Parcial 1.

1. La distancia media entre dos señales periodicas $x_1(t) \in \mathbb{R},\mathbb{C}$ y $x_2(t) \in \mathbb{R},\mathbb{C}$; se puede expresar a partir de las potencia media de la diferencia entre ellas:

$$d^2(x_1, x_2) = \bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\int |x_1(t) - x_2(t)|^2 dt$$

sea $x_1(t)$ y $x_2(t)$ dos señales definidas como:

$$\omega_0 = \frac{2\pi}{T} \qquad x_1(t) = A e^{-jn\omega_0 t} \qquad x_2(t) = B e^{jm\omega_0 t}$$

determinar la distancia entre las dos señales

S/n

$$x_1(t) - x_2(t) = Ae^{-jn\omega_0 t} - Be^{jm\omega_0 t}$$

$$|x_1(t) - x_2(t)|^2 = |Ae^{-jn\omega_0 t} - Be^{jm\omega_0 t}|^2$$

$$z = Ae^{-jn\omega_0 t} - Be^{jm\omega_0 t}$$

$$z^* = Ae^{jn\omega_0 t} - Be^{-jm\omega_0 t}$$

$$|x_1 - x_2|^2 = (Ae^{-jn\omega_0 t} - Be^{jm\omega_0 t})(Ae^{jn\omega_0 t} - Be^{-jm\omega_0 t})$$

$$= A^2 - ABe^{-jn\omega_0 t}e^{-jm\omega_0 t} - ABe^{jn\omega_0 t}e^{jm\omega_0 t} + B^2$$

$$= A^2 + B^2 - AB(e^{-j(n+m)\omega_0 t} + e^{j(n+m)\omega_0 t})$$

$$= A^2 + B^2 - 2AB\cos((n+m)\omega_0 t)$$

$$d^2 = \frac{1}{T}\int_0^T (A^2 + B^2 - 2AB\cos((n+m)\omega_0 t)$$

$$d^2 = A^2 + B^2 - \frac{2AB}{T}\int_0^T \cos(n+m)\omega_0 t \, dt$$

$k = n+m$

$$P\, x_1 - x_2 = \frac{1}{T_0}\left[\int_0^{T_0} (A^2+B^2)\,dt - 2AB\int_0^{T_0} \cos((k)w_0 t\right]$$

si $k=0 \rightarrow (n+m=0)$

$$\cos(0) = 1 - \frac{1}{T}\int_0^T dt = \frac{1}{T}T = 1$$

si $k \neq 0$:

$$\int_0^T \cos\left(\left(\frac{2\pi}{T}\right)kt\right)dt = \frac{T}{2\pi k}\left[sen(2\pi k) - sen(0)\right] = \frac{T}{2\pi k}\cdot 0 = 0$$

Si $n+m=0$

$$d^2 = A^2+B^2-2AB\cdot 1 = (A-B)^2$$

Si $n+m \neq 0$

$$d^2 = A^2+B^2-2AB\cdot 0 = A^2+B^2$$

$$\begin{cases} A-B \\ \sqrt{A^2+B^2} \end{cases}$$

2. $\omega_1 = 1000\pi$ $\quad F_1 = 1000\cancel{\pi}/2\cancel{\pi} = 500\,Hz$

$\omega_2 = 3000\pi$ $\quad F_2 = 3000\cancel{\pi}/2\cancel{\pi} = 1500\,Hz$

$\omega_3 = 11000\pi$ $\quad F_3 = 11000\cancel{\pi}/2\cancel{\pi} = 5500\,Hz$

$f_s = 5\,kHz = 5000\,Hz$

Segon Nyquist $\rightarrow f_s > 2F_{max}$

$2F_{max} = 11000\,Hz > f_s = 5000\,Hz$

No cumple Nyquist

$-f_e = |f - kf_s|$

$f_e = |5500\,Hz - 5000\,Hz| = 500\,Hz$

$x[n] = 3\cos\left(2\pi \cdot \frac{500}{5000} n\right) + 5\,sen\left(2\pi \cdot \frac{1500}{5000} n\right)$

$+ 10\cos\left(2\pi \cdot \frac{500}{5000} n\right)$

$= 3\cos(0{,}2\pi)n + 5\,sen(0{,}6\pi n) + 10\cos(2{,}2\pi)$

$\downarrow$

$10\cos(0{,}2\pi)$

$x[n] = 13\cos(0{,}2\pi n) + 5\,sen(0{,}6\pi n)$

Amplitud $= 13 + 5 = 18$

para 4 bits: $2^4 = 16$ niveles

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3. $$C_n = \frac{1}{(t_i - t_f)n^2 w_o^2} \int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt \quad n \in \mathbb{Z}$$

