Reinforcement Learning

Introduction

Antonio Ricciardi, PhD

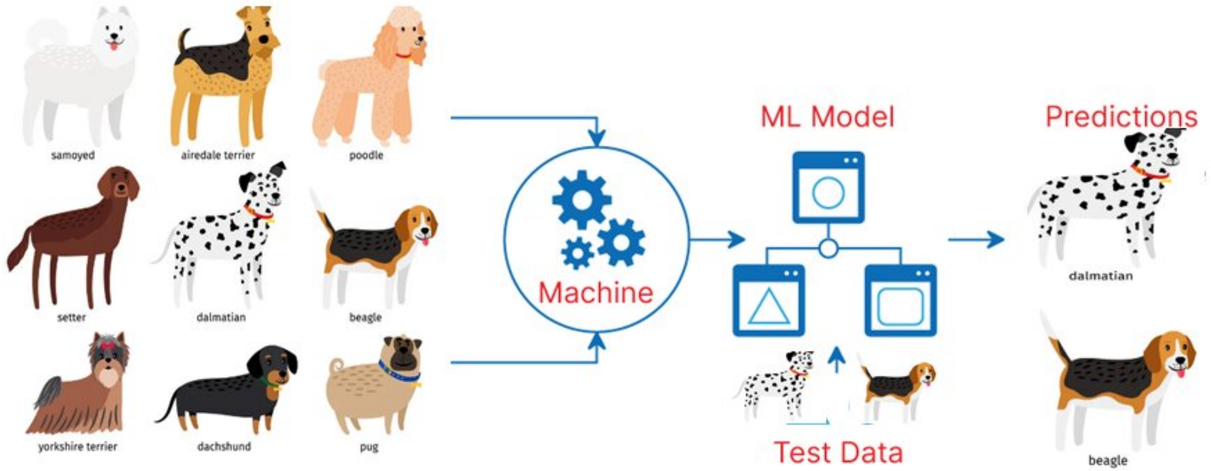
Machine Learning @ Mathematical Sciences for AI

Supervised Learning vs Reinforcement Learning

Supervised Learning

Learn using labeled data

Need pairs $\{x_i, y_i\}$



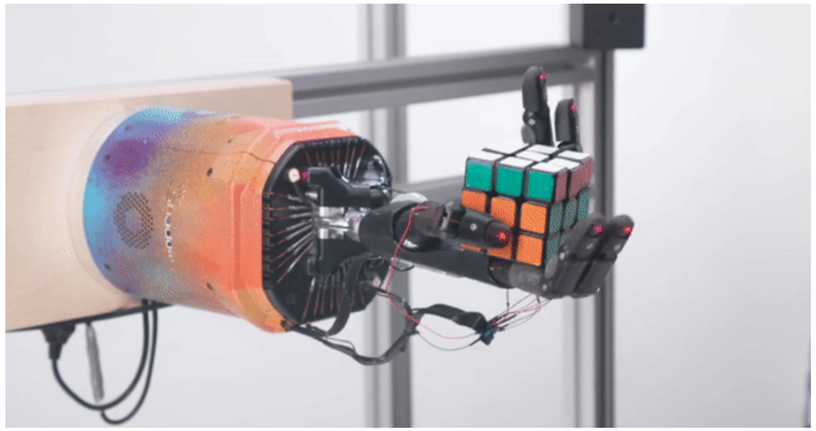
Reinforcement Learning

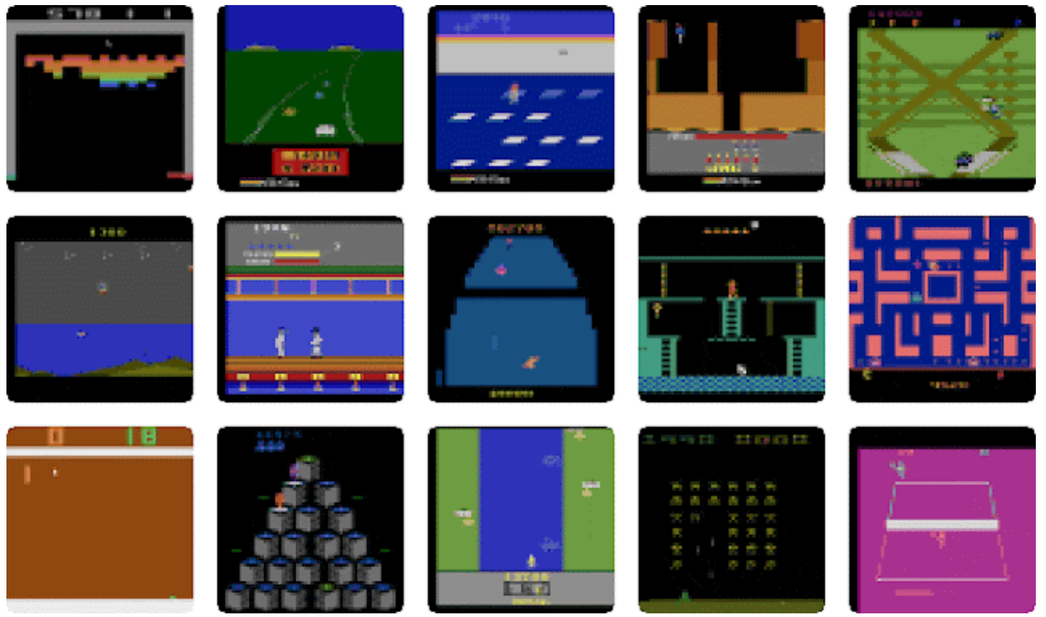
Framework for solving control tasks

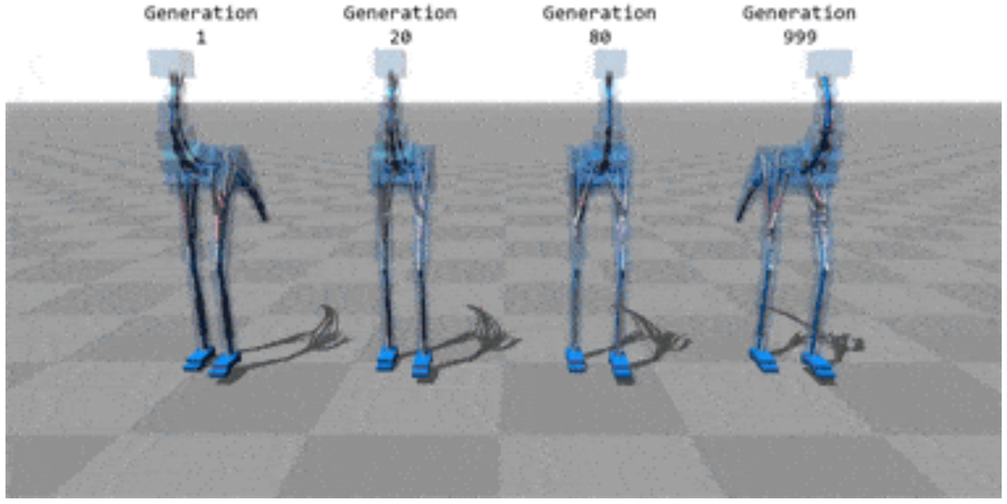
Learn from the environment through trial and error

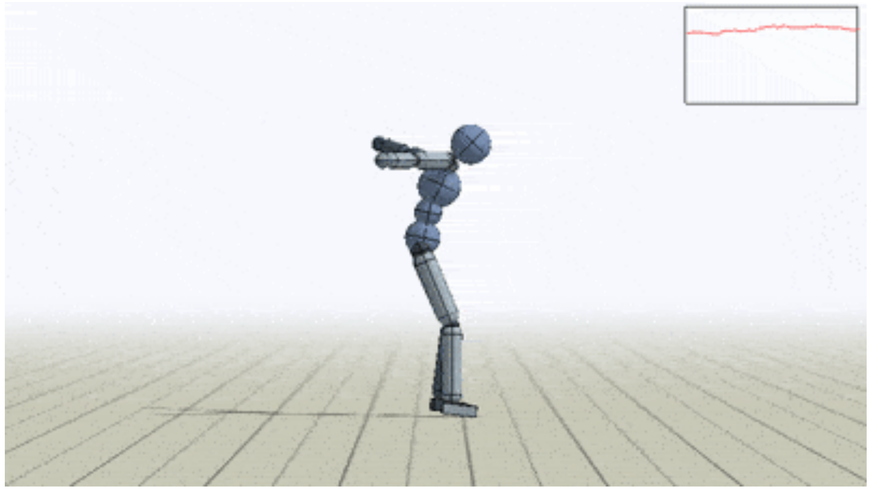
Receive rewards as feedback

Reinforcement Learning









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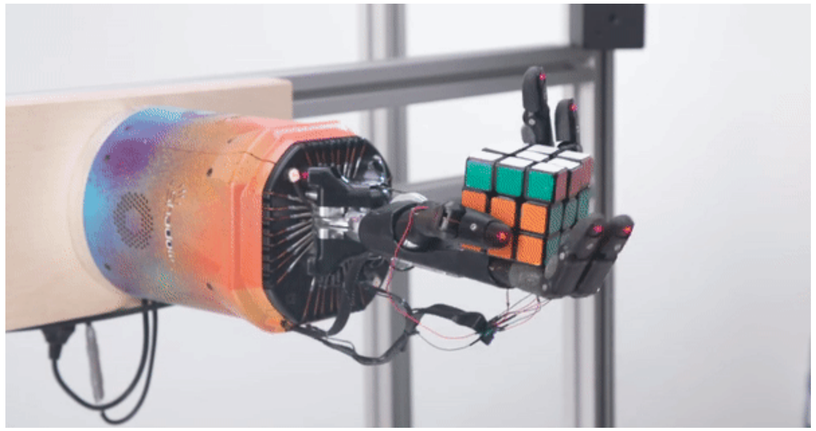

