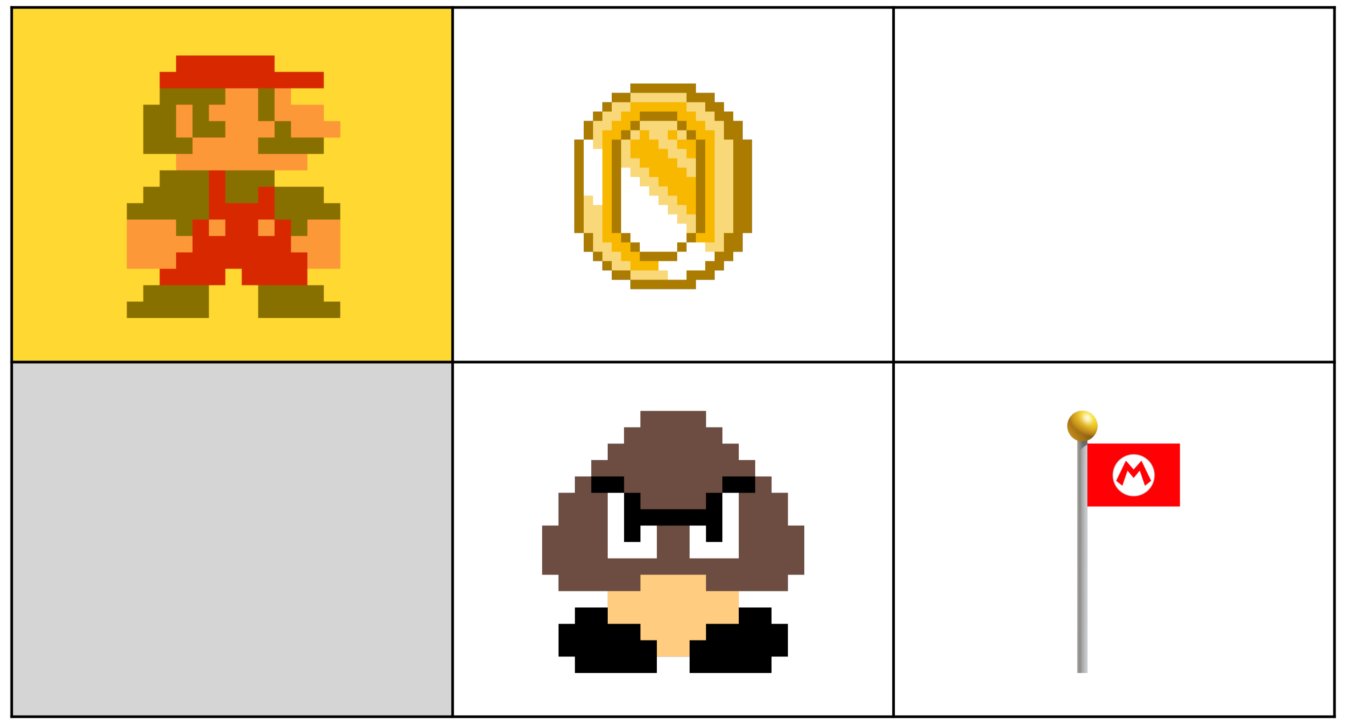
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning example

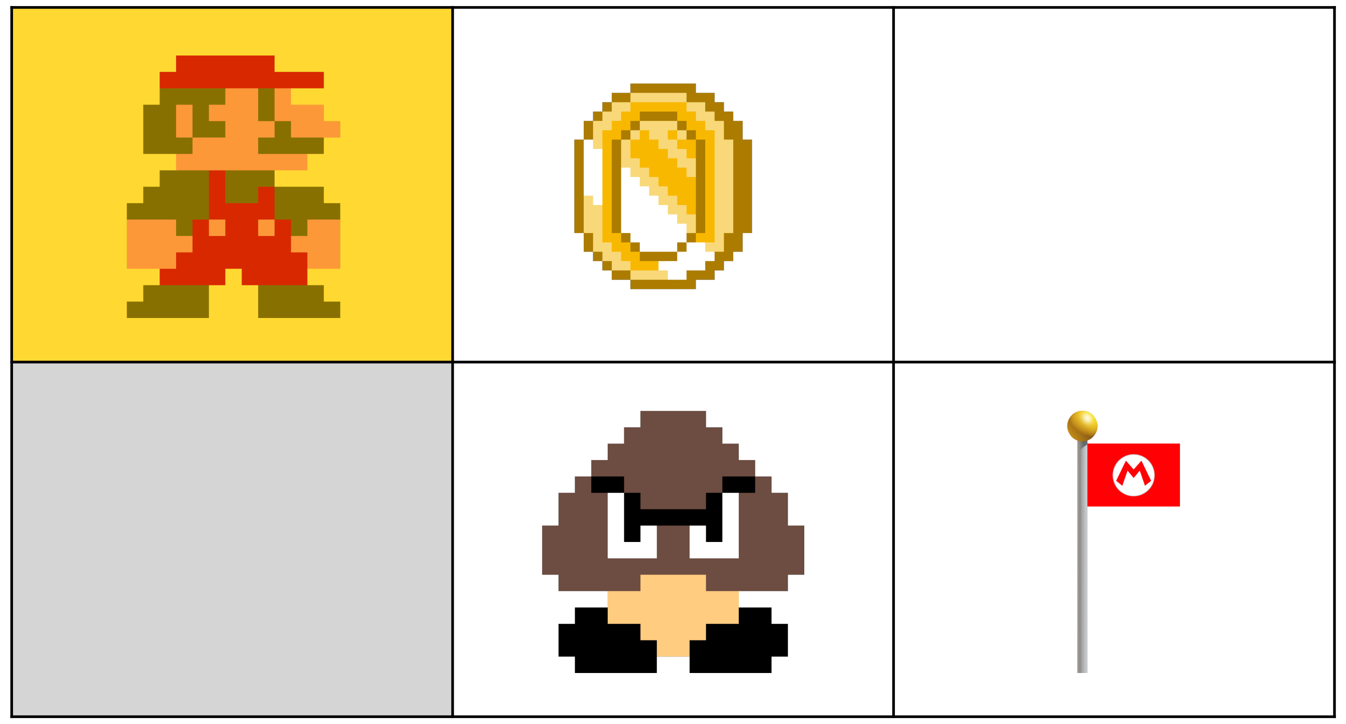
	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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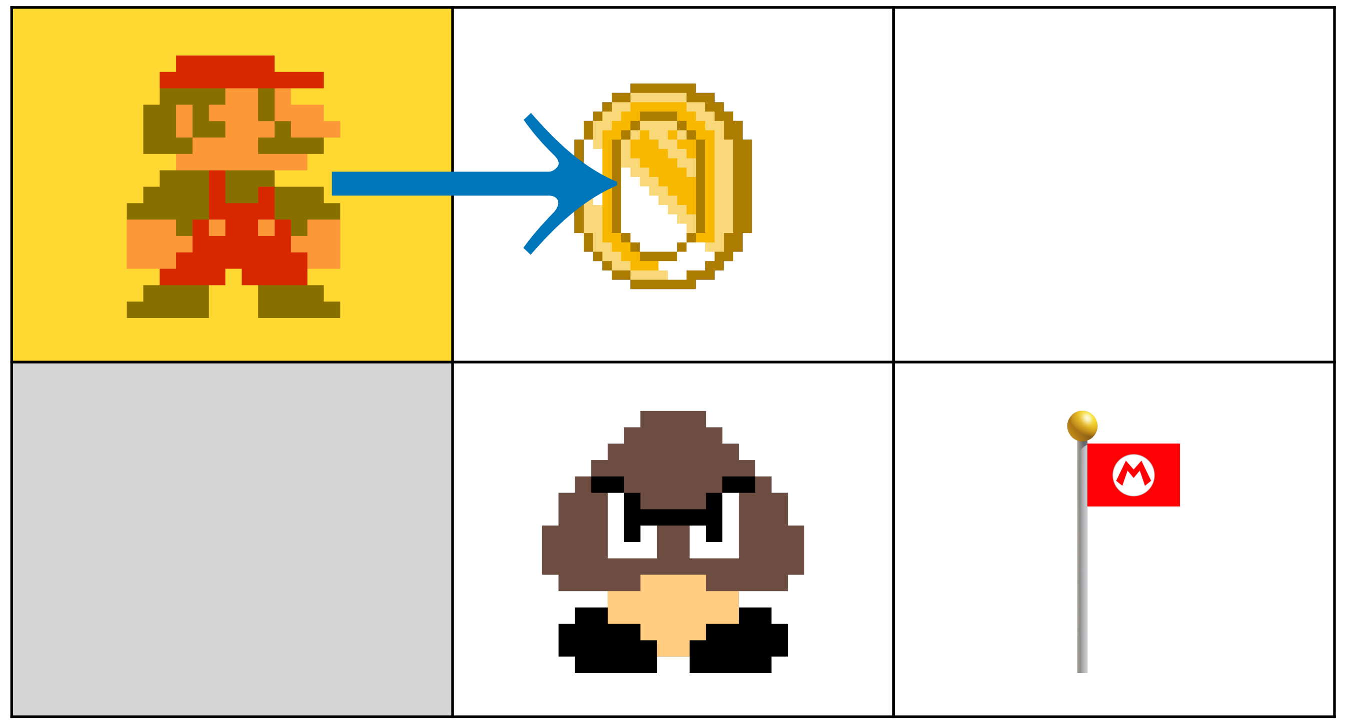
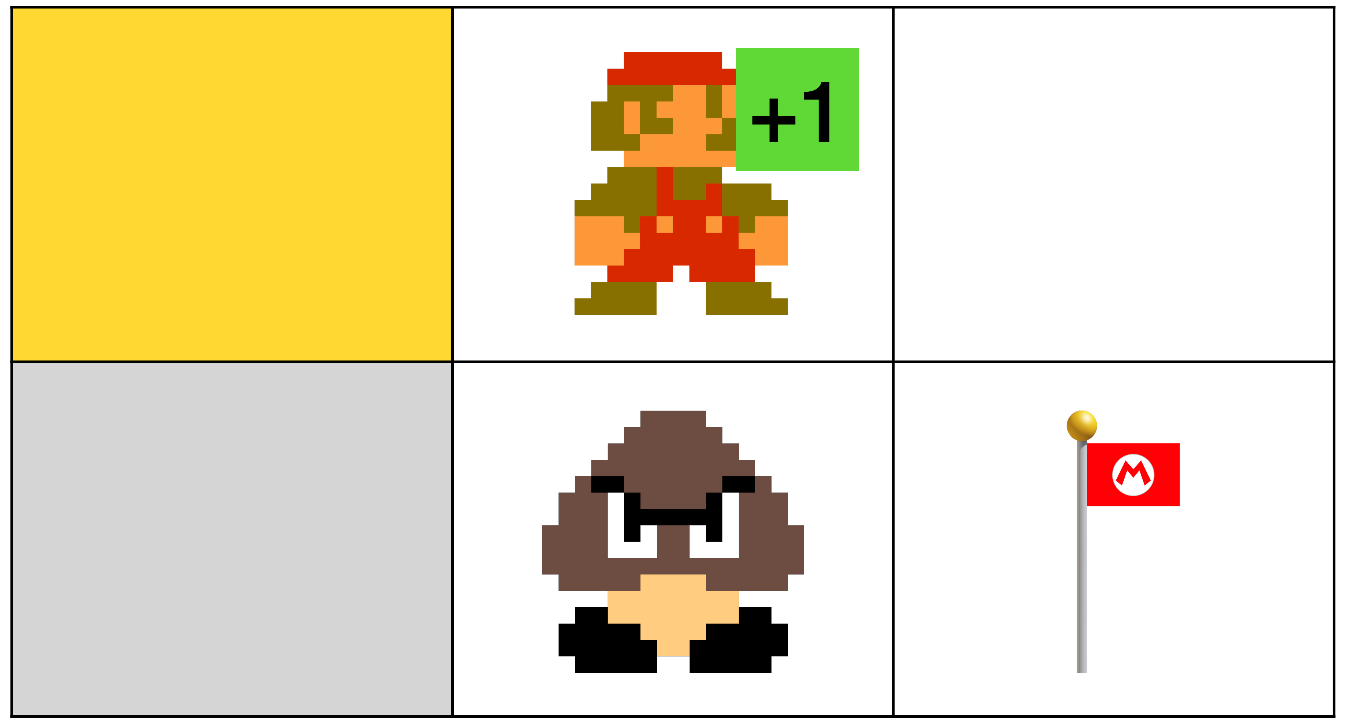


Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'

