

Fundamentals of Computer Graphics

Introduction
Shape representation

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SAPIENZA
UNIVERSITÀ DI ROMA

Logistics

- **Lecturer:** Prof. Emanuele Rodolà
- **When:** Mondays 14:00–16:30 and Thursdays 08:00–10:30
- **Where:** Aula Magna - Alfa - Sociologia, via Salaria 113
- **Office Hours:** Drop me an email
- **Official website:** <https://erodola.github.io/FundCG-s1-2018/>
Check frequently for **news** and **material** (code, papers, ...)!

Disclaimer

This is a new course! Some key **differences** with last year:

- More **geometry** (shape analysis, processing and modeling)
- Less rendering and pixels (lights, cameras, materials, etc.)

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Geometry



Pixels

Who am I?

- Had research positions at U Tokyo, TU Munich, U Lugano and visiting positions at Harvard, Stanford, Ecole polytechnique, Technion among others
- 3D computer vision, digital geometry processing, geometric deep learning
- Passionate about anything that is new, cool, and/or crazy
- Approach me for projects and theses!



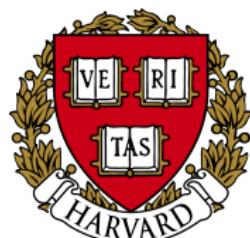
東京大学
THE UNIVERSITY OF TOKYO



Stanford
University



TECHNISCHE
UNIVERSITÄT
MÜNCHEN



Examples

- Can you reconstruct geometry by hearing instead of seeing?

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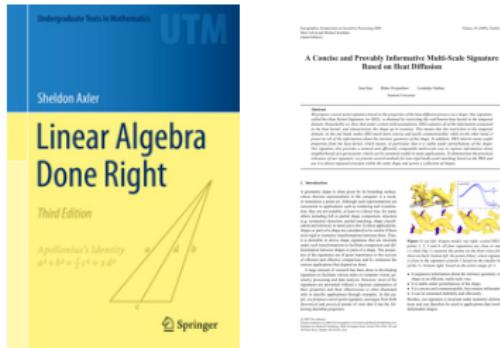
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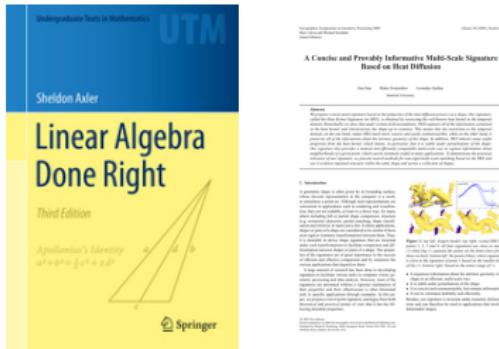
Pre-requisites and reading material

There is no official textbook. Specific references will be given throughout the course in the form of [book chapters](#) and [scientific articles](#).



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Pre-requisites:

- **Programming fundamentals.** Any language is alright, although the course will mostly use [C/C++/Matlab](#)
- Welcome (not mandatory): linear algebra, calculus, numerical methods, optimization

Grading and exams

Modality: Mid-term written exam + final project with oral examination.

- In-depth **survey** (possibly with **code**)
- **Software** project plus **report**

Topics and project descriptions will come later.

Original ideas are encouraged.

Lecture **exercises** are part of the oral exam.

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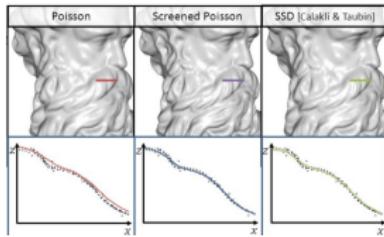
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Project examples:

- Survey: Review of shape reconstruction techniques



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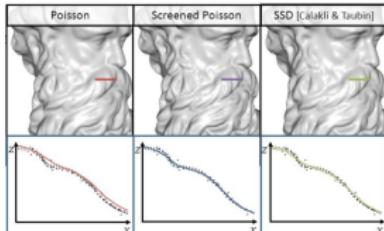
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Project examples:

- Survey: Review of shape reconstruction techniques
- Project: Implement one such technique and discuss results



Overall objective

What will you get out of this course?