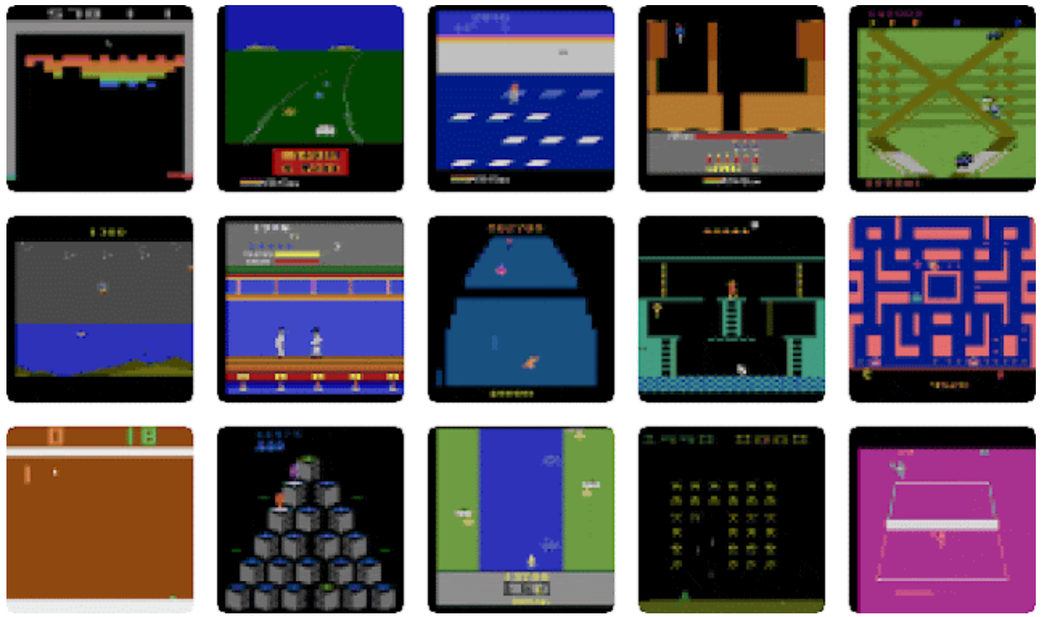

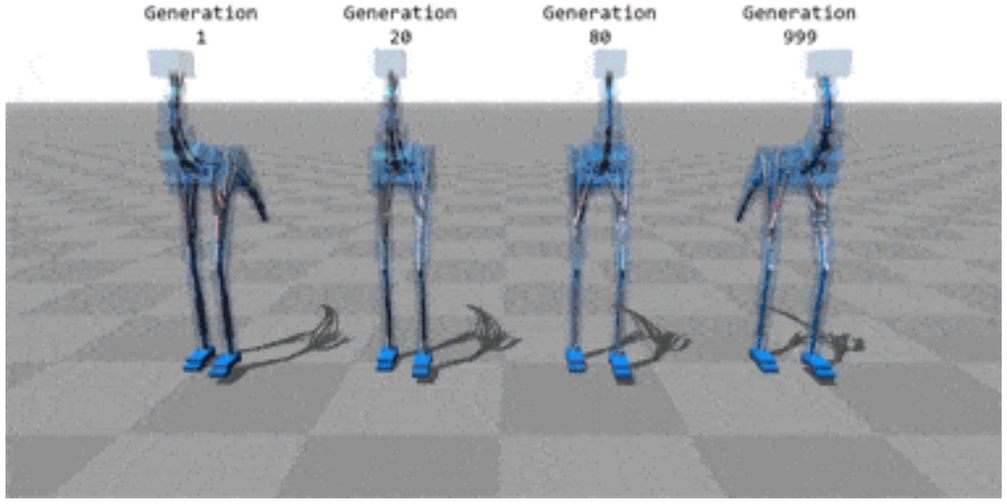

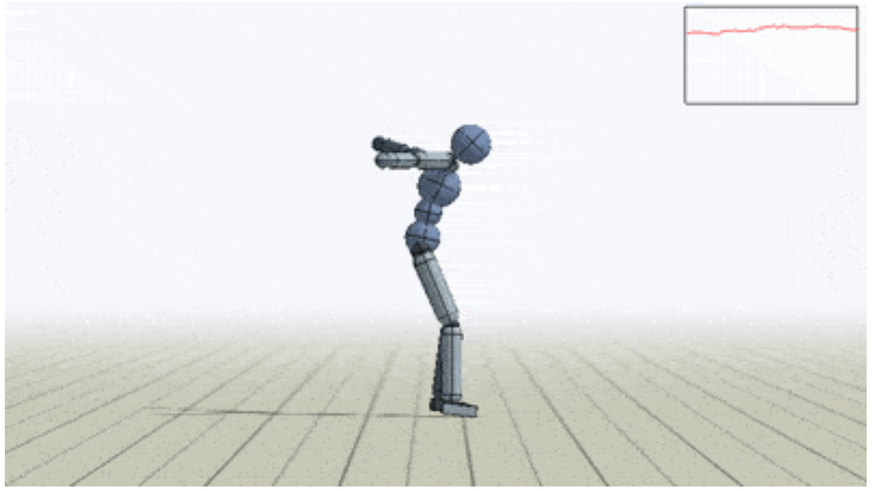

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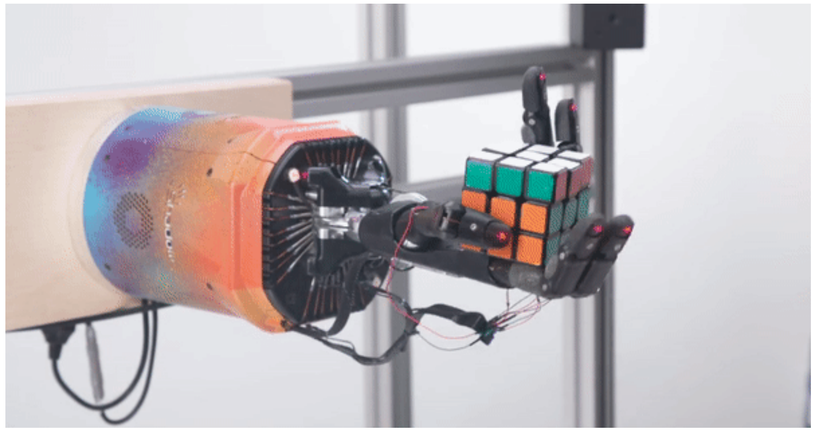

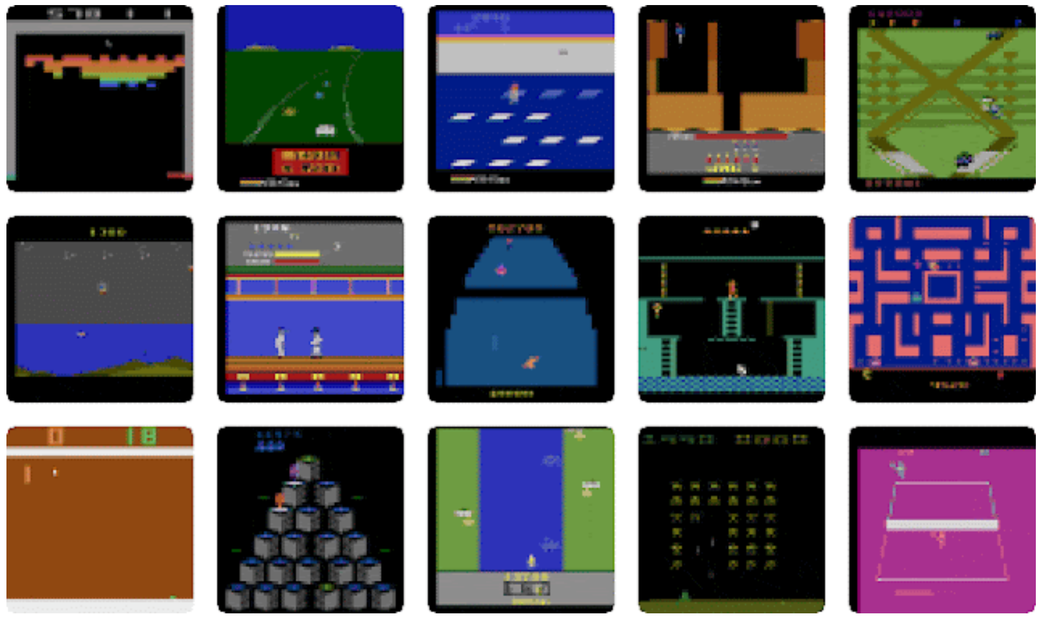

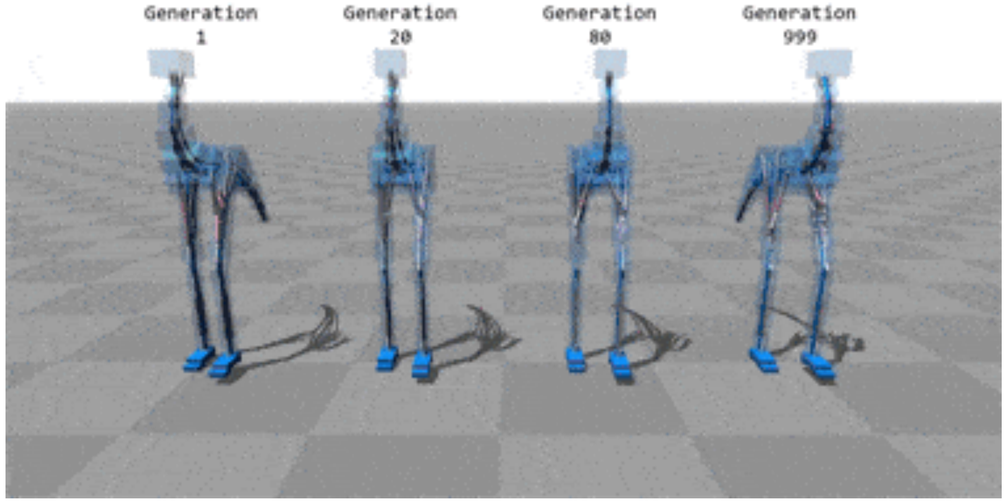

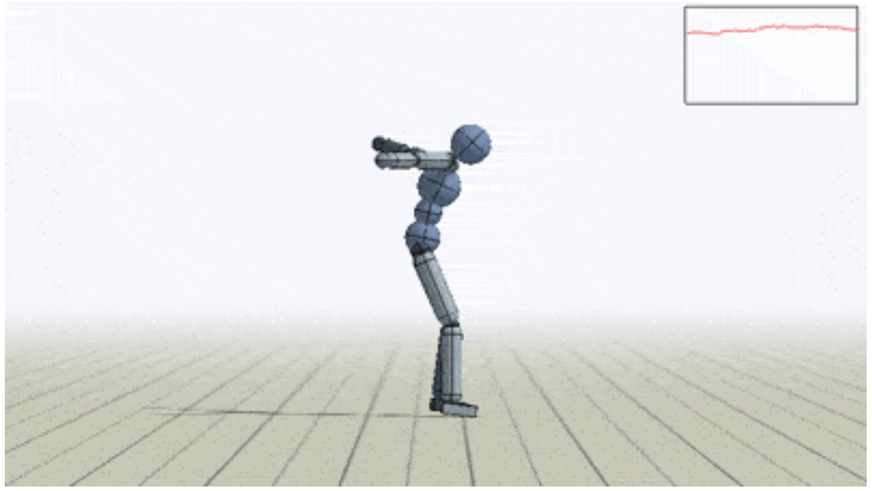

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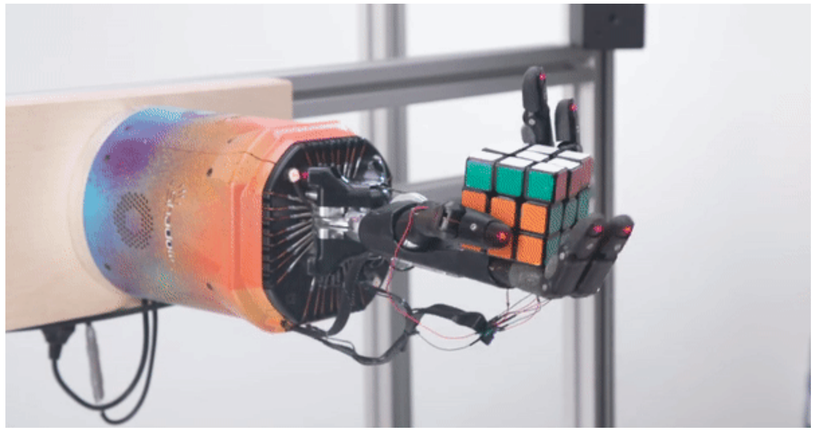