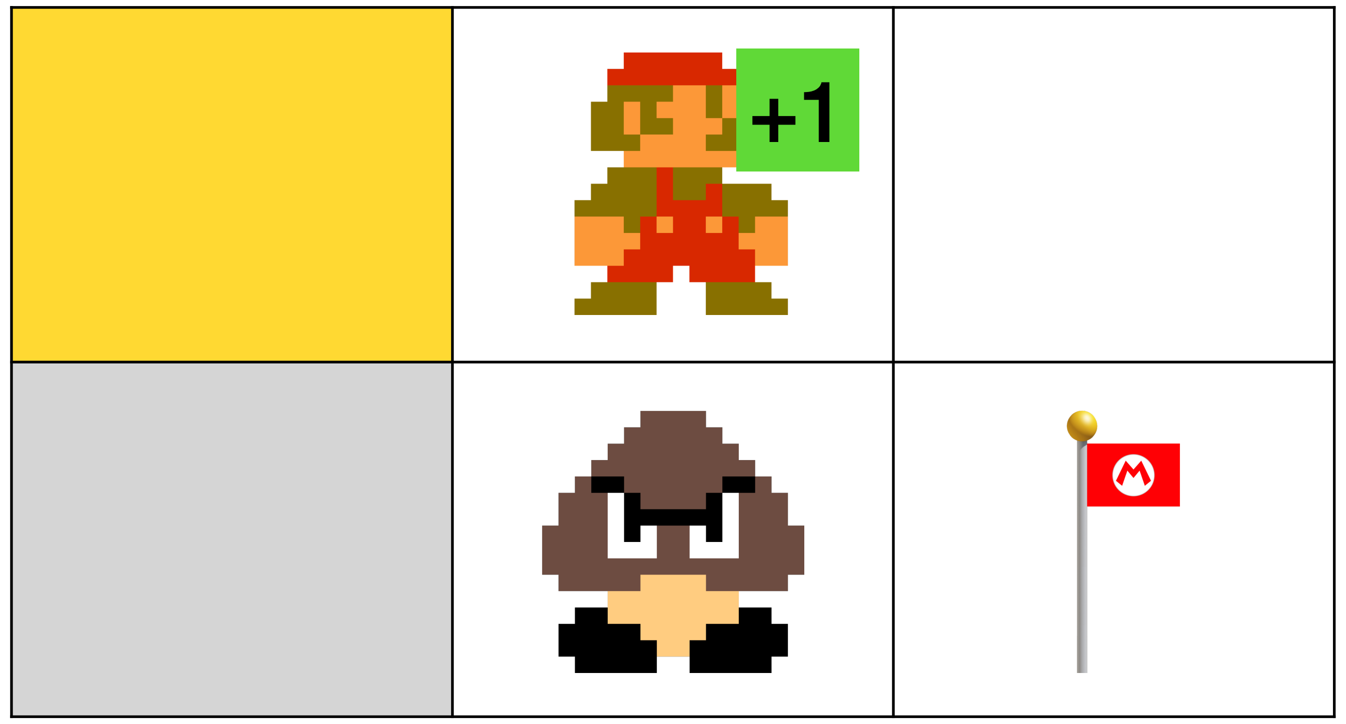


Q-Learning example

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

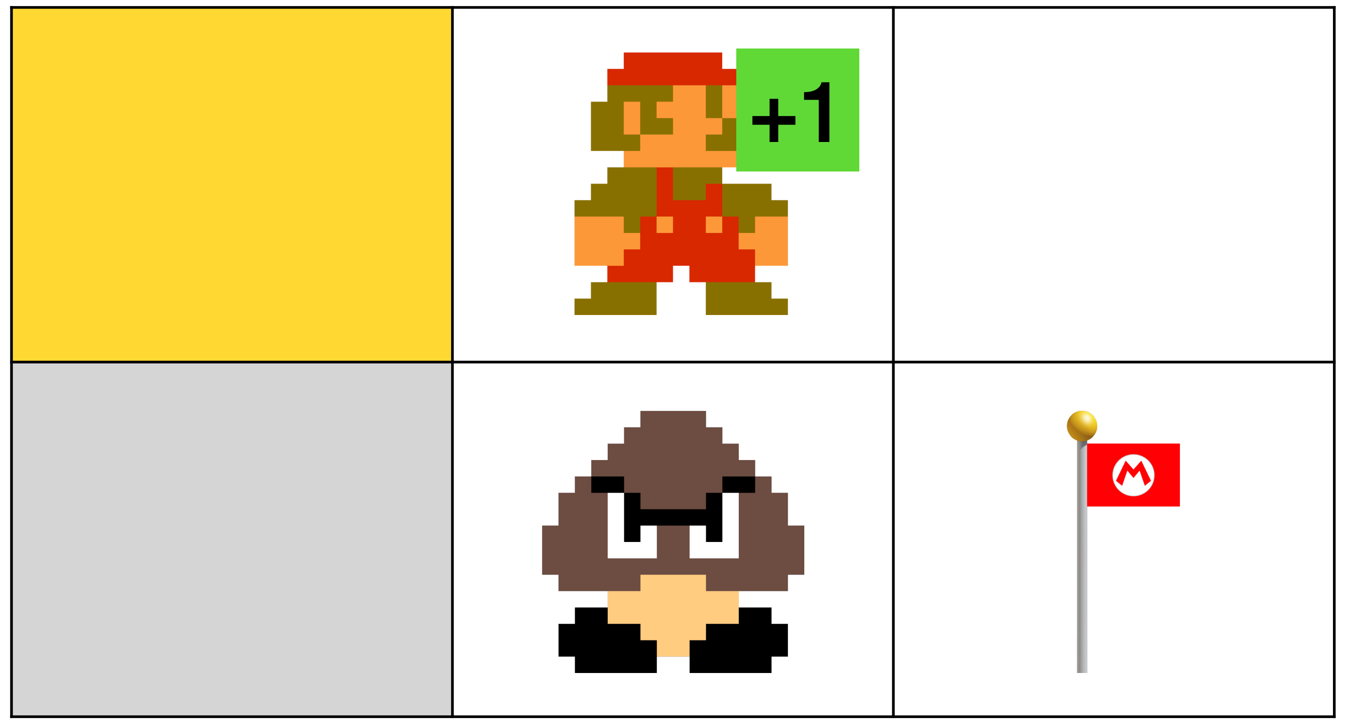
Take action A, observe R, S'



Q-Learning example

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\, max_{a'}\, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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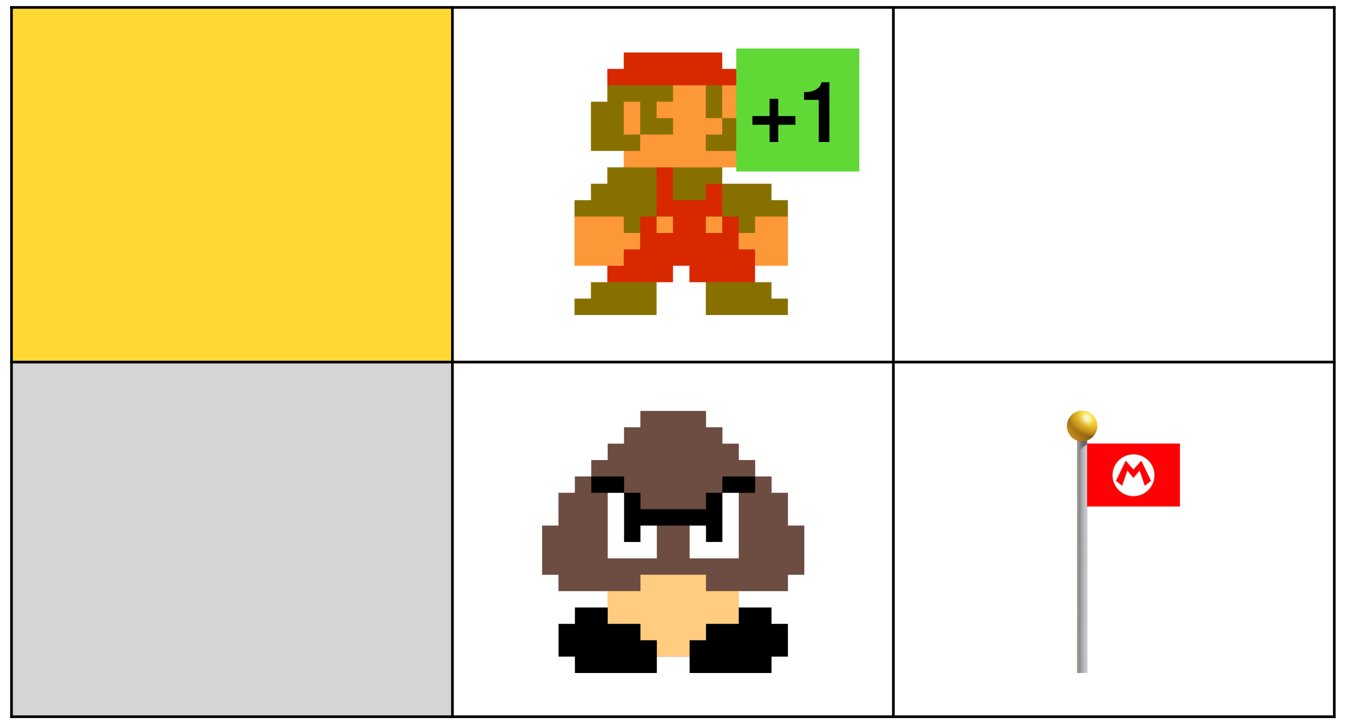
Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

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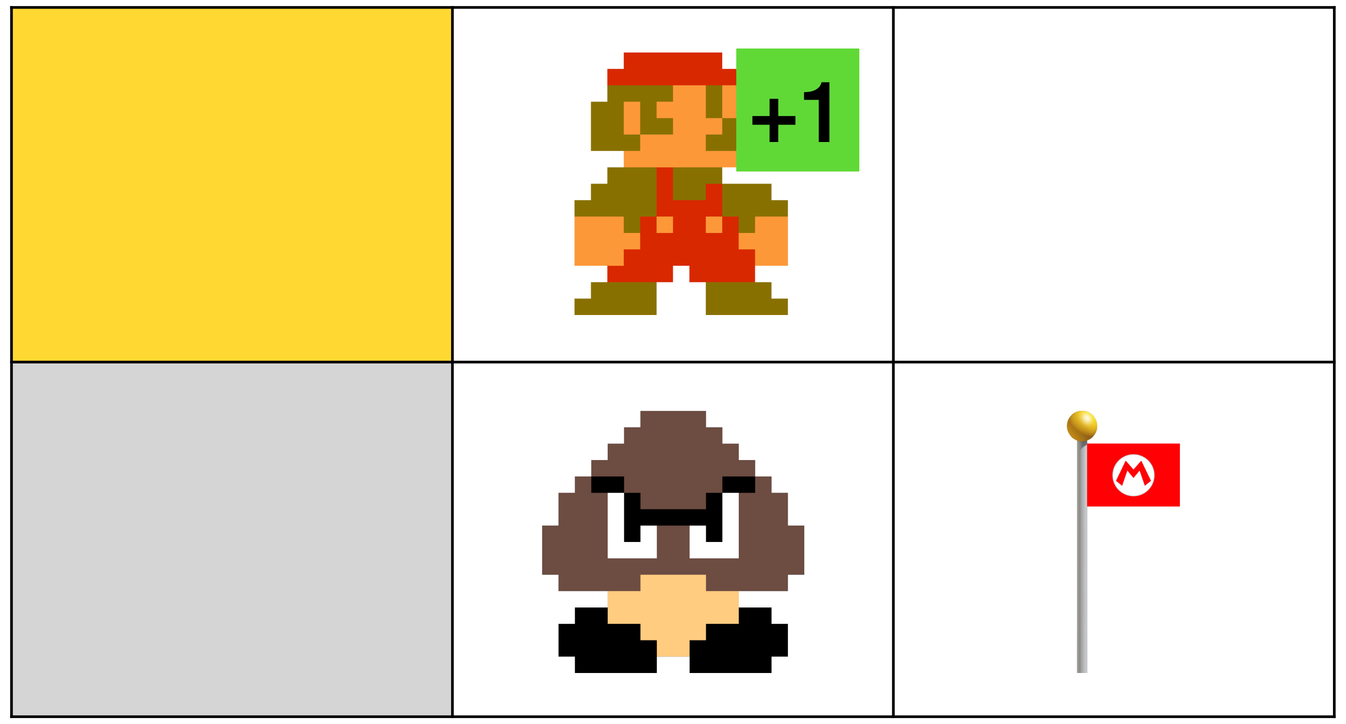
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q(initial, →) = 0 + 0.1*[1 + 0.99*0 - 0] = 0.1

Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

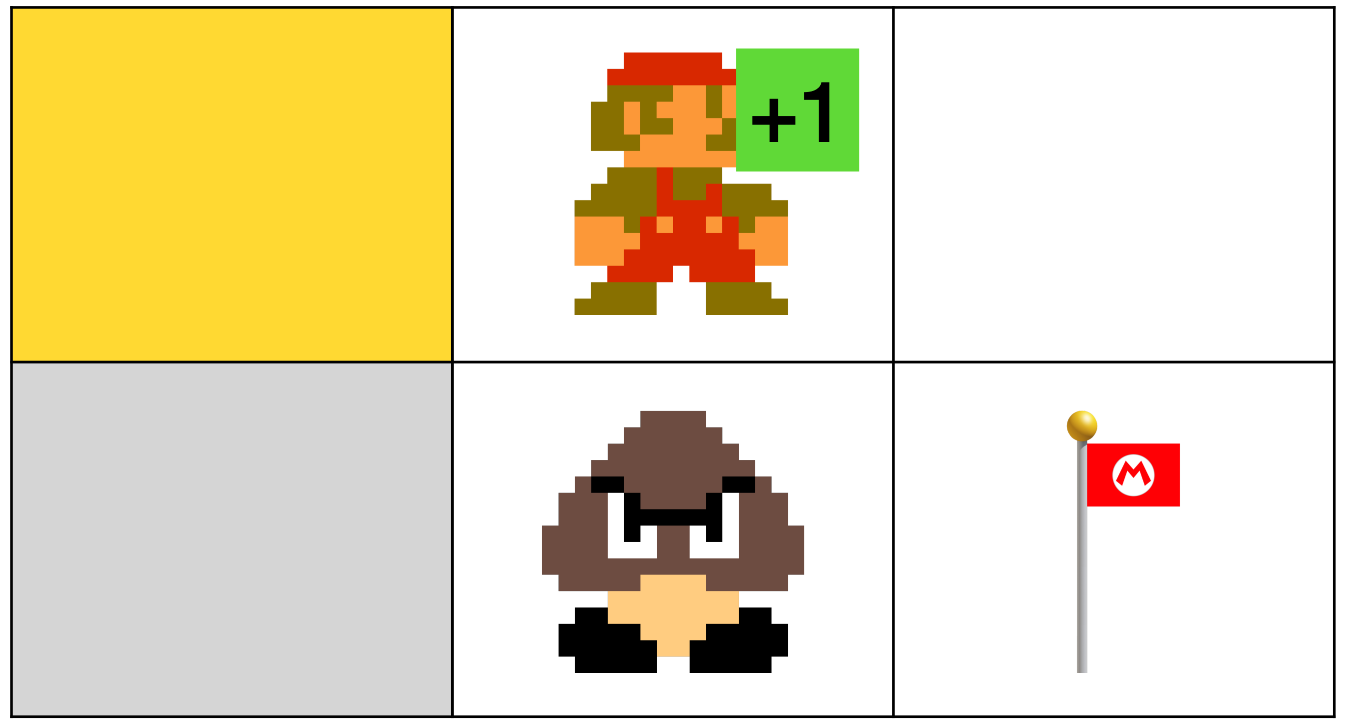
	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0 + 0.1*[1 + 0.99*0 - 0] = 0.1

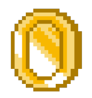
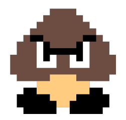
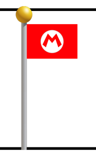
Take action A, observe R, S'



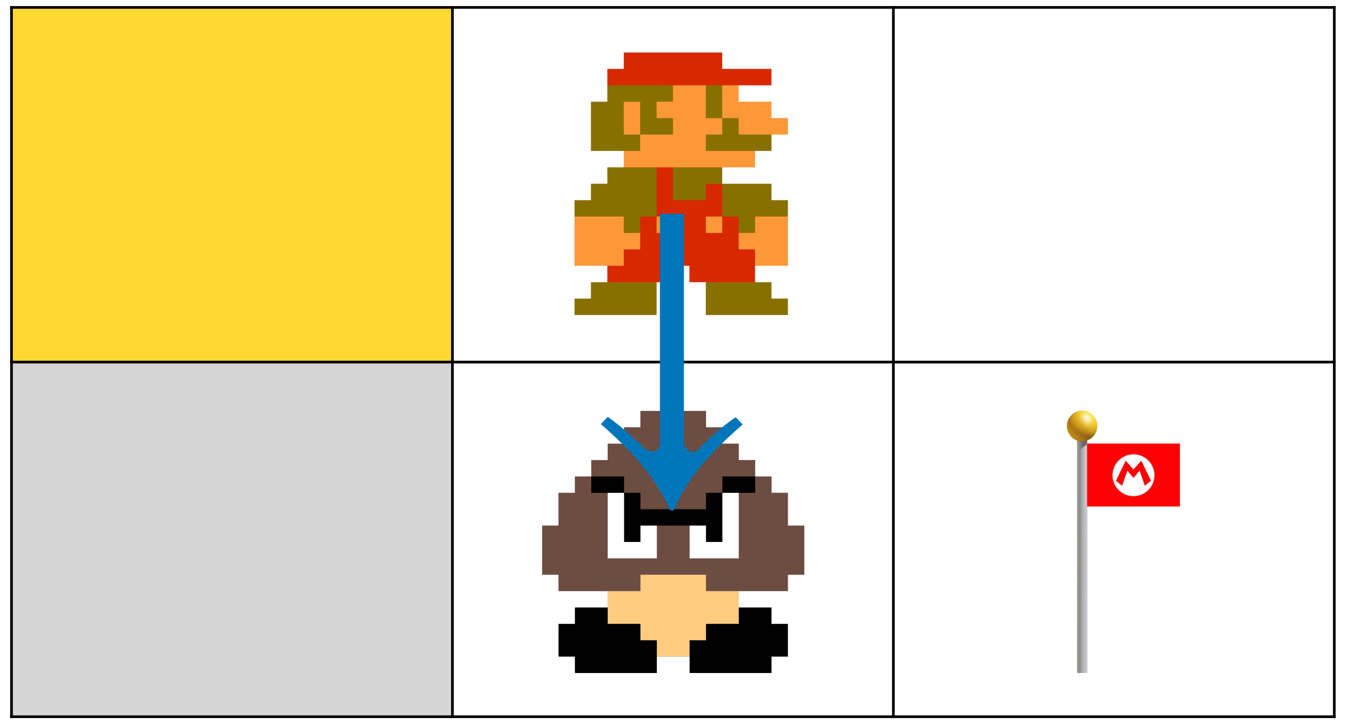
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