- You will understand next-generation technology dealing with 3D data
- You will understand scientific articles in the 3D area

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- You will understand next-generation technology dealing with 3D data
- You will understand scientific articles in the 3D area
- You will obtain unique skills



AUTODESK
MAYA®



Minority Report (20th Century Fox/Dreamworks) 2002



Microsoft Kinect 2010



>\$10K

2005



\$100

2010



\$20

2014

Deluge of geometric data



KINECT for XBOX 360
 SoftKinetic™
The Interface Is You

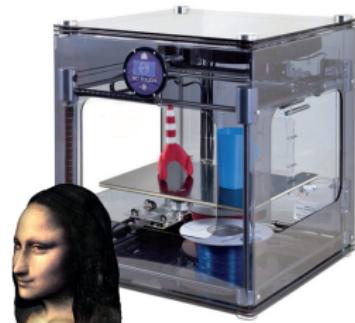


Google 3D warehouse

shapeways

3D sensors

Repositories



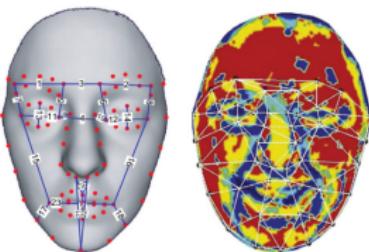
 Stratasys  3D SYSTEMS

3D printers

Applications



Reconstruction



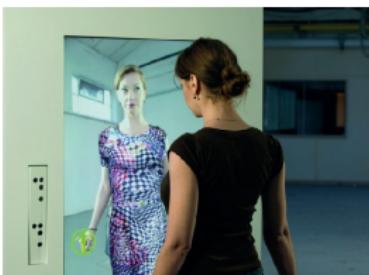
Recognition



Retrieval



Avatars



Virtual dressing



Gesture control

Images: Davison et al. 2011; Zaferiou et al. 2012; Kim et al. 2013; Faceshift; Fashion3D; Minority report



Microsoft

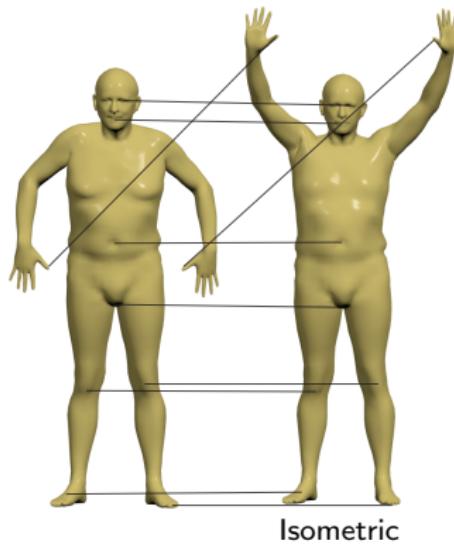


GDC

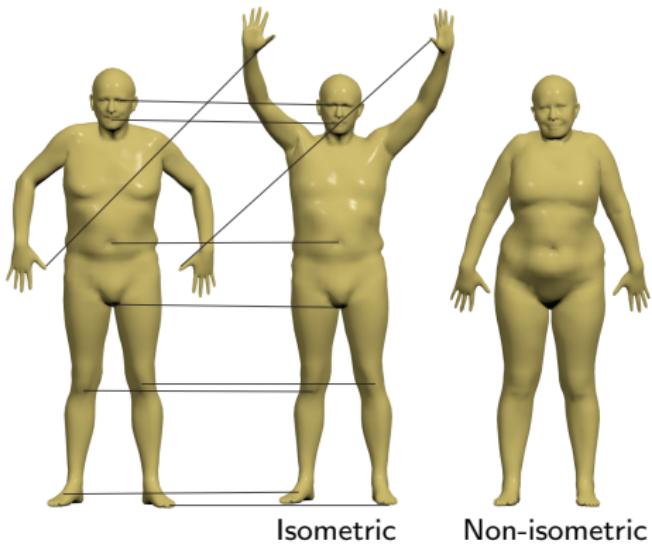


Faceshift (acquired by Apple in 2015)

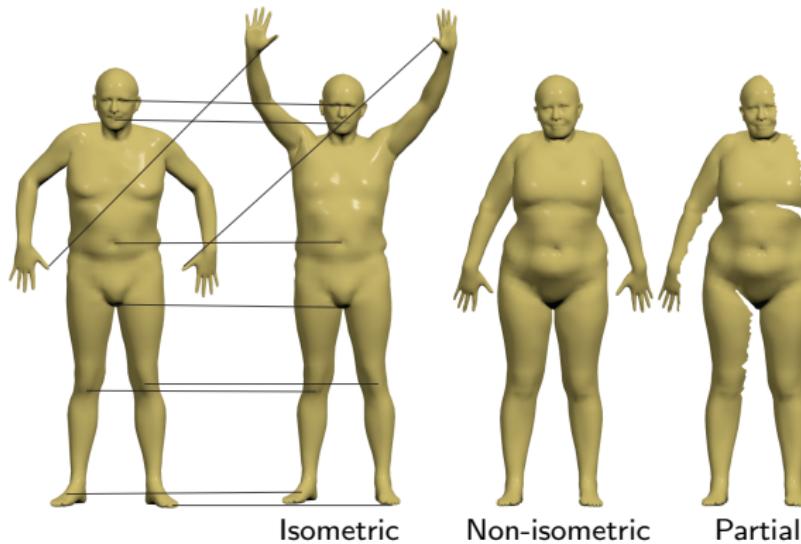
Basic problems: shape similarity and correspondence