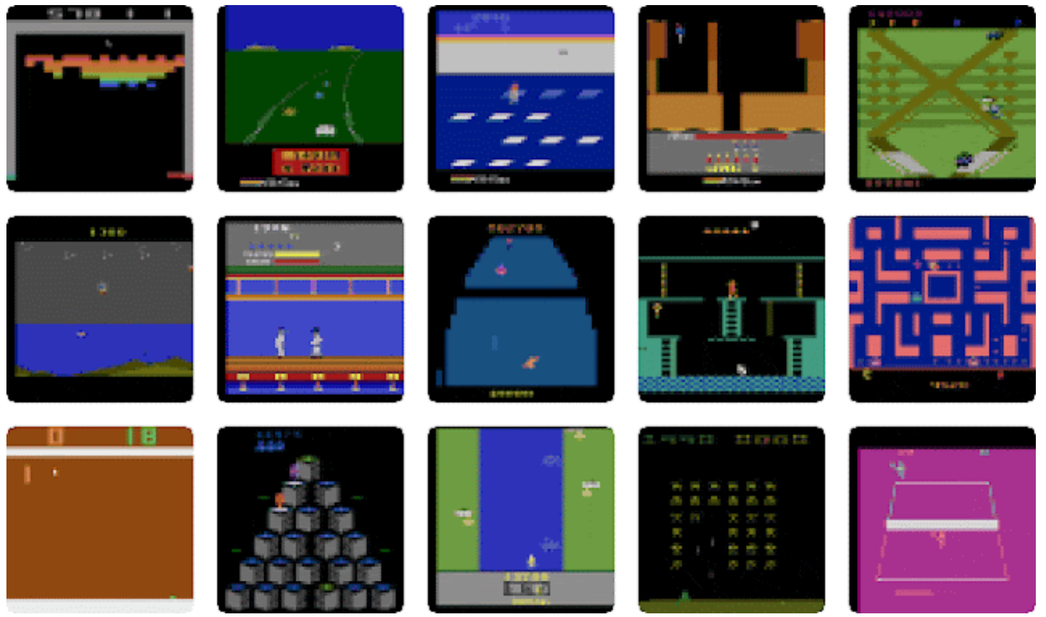

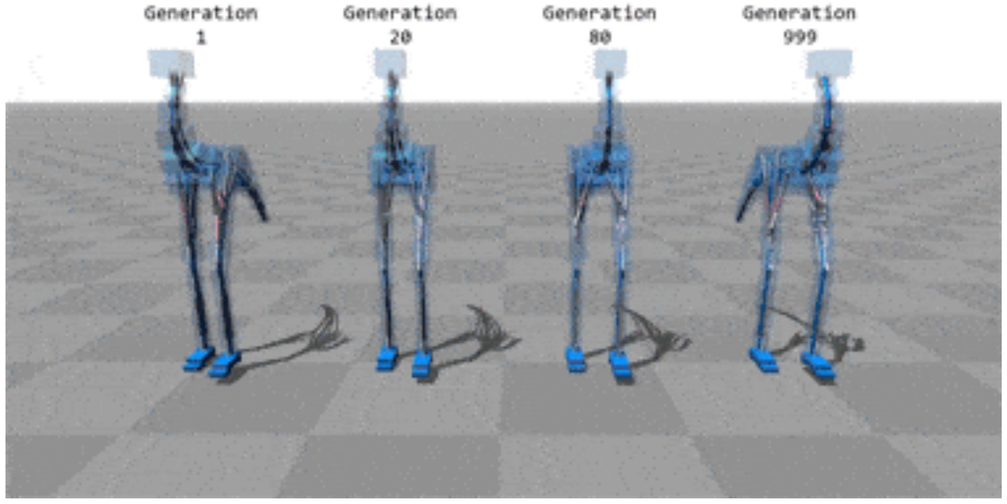

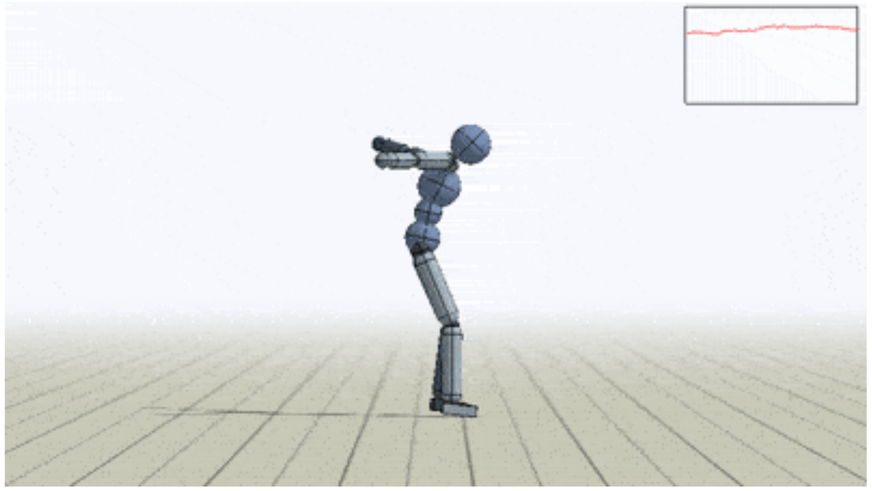

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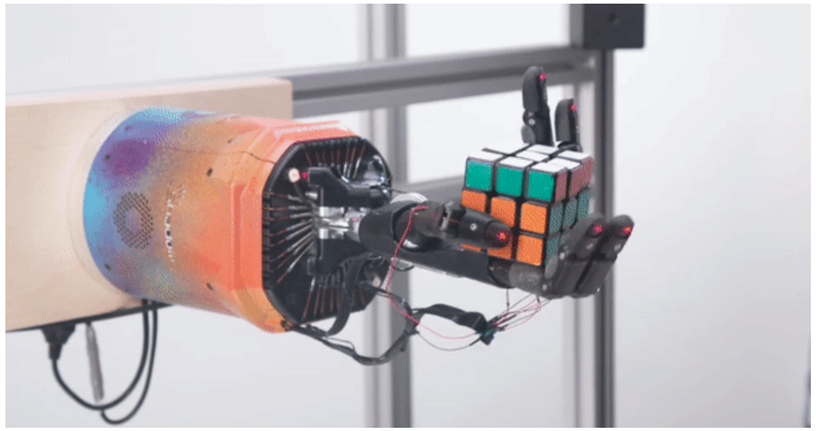

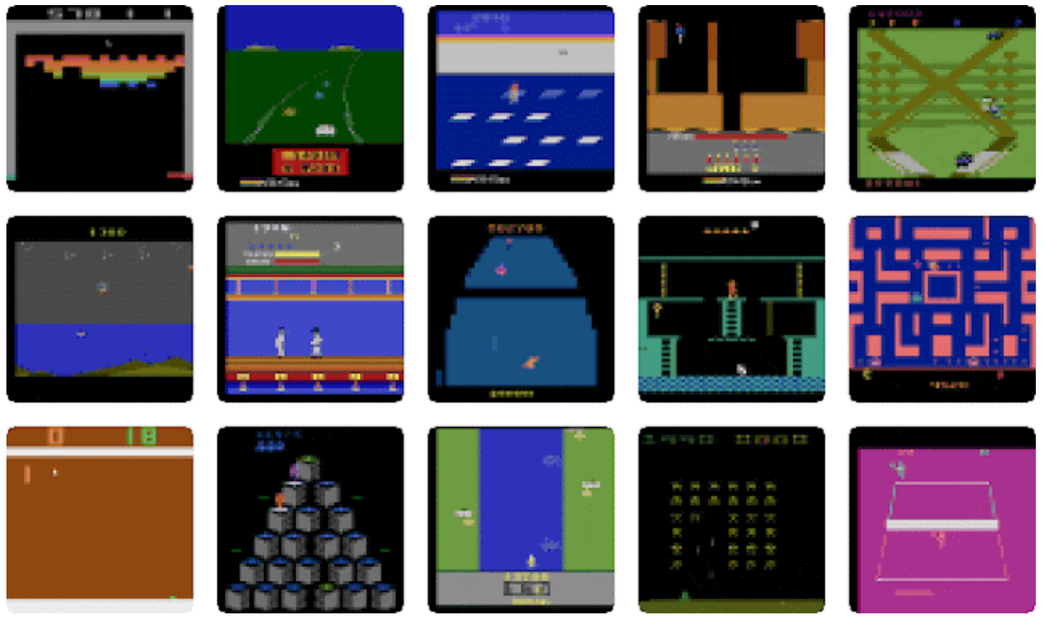

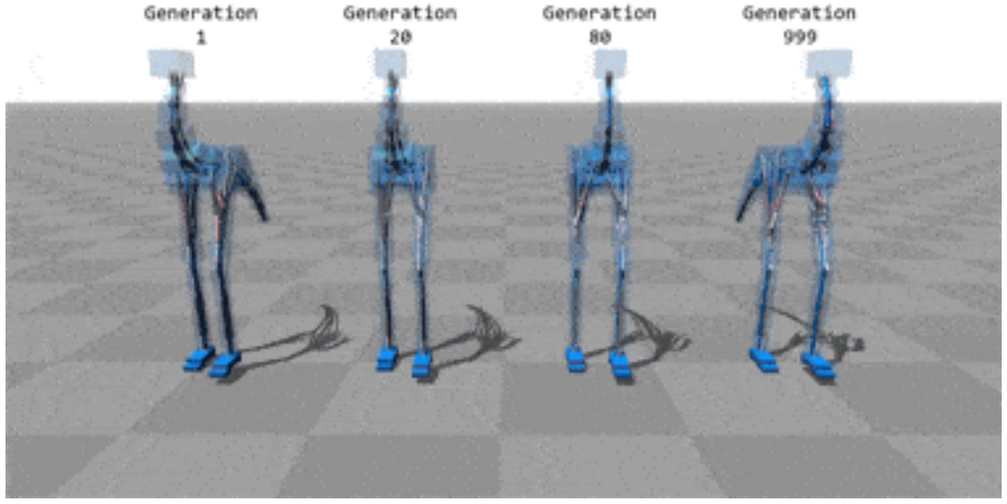

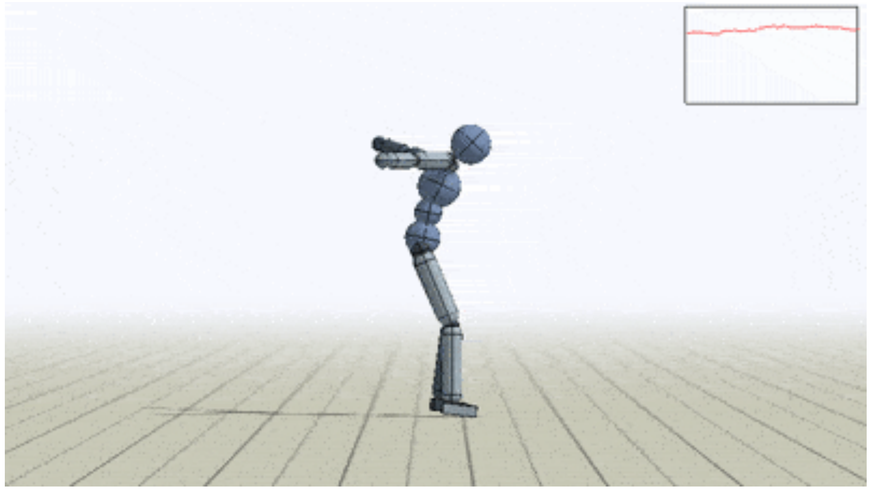

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Reinforcement Learning

https://www.youtube.com/watch?v=PxoG0A2QoFs

Reinforcement Learning

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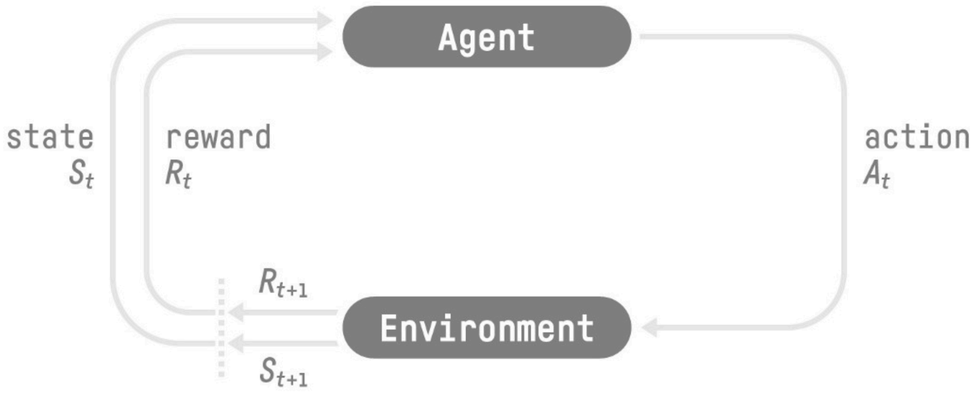
What you will learn

Basic concepts of RL

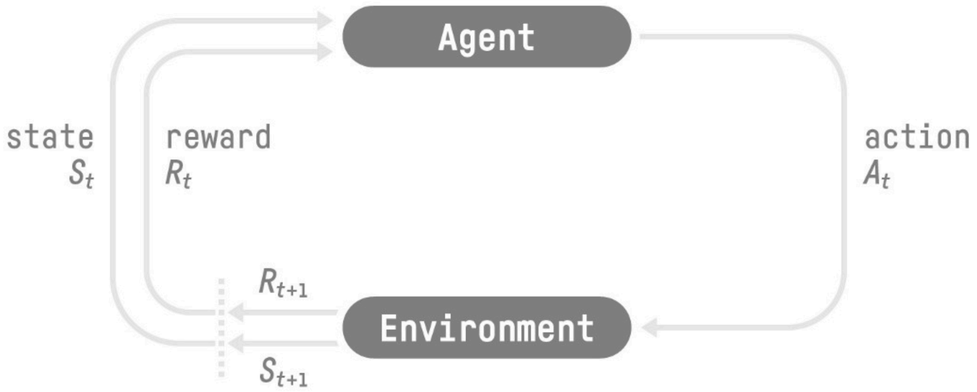
Q-Learning

Deep Q-Learning

Markov Decision Process (MDP) (finite)

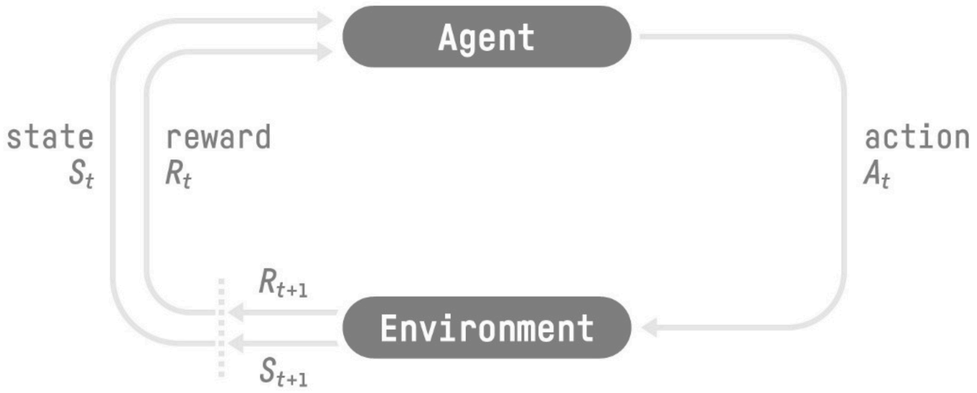


Markov Decision Process (MDP) (finite)



For t = 0, 1, 2, 3 …

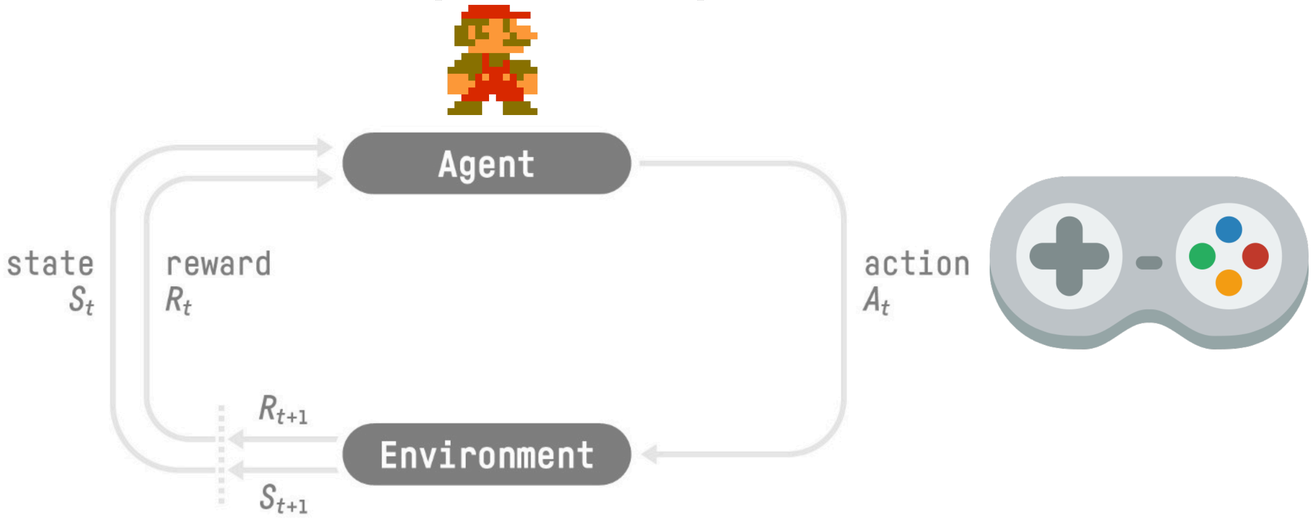
Markov Decision Process (MDP) (finite)