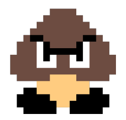
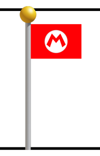
Choose A from S using policy derived from Q (e.g., ε-greedy)



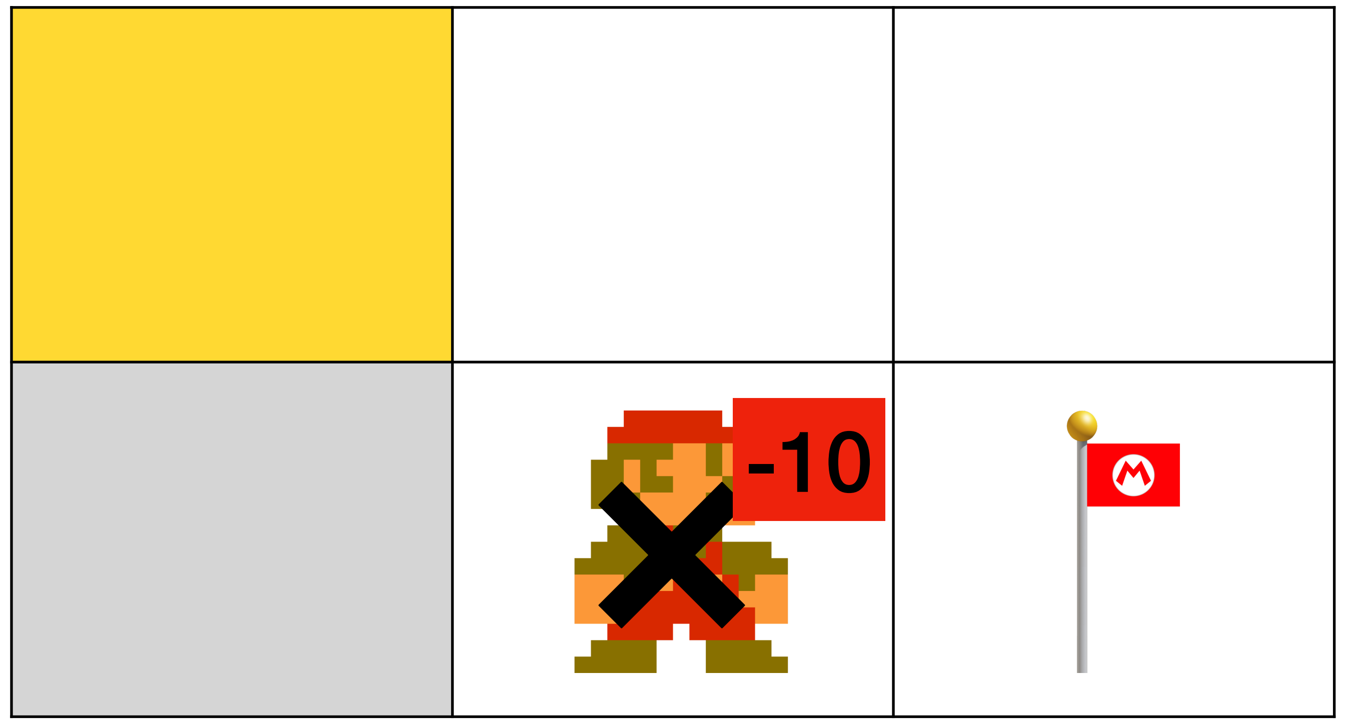
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

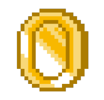
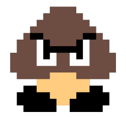
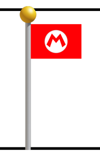
Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

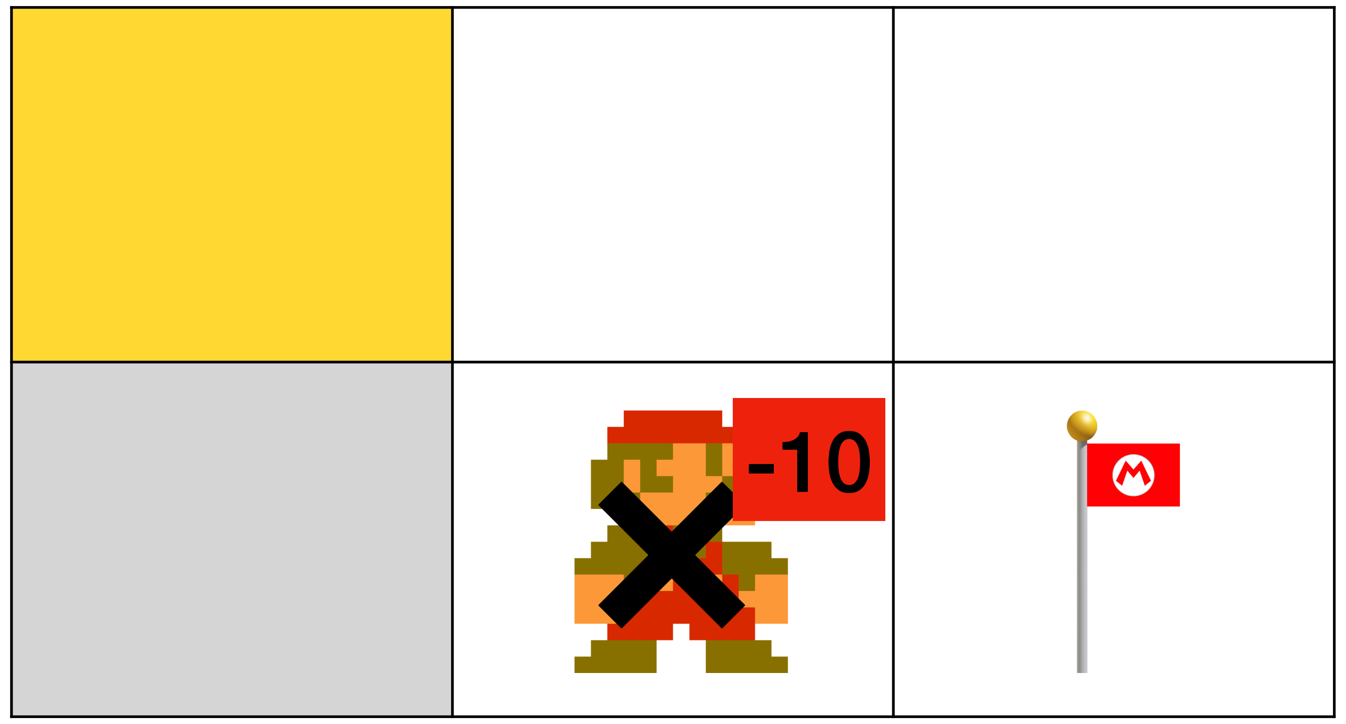
$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\ max_{a'}\ Q(S_{t+1}, a') - Q(S_t, A_t))$$

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

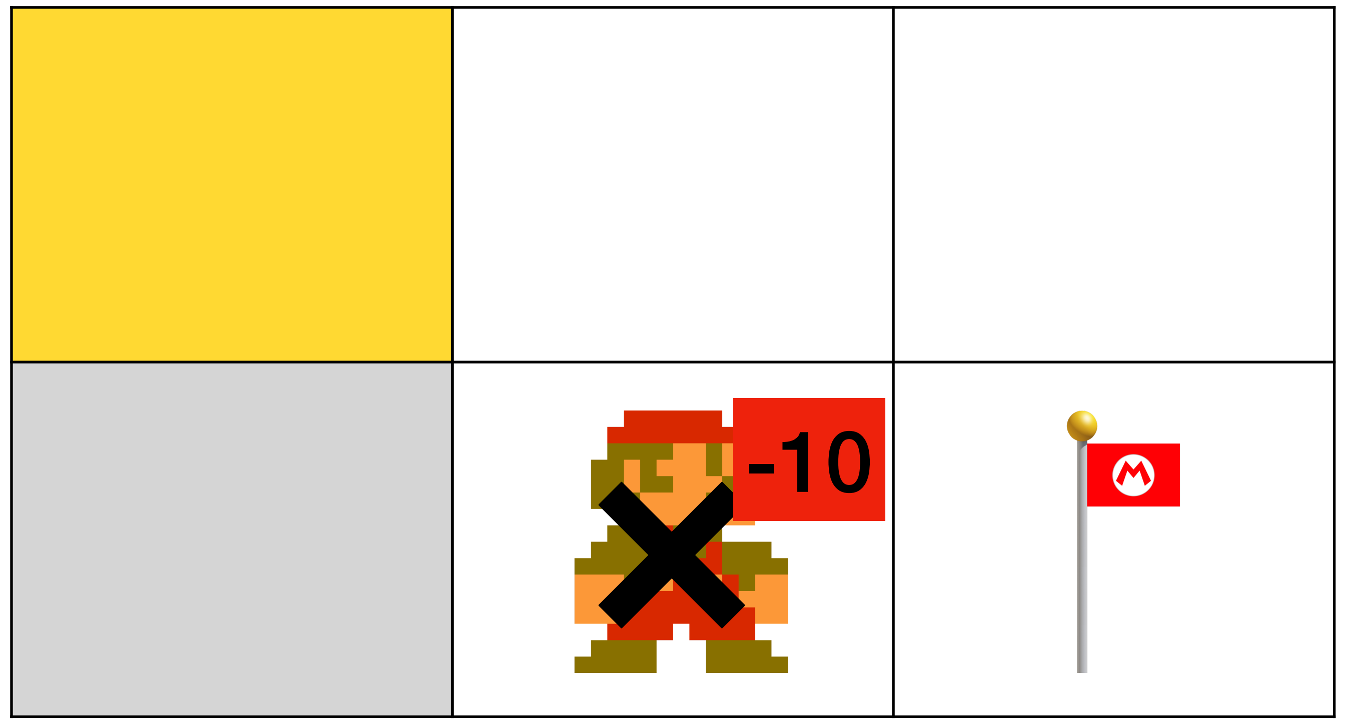
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\, max_{a'}\, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(state_2, ↓) = 0 + 0.1*[-10 + 0.99*0 - 0] = -1

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

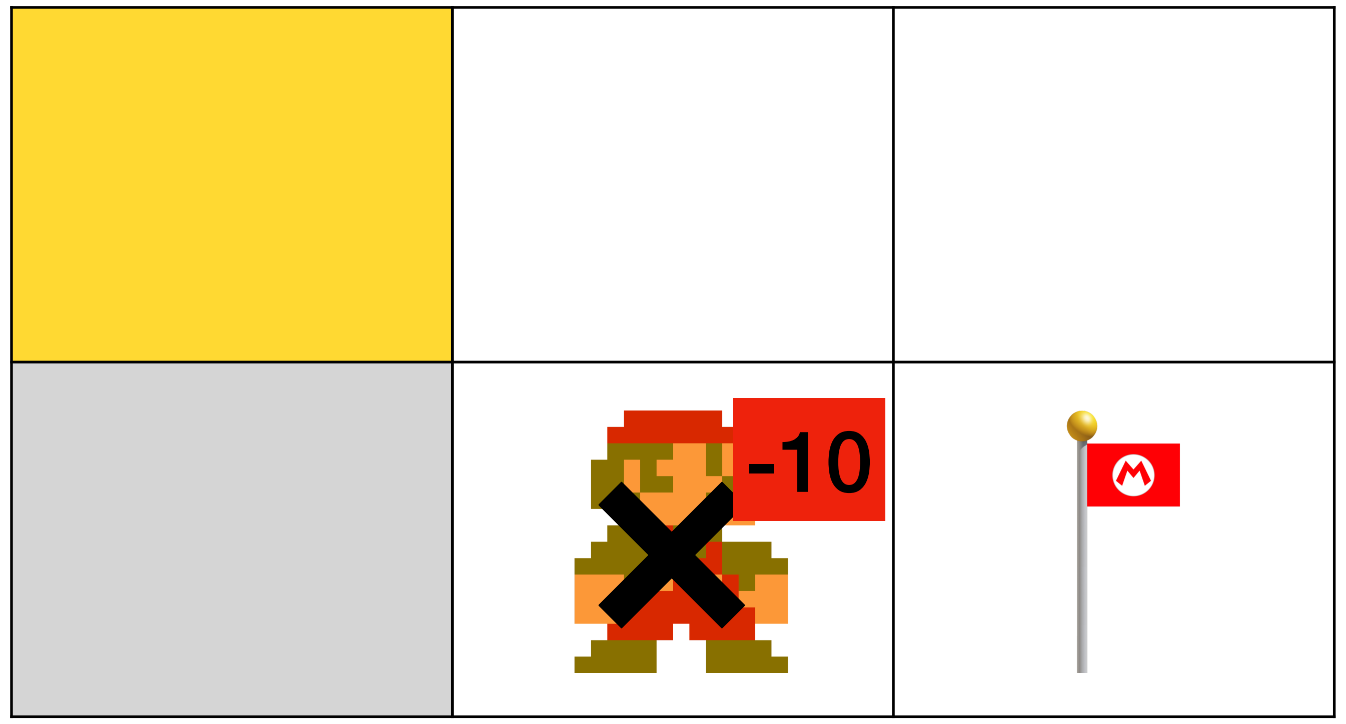
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \; max_{a'} \; Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(state_2, ↓) = 0 + 0.1*[-10 + 0.99*0 - 0] = -1

Take action A, observe R, S'