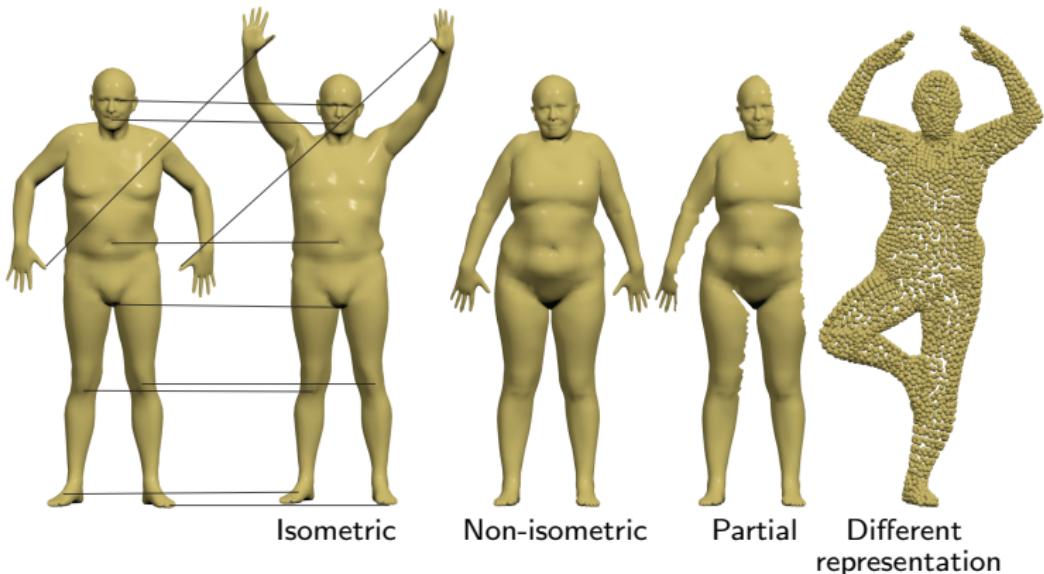
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Basic problems: shape similarity and correspondence



Basic problems: shape similarity and correspondence



Mathematical tools

- Linear algebra
- Metric spaces
- Differential geometry
- Partial differential equations
- Optimization

But...

Mathematical tools

- Linear algebra
- Metric spaces
- Differential geometry
- Partial differential equations
- Optimization

But...

90% of the time we will be able to
visualize what we are doing!

Example

“Vector-valued function on a 2-Riemannian manifold”

Example

“Vector-valued function on a 2-Riemannian manifold”
= A color for each point of a 3D shape



What is a shape?

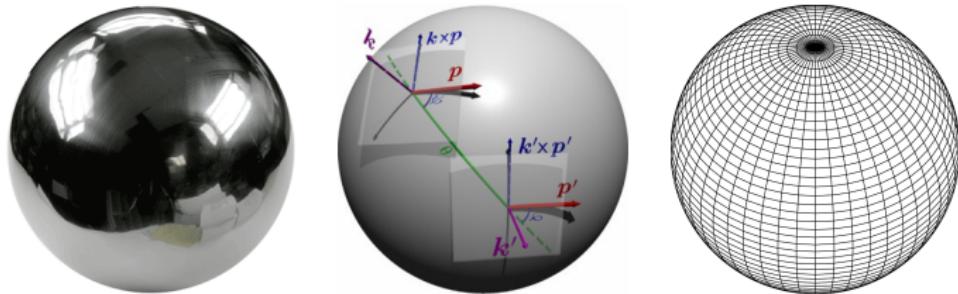
“There can be no such thing as a mathematical theory of shape. The very notion of shape belongs to the natural sciences.”¹

¹ J. Koenderink, “Solid Shape”. MIT Press 1990

What is a shape?

