Deep Learning & Applied AI

Data, features, and embeddings

Emanuele Rodolà
rodola@di.uniroma1.it

SAPIENZA
UNIVERSITÀ DI ROMA

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Data awareness

Machine learning involves dealing with data.

What do you do when you have a problem involving data?

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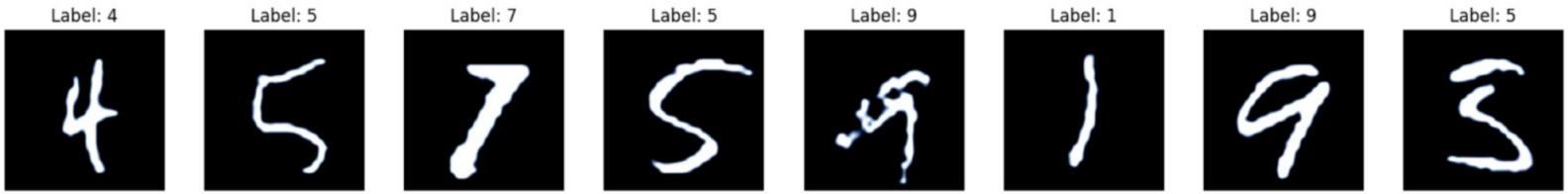
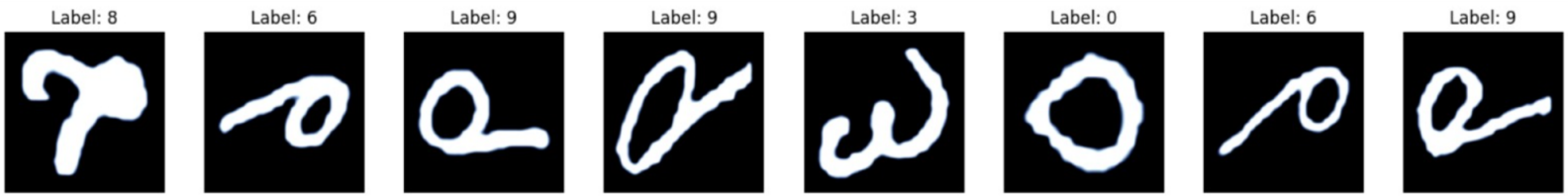
MNIST

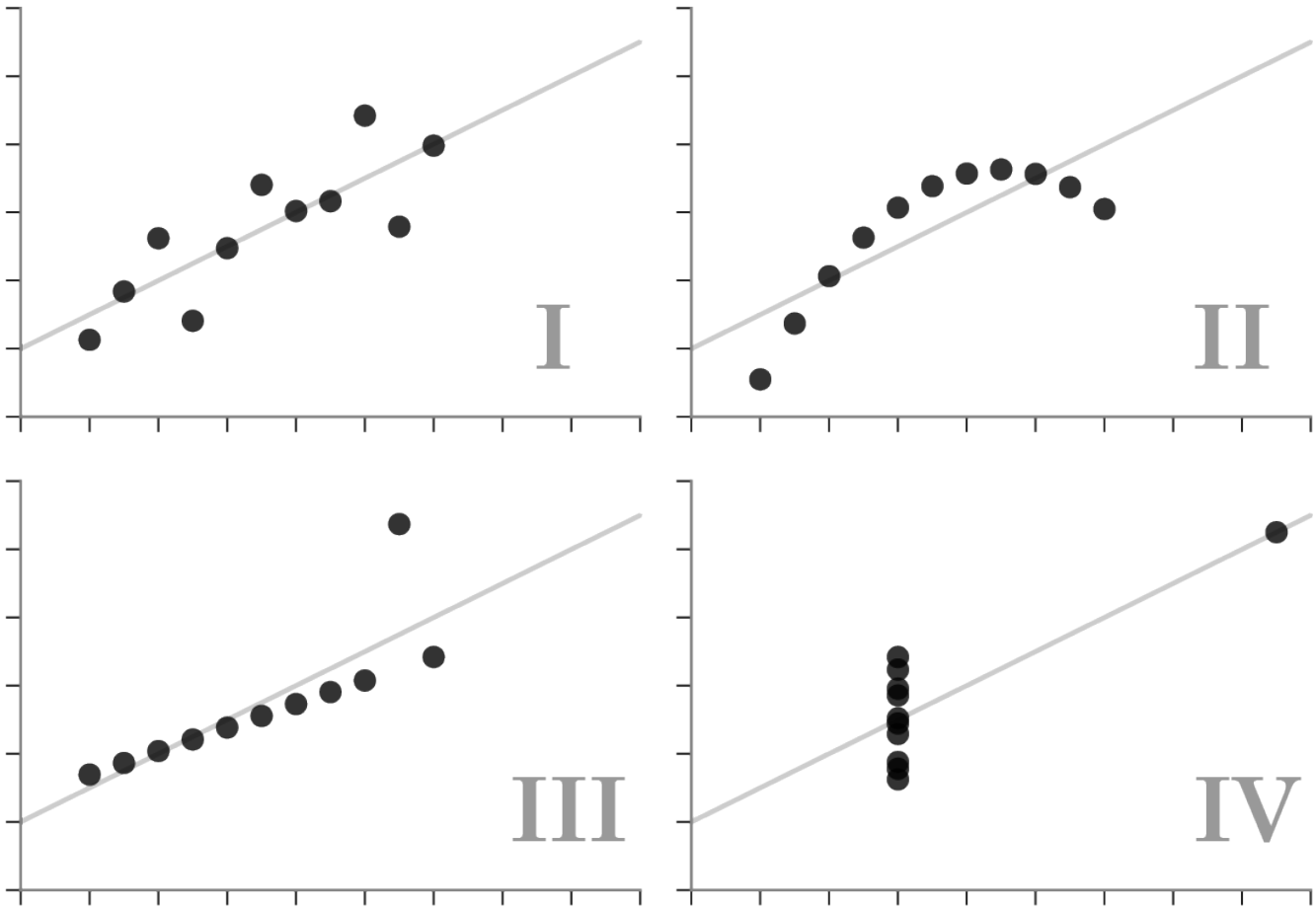


EMNIST



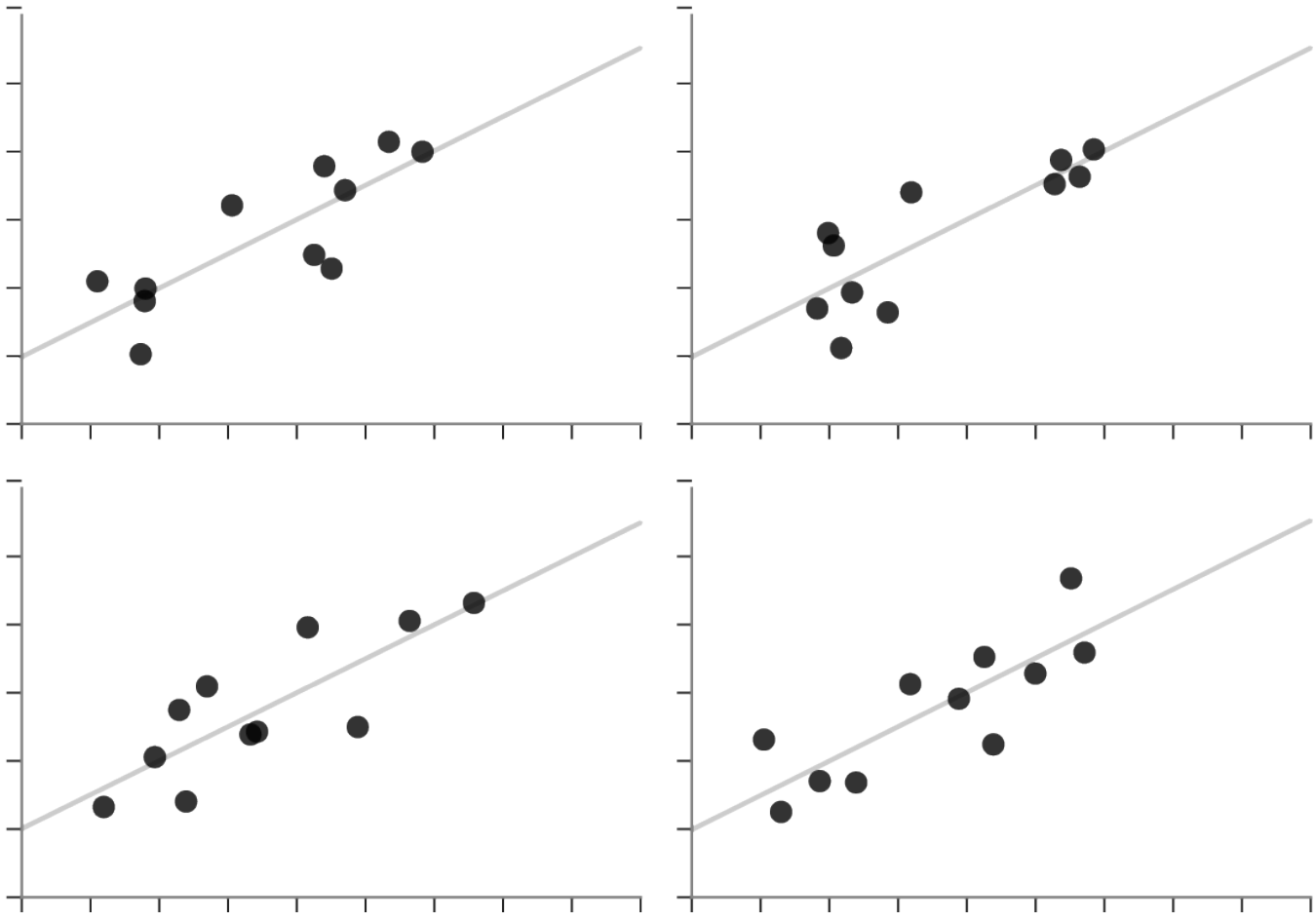
Anscombe's Quartet

Each dataset has the same summary statistics (mean, standard deviation, correlation), and the datasets are *clearly different*, and *visually distinct*.