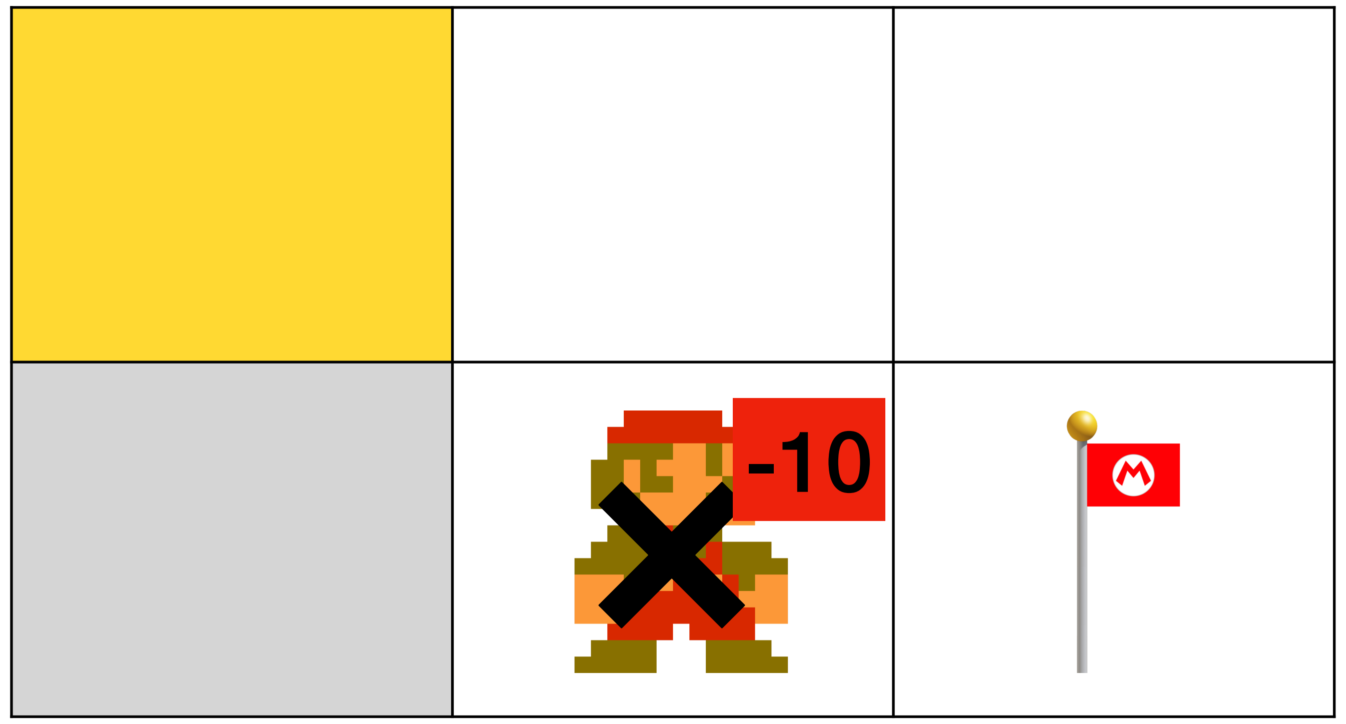
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Terminal state, Episode resets

Q-Learning example

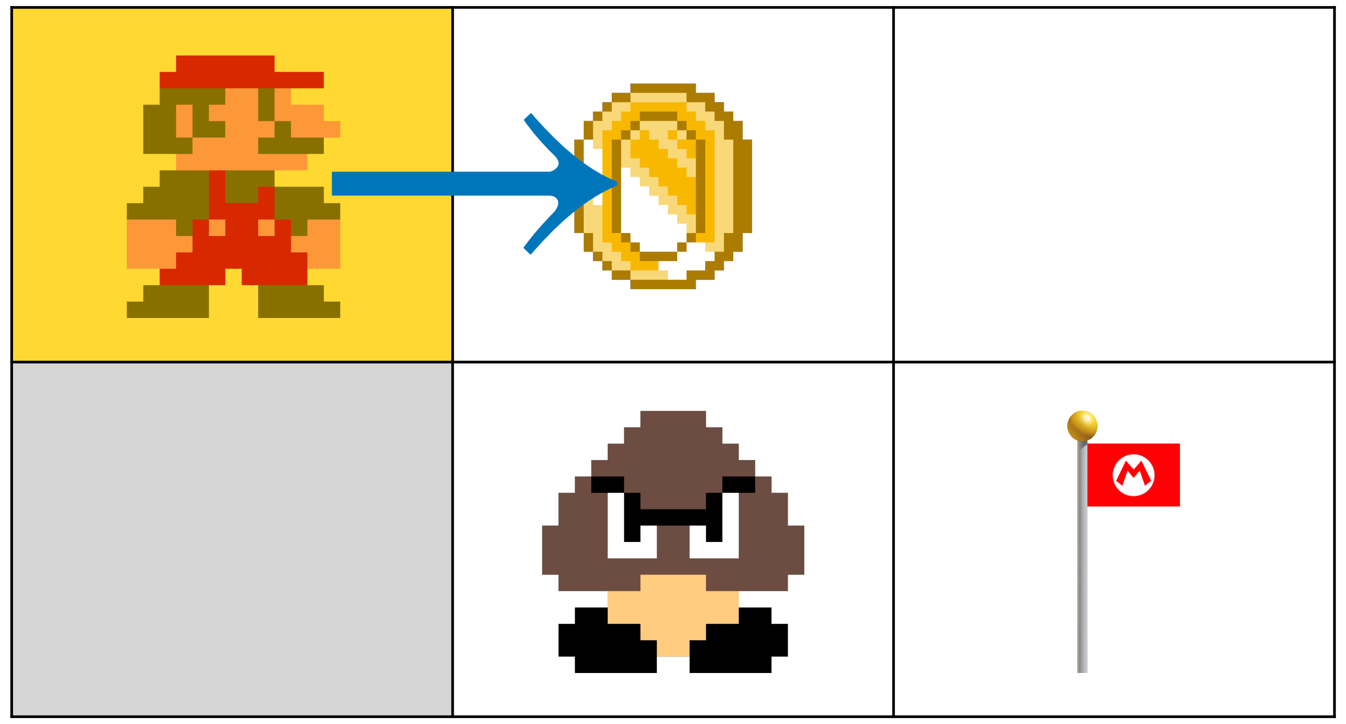
$\alpha = 0.1$

$\gamma = 0.99$

New episode

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

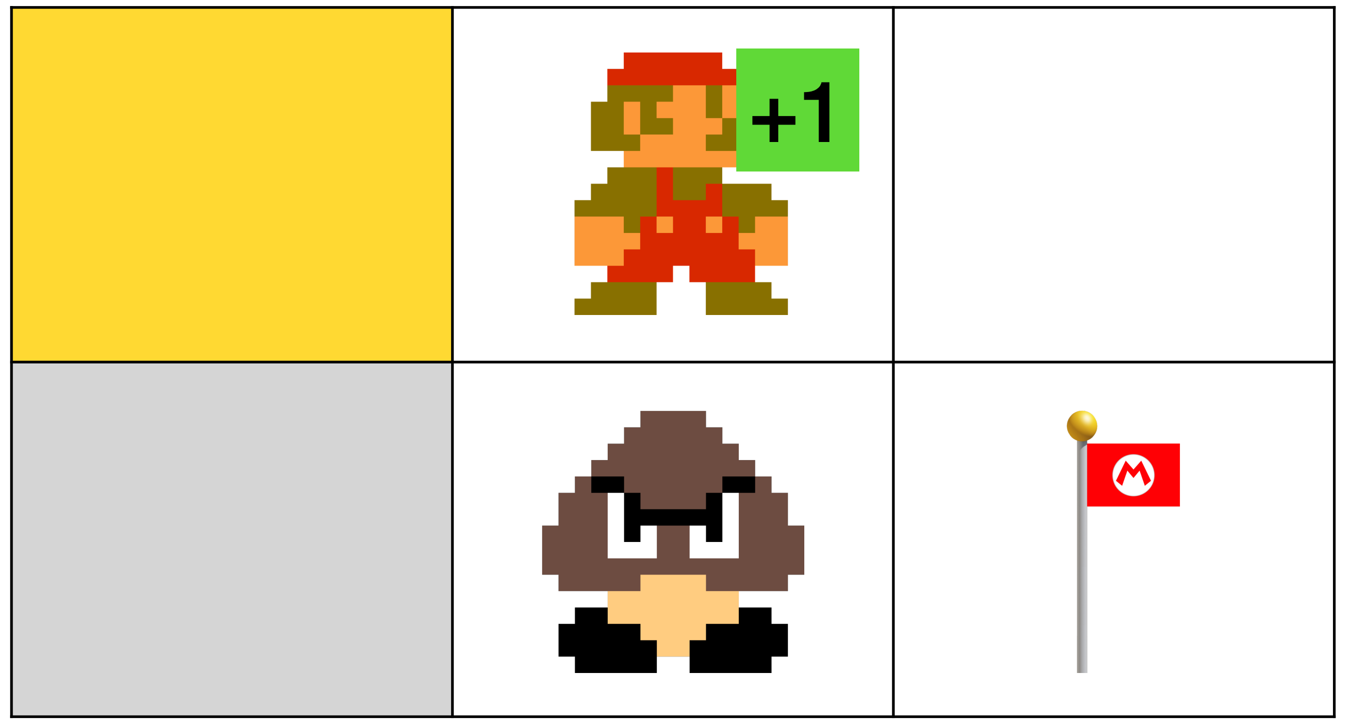
	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\ max_{a'}\ Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0.1 + 0.1*[1 + 0.99*0 - 0] = 0.2

Take action A, observe R, S'



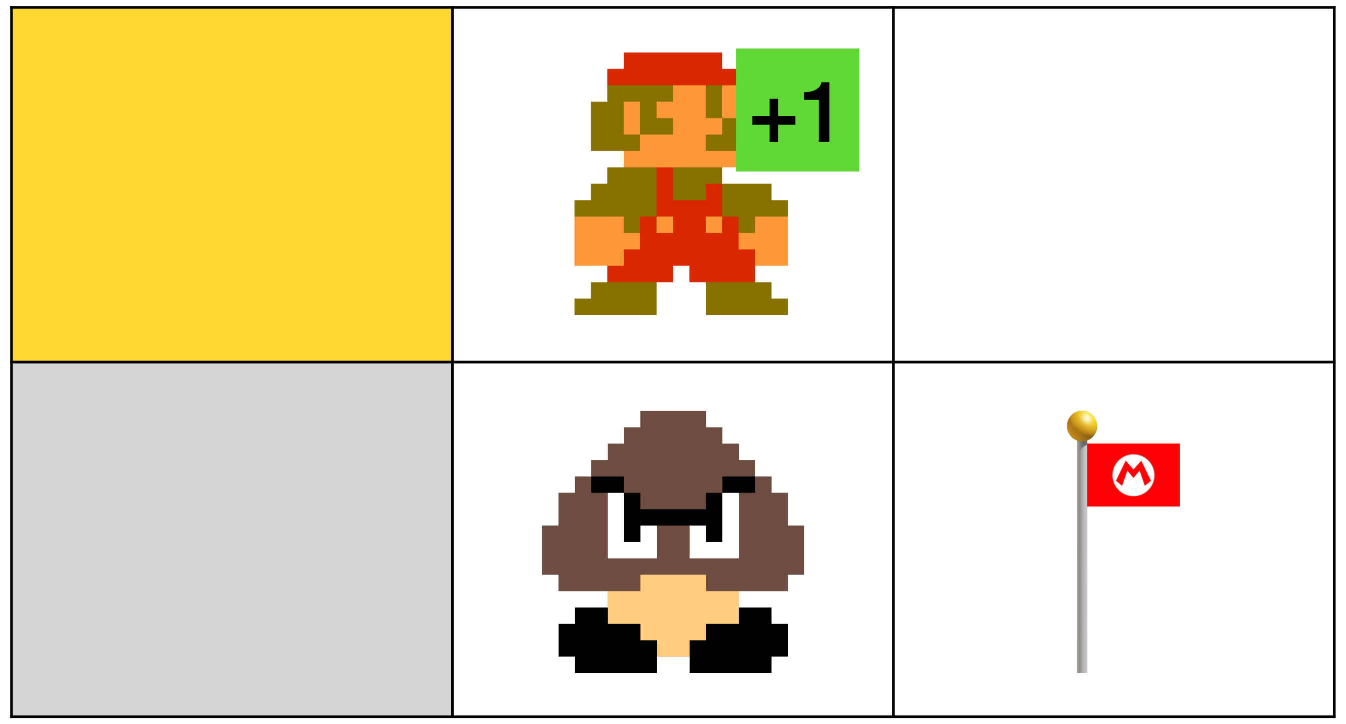
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \; max_{a'} \; Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0.1 + 0.1*[1 + 0.99*0 - 0] = 0.2

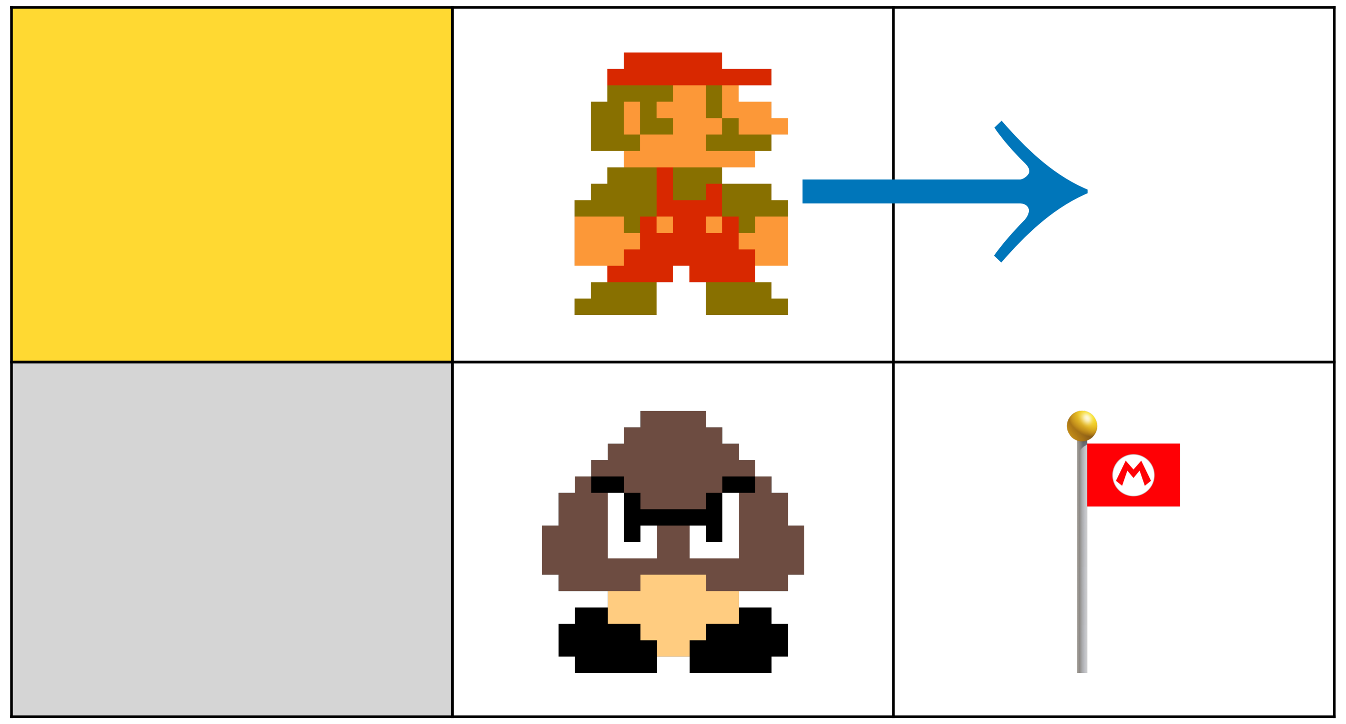
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