“There can be no such thing as a mathematical theory of shape. The very notion of shape belongs to the natural sciences.”¹

For us, shapes are **mathematical objects** (specifically, **manifolds**) with a precise and rigorous definition. We will model them mathematically in the **continuous** setting (“pen and paper”) and bring them to the digital world by translating to the **discrete** setting.

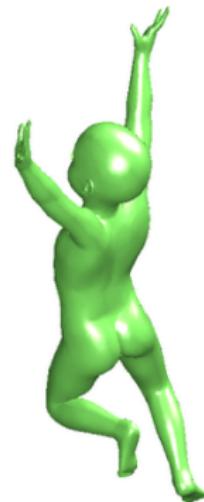


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Shapes vs Images: Domain

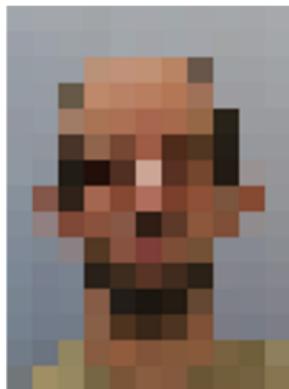


Euclidean (flat)



non-Euclidean (curved)

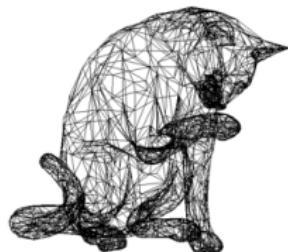
Shapes vs Images: Representation



Array of pixels (uniform grid)



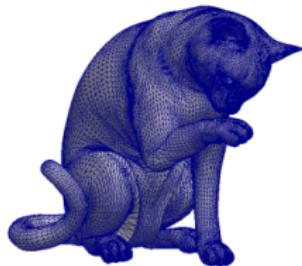
Splines



Graph



Point cloud

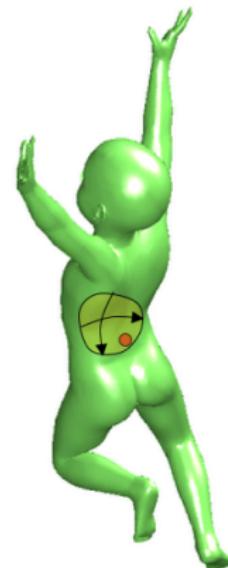


Triangle mesh

Shapes vs Images: Parametrization

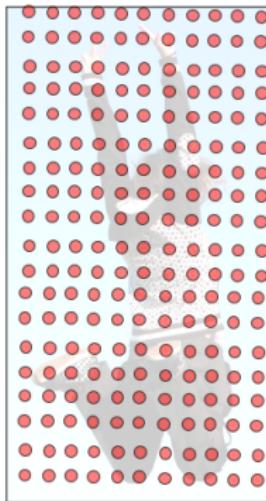


Global

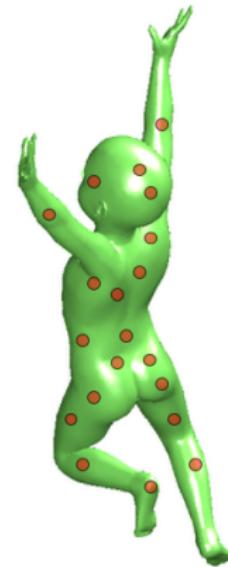


Local

Shapes vs Images: Sampling



Uniform



“Uniform” is not well defined

Shapes vs Images: Transformations



Perspective



Affine



General (non-rigid)

