Deep Learning & Applied AI

Adversarial learning

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SJ7ELPG9

Generative adversarial networks (GAN)

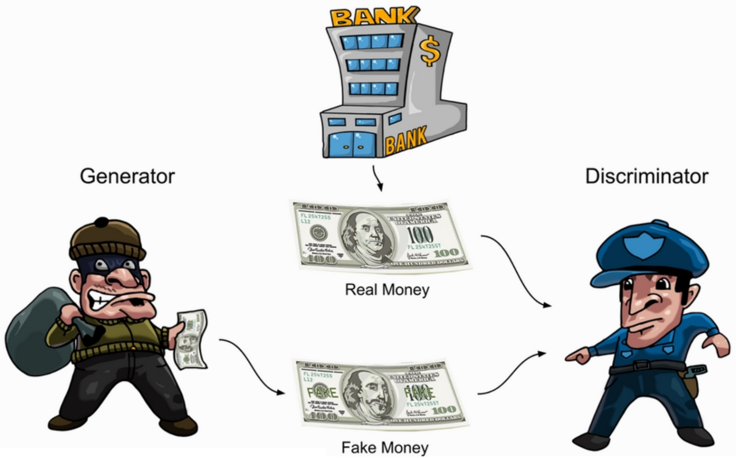

Figure: Rowel Atienza

Generative adversarial networks (GAN)

Is this idea mathematically grounded?

Generative adversarial networks (GAN)

- $\mathbf{x}$: sample from the real distribution
- $\mathbf{x}' = D_{\boldsymbol{\gamma}}(\mathbf{z})$: generated sample

A good discriminator should yield $\Delta_{\boldsymbol{\delta}}(\mathbf{x}) \approx 1$ and $\Delta_{\boldsymbol{\delta}}(\mathbf{x}') \approx 0$

Goodfellow et al, "Generative adversarial networks", NIPS 2014

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We train a classifier to distinguish generated from real:

$$\max_{\delta} \; \mathbb{E}_{\mathbf{x}} \log \Delta_\delta(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{z}} \log(1 - \Delta_\delta(D_\gamma(\mathbf{z})))$$

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In contrast, the generator tries to minimize the score:

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The generator competes against the adversarial discriminator and tries to minimize its success rate.

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Generative adversarial networks (GAN)

Consider p_g for the fake data, and p_{real} for the real data.

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Maximize the discriminator score given a generator G:

$$J(G) = \max_{\boldsymbol{\delta}} \; \mathbb{E}_{\mathbf{x} \sim p_{\text{real}}} \log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{x} \sim p_g} \log(1 - \Delta_{\boldsymbol{\delta}}(\mathbf{x}))$$

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&= \max_{\boldsymbol{\delta}} \; \int \left[\log \Delta_{\boldsymbol{\delta}}(\mathbf{x}) p_{\mathsf{real}}(\mathbf{x}) + \log(1 - \Delta_{\boldsymbol{\delta}}(\mathbf{x})) p_g(\mathbf{x})\right] d\mathbf{x}
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For any given $\mathbf{x}$, we want to maximize $\Delta_{\boldsymbol{\delta}}(\mathbf{x}) = a$; let's rename for simplicity $p_{\text{real}}(\mathbf{x}) \equiv p$ and $p_g(\mathbf{x}) \equiv q$, we get to:

$$\max_a \; p \log a + q \log(1-a)$$

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This is maximized when the derivative w.r.t. a is zero:

$$\frac{p}{a} - \frac{q}{1-a} = 0$$

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This is maximized when the derivative w.r.t. a is zero:

$$a = \frac{p}{p+q}$$

Goodfellow et al, "Generative adversarial networks", NIPS 2014

Generative adversarial networks (GAN)

Optimal discriminator in closed form:

$$a = \frac{p}{p + q}$$

Goodfellow et al, "Generative adversarial networks", NIPS 2014

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Optimal discriminator in closed form:

$$\Delta_\delta(\mathbf{x}) = \frac{p_{\mathtt{real}}(\mathbf{x})}{p_{\mathtt{real}}(\mathbf{x}) + p_g(\mathbf{x})}$$