For t = 0, 1, 2, 3 …

Trajectory: $S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$

Markov Decision Process (MDP)



For t = 0, 1, 2, 3 …

Paradigm

Paradigm

S_0

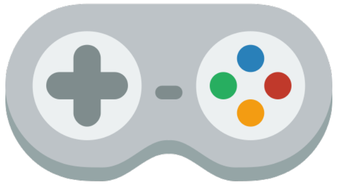
Paradigm

S_0

Policy π

Paradigm

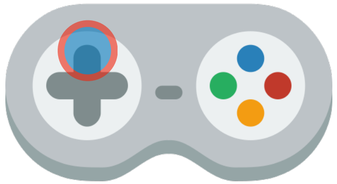
S_0



Policy π

Paradigm

S_0

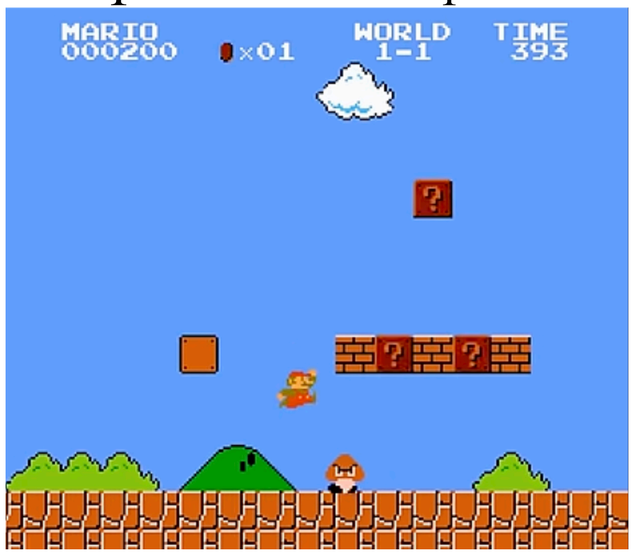
A_0 

Policy π

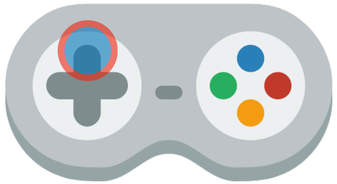
Paradigm

S_0

S_1 $r_1 = 0$



A_0



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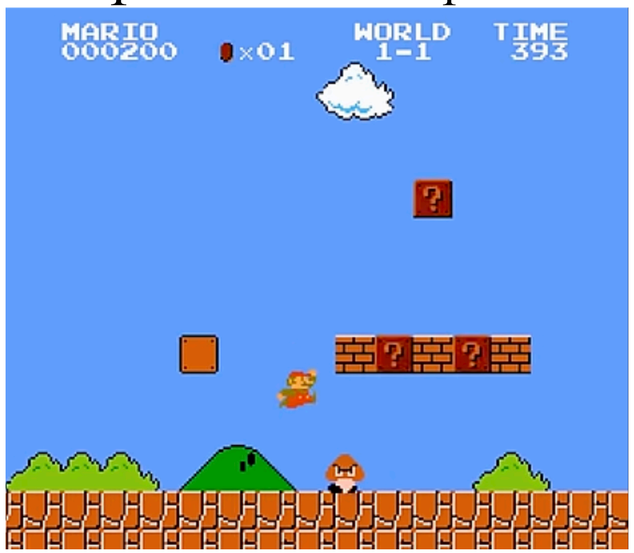
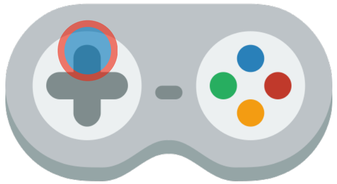
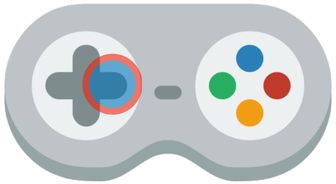


A_0



A_1

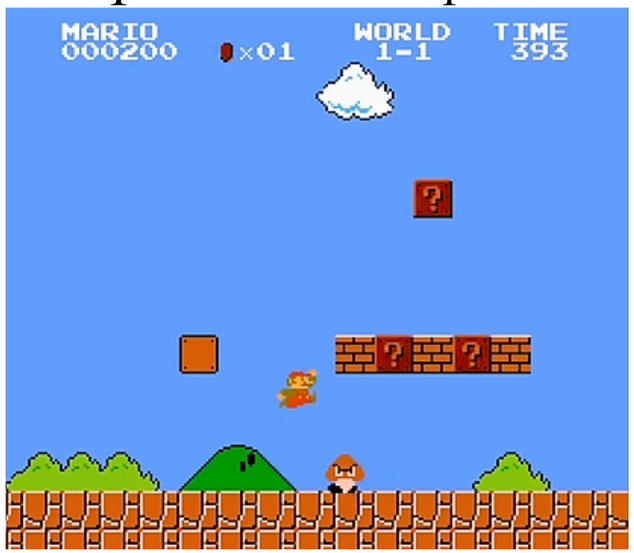


Policy π

Paradigm

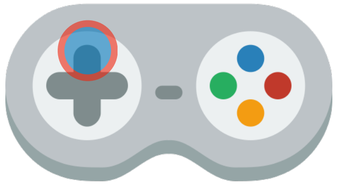
S_0

S_1 $r_1 = 0$

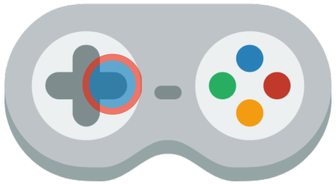


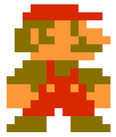
S_2 $r_2 = 100$

A_0



A_1





Policy π

Paradigm

S_0

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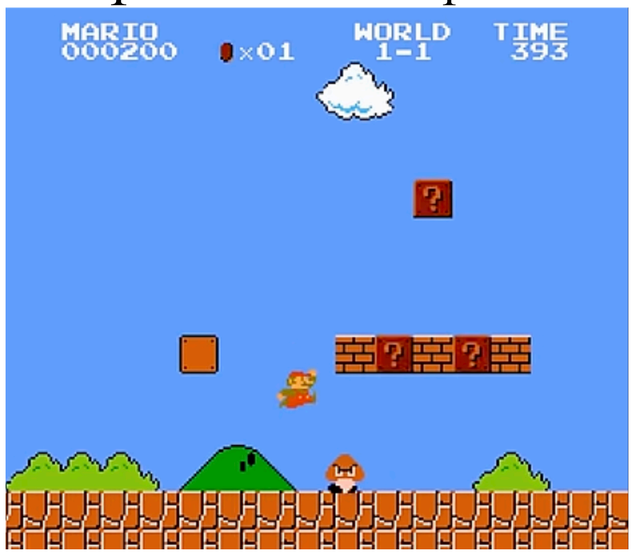
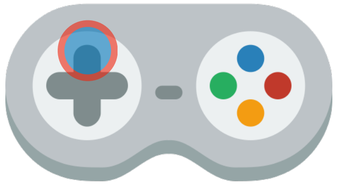
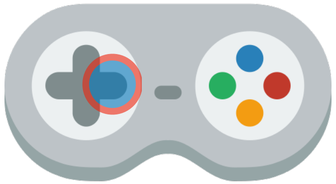
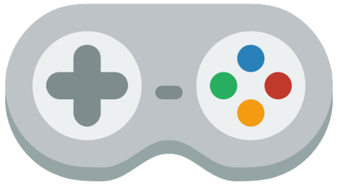


S_2 $r_2 = 100$

A_0 

A_1 

A_2 

Policy π

Agent Inputs

Agent Inputs

How do agents get information from the environment?

Agent Inputs

How do agents get information from the environment?

States

Agent Inputs

How do agents get information from the environment?

States

Observations

Agent Inputs

How do agents get information from the environment?

States

Complete description of the state of the world
(no hidden information)

Observations

Agent Inputs

How do agents get information from the environment?

States

Complete description of the state of the world
(no hidden information)

Observations

Agent Inputs

How do agents get information from the environment?

States

Complete description of the state of the world
(no hidden information)

Observations

Partial description of the state of the world

Agent Inputs

How do agents get information from the environment?

States

Complete description of the state of the world
(no hidden information)

Observations

Partial description of the state of the world

Agent Outputs

Agent Outputs

How do agents interact with the environment?

Agent Outputs

How do agents interact with the environment?

Action space

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

Discrete

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

Discrete

Continuous

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

Discrete

The number of possible actions is *finite*

Continuous

Agent Outputs

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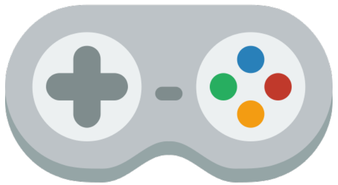

Continuous

Agent Outputs

How do agents interact with the environment?

Action space Set of all possible actions in an environment.

Discrete

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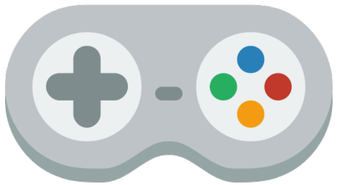

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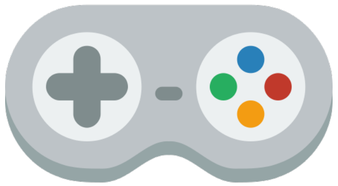

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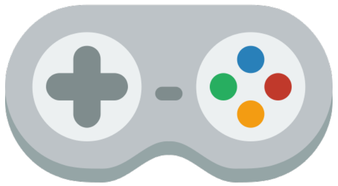

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$$G_t = R_{t+1} + R_{t+2} + R_{t+3}, \ldots + R_T$$

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Continuing tasks

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$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

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RL involves evaluating value functions

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Functions of states or state-action pairs

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RL is about changing the policy as a result of its experience

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Value function of state s under policy π

Expected return when starting in state s and following policy π

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$$q_\pi(s, a) \doteq \mathbb{E}_\pi[G_t \,|\, S_t = s, A_t = a] = \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,|\, S_t = s, A_t = a\Big], \quad \text{for all } s \in \mathcal{S}$$

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v_π and q_π can be estimated from experience

Q-Learning

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One of the first major *online* RL algorithms

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Bellman Equation for Q-Learning

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$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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Its behaviour policy can be anything that continues to visit all state actions pairs during learning (e.g. ε — greedy)

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
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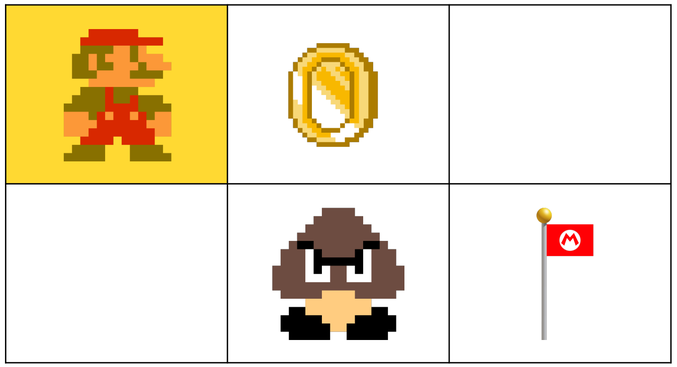
Bellman equation for state-action values

What is $Q(S, A)$?

Encode Q-values in a table, each cell corresponds to a state-action pair value.