Shapes vs Images: Calculus



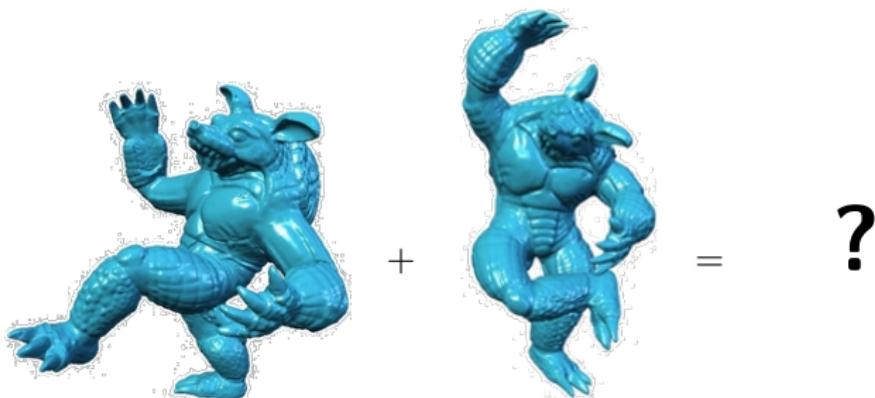
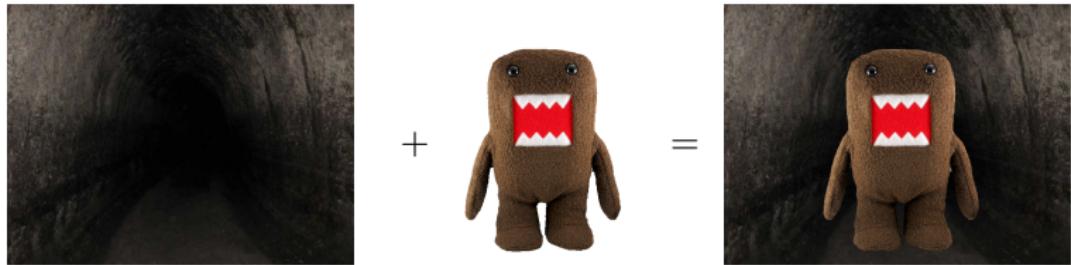
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=



Shapes vs Images: Calculus



Shape representation

We will use mainly two representations for shapes:

- Triangle mesh
- Point cloud

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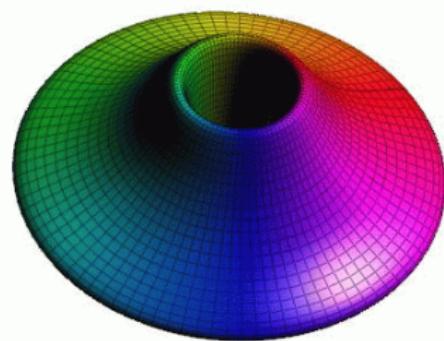
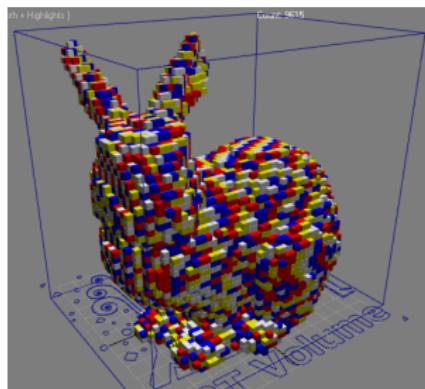
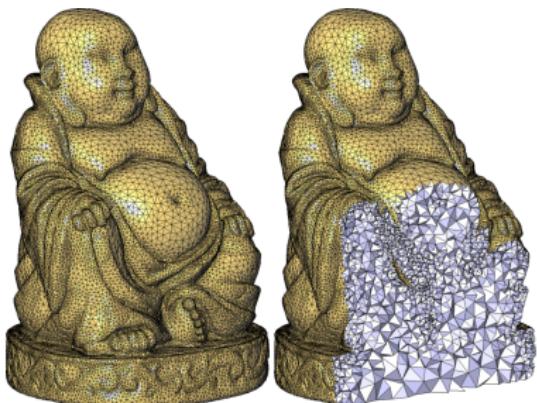
Other shape representations exist:

- Polygonal meshes (polygons are not restricted to triangles)
- Parametric surfaces
- Tet-meshes (**volumetric**)
- Voxel grids (**volumetric**)
- ...

The **volumetric** representations model the **interior** of the object, whereas the shape surface is the corresponding **boundary**.

However, these alternative representations will not be considered in the remainder of this course.

Example: Other representations



Shape representation: Key ingredients

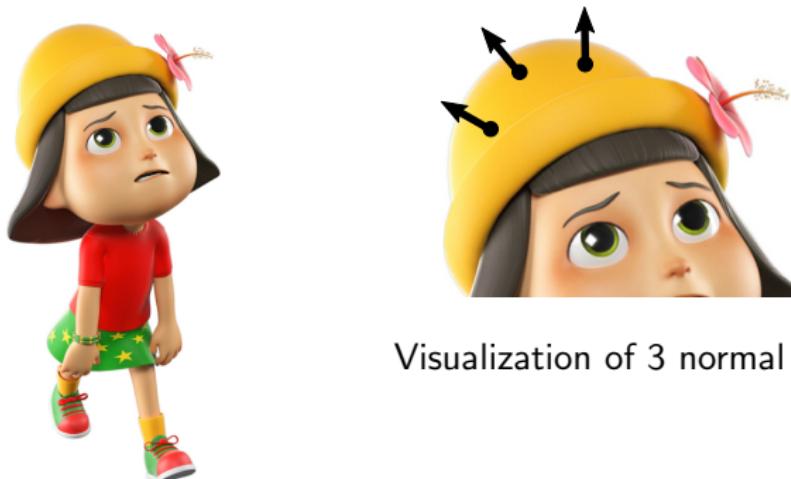
- We want to represent **surfaces**
- Each point can have **attributes** attached to it (e.g., RGB color)



Note: These notions will be covered more rigorously in the next lectures.