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Plugging it back into the main functional:

$$J(G) = \max_{\delta} \; \mathbb{E}_{\mathbf{x} \sim p_{\mathrm{real}}} \log \Delta_{\delta}(\mathbf{x}) \; + \; \mathbb{E}_{\mathbf{x} \sim p_g} \log(1 - \Delta_{\delta}(\mathbf{x}))$$

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Let us define the distribution $\rho = \frac{1}{2}p_{\text{real}} + \frac{1}{2}p_g$. We get:

$$J(G) = \mathbb{E}_{\mathbf{x}\sim p_{\text{real}}} \log \frac{p_{\text{real}}(\mathbf{x})}{2\rho(\mathbf{x})} + \mathbb{E}_{\mathbf{x}\sim p_g} \log \frac{p_g(\mathbf{x})}{2\rho(\mathbf{x})}$$

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Goodfellow et al, "Generative adversarial networks", NIPS 2014

Generative adversarial networks (GAN)

Therefore, the optimal GAN generator is found by minimizing:

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Therefore, the optimal GAN generator is found by minimizing:

$$\min_{p_g} \underbrace{KL(p_{\text{real}} \| \rho) + KL(p_g \| \rho)}_{\substack{2\times\text{Jensen-Shannon divergence} \\ \text{between } p_{\text{real}} \text{ and } p_g}}$$

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Therefore, the optimal GAN generator is found by minimizing:

$$\min_{p_g} \underbrace{KL(p_{\text{real}} \| \rho) + KL(p_g \| \rho)}_{\approx JS(p_{\text{real}} \| p_g)}$$

Property: $p_{\text{real}} = p_g \;\Leftrightarrow\; JS(p_{\text{real}} \| p_g) = 0$

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With GANs, the globally optimal generator has a data distribution equal to the real distribution of the data.

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Adversarial training

The generated data samples used for training are adversarial examples.

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Adversarial examples can be used maliciously.

"speed limit 50mph"

Adversarial attacks

Szegedy et al, "Intriguing properties of neural networks", 2013

Adversarial attacks

"truck"

"ostrich"

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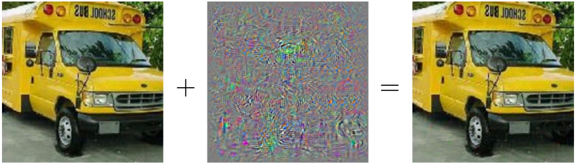
Adversarial attacks



"truck" perturbation "ostrich"

The perturbation can be explicitly optimized for.

Adversarial attacks can cause a system to take unwanted actions.

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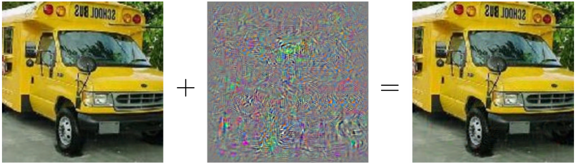

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How to construct undetectable adversarial examples?

Szegedy et al, "Intriguing properties of neural networks", 2013

Types of attack

Distinction based on the amount of information available to the attacker:

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- Black-box attack:

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- Gray-box attack:

  Access to partial information (only the features, architecture, etc.).

- White-box attack:

  Complete access to the network (architecture, parameters, etc.).

  You are not allowed to touch the network weights.

Targeted attacks

Given an input sample **x**, a classifier C, and a target class t, consider:

$$\begin{aligned} \min_{\mathbf{x}' \in [0,1]^n} \quad & \|\mathbf{x} - \mathbf{x}'\|_2^2 \\ \text{s.t.} \quad & C(\mathbf{x}') = t \end{aligned}$$

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$$\text{s.t. } C(\mathbf{x}') = t$$

Relax the difficult constraint to a penalty term:

$$\min_{\mathbf{x}' \in [0,1]^n} \|\mathbf{x} - \mathbf{x}'\|_2^2 + c\, L(\mathbf{x}', t)$$

where L is the cross-entropy loss.