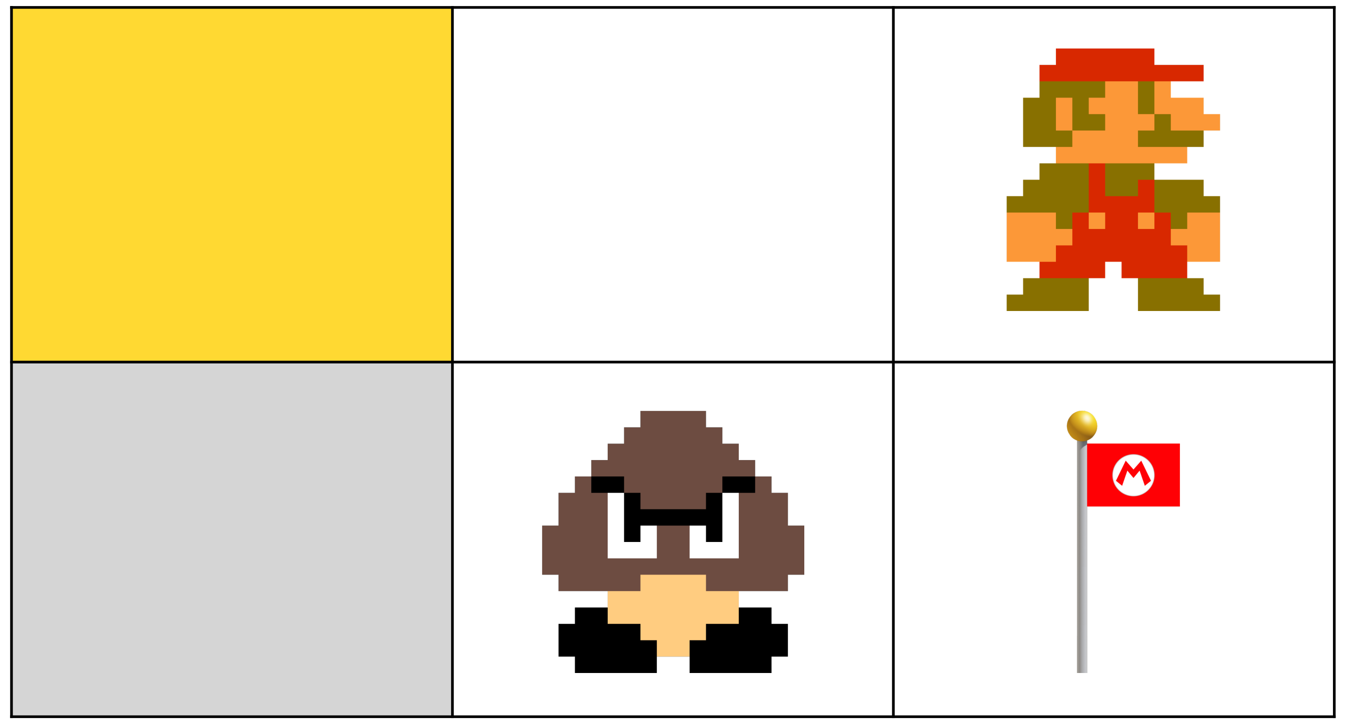
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

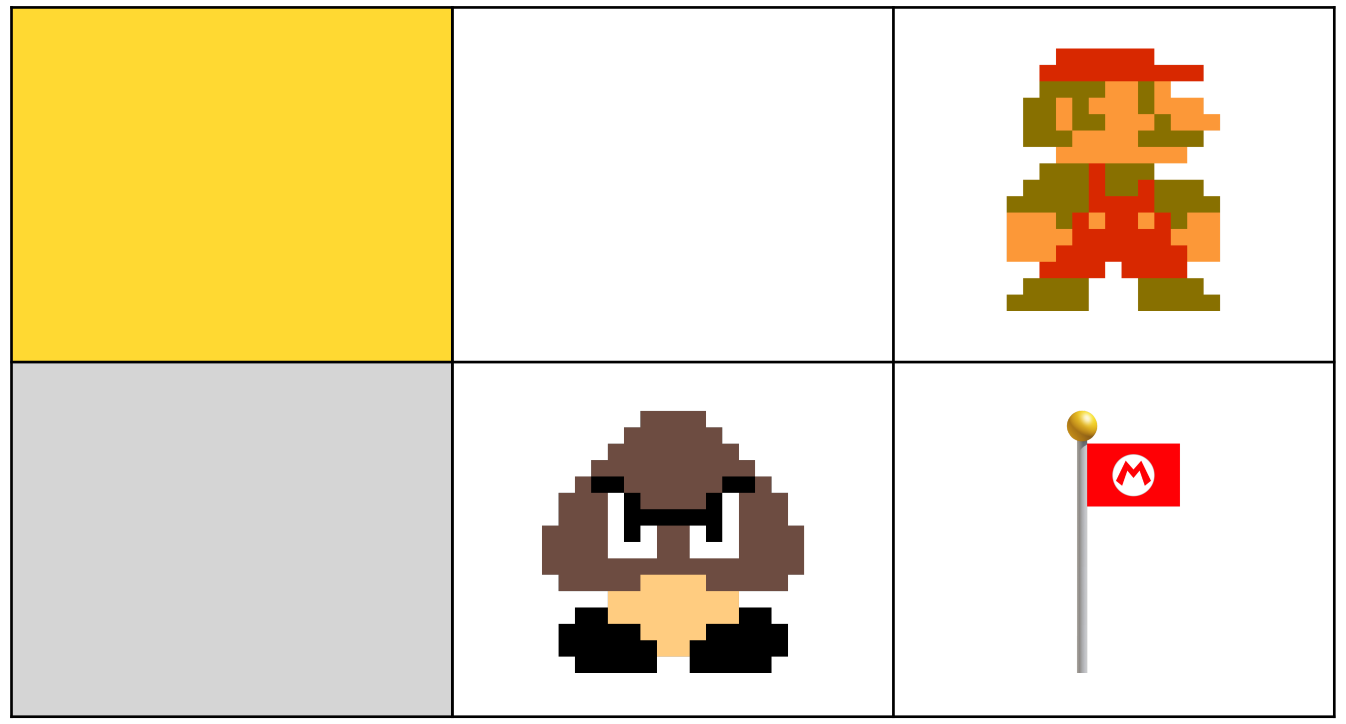
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(, →) = 0 + 0.1*[0 + 0.99*0 - 0] = 0

Take action A, observe R, S'



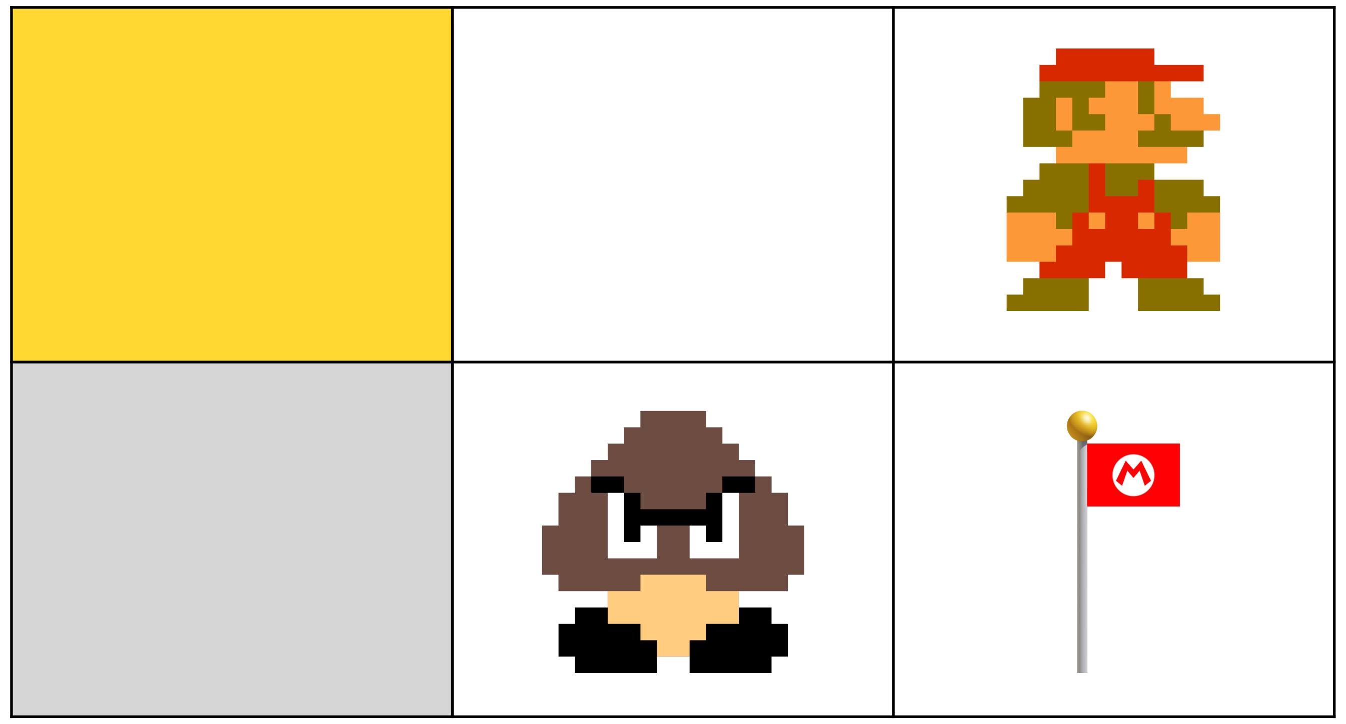
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q(, →) = 0 + 0.1*[0 + 0.99*0 - 0] = 0

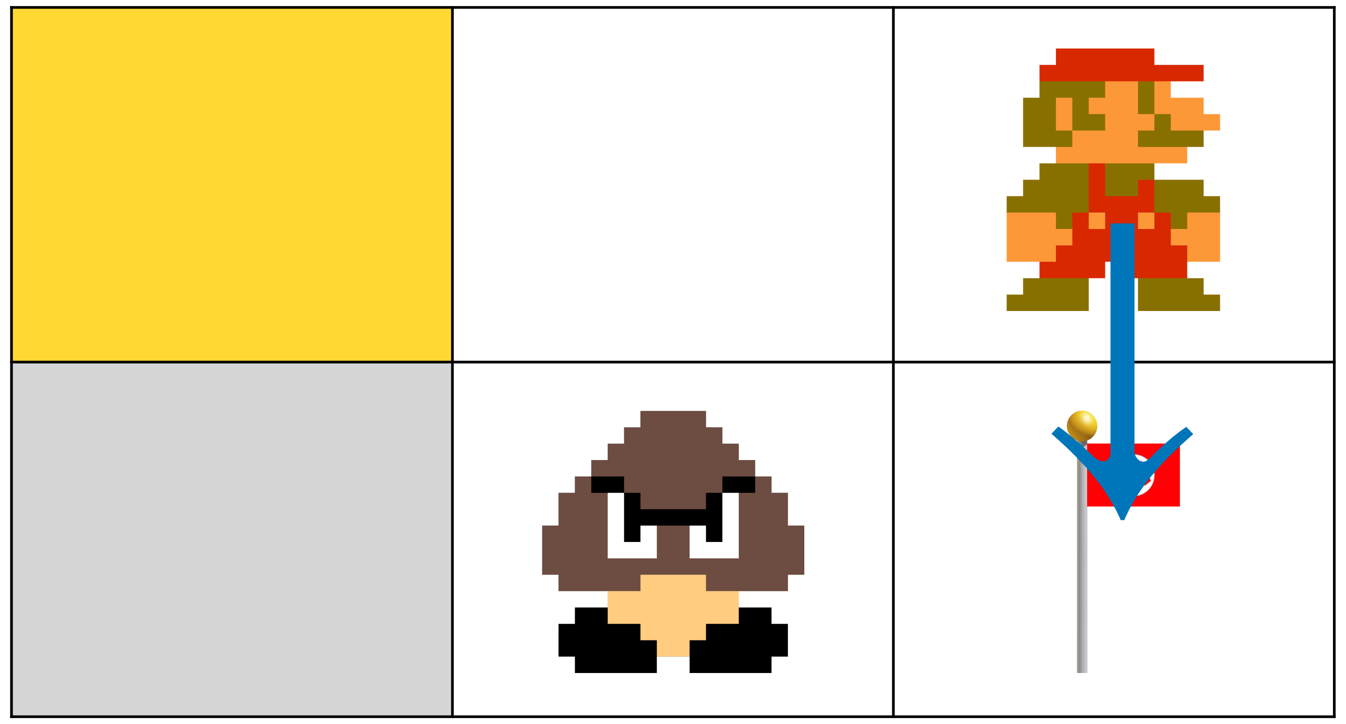
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



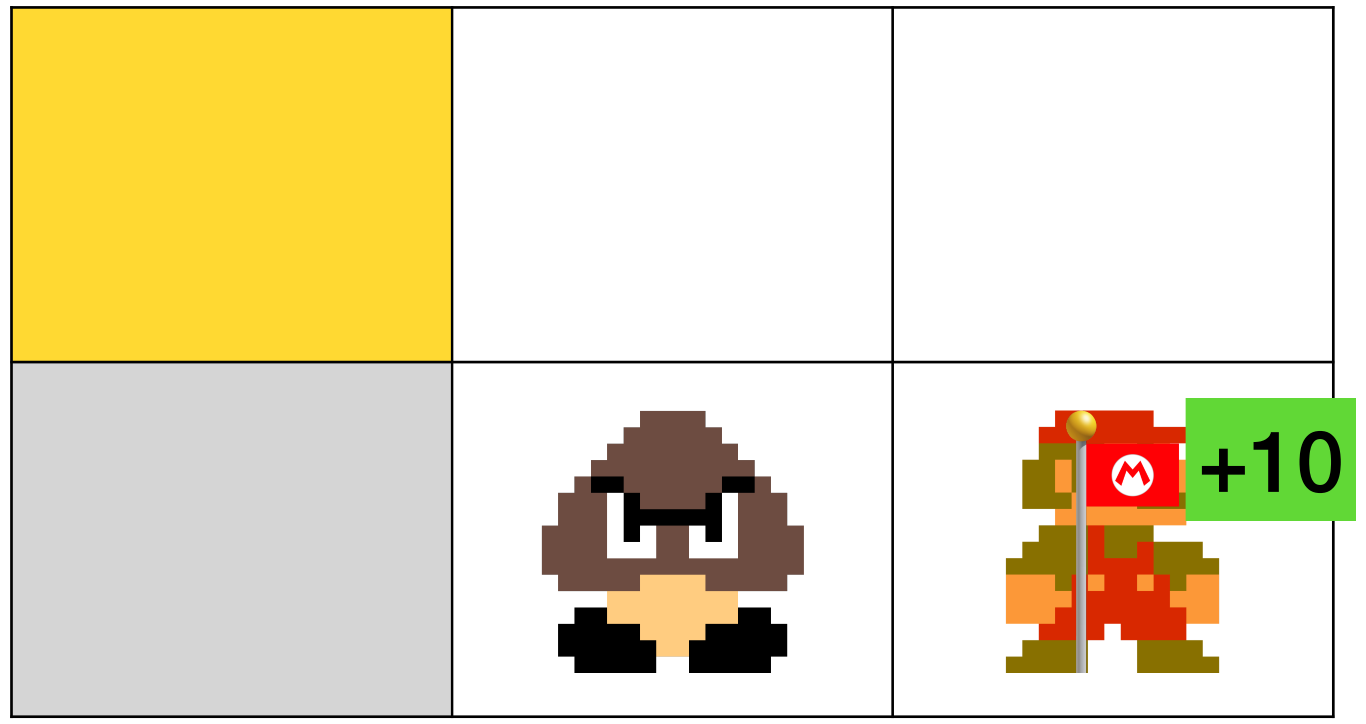
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