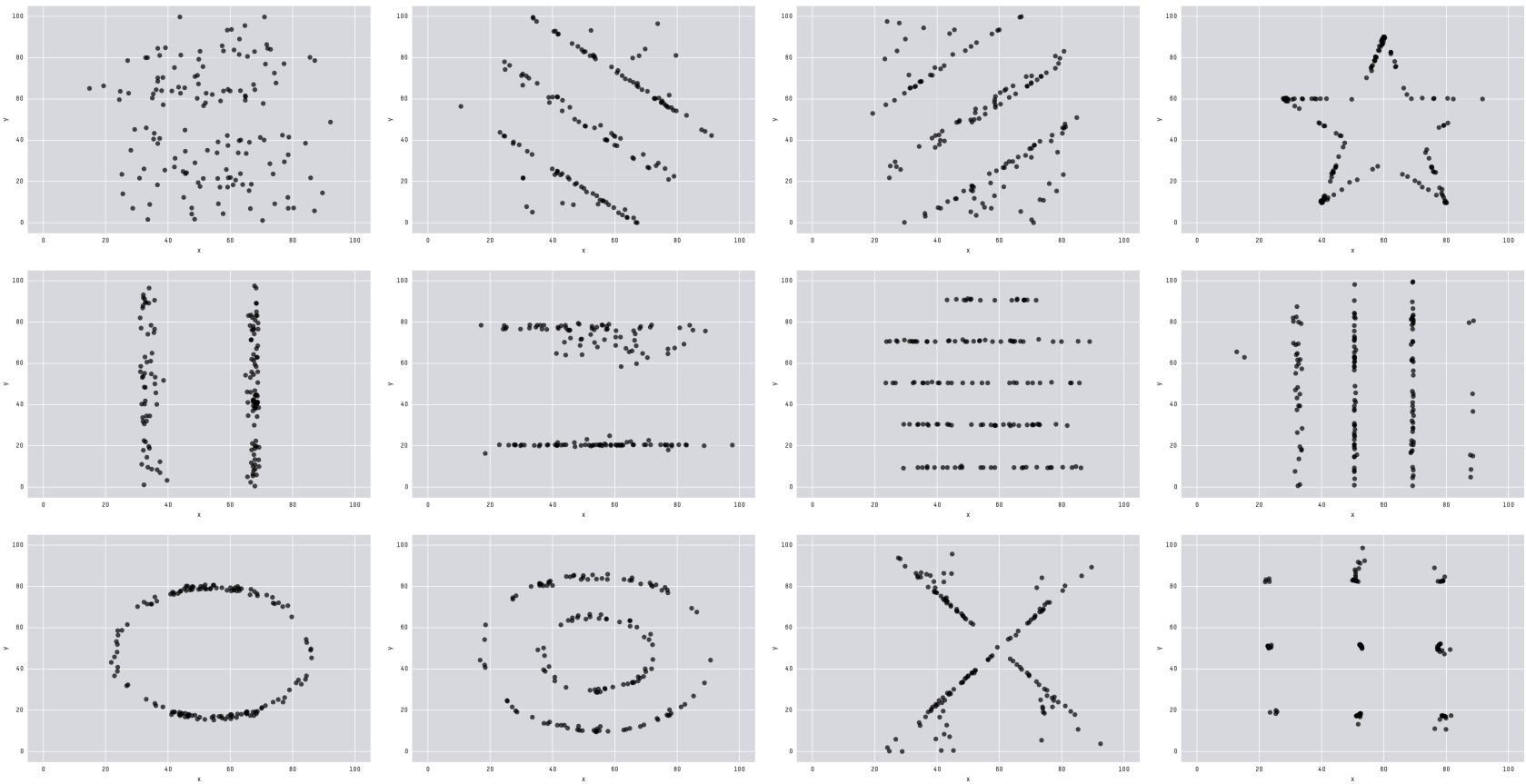
✗ Unstructured Quartet

Each dataset here also has the same summary statistics. However, they are not *clearly different* or *visually distinct*.



The datasaurus dozen

All these datasets have the same summary stats to 2 decimal places:



Matejka and Fitzmaurice, SIGCHI 2017

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It will not always be easy to visualize.

Difficult cases: high-dimensional data, no physical access to data, implicit access to data (e.g. latent spaces).

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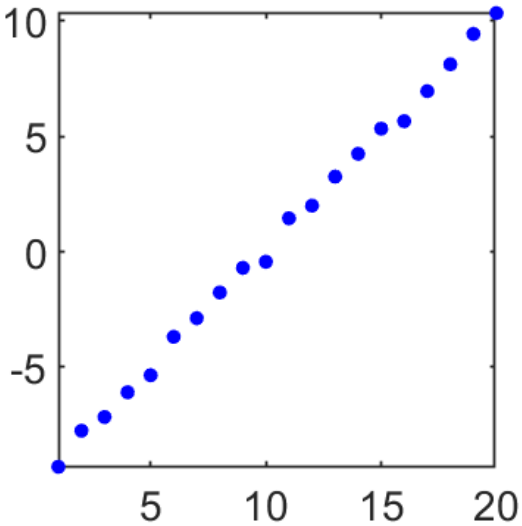

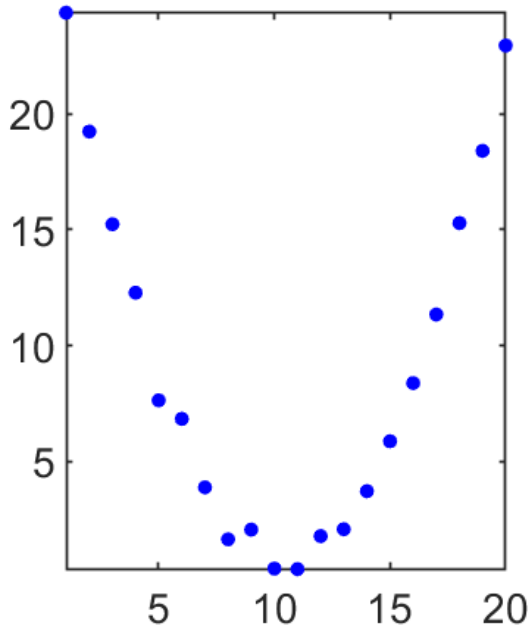

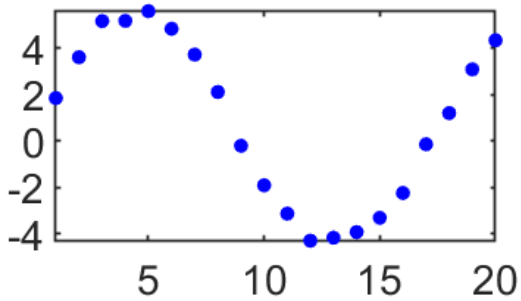

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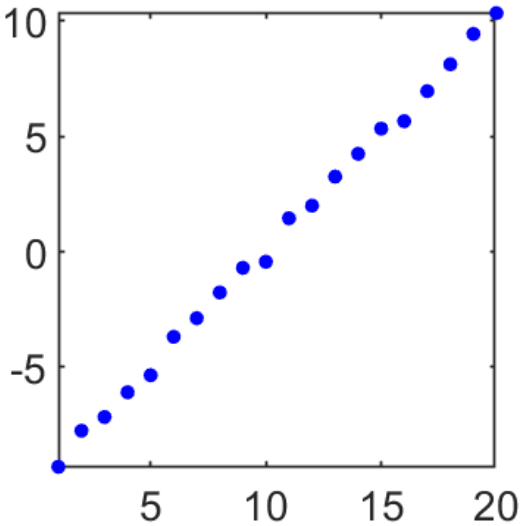
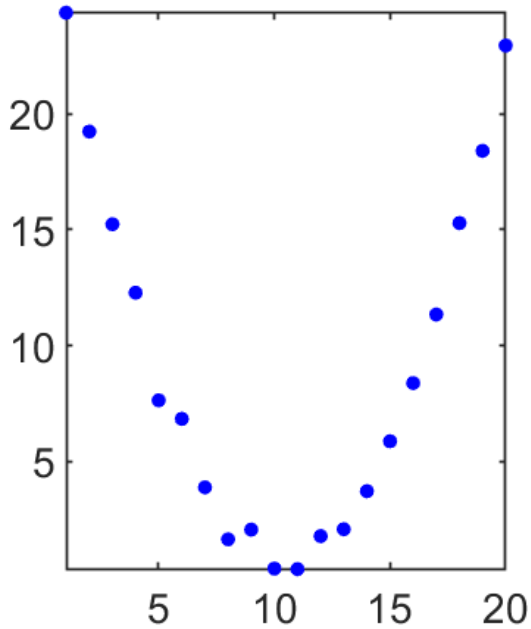
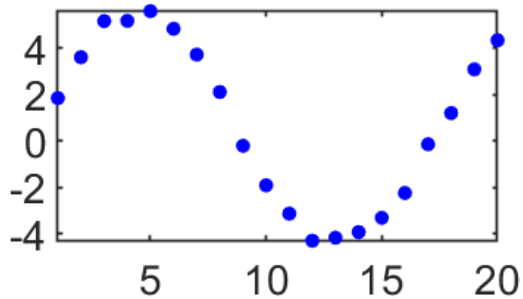


$y = ax + b$



$y = ax^2 + bx + c$



$y = ax^3 + bx^2 + cx + d$

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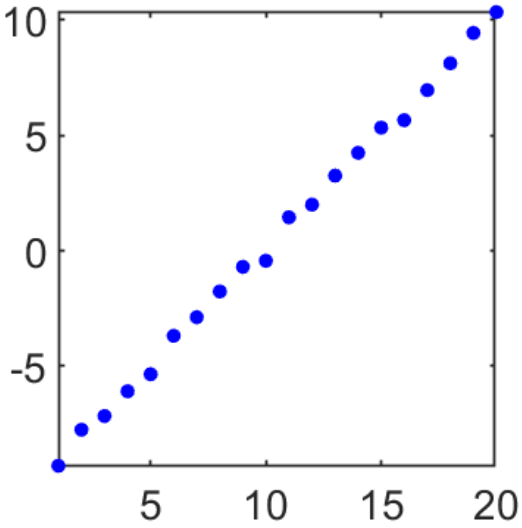