Shape representation: Key ingredients

- We want to represent **surfaces**
- Each point can have **attributes** attached to it (e.g., RGB color)
- Surfaces have an **orientation** defined by **normal vectors**



Visualization of 3 normal vectors

Note: These notions will be covered more rigorously in the next lectures.

Shape representation: Triangle mesh

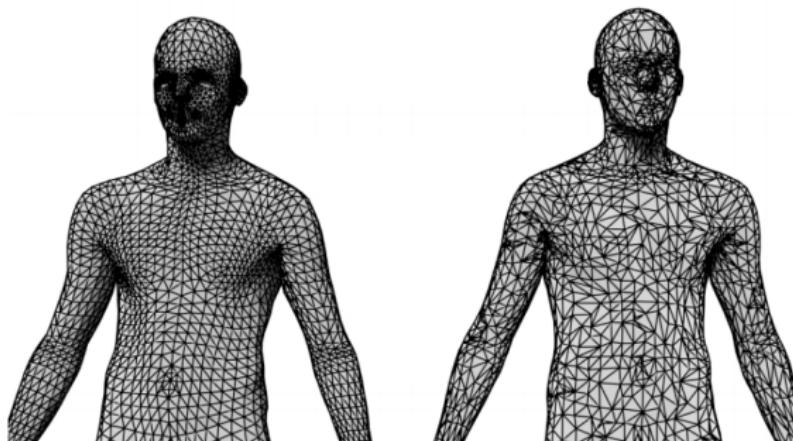
A **triangle mesh** is a collection of connected triangles.

Shape representation: Triangle mesh

A **triangle mesh** is a collection of connected triangles.

The incidence relations of triangles defines the mesh **connectivity** (also referred to as **mesh topology**).

In this example, the same underlying surface is discretized with meshes having different connectivity:

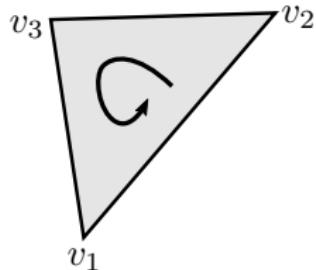


Shape representation: Triangle mesh

A **triangle mesh** is a collection of connected triangles.

We will only consider **oriented** manifold meshes.

- Each triangle has an **orientation**

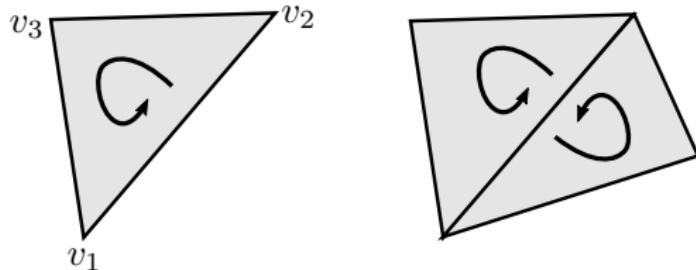


Shape representation: Triangle mesh

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- Each triangle has an **orientation**
- All triangles should be **consistently** oriented (e.g. counter-clockwise)

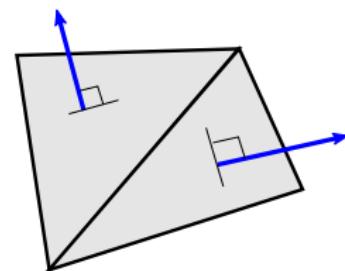
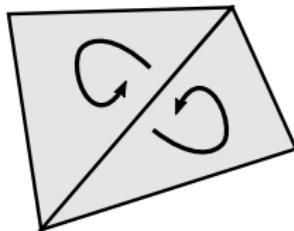
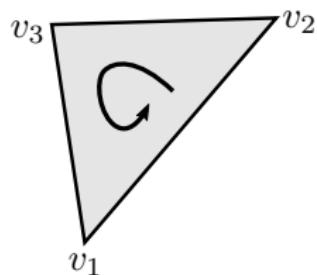


Shape representation: Triangle mesh

A **triangle mesh** is a collection of connected triangles.