Q-Learning example



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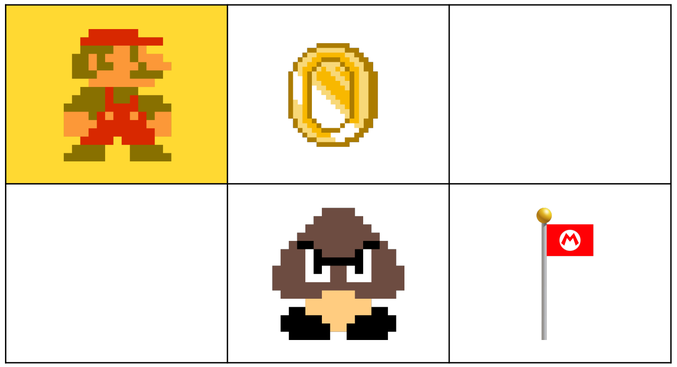
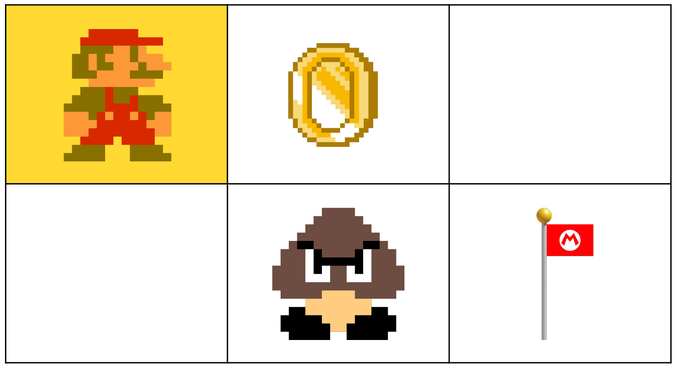


Policy π

Q-Learning example



Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

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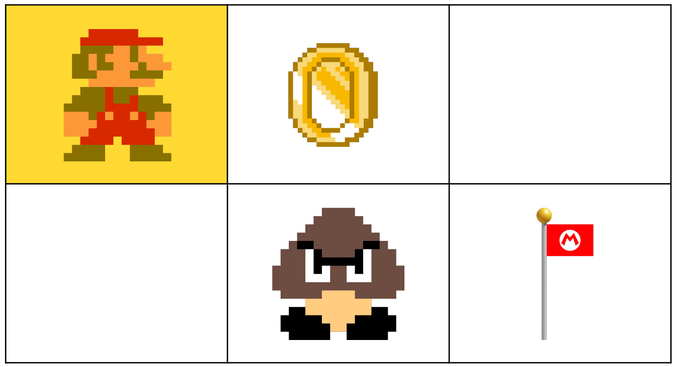

Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

+1

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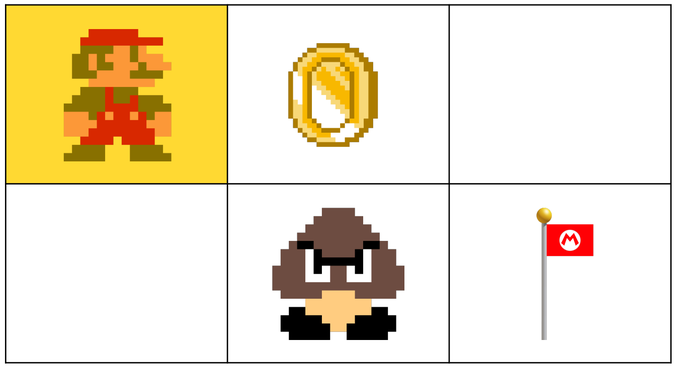

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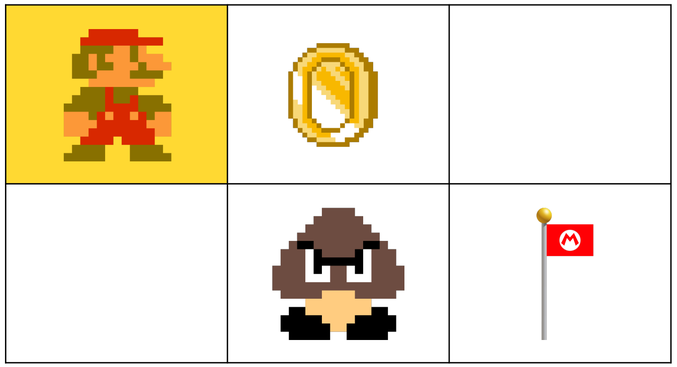
+1

-10

Q-Learning example



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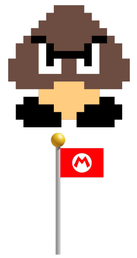
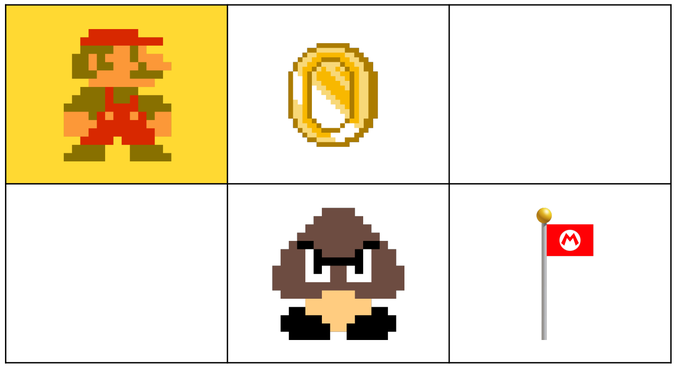


-10

+10
(terminal)

Q-Learning example



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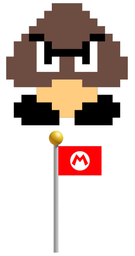

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$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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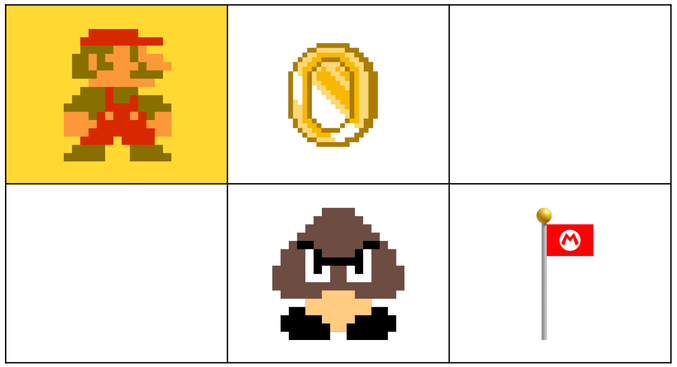

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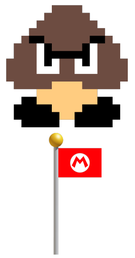

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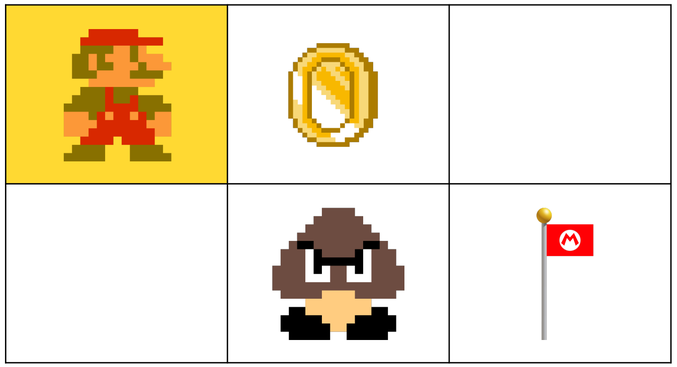

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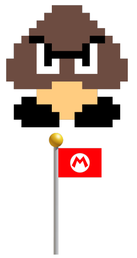

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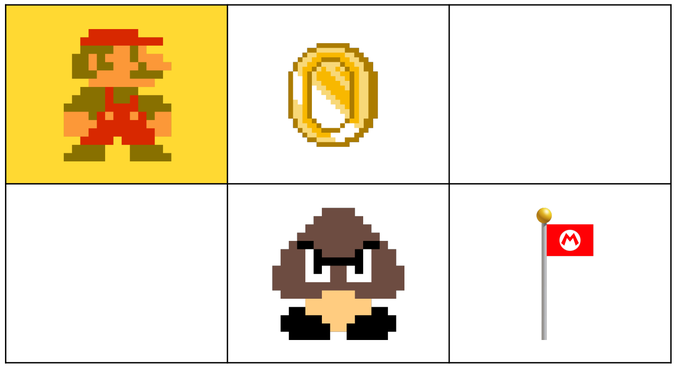
How to represent the Q-Table?

Q-Learning example



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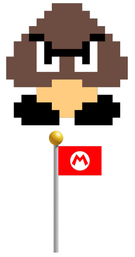

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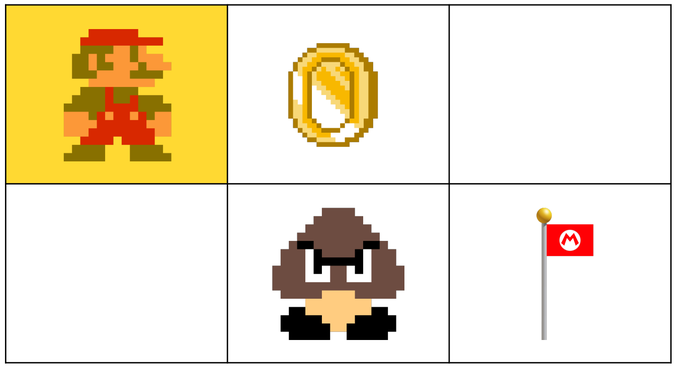
	←	→	↑	↓
				
				
				

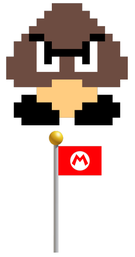
Q-Learning example



Policy π

Actions $\uparrow \downarrow \leftarrow \rightarrow$

+1



-10

+10
(terminal)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

$$\alpha = 0.1$$

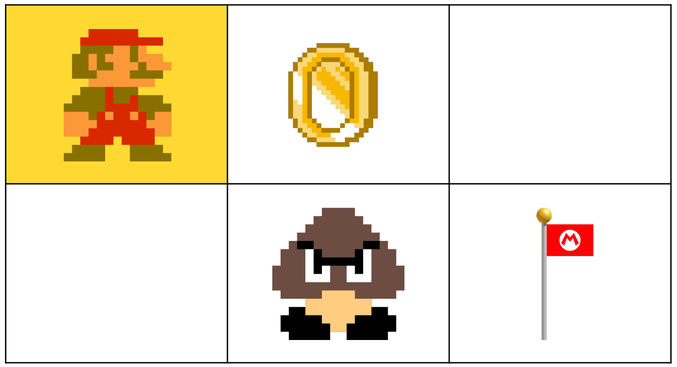
$$\gamma = 0.99$$

How to represent the Q-Table?

	←	→	↑	↓
				
				
				

How to move?

Q-Learning example



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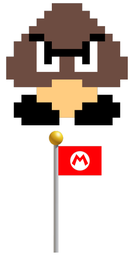
Actions $\uparrow \downarrow \leftarrow \rightarrow$

+1



-10

+10
(terminal)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

$$\alpha = 0.1$$

$$\gamma = 0.99$$

How to represent the Q-Table?

	←	→	↑	↓
				
				
				

How to move?

$$\pi = \text{argmax}_A Q(S, A)$$

Q-Learning example

Q-Learning example

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Q-Learning example

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

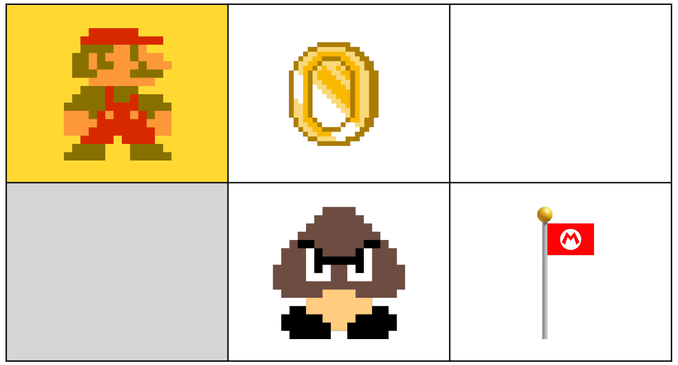
	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Q-Learning example

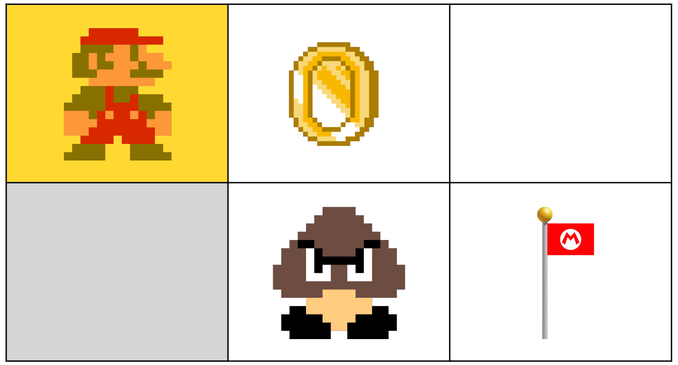
	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

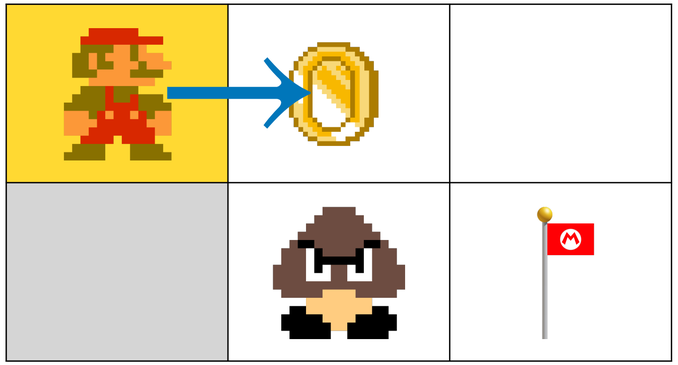
Choose A from S using policy derived from Q (e.g., ε-greedy)



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

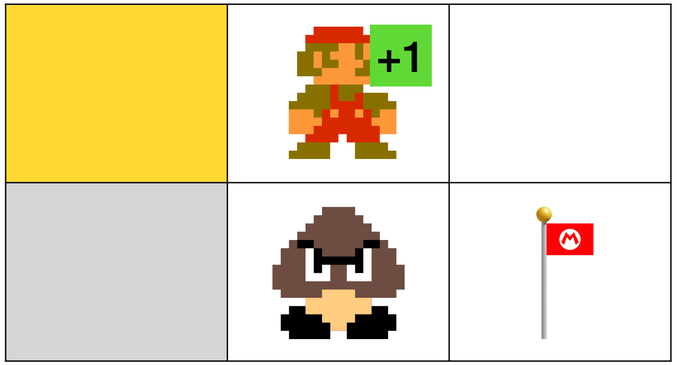
Choose A from S using policy derived from Q (e.g., ε-greedy)



Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'