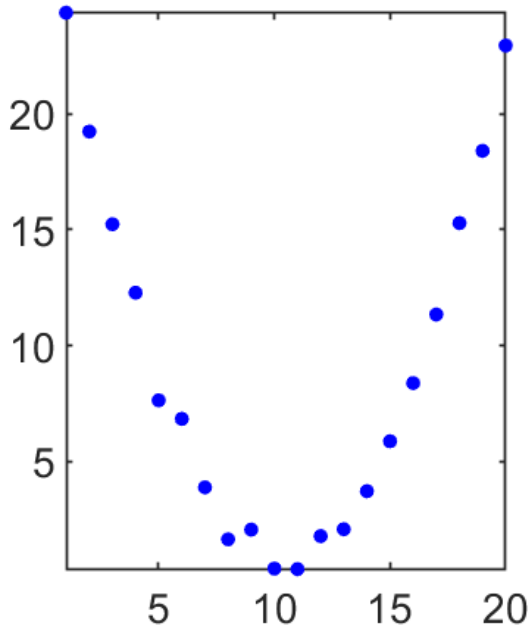

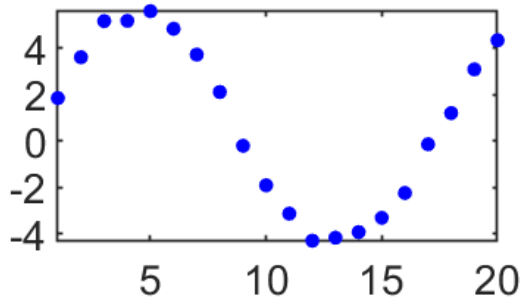

$y = ax + b$ $y = ax^2 + bx + c$ $y = a\,\mathtt{sin}(x) + bx + c$

Our model might use prior knowledge on the data.

Third plot: the data comes from a periodic process.

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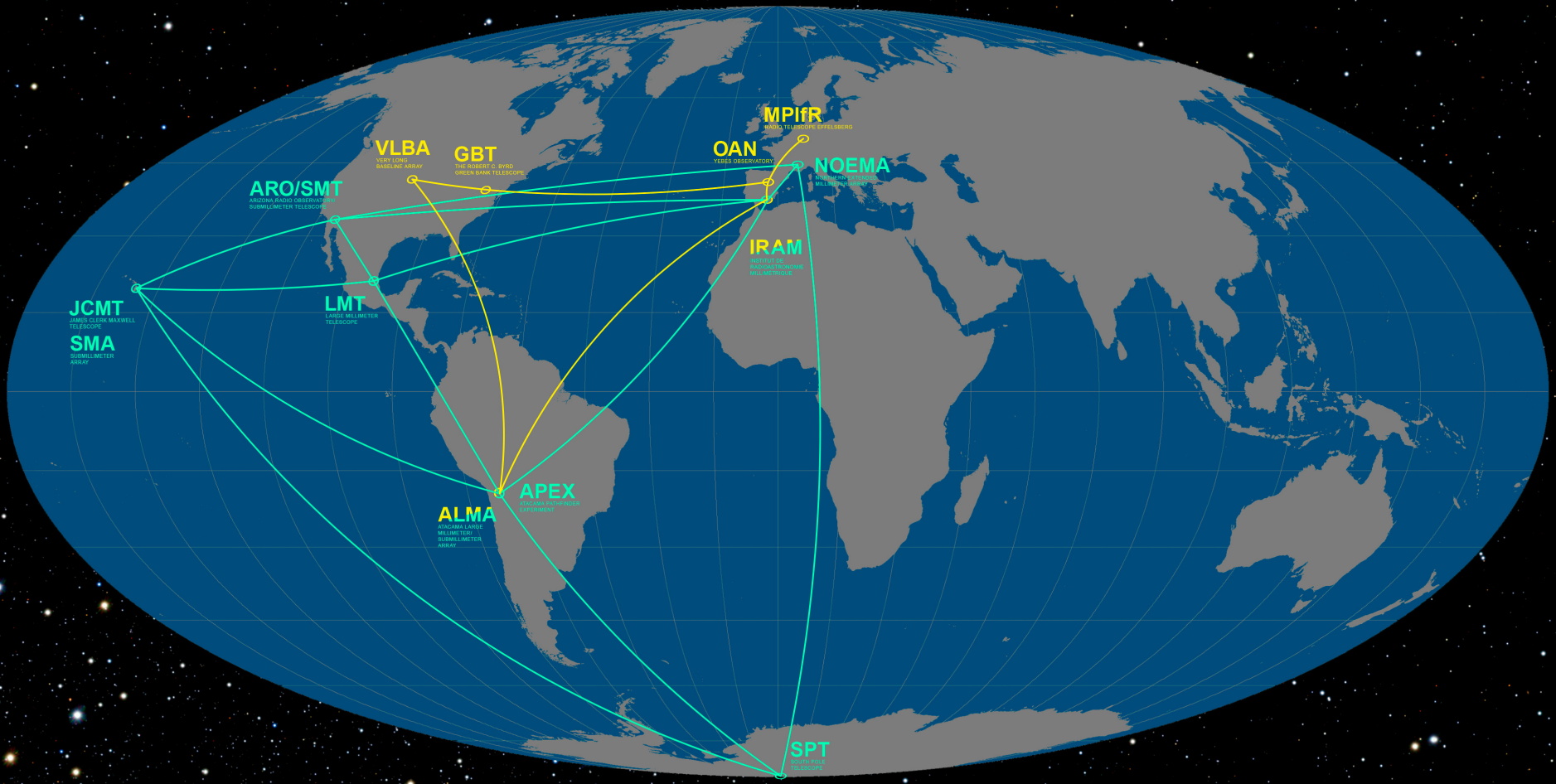
All these encode some expected behavior.

Event Horizon Telescope



Reliability of the prior: imaging the black hole

Problem: reconstruct an image from a sparse set of spectral measurements (VLBI imaging).

Bouman et al, "Computational imaging for VLBI image reconstruction", CVPR 2016

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It is an ill-posed inverse problem:

- Infinite number of possible images explain the data

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It is an ill-posed inverse problem:

- Infinite number of possible images explain the data
- Optimization heavily relies on priors.
  Find an explanation that respects prior assumptions about the "visual" universe while still satisfying the observed data.

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Reliability of the prior: imaging the black hole

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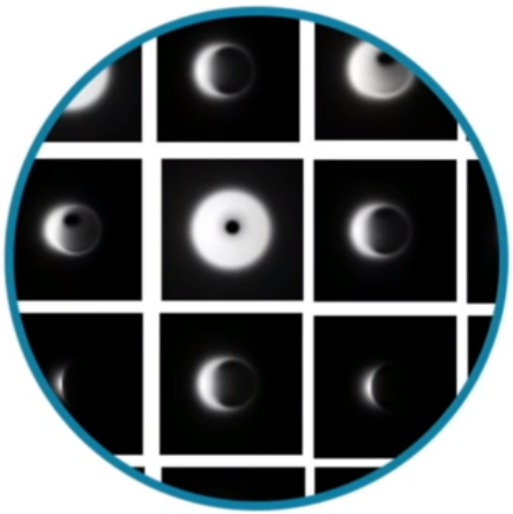
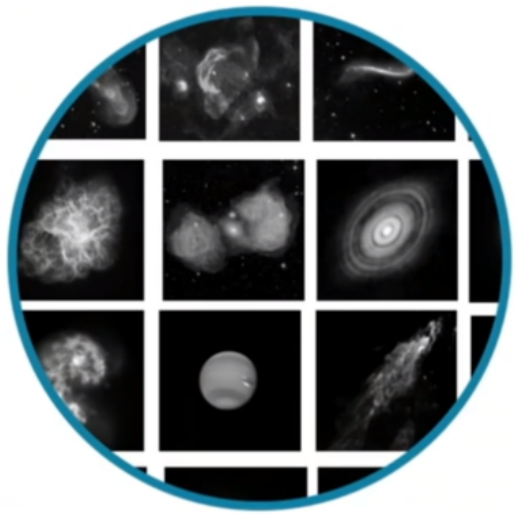
Which of these datasets would you use?



black holes



astronomy

everyday

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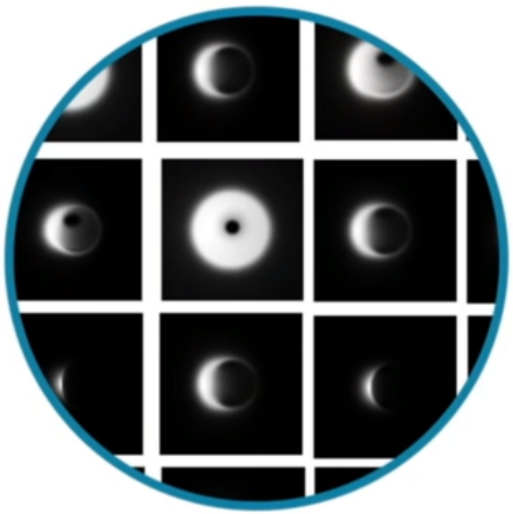
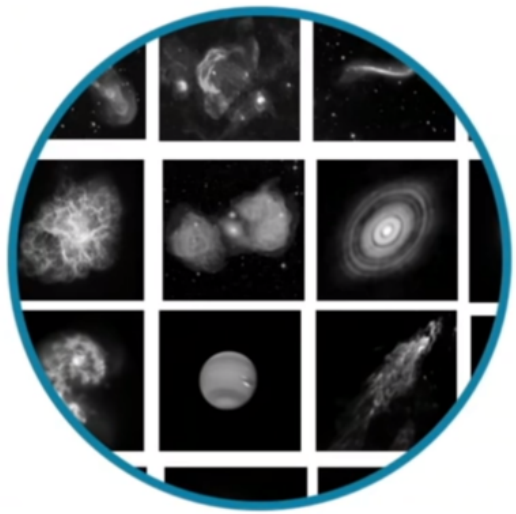


black holes
unreliable



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Black holes are dangerous! They will yield what one expects to obtain.

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Reliability of the prior: fairness

AI learns what humans teach.

The data provided by human can be highly biased.