We will only consider **oriented** manifold meshes.

- Each triangle has an **orientation**
- All triangles should be **consistently** oriented (e.g. counter-clockwise)
- Each triangle has a **normal** (i.e. orthogonal) vector consistent with the triangle orientation

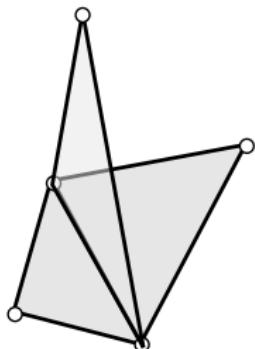


Shape representation: Triangle mesh

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- All edges have **at most** two incident triangles



3 triangles incident to 1 edge

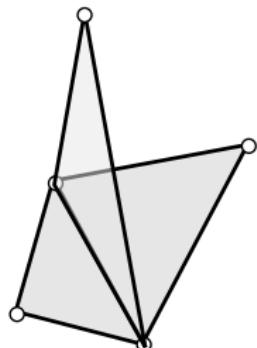
⇒ **non-manifold**

Shape representation: Triangle mesh

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- Edges with only one incident triangle form the mesh **boundary**



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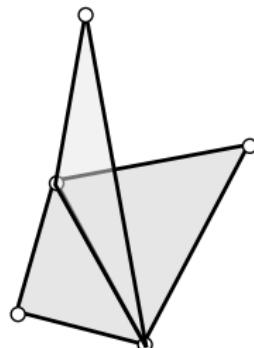
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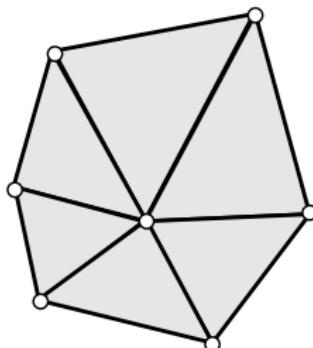
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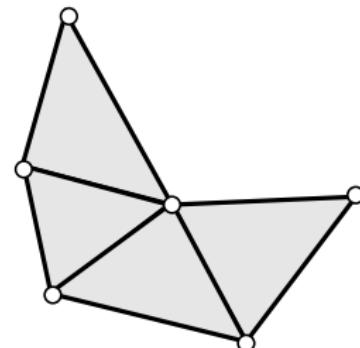
- All edges have **at most** two incident triangles
- Edges with only one incident triangle form the mesh **boundary**
- The faces incident to a vertex form a **closed** or an **open** fan



3 triangles incident to 1 edge
⇒ **non-manifold**

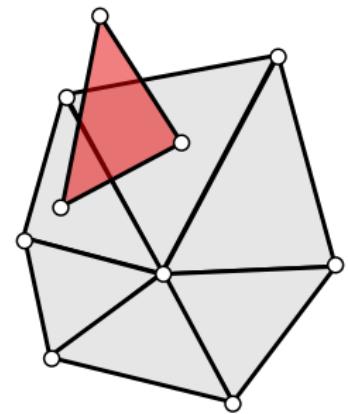
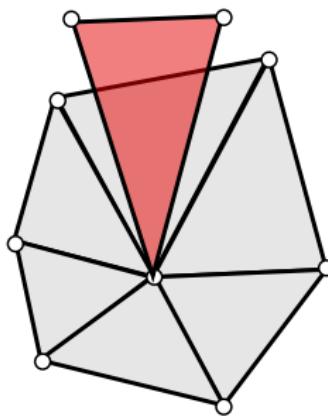
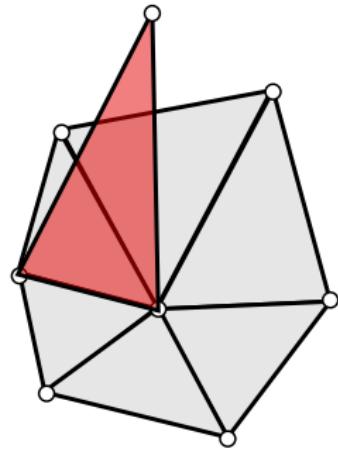


closed fan



open fan

Example: Non-manifold meshes



Shape representation: Point cloud

A **point cloud** is a collection of points in 3D space.