Expandiendo:

$$x(t) = \sum_n C_n e^{jnw_o t}$$

$$x'(t) = \frac{d}{dt}\left(\sum_n C_n e^{jnw_o t}\right) = \sum C_n jnw_o e^{jnw_o t}$$

$$x''(t) \frac{d}{dt}\left(\sum_n C_n jnw_o e^{jnw_o t}\right) \sum_n C_n (jnw_o)^2 e^{jnw_o t}$$

calculamos C_n

$$\int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt = \int_{t_i}^{t_f} \sum_n C_n (jnw_o)^2 e^{jnw_o t} e^{-jnw_o t} dt$$

$$\int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt = C_n (jnw_o)^2 (t_f - t_i)$$

$$C_n = \frac{1}{(t_f - t_i)(jnw_o)^2} \int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt$$

con $j^2 = -1$

$$C_n = \frac{1}{(t_f - t_i)n^2 w_o^2} \int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt$$

$$x(t) = a_o + \sum_{n=1}^{\infty} [a_n \cos(nw_o t) + b_n \operatorname{sen}(nw_o t)]$$

$$x'(t) = \sum_{n=1}^{\infty} \left[-a_n n w_0 \operatorname{sen}(n w_0 t) + b_n n w_0 \cos(n w_0 t) \right]$$

$$x''(t) = \sum_{n=1}^{\infty} \left[-a_n (n w_0)^2 \cos(n w_0 t) - b_n (n w_0)^2 \operatorname{sen}(n w_0 t) \right]$$

ortogonalidad Para a_n:

$$a_n = \frac{-2}{T n^2 w_0^2} \int_{t_i}^{t_f} x''(t) \cos(n w_0 t)\, dt$$

Para b_n

$$b_n = \frac{-2}{T n^2 w_0^2} \int_{t_i}^{t_f} x''(t) \operatorname{sen}(n w_0 t)\, dt$$

4.

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn w_0 t}\, dt = \frac{2}{T}\int_0^T x(t)\cos(n w_0 t)\, dt$$

$$= \frac{2}{T}\int_0^{2a} x(t)\cos(kt)\, dt$$

$$I_1 = \int_0^a \frac{A}{a} t\cos(kt)\, dt \qquad I_2 = \int_a^{2a} \frac{A}{a}(2a-t)\cos(kt)\, dt$$

I_1:

$$\int t\cos(kt)\, dt \qquad u = t \quad dy = \cos(kt)$$

$$\int t\cos(kt)\, dt = \frac{t}{k}\sin(kt) + \frac{1}{k^2}\cos(kt) + C$$

$$I_1 = \frac{A}{a}\left[\frac{a}{k}\sin(ka) + \frac{1}{k^2}(\cos(ka) - 1)\right]$$

I_2:

$$u = 2a - t \qquad dv = \cos(kt) + C$$

$$\int (2a-t)\cos(kt)\, dt =$$

$$= \frac{2a-t}{k}\sin(kt) - \frac{1}{k^2}\cos(kt) + C$$

$$I_2 = \frac{A}{a}\left[-\frac{a}{k}\sin(ka) + \frac{1}{k^2}(\cos(ka) - \cos(2ka))\right]$$

$$I_1 + I_2 = \frac{A}{a} \cdot \frac{1}{k^2}\left(2\cos(ka) - \cos(2ka) - 1\right)$$

Para $n \neq 0$

$$c_n = \frac{2}{T}(I_1 + I_2) = \frac{2}{T} \cdot \frac{A}{a} \cdot \frac{1}{k^2}\left(2\cos(ka) - \cos(2ka) - 1\right)$$

$$k = n\omega_0 \quad k^2 = (n\omega_0)^2 = (n \cdot 2\pi/T)^2 = \frac{4\pi^2 n^2}{T^2}$$

$$\frac{2}{T} \cdot \frac{1}{k^2} = \frac{2}{T} \cdot \frac{T^2}{4\pi^2 n^2} = \frac{T}{2\pi^2 n^2}$$

$$c_n = \frac{AT}{2\pi^2 a n^2}\left(2\cos(n\omega_0 a) - \cos(2n\omega_0 a) - 1\right) \quad n \neq 0$$

$$c_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt = \frac{2}{T}\int_0^{2a} x(t)\, dt$$

$$\int_0^a \frac{A}{a} t\, dt = \frac{Aa}{2}, \quad \int_a^{2a} \frac{A}{a}(2a - t)\, dt = \frac{Aa}{2}$$

$$c_0 = \frac{2}{T} Aa = \frac{2Aa}{T} = \frac{AT}{T}$$

$$x(t) = \sum_n c_n e^{jn\omega_0 t}$$

$$x''(t) = \sum_n c_n (jn\omega_0)^2 e^{jn\omega_0 t} = -\sum_n c_n (n\omega_0)^2 e^{jn\omega_0 t}$$

$$d_n = -(n\omega_0)^2 c_n$$

$$d_n = -k^2 \cdot \frac{2}{T} \cdot \frac{A}{a} \cdot \frac{1}{k^2} \left(2\cos(ka) - \cos(2ka) - 1\right)$$

$$= -\frac{2A}{Ta}\left(2\cos(ka) - \cos(2ka) - 1\right)$$

$$2\cos\theta - \cos 2\theta - 1 = 4\cos\theta \sin^2\left(\frac{\theta}{2}\right) \qquad \theta = n\omega_0 a$$

$$d_n = -\frac{8A}{Ta}\cos(n\omega_0 a)\sin^2\left(\frac{n\omega_0 a}{2}\right) \qquad n \neq 0$$

$$d_0 = 0$$

$$|d_n| = \frac{8|A|}{Ta}\left|\cos(n\omega_0 a)\right| \sin^2\left(\frac{n\omega_0 a}{2}\right)$$

$$\angle d_n = \begin{cases} 0 & d_n > 0 \\ \pi & d_n < 0 \\ \text{indefinida} & d_n = 0 \end{cases}$$

como $d_{-n} = d_n \rightarrow n = 1, 2, 3, 4, 5$

$$d_0 = 0$$

$$d_n = -\frac{8A}{Ta}\cos(\theta_n)\sin^2\left(\frac{\theta_n}{2}\right)$$

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