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	WHITE	AFRICAN AMERICAN
Labeled Higher Risk, But Didn't Re-Offend	23.5%	44.9%
Labeled Lower Risk, Yet Did Re-Offend	47.7%	28.0%

Overall, Northpointe's assessment tool correctly predicts recidivism 61 percent of the time. But blacks are almost twice as likely as whites to be labeled a higher risk but not actually re-offend. It makes the opposite mistake among whites: They are much more likely than blacks to be labeled lower risk but go on to commit other crimes. *(Source: ProPublica analysis of data from Broward County, Fla.)*

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Search query	Work experience	Education experience	Profile views	Candidate	Xing ranking
Brand Strategist	146	57	12992	male	1
Brand Strategist	327	0	4715	female	2
Brand Strategist	502	74	6978	male	3
Brand Strategist	444	56	1504	female	4
Brand Strategist	139	25	63	male	5
Brand Strategist	110	65	3479	female	6
Brand Strategist	12	73	846	male	7
Brand Strategist	99	41	3019	male	8
Brand Strategist	42	51	1359	female	9
Brand Strategist	220	102	17186	female	10

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Bias in the training dataset is still an open problem!

Some possible causes:

- **Skewed sample**: a tiny initial bias grows over time, since future observations confirm prediction.

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Assessing data and prior reliability is crucial for any learning-based system.

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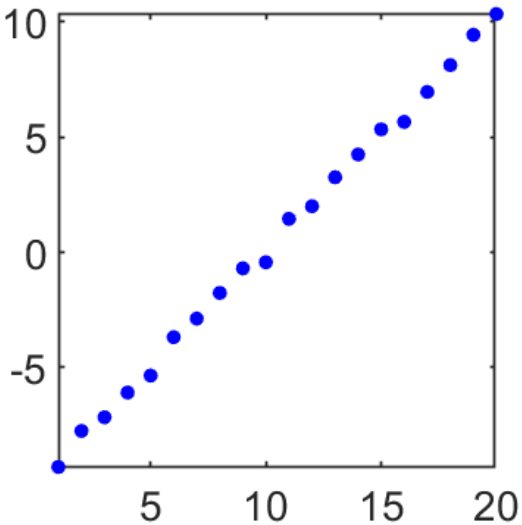

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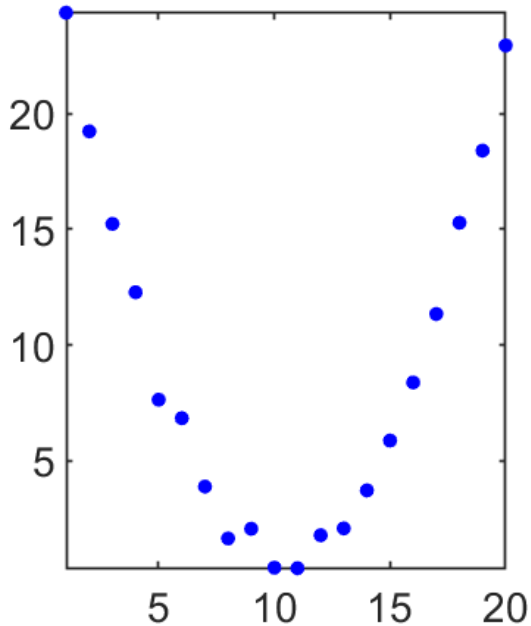
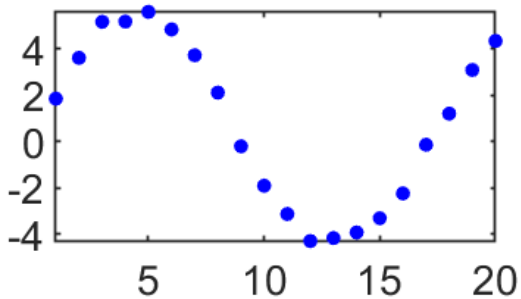


$y = ax^2 + bx + c$



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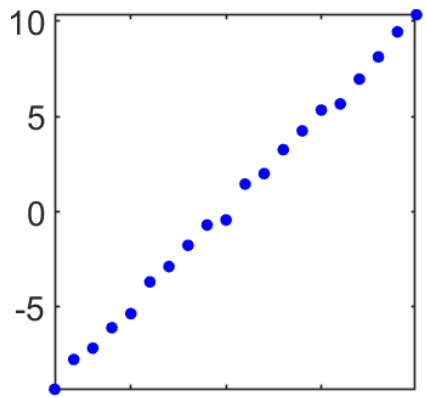

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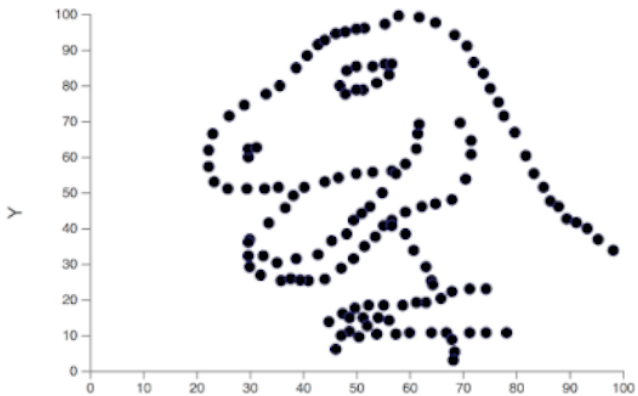

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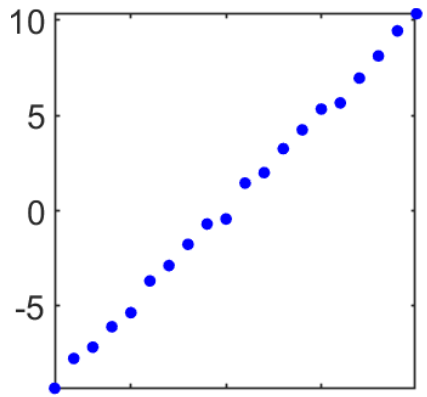
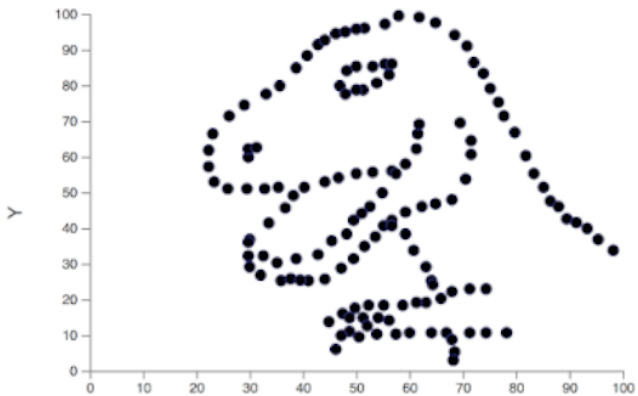


$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} t + \begin{pmatrix} b_x \\ b_y \end{pmatrix}$$



(not a function in 1D)

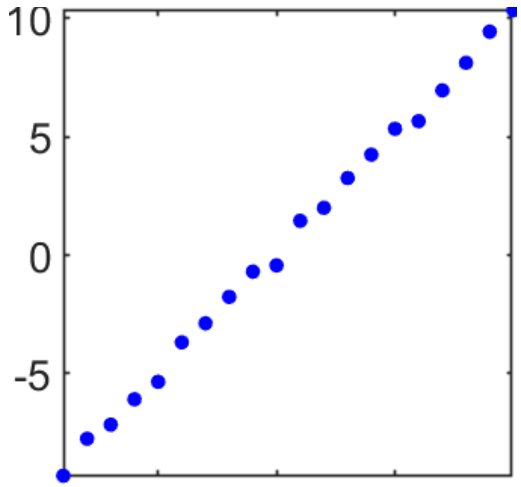
Clearly, data is not always one-dimensional.

Choosing a representation

The same data can be described in different ways.

What is the "right" way?

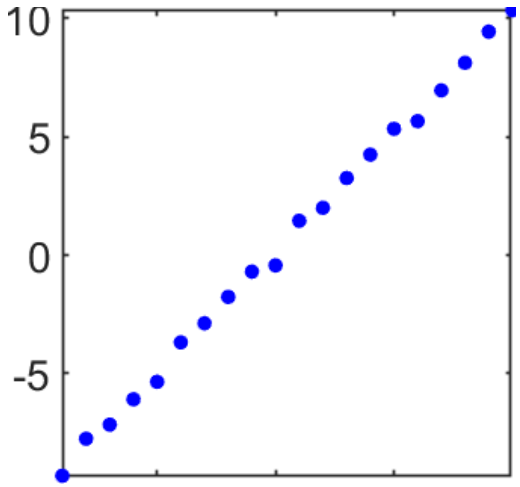
as a **function**



$$y = ax + b$$

$=$

as a **parametric curve**



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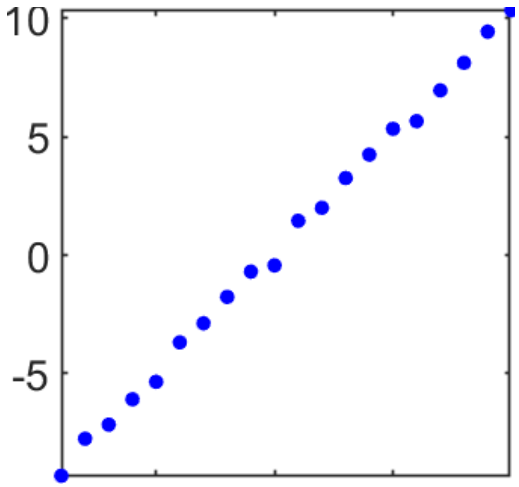

$y = ax + b$

2 weights

$=$

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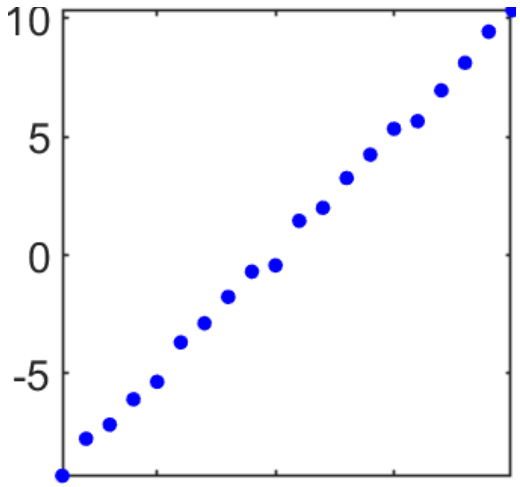