Shape representation: Point cloud

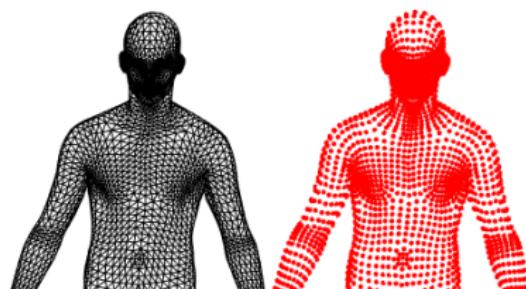
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- An **oriented** point cloud also has a normal vector for each point

Shape representation: Point cloud

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- Point clouds are interpreted as point-wise **samplings** of an underlying **unknown** continuous surface...

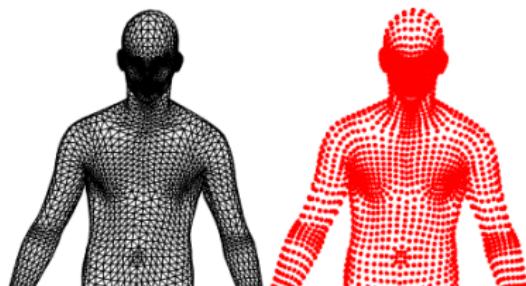


Synthetic point cloud
(obtained by removing mesh connectivity)

Shape representation: Point cloud

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- An **oriented** point cloud also has a normal vector for each point
- Point clouds are interpreted as point-wise **samplings** of an underlying **unknown** continuous surface...
- ...in practice, they come from depth sensors and can be **very noisy!**



Synthetic point cloud
(obtained by removing mesh connectivity)



Real-world Kinect scan

Exercise: Triangle mesh data structure

Create a data structure to represent a generic triangle mesh. The data structure must contain the following information:

- A collection of (x, y, z) coordinates for all the triangle vertices;
Example: A matrix of size $n \times 3$, where n is the total number of vertices
- A collection of triangles (v_1, v_2, v_3) ; each v_1, v_2, v_3 is an index to the set of vertices;
Example: A matrix of size $m \times 3$, where m is the total number of triangles

Additionally, write the following functions:

- `read_off ()`: reads from disk a triangle mesh in .off format
- `write_off ()`: writes to disk a given triangle mesh in .off format
- `calc_tri_areas ()`: computes the area for each triangle in the mesh
- `calc_boundary_edges ()`: identifies the boundary edges (if any) of the mesh

Test your code by loading and visualizing `cat0.off` (download from the course website)

Exercise: Point cloud data structure

Create a data structure to represent a generic point cloud. The data structure must contain the following information:

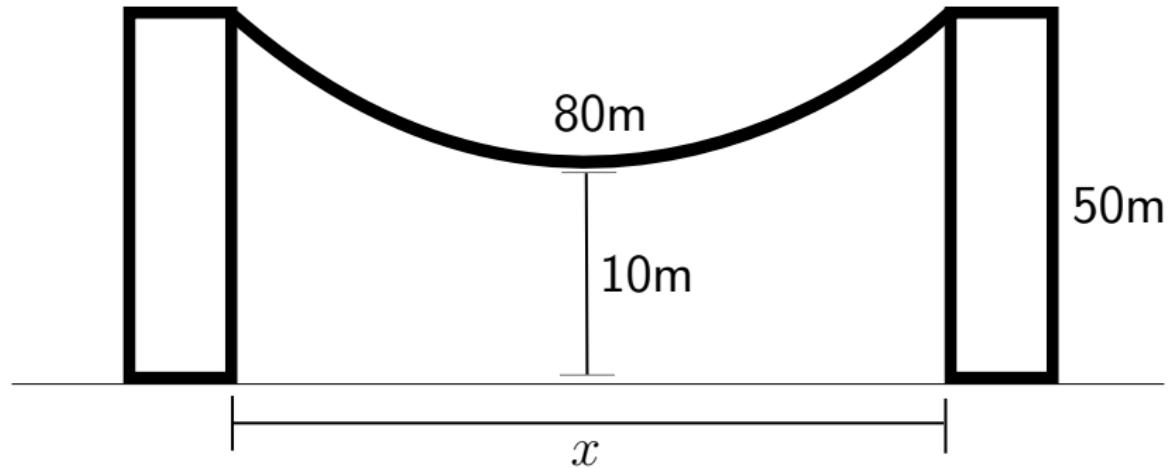
- A collection of (x, y, z) coordinates for all the point vertices;

Example: A matrix of size $n \times 3$, where n is the total number of vertices

Test your code by encoding the mesh from the previous example simply as a point cloud, and by visualizing it

Teaser exercise: Metric spaces

Consider the following design for a bridge:



What is x ?