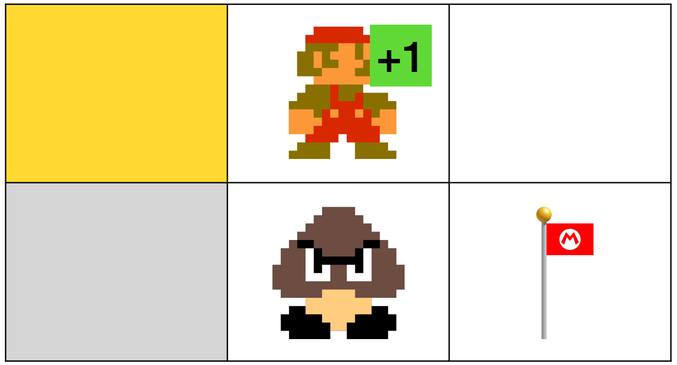


Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

Take action A, observe R, S'



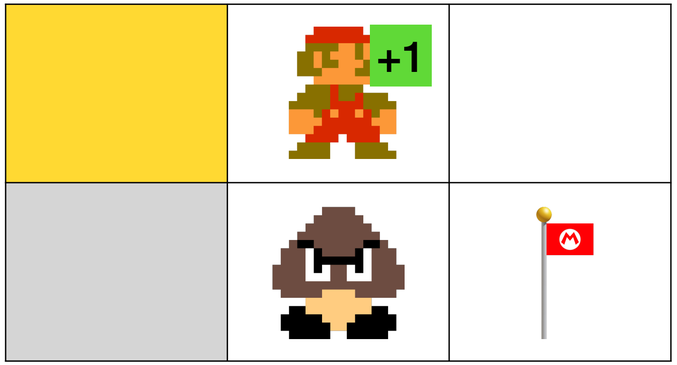
Q-Learning example

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Take action A, observe R, S'



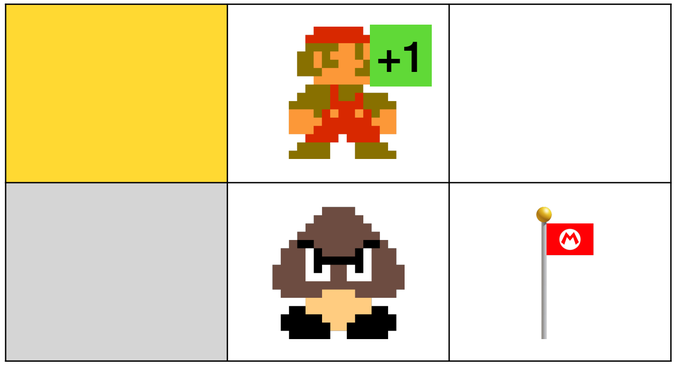
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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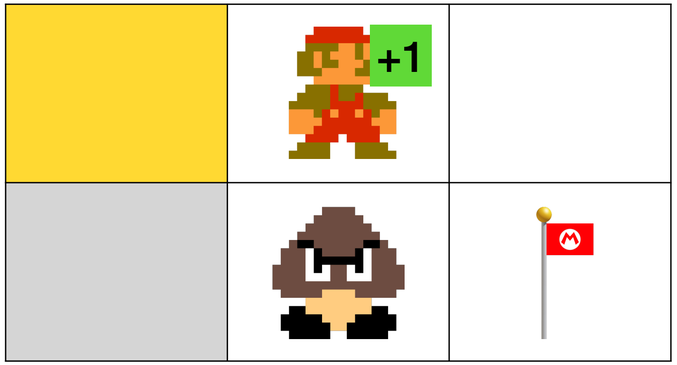
$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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Q(initial, →) = 0 + 0.1*[1 + 0.99*0 - 0] = 0.1

Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

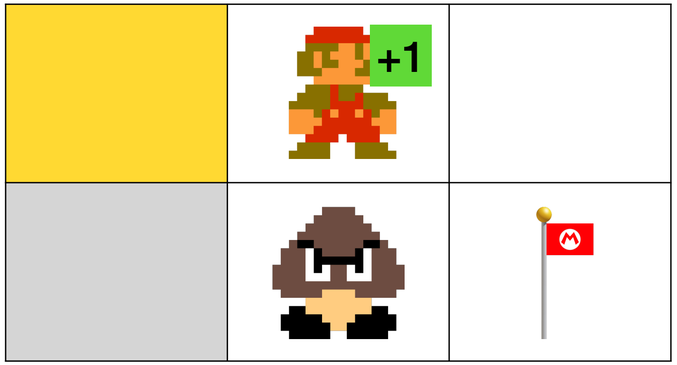
	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0 + 0.1*[1 + 0.99*0 - 0] = 0.1

Take action A, observe R, S'



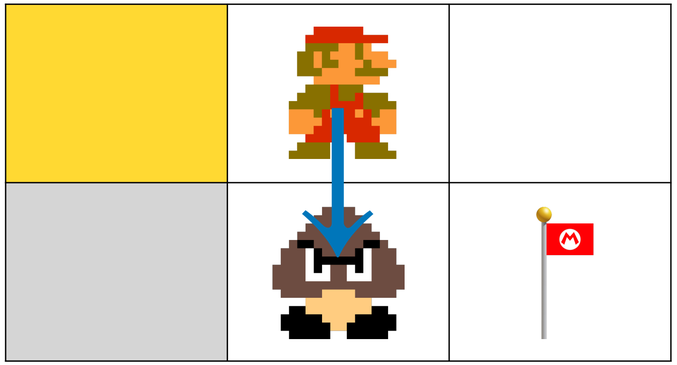
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



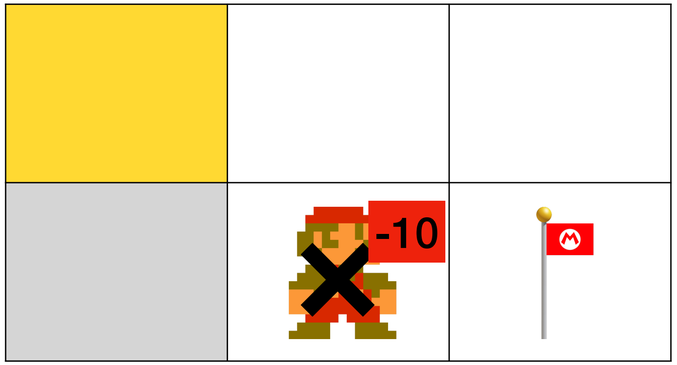
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	$\leftarrow$	$\rightarrow$	$\uparrow$	$\downarrow$
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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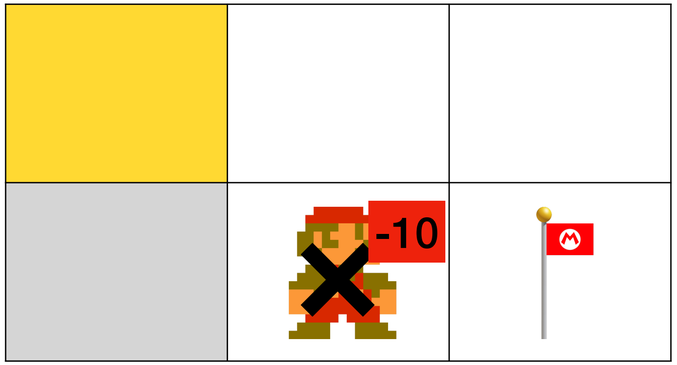

Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

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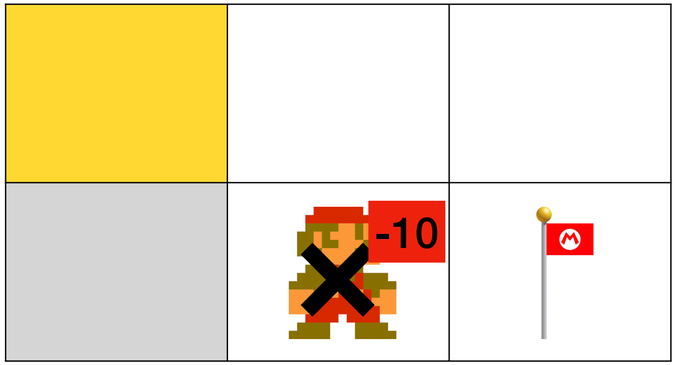
Q(state_2, ↓) = 0 + 0.1*[-10 + 0.99*0 - 0] = -1

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

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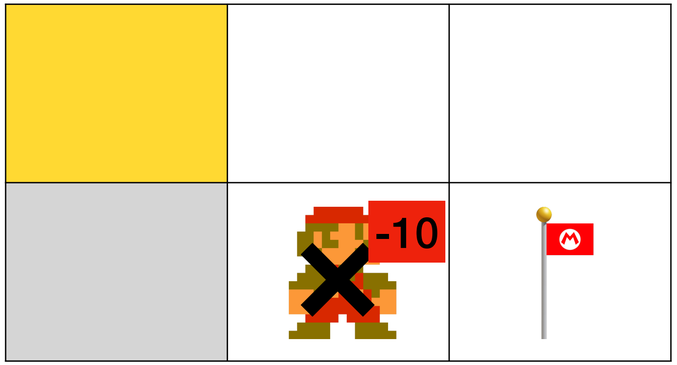
Take action A, observe R, S'



Q-Learning example

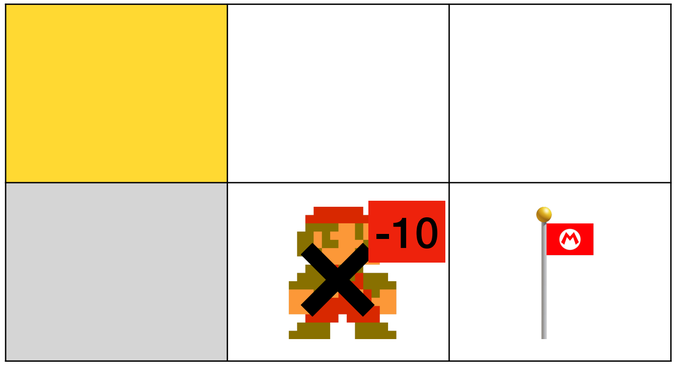
$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Terminal state, Episode resets

Take action A, observe R, S'



Q-Learning example

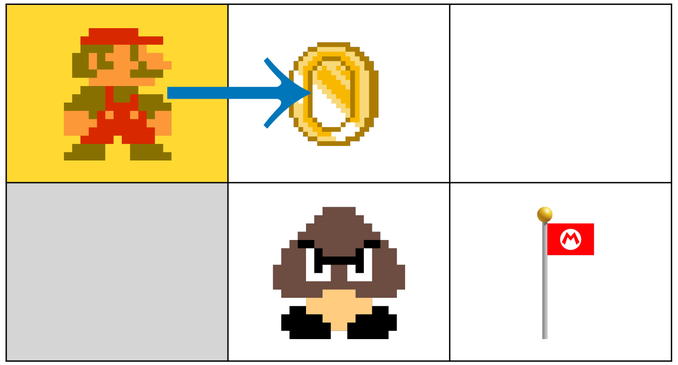
$\alpha = 0.1$

$\gamma = 0.99$

New episode

	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

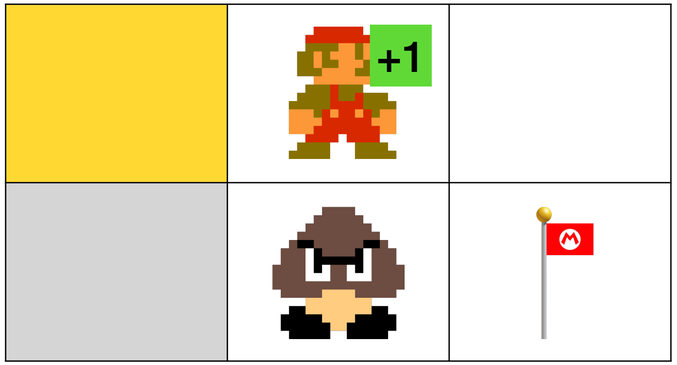
	←	→	↑	↓
	0	0.1	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0.1 + 0.1*[1 + 0.99*0 - 0] = 0.2

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

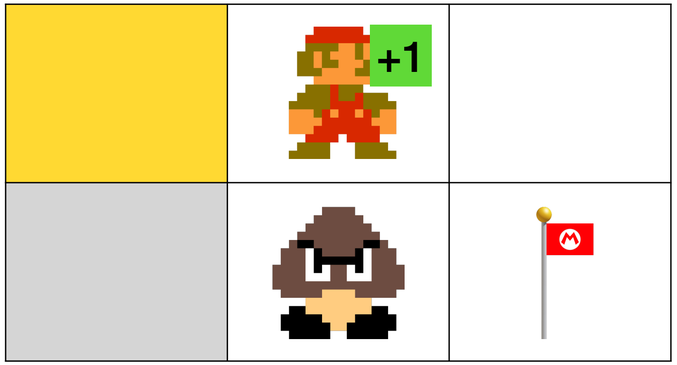
	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(initial, →) = 0.1 + 0.1*[1 + 0.99*0 - 0] = 0.2

Take action A, observe R, S'



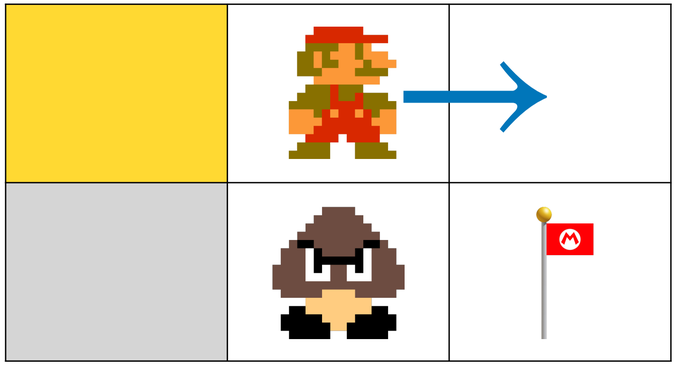
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