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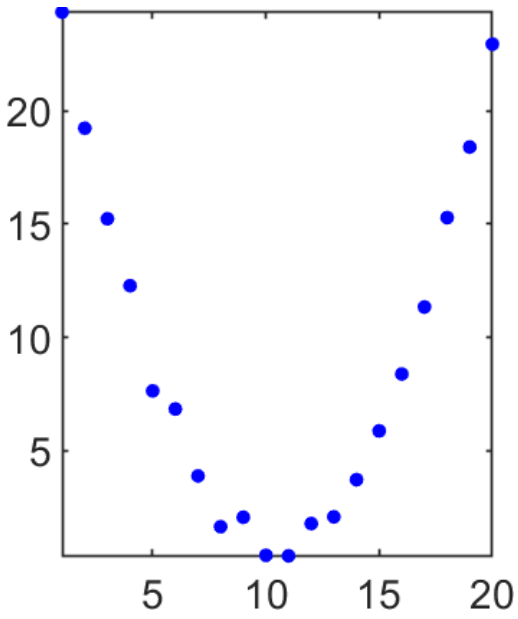

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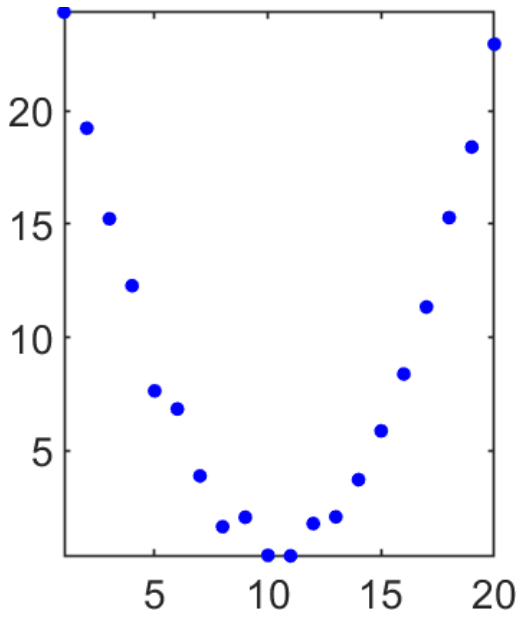

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} t^2 + \begin{pmatrix} b_x \\ b_y \end{pmatrix} t + \begin{pmatrix} c_x \\ c_y \end{pmatrix}$$

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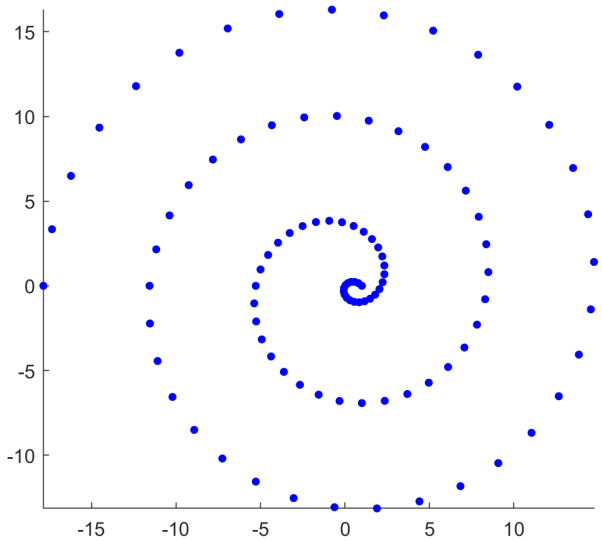

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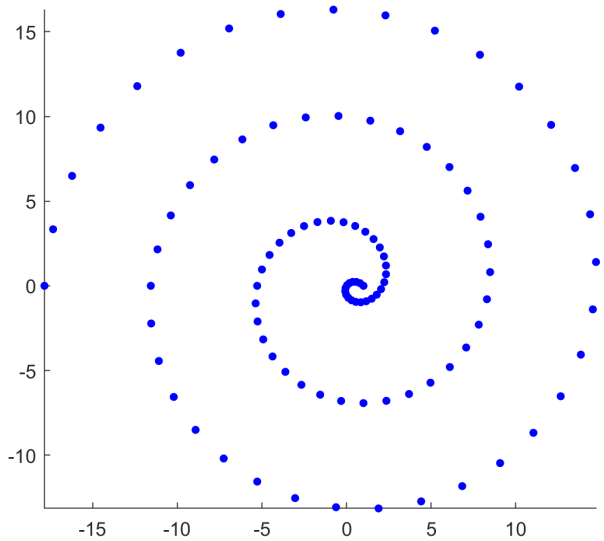
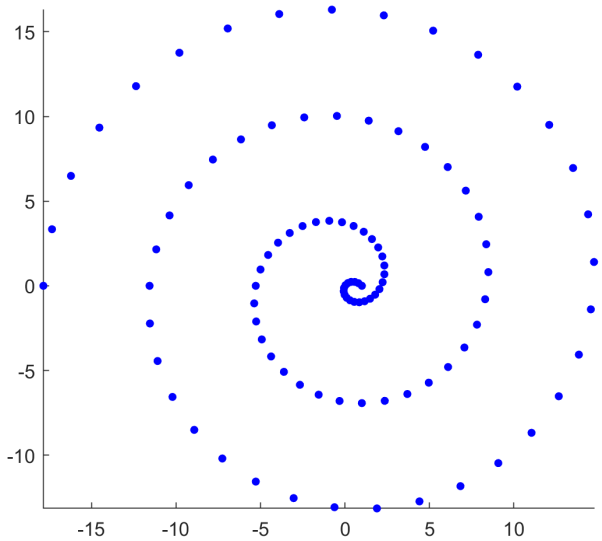


Choosing a representation

The same data can be described in different ways.

What is the "right" way?

as a **function**



y is not a function of x

$=$

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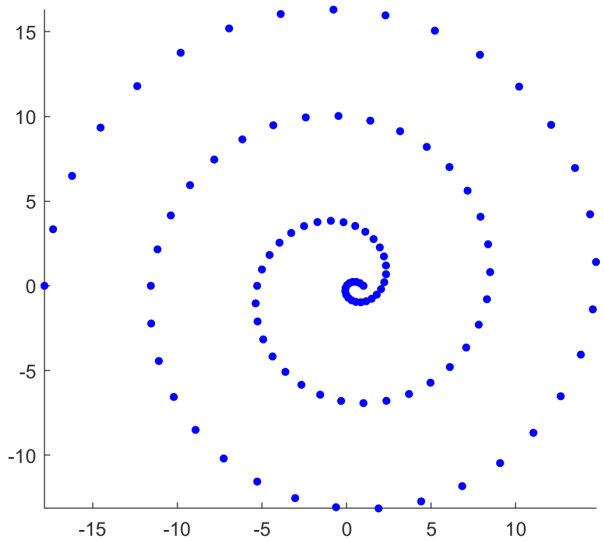

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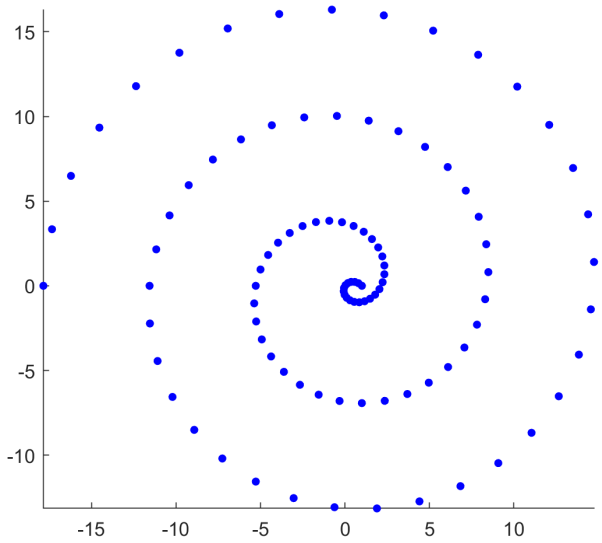

$r = a\theta$
(polar coordinates)

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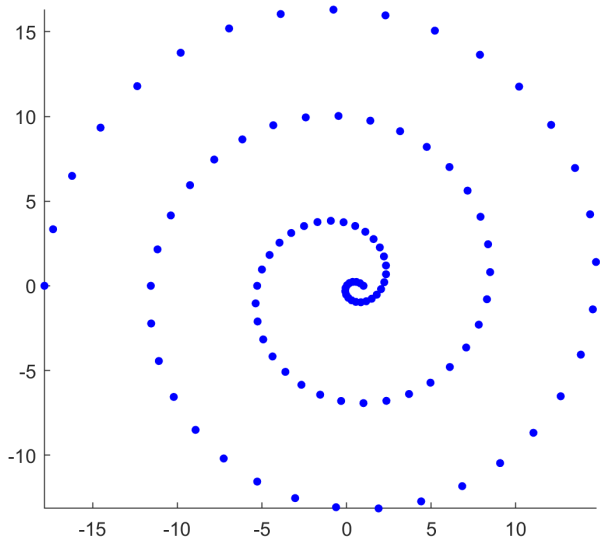

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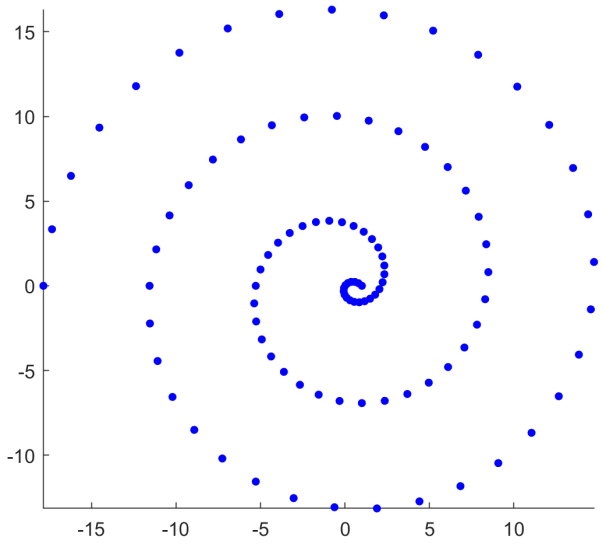

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linear!