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$c > 0$ is a trade-off parameter that is chosen as small as possible; it can be found via line search.

Szegedy et al, "Intriguing properties of neural networks", 2013

Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \; d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta})$$
$$\text{s.t. } C(\mathbf{x} + \boldsymbol{\delta}) = t$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d depends on the specific task.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

Targeted attacks

A more general approach is given by:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \; d(\mathbf{x}, \mathbf{x} + \boldsymbol{\delta})$$
$$\text{s.t. } f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$$

where the perturbation $\boldsymbol{\delta}$ appears explicitly, and d depends on the specific task.

f is such that $C(\mathbf{x} + \boldsymbol{\delta}) = t$ if and only if $f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$.

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f is such that $C(\mathbf{x} + \boldsymbol{\delta}) = t$ if and only if $f(\mathbf{x} + \boldsymbol{\delta}) \leq 0$.

For example:

$$\min_{\boldsymbol{\delta} \in [0,1]^n} \|\boldsymbol{\delta}\|_p + c\, \left(\max\{F(\mathbf{x} + \boldsymbol{\delta})_i \,:\, i \neq t\} - F(\mathbf{x} + \boldsymbol{\delta})_t\right)^+$$

where $F : \mathbf{x} \mapsto [0,1]^k$ is the NN yielding a probability distribution over all k classes, and $(a)^+ = \max(a, 0)$.

Carlini and Wagner, "Towards Evaluating the Robustness of Neural Networks", 2016

Untargeted attacks

If there is no specific target, consider for a given input $\mathbf{x}$ with ground-truth label ℓ_{gt}:

$$\mathbf{x}' = \mathbf{x} + \alpha \underbrace{\operatorname{sign}\left(\nabla L(\mathbf{x}, \ell_{\text{gt}})\right)}_{\text{perturbation}}$$

which adds a perturbation maximizing the cost.

Kurakin et al, "Adversarial examples in the physical world", 2016

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which adds a perturbation maximizing the cost.

For better results, iterate:

$$\mathbf{x}'_{(i)} = \operatorname{clip}_{\epsilon}\left(\mathbf{x}'_{(i-1)} + \alpha \operatorname{sign}\left(\nabla L(\mathbf{x}'_{(i-1)}, \ell_{\text{gt}})\right)\right)$$

with $\mathbf{x}'_{(0)} = \mathbf{x}$.

The clip operation projects to an ϵ-neighborhood from $\mathbf{x}$.

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The clip operation projects to an ϵ-neighborhood from $\mathbf{x}$.

Designed to be fast: 1 iteration ≈ 1 backprop step.

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Example: Adversarial training

Using adversarial examples as training data improves the robustness of the attacked learning model:

original

target

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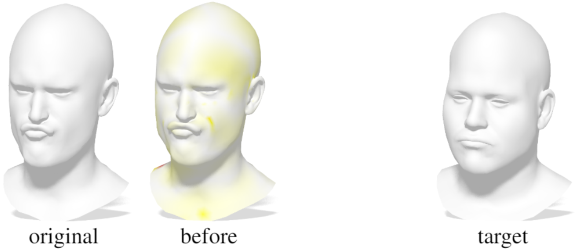

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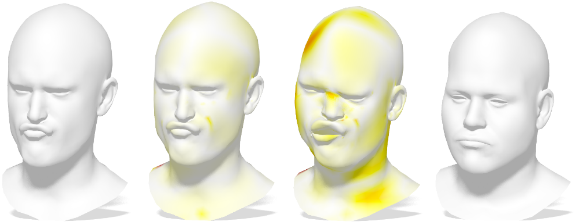
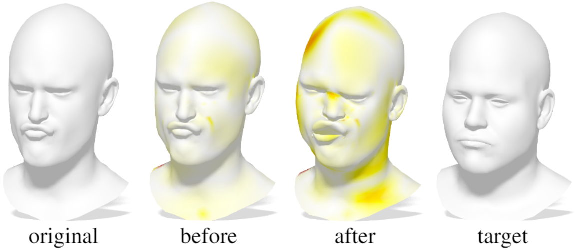
Using adversarial examples as training data improves the robustness of the attacked learning model:



original before after target

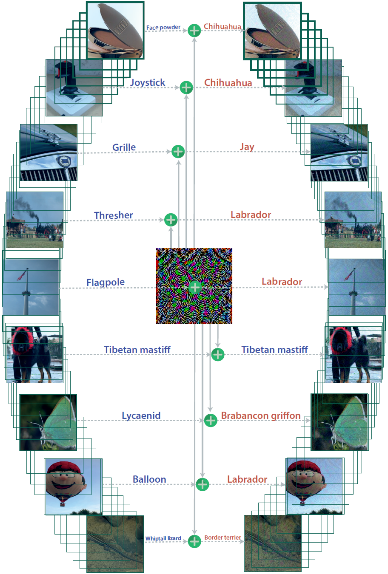
Example: Adversarial training

Using adversarial examples as training data improves the robustness of the attacked learning model:



Still, it can be proven that classifiers are always vulnerable!

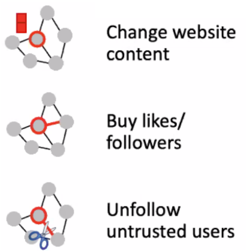
Universal perturbations

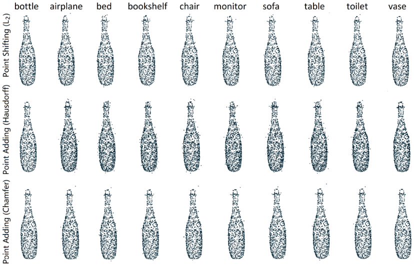


Moosavi-Dezfooli et al, "Universal adversarial perturbations", 2017

Non-Euclidean domains

Adversarial training can also be done on geometric domains.





Zügner et al, "Adversarial attacks on neural networks for graph data", 2018; Xiang et al, "Generating 3D Adversarial Point Clouds", 2018

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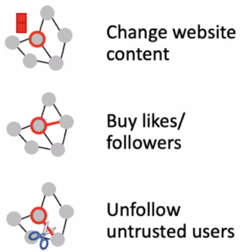
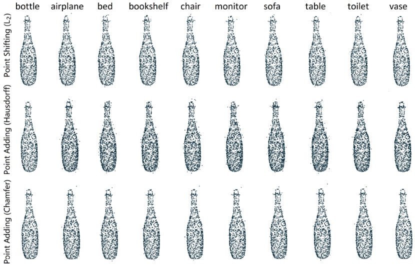
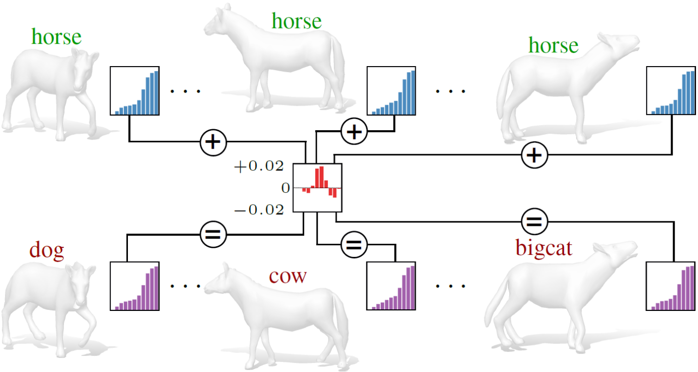




- The notion of perceptible is different than with images.
- Can alter the domain (e.g. the graph connections) rather than just the features (e.g. the values stored at the nodes).

Zügner et al, "Adversarial attacks on neural networks for graph data", 2018; Xiang et al, "Generating 3D Adversarial Point Clouds", 2018

Universal perturbations on 3D data



Rampini, Pestarini, Cosmo, Melzi, Rodolà, "Universal Spectral Adversarial Attacks for Deformable Shapes", 2021