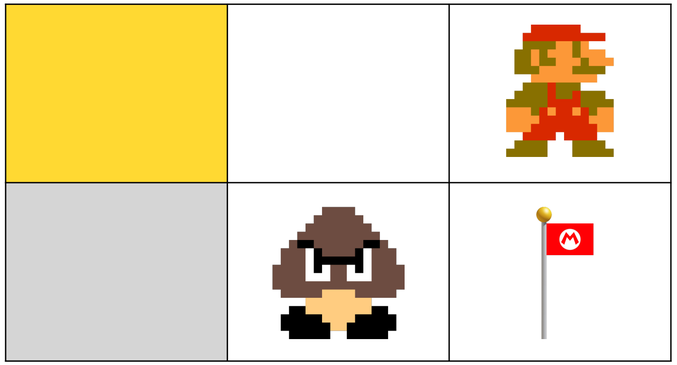
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

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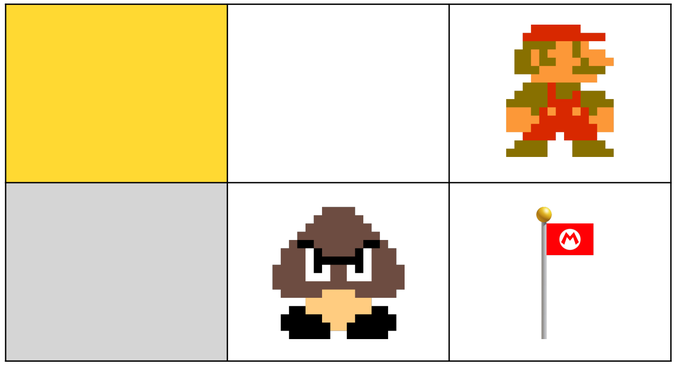

Q-Learning example

$\alpha = 0.1$

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	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

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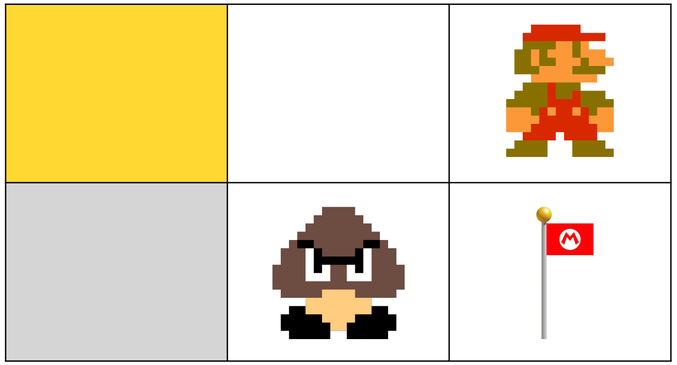
Take action A, observe R, S'



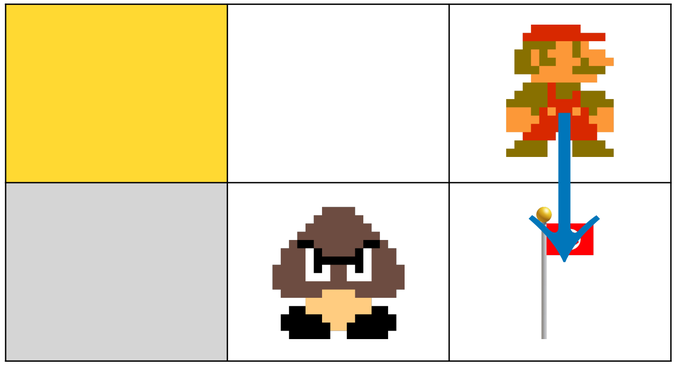
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Choose A from S using policy derived from Q (e.g., ε-greedy)



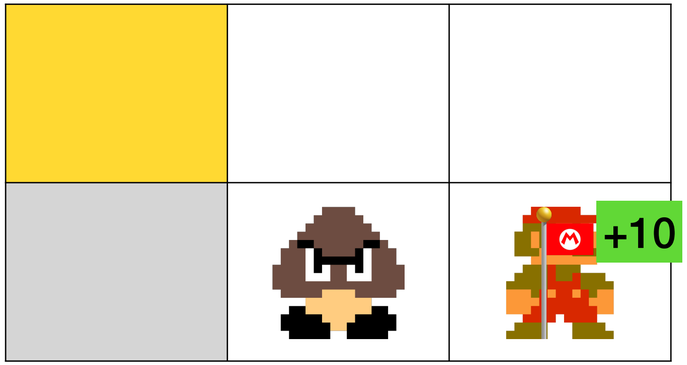
Q-Learning example

$\alpha = 0.1$

$\gamma = 0.99$

	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Take action A, observe R, S'



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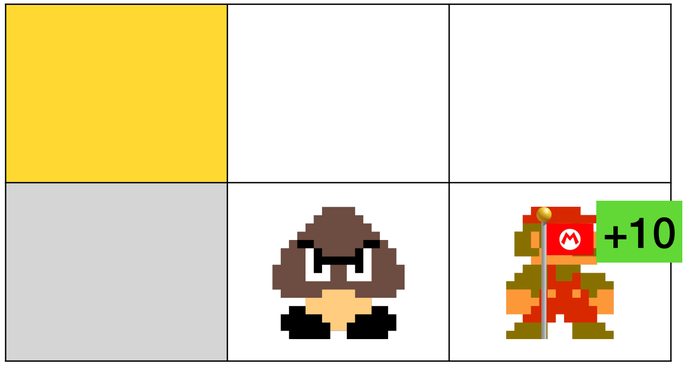
	←	→	↑	↓
	0	0.2	0	0
	0	0	0	-1
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

Update $Q(S, A)$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Q(, ↓) = 0 + 0.1*[10 + 0.99*0 - 0] = 1

Take action A, observe R, S'



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	0	0	0	0
	0	0	0	0
	0	0	0	0

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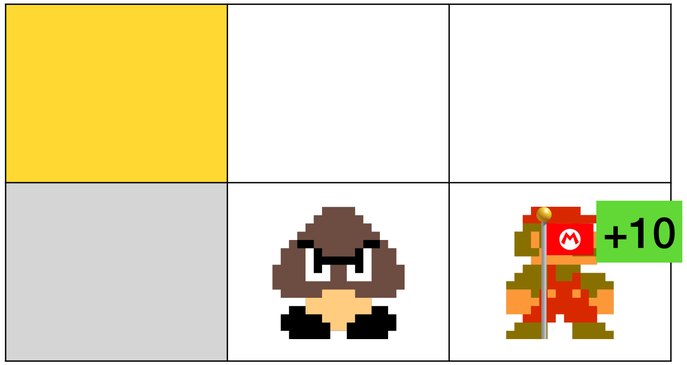

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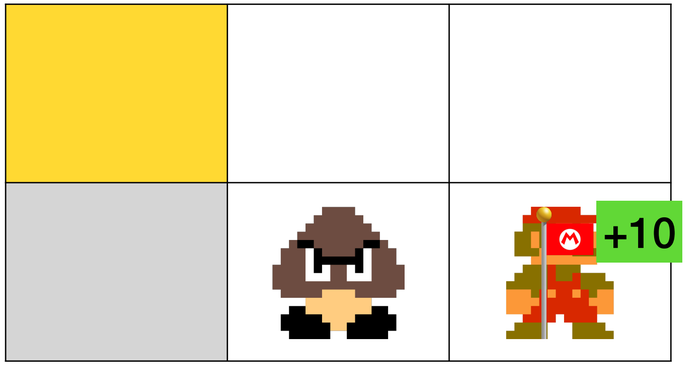

Episode won! Terminal state

Why ε-greedy?

Why ε-greedy?

Exploration-exploitation tradeoff

Why ε-greedy?

Exploration-exploitation tradeoff

- **Take greedy actions with probability $1 - \varepsilon$**

Why ε-greedy?

Exploration-exploitation tradeoff

Take greedy actions with probability $1 - \varepsilon$

Exploit what the agent already knows

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Explore with probability ε

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Decrease ε with time

Why ε-greedy?

Exploration-exploitation tradeoff

Take greedy actions with probability $1 - \varepsilon$

Exploit what the agent already knows

Explore with probability ε

Take random actions to visit new states you would not visit otherwise

Decrease ε with time

$0 \leq \varepsilon \leq 1$

Why ε-greedy?

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

If we take only greedy actions…

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-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

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We end up with a suboptimal solution

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

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Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

G = 10

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

If we explore…

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

If we explore…

We might discover more valuable states

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

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-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

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If we explore…

We might discover more valuable states

That lead to higher returns

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

If we explore…

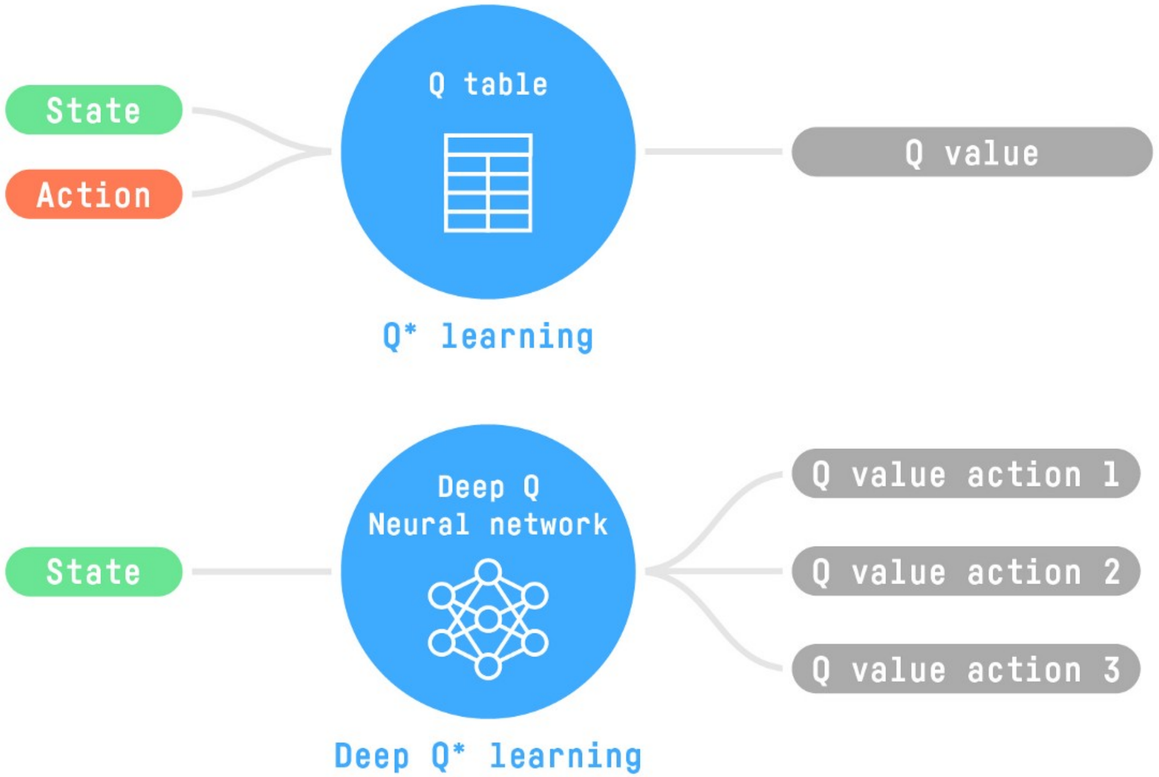
We might discover more valuable states

That lead to higher returns

G = 22

Start	0	0	0	0	0	0
-1		0				0
-1	1	0				0
0						0
0	10	1	1	1	1	Goal (10)

Deep Q-Learning


State
Action
Q table
Q value
Q* learning
State
Deep Q
Neural network
Q value action 1
Q value action 2
Q value action 3
Deep Q* learning

Deep Q-Learning

Deep Q-Learning

Q-Learning works well for small state spaces

Deep Q-Learning

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Atari games, however, have observation space of shape (210, 160, 3), values in [0, 255]

Deep Q-Learning

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This gives us $256^{210 \times 160 \times 3} = 256^{100800}$ possible observations

Deep Q-Learning

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Approximate Q-values using a neural network

Deep Q-Learning

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Approximate Q-values using a neural network

Parametrized Q-function $Q_\theta(s, a)$

Deep Q-Learning

CONVOLUTIONAL LAYERS

FULLY CONNECTED LAYERS

OUTPUT LAYER

Deep Q-Learning

CONVOLUTIONAL LAYERS

FULLY CONNECTED LAYERS

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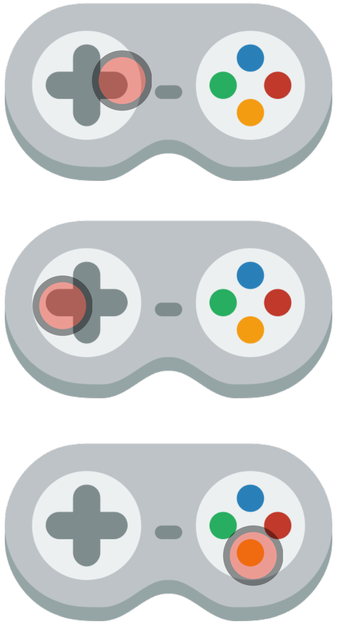