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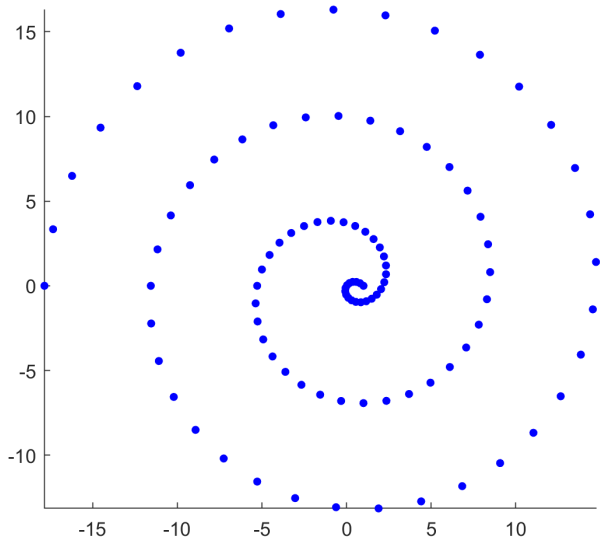

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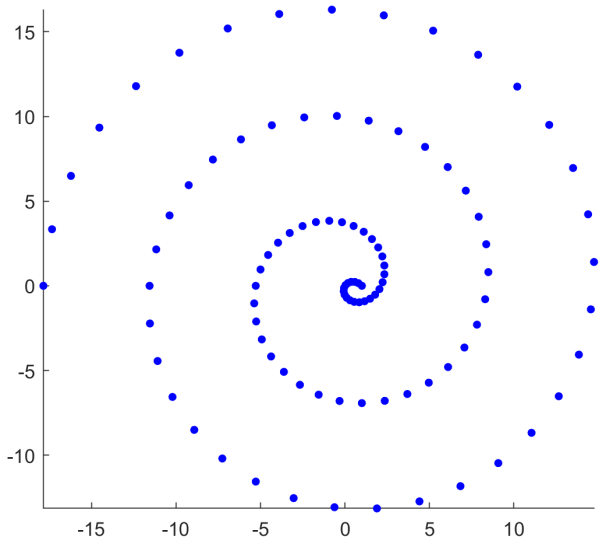

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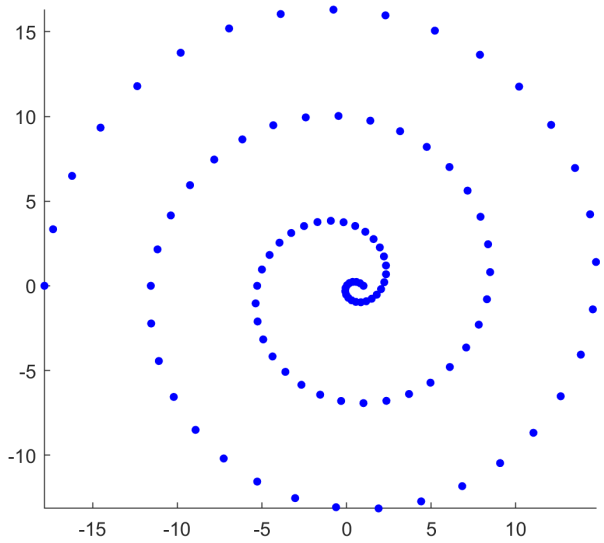

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Trade-off between #weights and simplicity

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Example: $\sim$1 megapixel photo (grayscale) has $\sim 10^6$ dimensions.

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For example, a $w \times h$ image has wh dimensions.

The value of each coordinate is given by the gray value at that pixel. Then, the entire image is one point in a wh-dimensional space.

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Example: $\sim$1 megapixel photo (grayscale) has $\sim 10^6$ dimensions.

Are all those dimensions significant?

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For simplicity, consider 1×1 images, i.e., consisting of one single pixel.

Each image is a point in one dimension.

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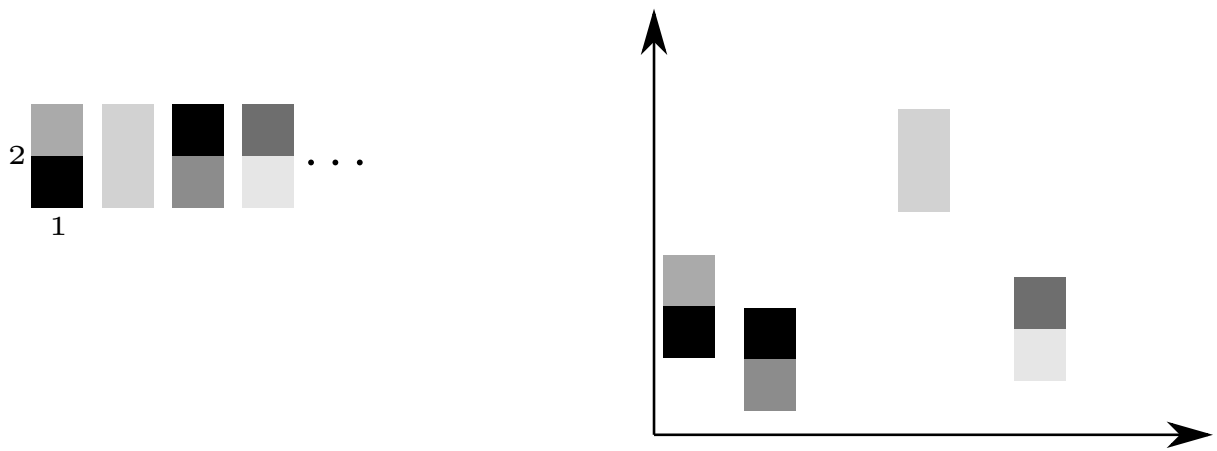

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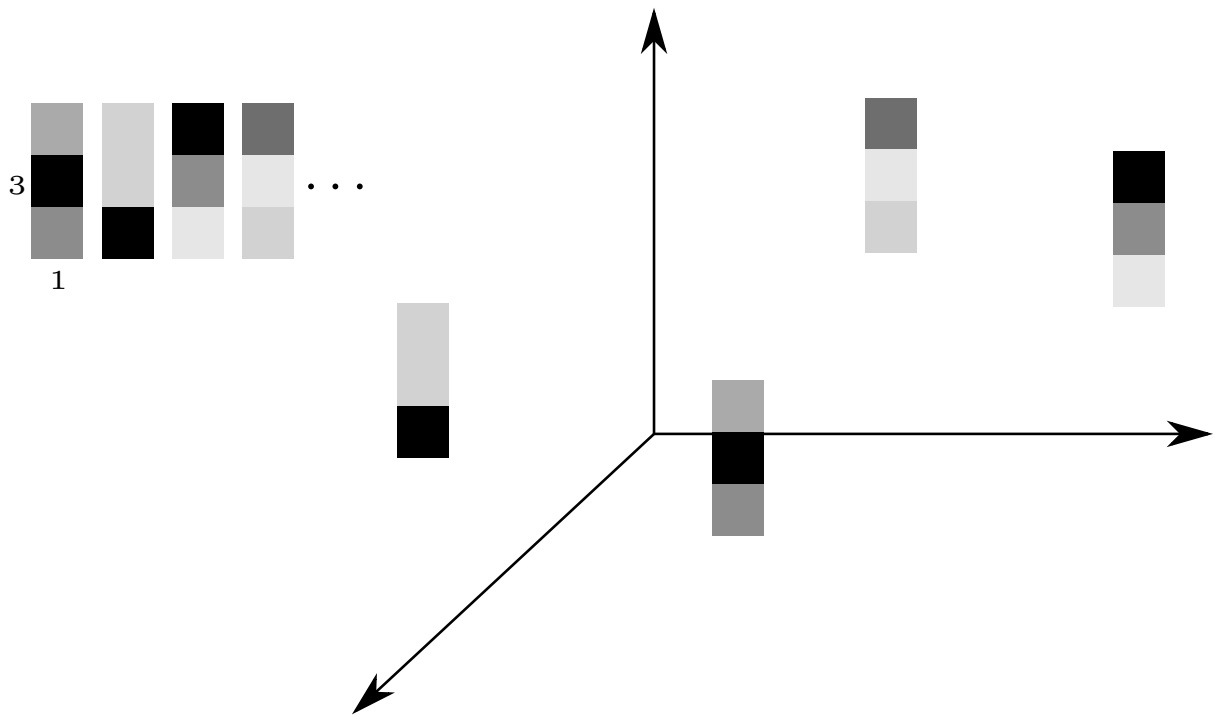

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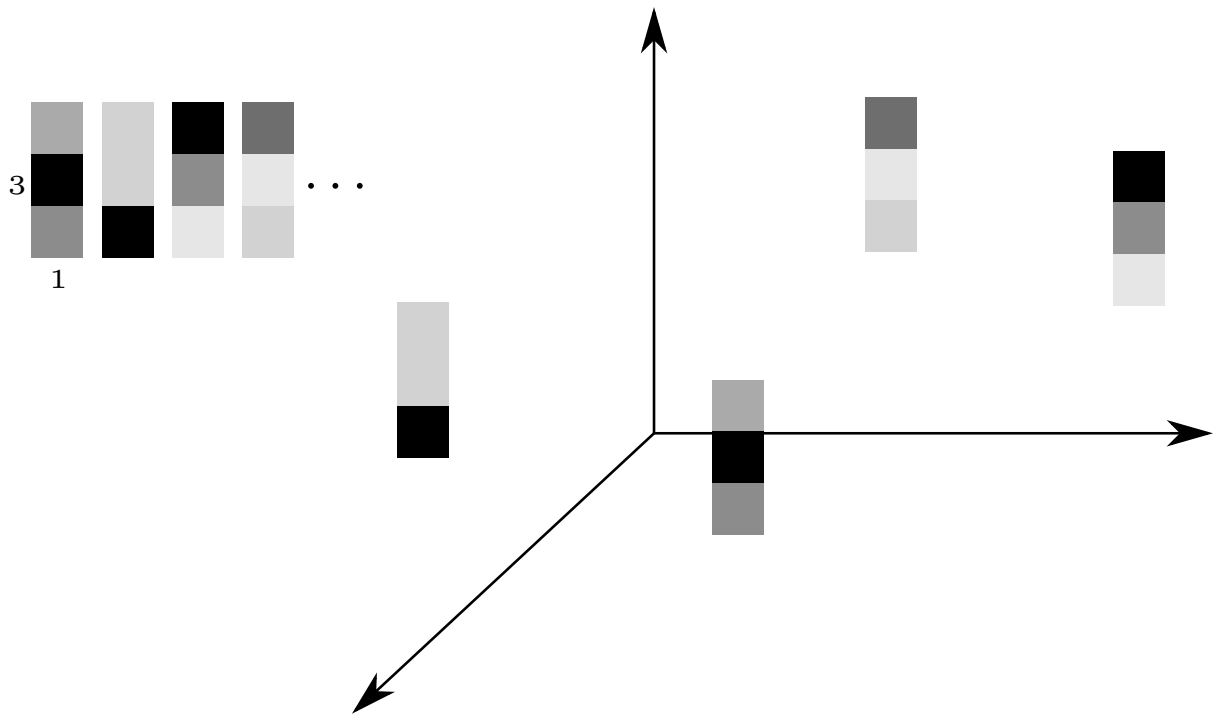

Each new dimension increases sparsity of the point cloud.