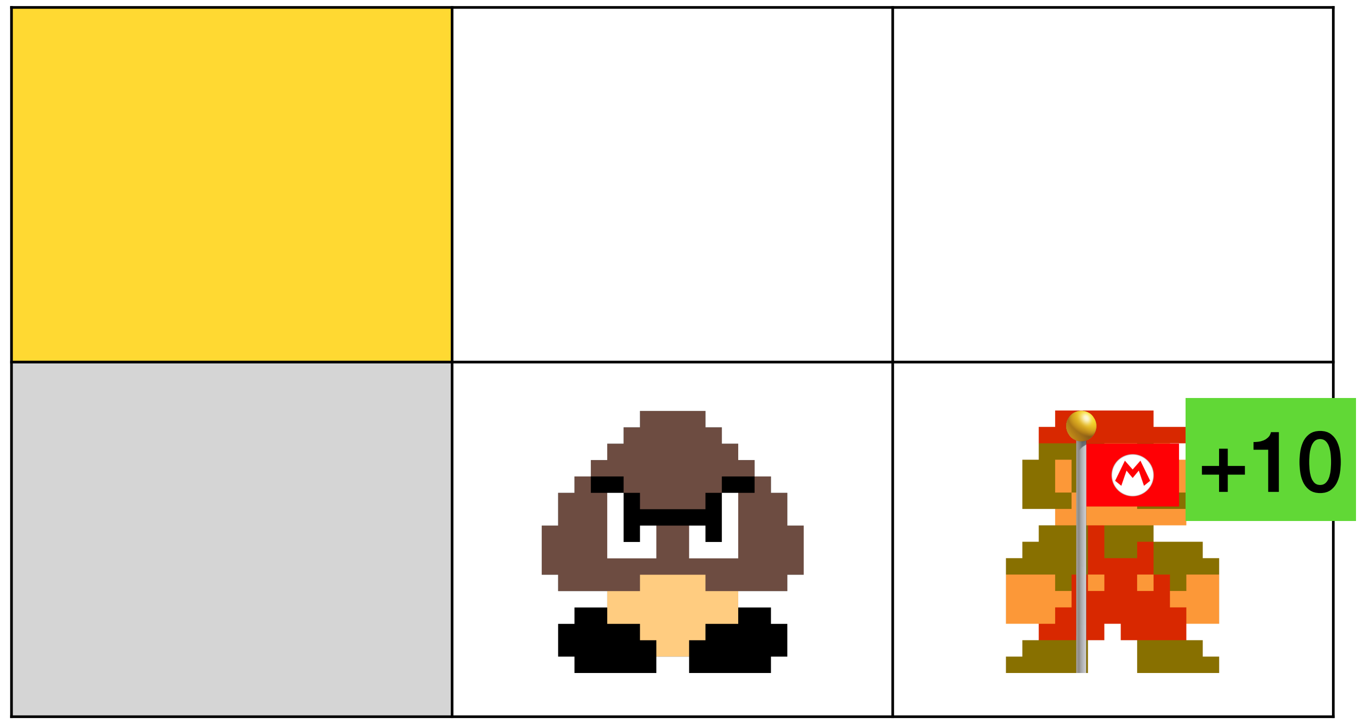
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \; max_{a'} \; Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(, ↓) = 0 + 0.1*[10 + 0.99*0 - 0] = 1

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

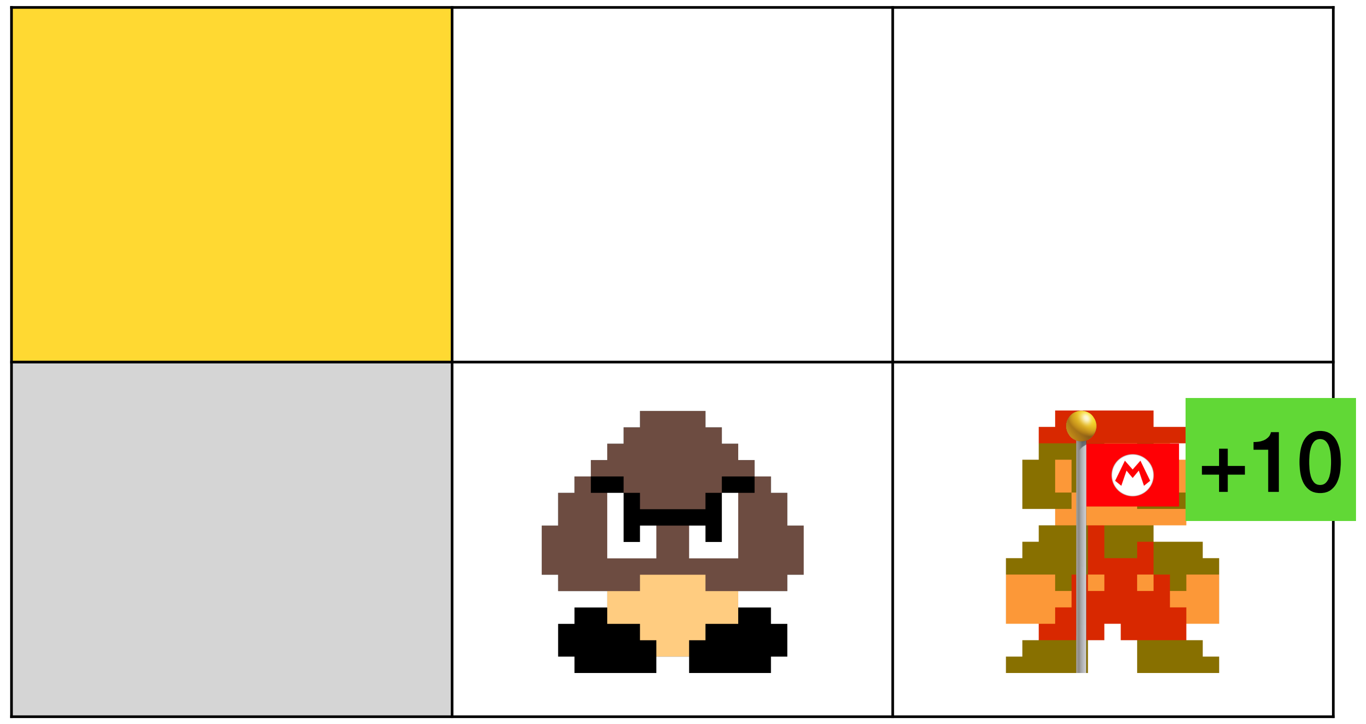
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	1
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \; max_{a'} \; Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(, ↓) = 0 + 0.1*[10 + 0.99*0 - 0] = 1

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

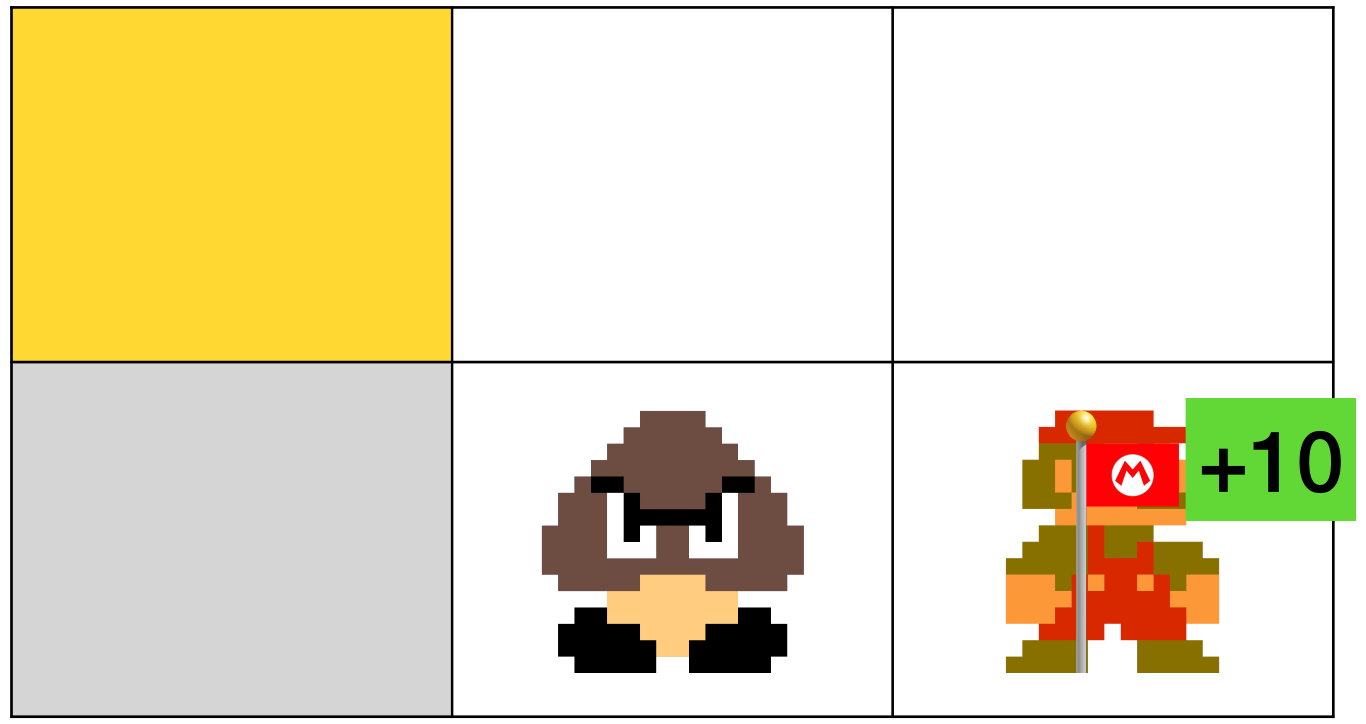
	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.2	0	0
	0	0	0	-1
	0	0	0	1
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\ max_{a'}\ Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(, ↓) = 0 + 0.1*[10 + 0.99*0 - 0] = 1

Take action A, observe R, S'



Episode won! Terminal state

Why ε-greedy?

Why ε-greedy?

Exploration-exploitation tradeoff

Why ε-greedy?

Exploration-exploitation tradeoff

Take greedy actions with probability $1 - \varepsilon$

Why ε-greedy?

Exploration-exploitation tradeoff

Take greedy actions with probability $1 - \varepsilon$

Exploit what the agent already knows

Why ε-greedy?

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Exploit what the agent already knows

Explore with probability ε

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Take greedy actions with probability $1 - \varepsilon$

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Take random actions to visit new states you would not visit otherwise

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Exploration-exploitation tradeoff

Take greedy actions with probability $1 - \varepsilon$

Exploit what the agent already knows

Explore with probability ε

Take random actions to visit new states you would not visit otherwise

Decrease ε with time

Why ε-greedy?

Exploration-exploitation tradeoff

Take greedy actions with probability $1-\varepsilon$

Exploit what the agent already knows

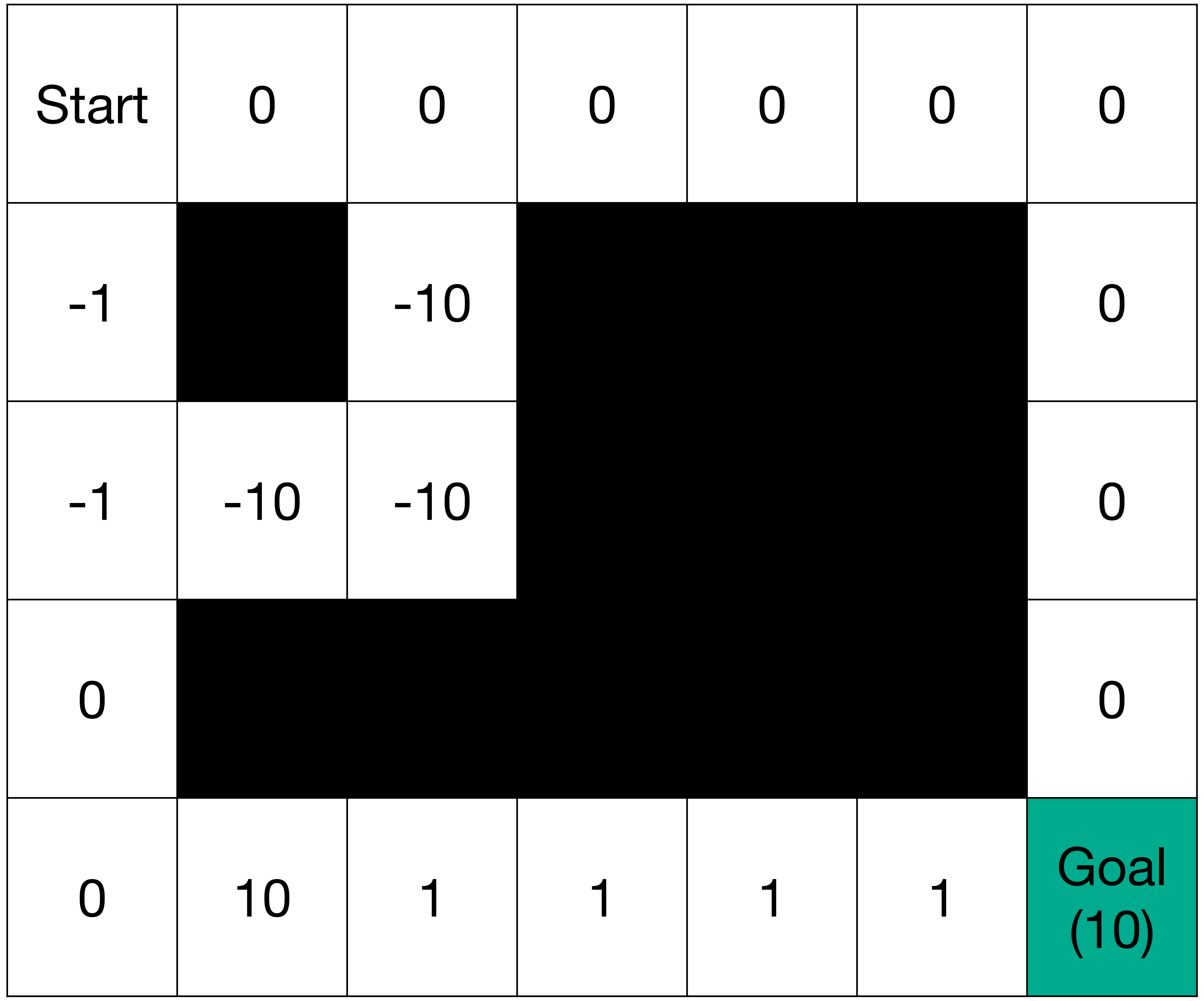
Explore with probability ε

Take random actions to visit new states you would not visit otherwise

Decrease ε with time

$0 \le \varepsilon \le 1$

Why ε-greedy?



Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

If we take only greedy actions…

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

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$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

If we take only greedy actions…

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

If we take only greedy actions…

We end up with a suboptimal solution

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma \; max_{a'} \; Q(s',a') - Q(s,a)]$$

If we take only greedy actions…

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

We end up with a suboptimal solution

G = 10

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma \, max_{a'} \, Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

If we explore…

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

If we explore…

We might discover more valuable states

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

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We might discover more valuable states

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma \; max_{a'} \; Q(s',a') - Q(s,a)]$$

If we explore…

We might discover more valuable states

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$

If we explore…

Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

We might discover more valuable states

That lead to higher returns

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma \, max_{a'} \, Q(s',a') - Q(s,a)]$$

If we explore…

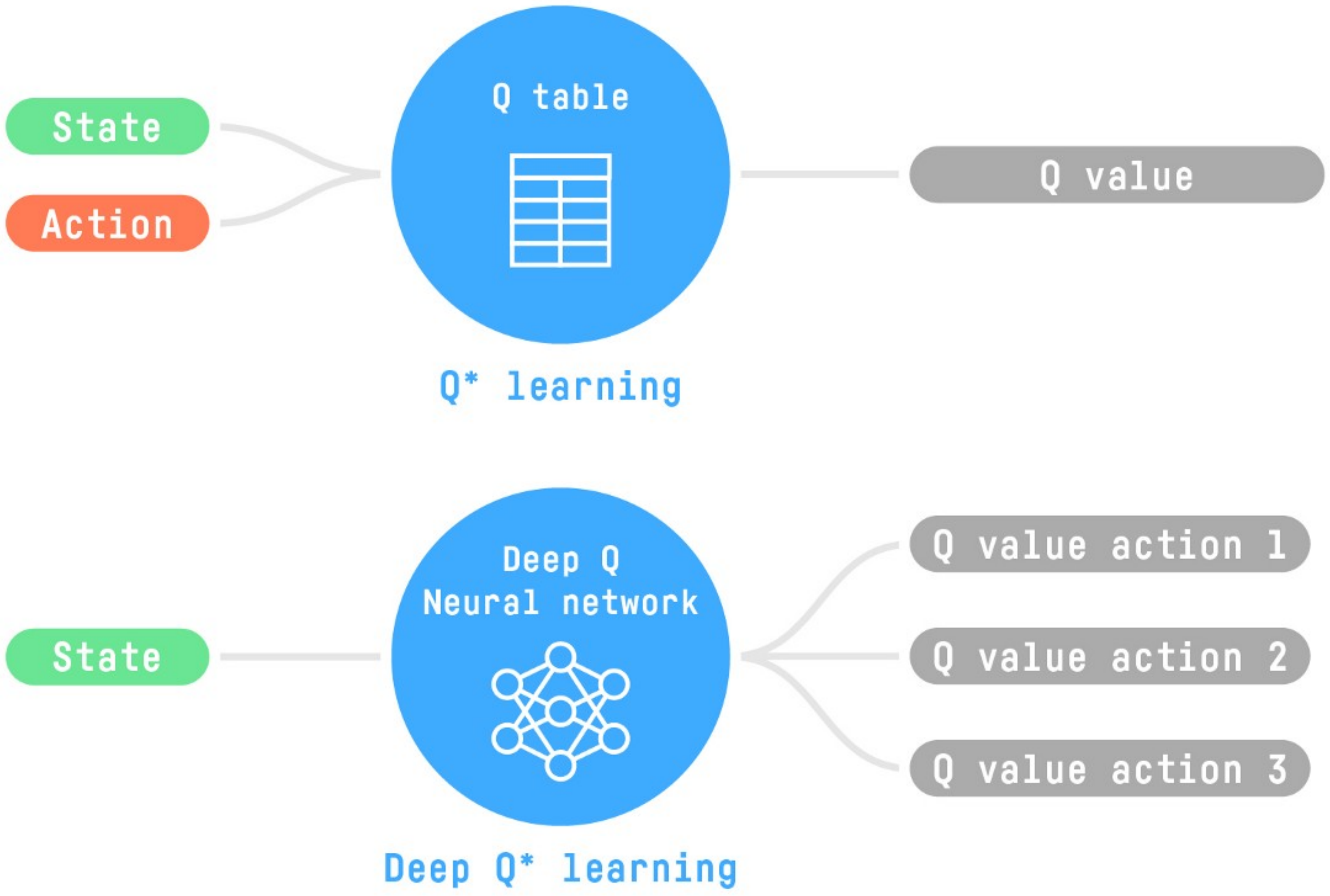
Start	0	0	0	0	0	0
-1		-10				0
-1	-10	-10				0
0						0
0	10	1	1	1	1	Goal (10)

We might discover more valuable states

That lead to higher returns

G = 22

Deep Q-Learning


State
Action
Q table
Q value
Q* learning
State
Deep Q
Neural network
Q value action 1
Q value action 2
Q value action 3
Deep Q* learning

Deep Q-Learning

Deep Q-Learning

Q-Learning works well for small state spaces

Deep Q-Learning

Q-Learning works well for small state spaces

Atari games, however, have observation space of shape (210, 160, 3), values in [0, 255]

Deep Q-Learning

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This gives us $256^{210 \times 160 \times 3} = 256^{100800}$ possible observations

Deep Q-Learning

Q-Learning works well for small state spaces

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Approximate Q-values using a neural network

Deep Q-Learning

Q-Learning works well for small state spaces

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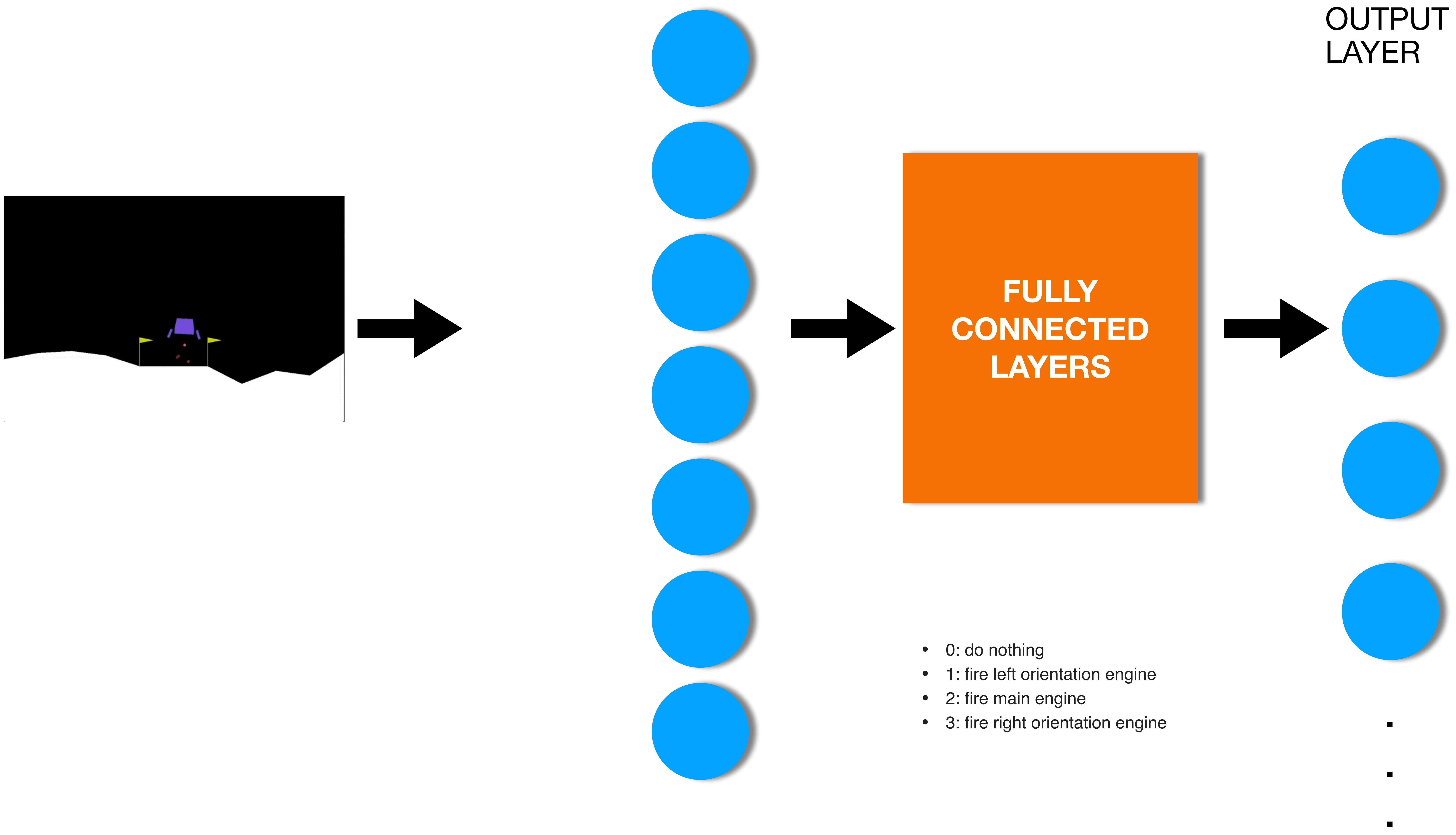
Approximate Q-values using a neural network

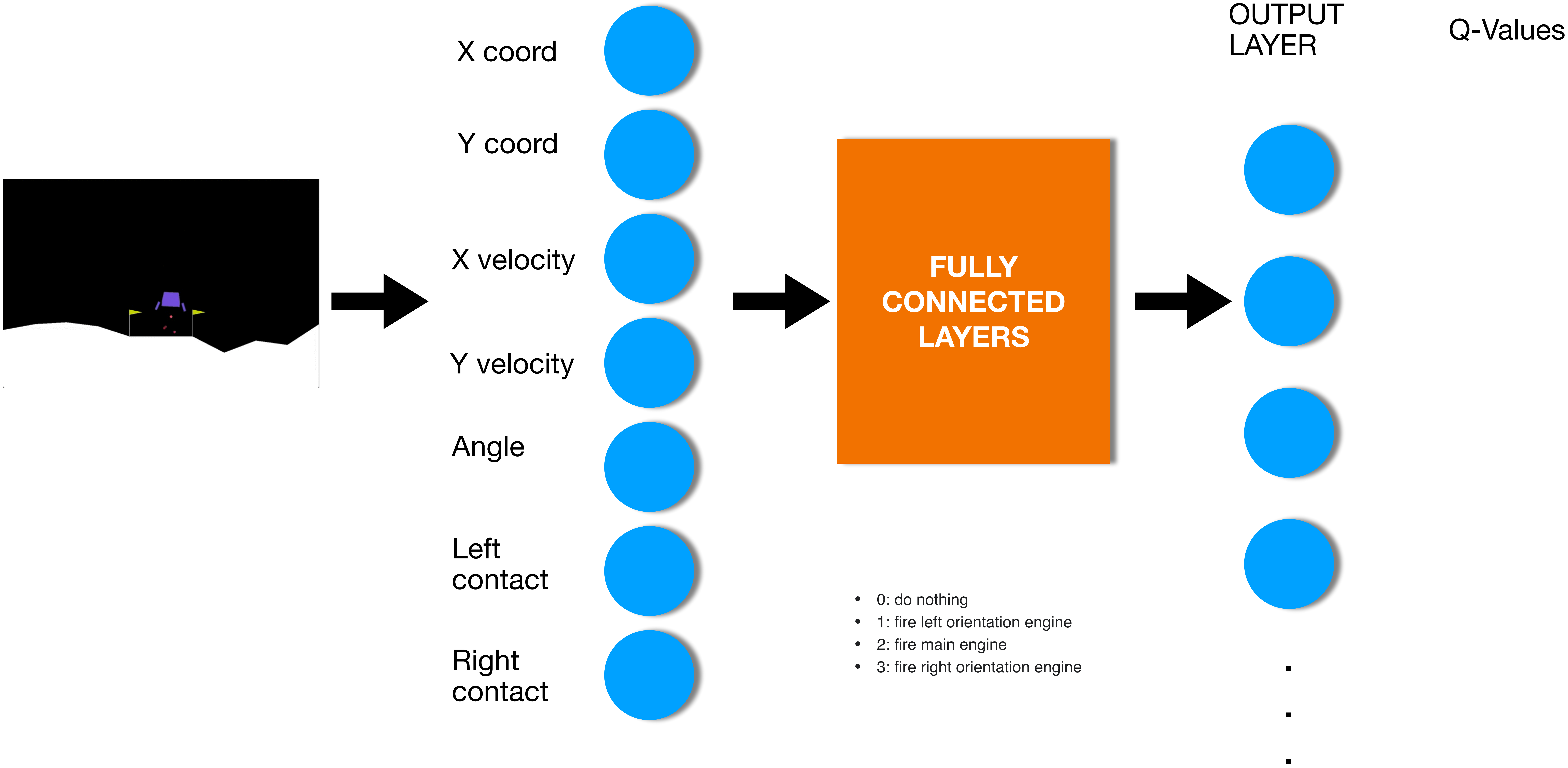
Parametrized Q-function $Q_\theta(s, a)$

Deep Q-Learning

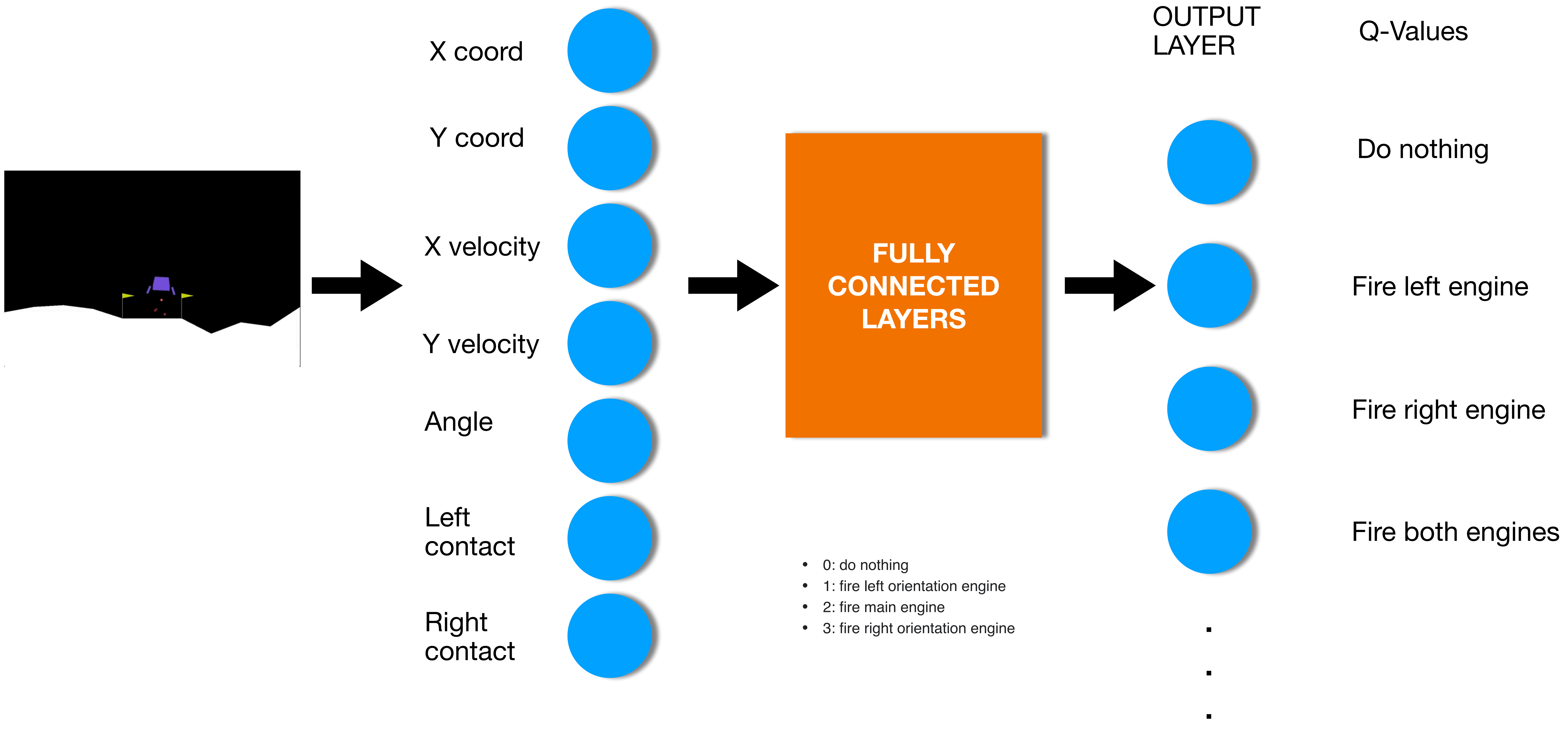


- 0: do nothing
- 1: fire left orientation engine
- 2: fire main engine
- 3: fire right orientation engine