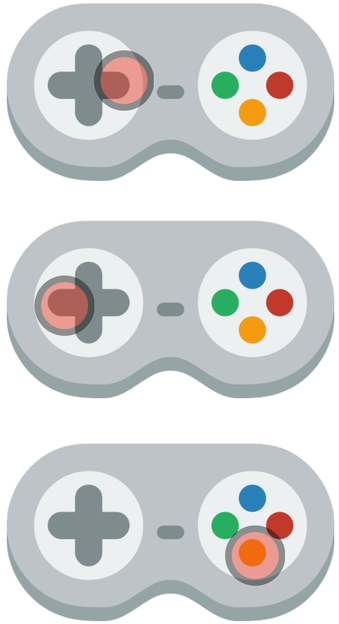
Deep Q-Learning

CONVOLUTIONAL LAYERS

FULLY CONNECTED LAYERS

OUTPUT LAYER

Q-Values



Deep Q-Learning algorithm

For episode = 1, M **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1$,T **do**

With probability ε select a random action a_t

otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $\left(\phi_t, a_t, r_t, \phi_{t+1}\right)$ in D

Sample random minibatch of transitions $\left(\phi_j, a_j, r_j, \phi_{j+1}\right)$ from D

Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{otherwise} \end{cases}$

Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

End For

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ϕ represents the NN

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End For

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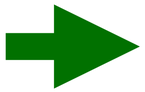

ϕ represents the NN

Sampling (interaction with the environment)

Deep Q-Learning algorithm

For episode = 1, M **do**

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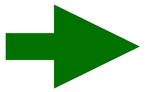
Every C steps reset $\hat{Q} = Q$

End For

End For



ϕ represents the NN

Sampling (interaction with the environment)

Training

Deep Q-Learning algorithm

Tabular Q-Learning

Deep Q-Learning

Deep Q-Learning algorithm

Tabular Q-Learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

Deep Q-Learning

Deep Q-Learning algorithm

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Deep Q-Learning

$$y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{for non-terminal } \phi_{j+1} \end{cases}$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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New
Q-value

Deep Q-Learning

$$y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{for non-terminal } \phi_{j+1} \end{cases}$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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New Q-value

Former Q-value

Deep Q-Learning

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Deep Q-Learning algorithm

Tabular Q-Learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \, max_{a'} \, Q(S_{t+1}, a') - Q(S_t, A_t))$$

New Q-value

Former Q-value

step size

Deep Q-Learning

$$y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{for non-terminal } \phi_{j+1} \end{cases}$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

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Reward

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Deep Q-Learning algorithm

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- New Q-value
- Former Q-value
- step size
- Reward
- Discounted Estimate (optimal Q-value of next state)

Deep Q-Learning

$$y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{for non-terminal } \phi_{j+1} \end{cases}$$

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- $Q(S_t, A_t)$: Former Q-value

Deep Q-Learning

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

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Deep Q-Learning

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

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Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

Tabular Q-Learning

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- r_j: Reward

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

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Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

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Q-Loss

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Deep Q-Learning algorithm

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- $- Q(S_t, A_t)$: Former Q-value
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Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

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- $\gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$: Discounted Estimate (optimal Q-value of next state)
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Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

Deep Q-Learning algorithm

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Deep Q-Learning

Q-Target

$$y_j = r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$$

- r_j: Reward
- $\gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$: Discounted Estimate (optimal Q-value of next state)
- TD-target: $r_j + \gamma \, \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right)$

Q-Loss

$$\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$$

- TD-error: $y_j - Q\left(\phi_j, a_j; \theta\right)$

To the notebook!

Deep Q-Learning algorithm

For episode = 1, M **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1$,T **do**

With probability ε select a random action a_t

otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $\left(\phi_t, a_t, r_t, \phi_{t+1}\right)$ in D

Sample random minibatch of transitions $\left(\phi_j, a_j, r_j, \phi_{j+1}\right)$ from D

Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^-\right) & \text{otherwise} \end{cases}$

Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

End For

End For

Sample-Based RL Methods

Only learn from experience - lots of trial-error interactions

No need for a model of the world - only data collected while interacting

Temporal Difference Learning

Based on value functions and dynamic programming ideas

Monte Carlo methods for prediction and control

Often used for estimation that relies on repeated random sampling

Monte Carlo methods

Monte Carlo for prediction and control

Often used for estimation that relies on repeated random sampling

Monte Carlo in RL

Estimate values directly from experience (sequence of S, A, R)

Learning from experience

Agent can accurately estimate a value function without prior knowledge of environment dynamics

No need to know transition probability p

Monte Carlo for policy evaluation

Value functions represent expected returns

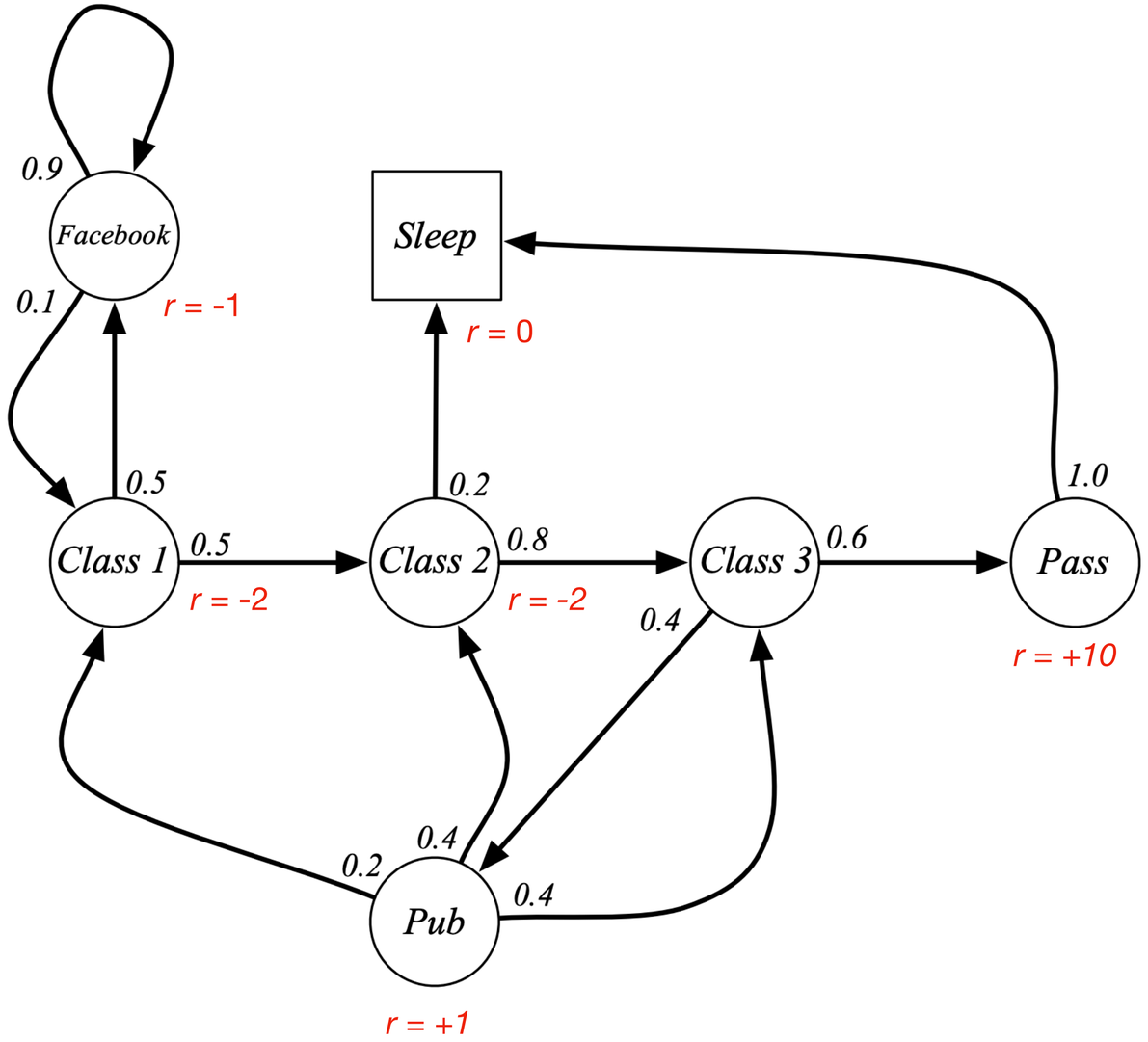
Observe multiple returns from the same state

Then average those to estimate te expected return from that state

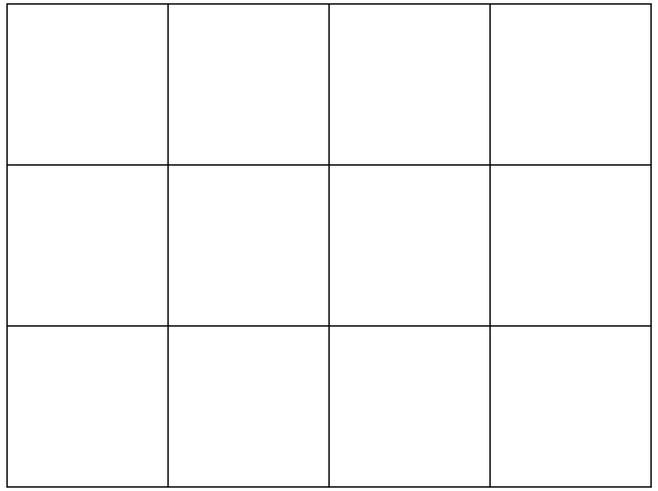
---

As the number of samples increases, the expected terutn gets closer to real avg

Example


0.9
Facebook
0.1
r = -1
Sleep
r = 0
0.5
0.2
1.0
Class 1
0.5
r = -2
Class 2
0.8
r = -2
Class 3
0.6
Pass
r = +10
0.4
0.4
0.2
0.4
Pub
r = +1

Temporal-Difference Learning



$$V(S_t) = V(S_t) + \alpha\left[V(S_{t+1} - V(S_t)\right]$$

Finite Markov Decision Process (MDP)

Bellman Equation

Bellman Equation

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

Off-policy:

Bellman Equation

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\, max_{a'}\, Q(s',a') - Q(s,a)]$$

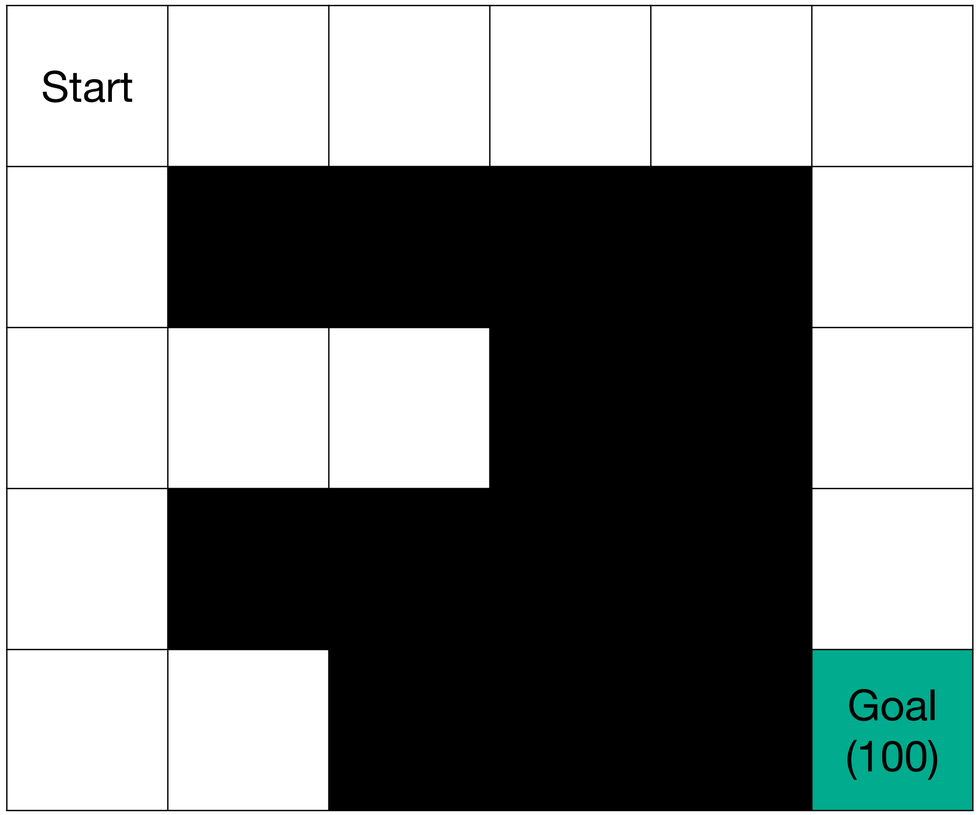
Q-Learning algorithm

Algorithm parameters: step size $\alpha \in (0,1]$, small $\varepsilon > 0$
Initialize $Q(s,a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal,\cdot) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
        $S \leftarrow S'$
    until S is terminal

Why ε-greedy?

$$Q(s,a) = Q(s,a) + \alpha[r + \gamma\ max_{a'}\ Q(s',a') - Q(s,a)]$$



Updating the Q Table

Da controllare

Quando MDP non è a finite states.
Come si definisce nel caso di una NN (che ha immagini in input)?
Assumiamo la stessa cosa?