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A dataset of natural images will be extremely sparse in $\mathbb{R}^{w \times h}$, since each region of space is observed very infrequently.

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More data points make interesting structures emerge

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Two options:

1. **Increase** the dataset
2. **Decrease** the dimensions

Favor simplicity

Let's play a game:

$$2, 4, 8, \ldots$$

Rules:

- **Task:** Discover the rule I used to produce the sequence
- Give me a number: I'll tell you if it's next in sequence or not
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Also: when feasible, add more data!

Features

Assume each data point $x \in \mathcal{D} \subset \mathbb{R}^n$ is the result of a synthesis process:

$$\sigma : F \mapsto x$$

which takes a set of features F and composes them into x.

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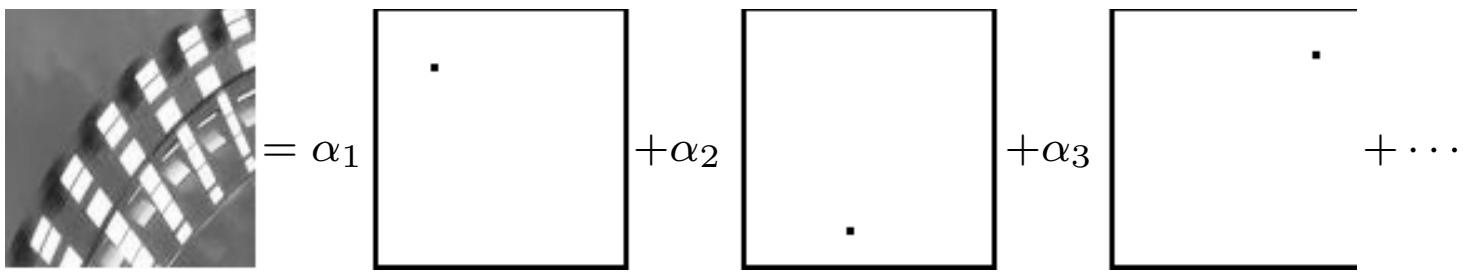
An image $x \in \mathbb{R}^{w \times h}$ is composed by pixels.
If each pixel of x is a feature, then σ simply sums them up:



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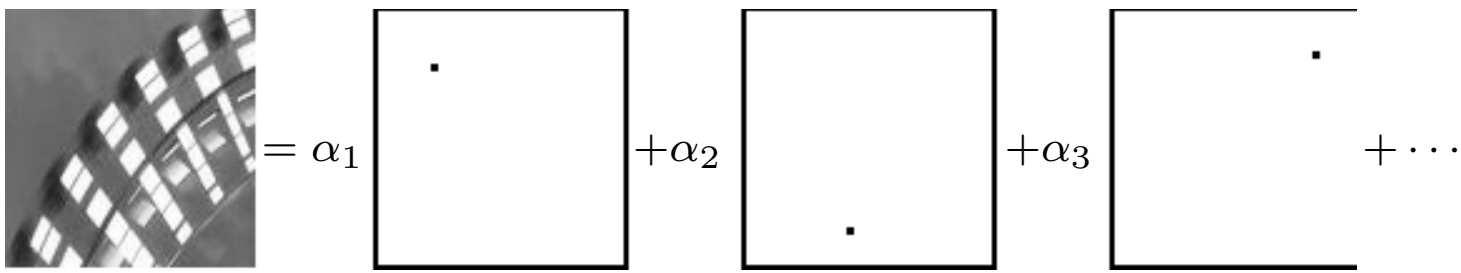

In this case, the feature space F is spanned by individual pixels.
Each feature (each pixel) represents a dimension.

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In this **particular case**, the feature space is a vector space and σ is linear.

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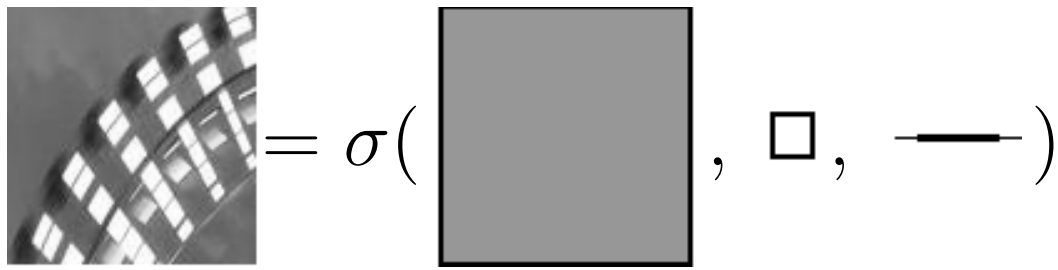

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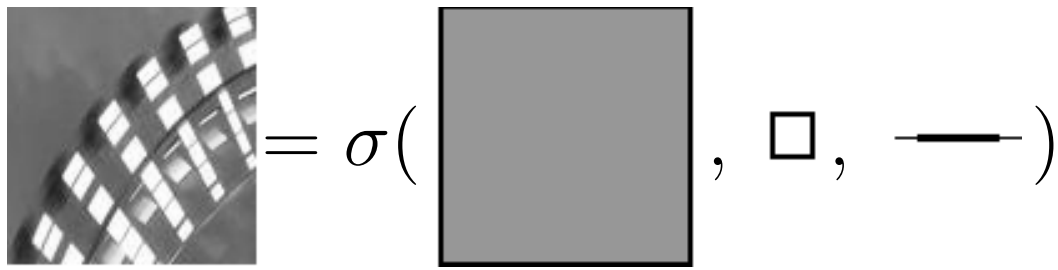

In general, the transformation σ acts nonlinearly on the features.

The output of σ is called an embedding of the data point.

For the data point $x \in \mathcal{D} \subset \mathbb{R}^n$, the embedding space is $\mathbb{R}^n$.

Invariances

In general, a given data point admits many possible embeddings.

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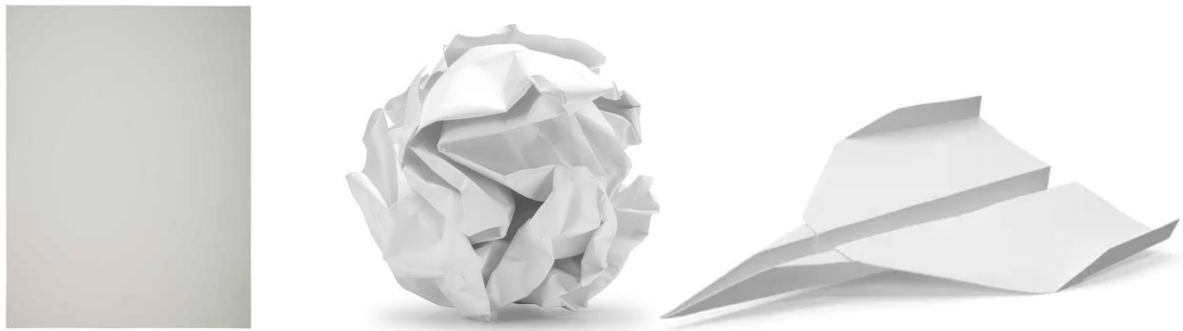

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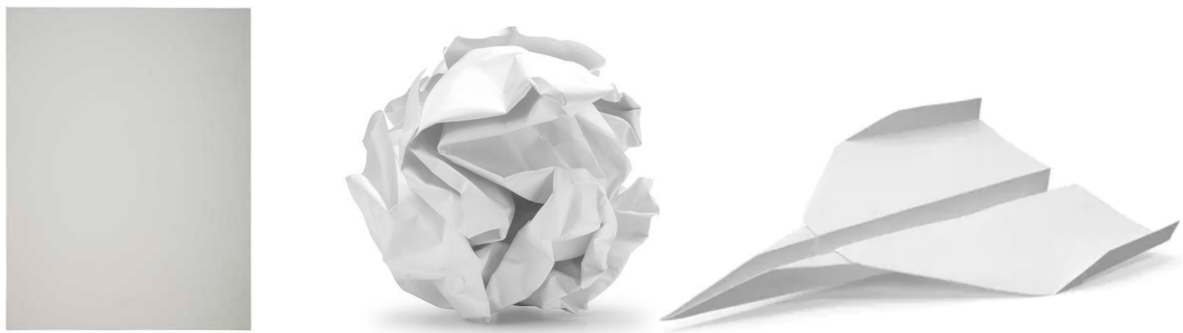

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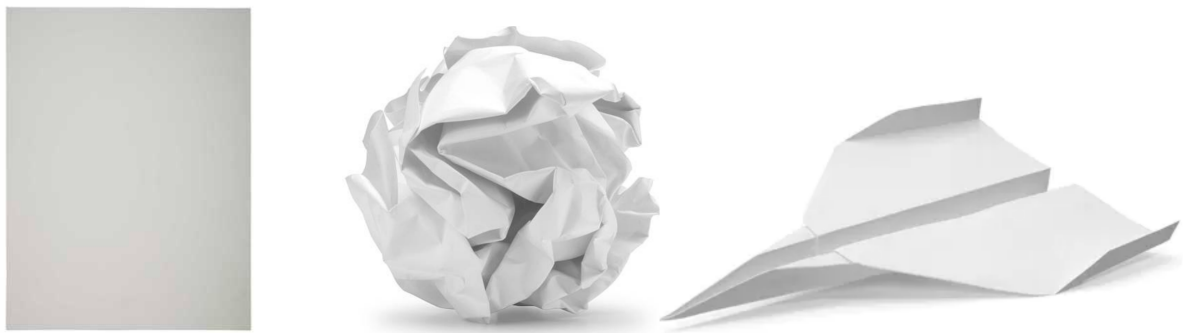