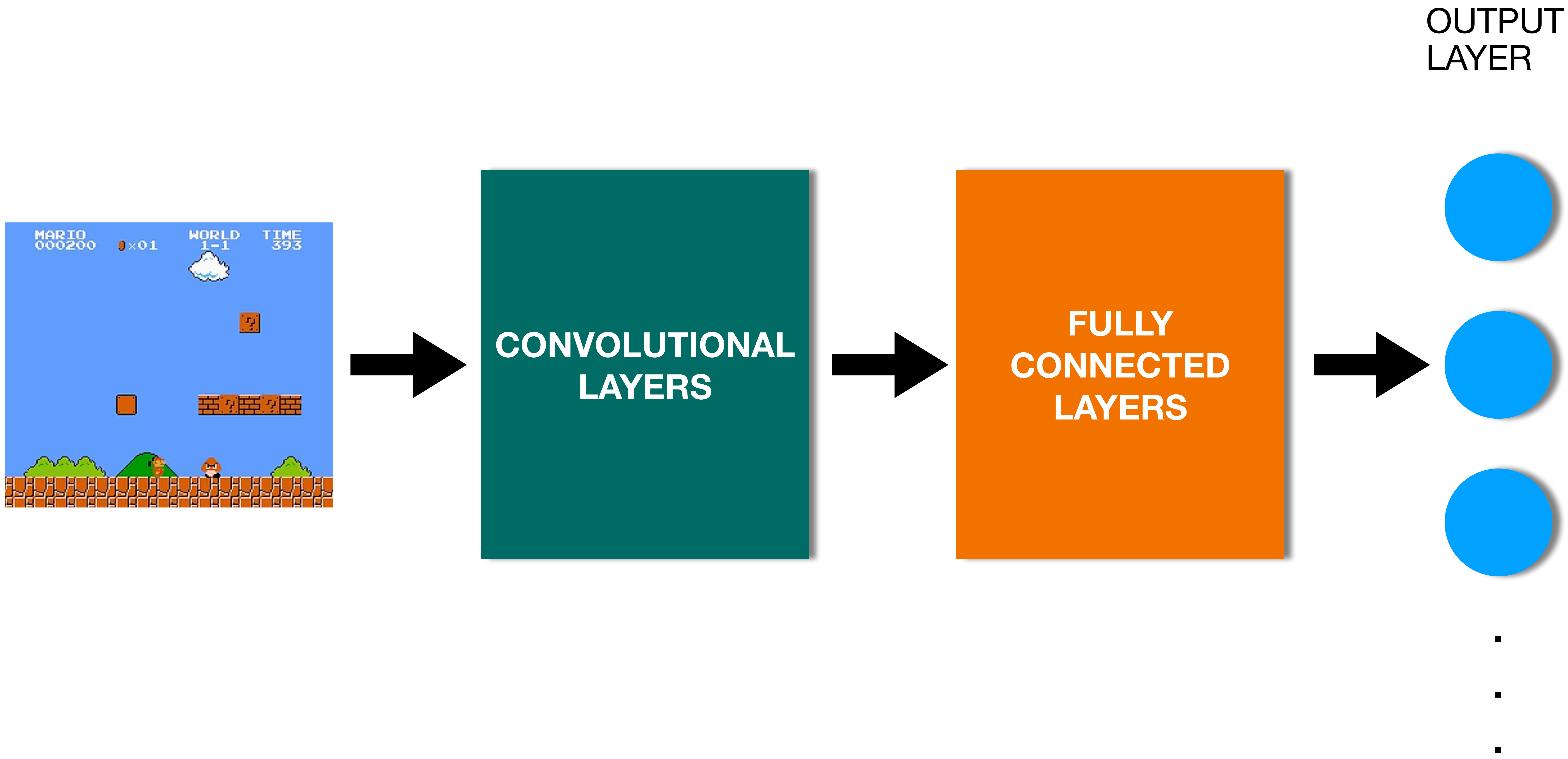
Deep Q-Learning



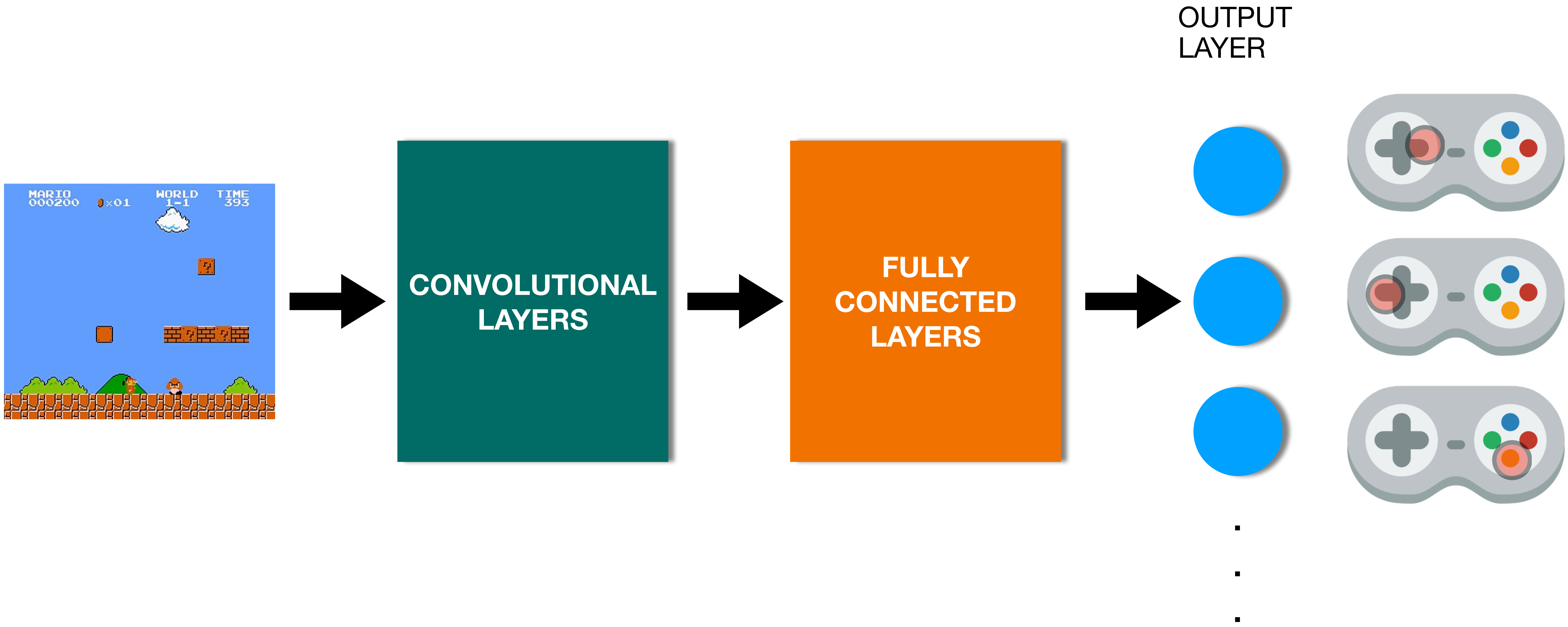
Deep Q-Learning



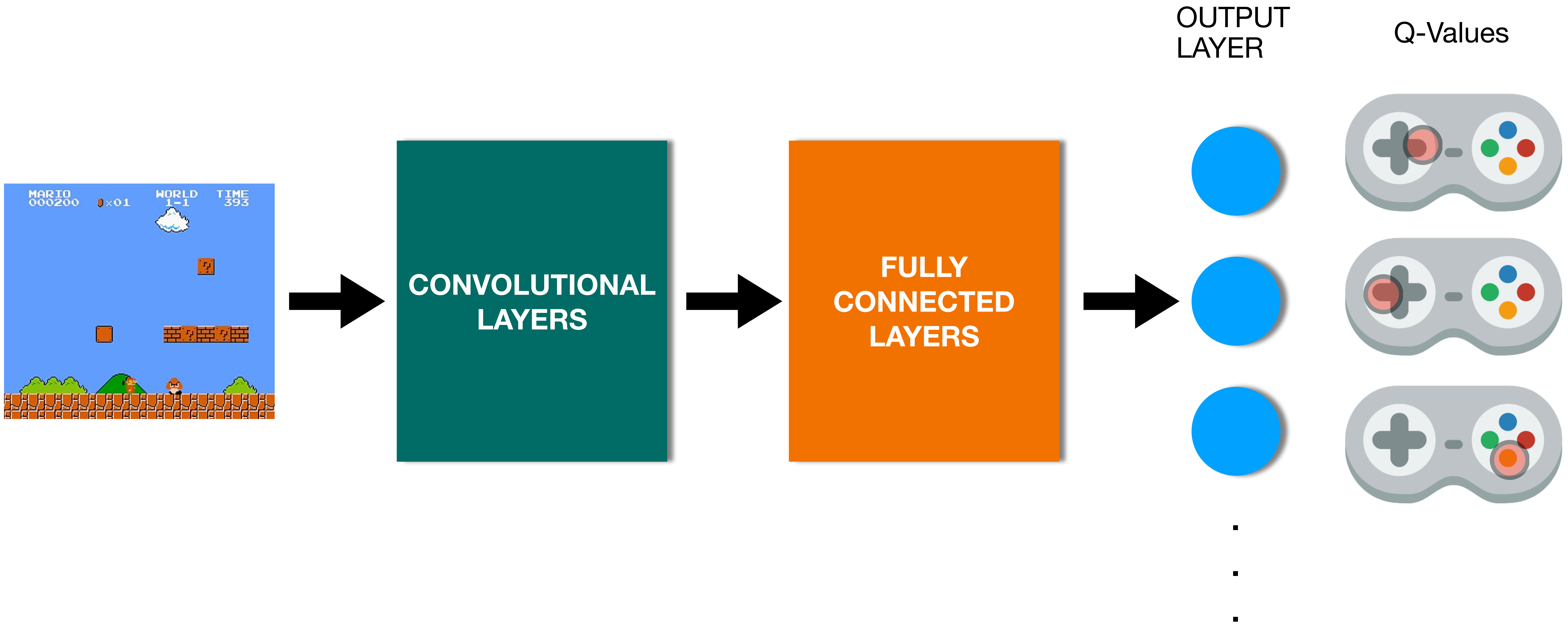
Deep Q-Learning



Deep Q-Learning



Deep Q-Learning



Deep Q-Learning algorithm

For episode $= 1, M$ **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1$,T **do**

With probability ε select a random action a_t

otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

Sample random minibatch of transitions $\left(\phi_j, a_j, r_j, \phi_{j+1}\right)$ from D

$$\text{Set } y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{otherwise} \end{cases}$$

Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

End For

End For

Deep Q-Learning algorithm

ϕ represents the NN

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ϕ represents the NN

Sampling (interaction with the environment)

Deep Q-Learning algorithm

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ϕ represents the NN

Sampling (interaction with the environment)

Training

Deep Q-Learning algorithm

Tabular Q-Learning

Deep Q-Learning

Deep Q-Learning algorithm

Tabular Q-Learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\, max_{a'}\, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Deep Q-Learning

Deep Q-Learning algorithm

Tabular Q-Learning

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Deep Q-Learning

$$y_j = \quad r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^{-}\right)$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma\ max_{a'}\ Q(S_{t+1}, a') - Q(S_t, A_t))$$

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Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \alpha(R_{t+1} + \gamma\ max_{a'}\ Q(S_{t+1}, a') - Q(S_t, A_t))$$

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Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}}(\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma\ max_{a'}\ Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}} - Q(S_t, A_t))$$

Deep Q-Learning

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^{-}\right)$$

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Deep Q-Learning

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Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}} (\underbrace{\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma \ max_{a'} \ Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}}}_{\text{TD-target}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}})$$

Deep Q-Learning

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

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Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}} \underbrace{(\underbrace{\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma \; max_{a'} \, Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}}}_{\text{TD-target}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}})}_{\text{TD-error}}$$

Deep Q-Learning

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}}(\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma\ max_{a'}\ Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}})$$

New Q-value

Former Q-value

step size

Reward

Discounted Estimate (optimal Q-value of next state)

Former Q-value

TD-target

TD-error

Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}}(\underbrace{\underbrace{\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma\ max_{a'}\ Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}}}_{\text{TD-target}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}}}_{\text{TD-error}})$$

Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}}\Bigg(\overbrace{\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma\ max_{a'}\ Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}}}^{\text{TD-target}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}}\Bigg)$$

TD-error

Deep Q-Learning

Q-Target

$$y_j = \underbrace{r_j}_{\text{Reward}} + \gamma\ \max_{a'}\ \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

$$\underbrace{Q(S_t, A_t)}_{\text{New Q-value}} \leftarrow \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}} + \underbrace{\alpha}_{\text{step size}}(\underbrace{R_{t+1}}_{\text{Reward}} + \underbrace{\gamma\, max_{a'}\, Q(S_{t+1}, a')}_{\text{Discounted Estimate (optimal Q-value of next state)}} - \underbrace{Q(S_t, A_t)}_{\text{Former Q-value}})$$

TD-target: $R_{t+1} + \gamma\, max_{a'}\, Q(S_{t+1}, a')$

TD-error: $R_{t+1} + \gamma\, max_{a'}\, Q(S_{t+1}, a') - Q(S_t, A_t)$

Deep Q-Learning

Q-Target

$$y_j = \underbrace{r_j}_{\text{Reward}} + \gamma\, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

Discounted Estimate (optimal Q-value of next state)

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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New Q-value

Former Q-value

step size

Reward

Discounted Estimate (optimal Q-value of next state)

Former Q-value

TD-target

TD-error

Deep Q-Learning

Q-Target

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Discounted Estimate (optimal Q-value of next state)

TD-target

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Deep Q-Learning algorithm

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New Q-value

Former Q-value

step size

Reward

Discounted Estimate (optimal Q-value of next state)

Former Q-value

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Deep Q-Learning

Q-Target

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Reward

Discounted Estimate (optimal Q-value of next state)

TD-target

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

TD-error

Suggested reads

Reinforcement Learning: An Introduction (by Sutton, Barto)

http://incompleteideas.net/book/the-book-2nd.html

Human-level control through deep reinforcement learning (Mnih et al., 2015)

https://www.nature.com/articles/nature14236

Deep Reinforcement Learning with Double Q-learning

https://arxiv.org/abs/1509.06461

To the notebook!