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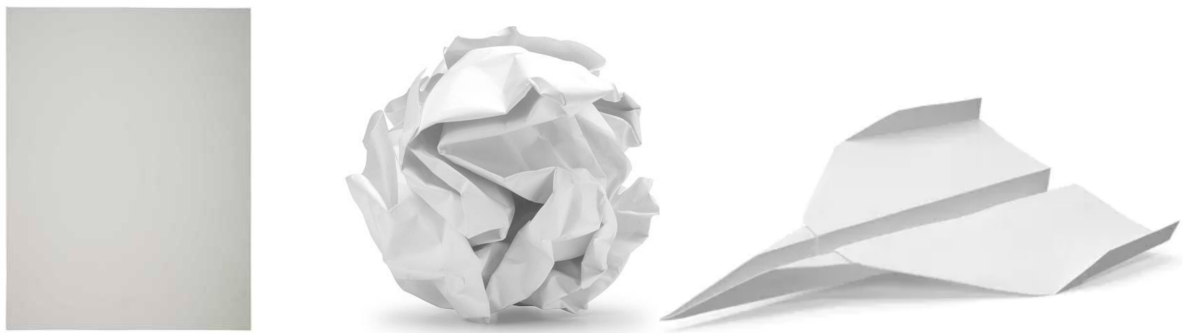

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Challenge: discover what intrinsic properties are preserved; these properties characterize the data.

Latent features

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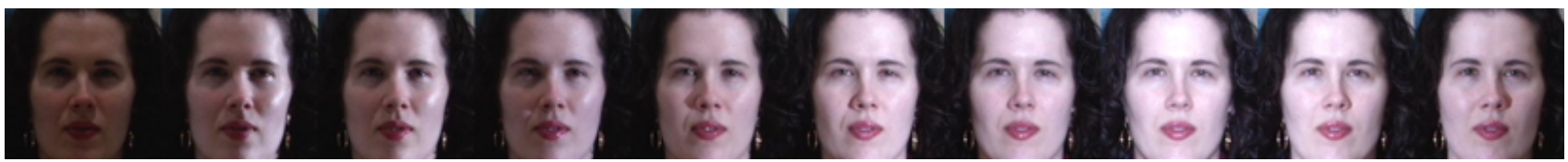

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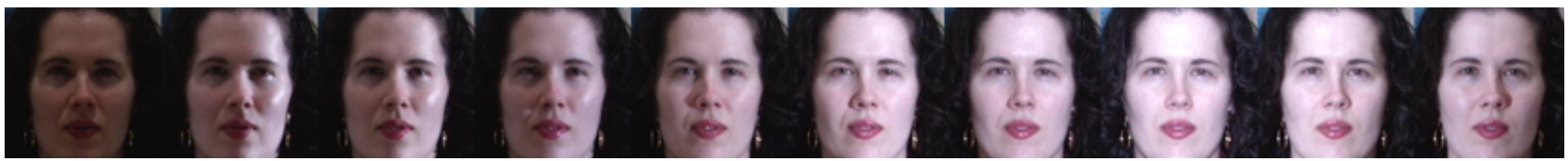

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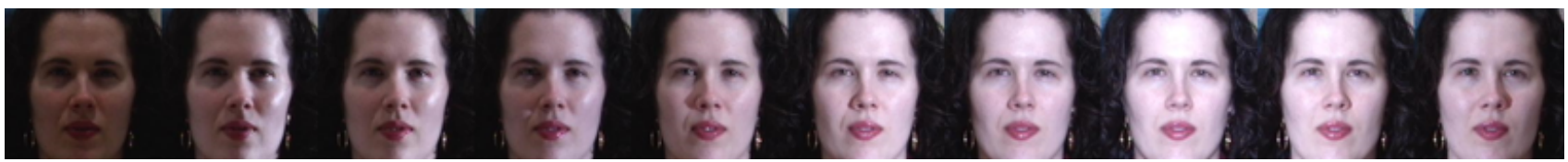

Latent feature: directional illumination

3 params for the light source position + **1** param for light intensity

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Discovering latent features involves discovering:

the "true" embedding space for the data
+
the transformation between the two spaces

General idea: find & discard non-informative dimensions.

Optimal dimensionality

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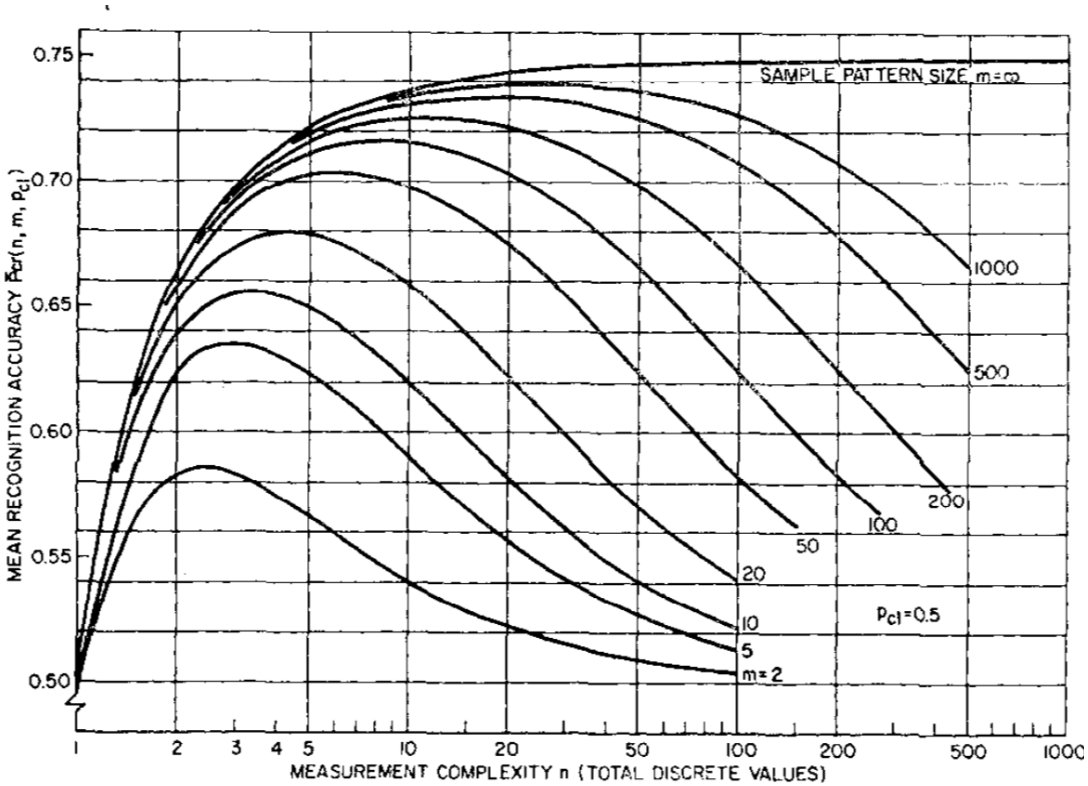
The effect of different dimensions, captured by Hughes phenomenon:



There is an optimal dimension which maximizes accuracy.

Hughes, "On the mean accuracy of statistical pattern recognizers", IEEE TIT 1968

The manifold hypothesis

Deep learning assumes that the input data lives on some underlying non-Euclidean structure called a manifold.

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Is color important?

Rank, suit, and color are generic features, but specific problems determine what features are important for that task.

What counts in spades, does not count in poker.

Deep learning is a task-driven paradigm to extract patterns and latent features from given observations

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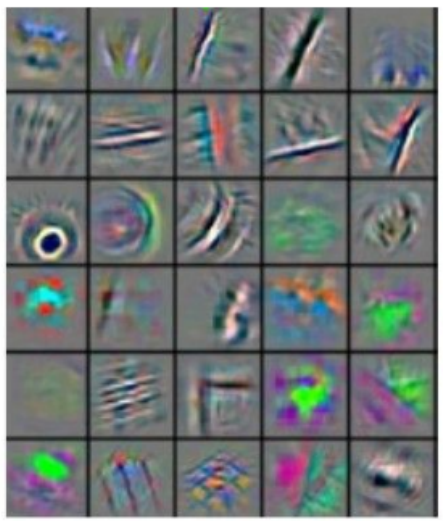
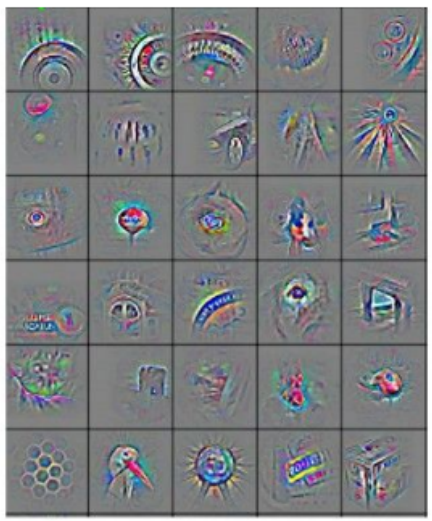
However, features are not always the focus of deep learning; rather, they are instrumental for the given task.

Example: Visual classification

$\rightarrow$ low-level features $\rightarrow$ mid-level features $\rightarrow$ hi-level features $\rightarrow$ "car"





Suggested reading

Blog post on the datasaurus:
https://www.autodeskresearch.com/publications/samestats

TED talk on the idea behind imaging the black hole:
https://www.youtube.com/watch?v=BIvezCVcsYs

VLBI reconstruction dataset:
http://vlbiimaging.csail.mit.edu/

Paper on the black hole imaging technique:
https://arxiv.org/pdf/1512.01413.pdf

Tutorial video and slides on ML fairness:
https:
//nips.cc/Conferences/2017/Schedule?showEvent=8734

Distill post on t-SNE:
https://distill.pub/2016/misread-tsne/