1.3 a) $x(t) = e^{-a|t|}, \; a > 0$

Separaren $t \geq 0$ y $t < 0$:

$$x(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t}\, dt + \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt$$

$$\int e^{-(a+j\omega)t}\, dt = \frac{1}{a+j\omega}, \quad \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt$$

$$\int_{-\infty}^{0} e^{(a-j\omega)t}\, dt = \frac{1}{a-j\omega}$$

$$X(\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2}$$

b) $x(t) = \cos(\omega_c t)$

$$\cos(\omega_c t) = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$$

$$X(\omega) = \frac{1}{2}\left(2\pi\delta(\omega-\omega_c) + 2\pi\delta(\omega+\omega_c)\right)$$

$$= \pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

c) $x(t) = \sin(\omega_s t)$

$$\sin(\omega_s t) = \frac{1}{2j}\left(e^{j\omega_s t} - e^{-j\omega_s t}\right)$$

$$X(\omega) = \frac{1}{2j}\left(2\pi\delta(\omega-\omega_s) - 2\pi\delta(\omega+\omega_s)\right) = \frac{\pi}{j}\left[\delta(\omega-\omega_s) - \delta(\omega+\omega_s)\right]$$

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d) $x(t) = f(t)\cos(w_c t)$

$$f(t)\cos(w_c t) = \frac{1}{2} f(t) e^{jw_c t} + \frac{1}{2} f(t) e^{-jw_c t}$$

$$X(w) = \frac{1}{2} F(w - w_c) + \frac{1}{2} F(w + w_c)$$

e) $x(t) = e^{-at^2}$, $a > 0$

$$X(w) = \int_{-\infty}^{\infty} e^{-at^2} e^{-jwt} dt = \sqrt{\frac{\pi}{a}}\, e^{-\frac{w^2}{4a}}$$

f) $x(t) = A\,\mathrm{rect}_d(t) \rightarrow X(w) = A\int_{-d/2}^{d/2} e^{-jwt} dt = A\left[\frac{e^{-jwt}}{-jw}\right]_{-d/2}^{d/2} = \frac{A(e^{-jwd/2} - e^{jwd/2})}{-jw}$

$$X(w) = A\frac{2\sin(wd/2)}{w} = Ad\,\frac{\sin(wd/2)}{wd/2}$$

$$= Ad\,\mathrm{sinc}\left(\frac{wd}{2}\right)$$

- $\mathrm{sinc}(x) = \frac{\sin x}{x}$

7.4) a. $\mathcal{F}\{e^{-j\omega_1 t}\cos(\omega_c t)\}$

$$\mathcal{F}\{e^{j\omega_0 t}x(t)\} = X(\omega-\omega_0)$$

$$\mathcal{F}\{\cos(\omega_c t)\} = \pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$$

$$e^{-j\omega_1 t} = e^{j(-\omega_1)t} \rightarrow \omega_0 = -\omega_1$$

$$\mathcal{F}\{e^{-j\omega_1 t}\cos(\omega_c t)\} = \pi[\delta(\omega+\omega_1-\omega_c)+\delta(\omega+\omega_1+\omega_c)]$$

b. $\mathcal{F}\{u(t)\cos^2(\omega_c t)\}$ ($u(t)$ = escalon unitario)

$$\rightarrow \cos^2(\omega_c t) = \frac{1}{2}+\frac{1}{2}\cos(2\omega_c t)$$

$$u(t)\cos^2(\omega_c t) = \frac{1}{2}u(t)+\frac{1}{2}u(t)\cos(2\omega_c t)$$

$$\mathcal{F}\{u(t)\} = \pi\delta(\omega)+\frac{1}{j\omega}$$

$$\rightarrow \mathcal{F}\{u(t)e^{j\omega_0 t}\} = U(\omega-\omega_0)$$

$$\rightarrow \cos(2\omega_c t) = \frac{1}{2}(e^{j2\omega_c t}+e^{-j2\omega_c t})$$

$$\mathcal{F}\{u(t)\cos^2(\omega_c t)\} = \frac{1}{2}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)$$
$$+\frac{1}{4}\left(\pi\delta(\omega-2\omega_c)+\frac{1}{j(\omega-2\omega_c)}\right)+\frac{1}{4}\left(\pi\delta(\omega+2\omega_c)+\frac{1}{j(\omega+2\omega_c)}\right)$$

c) $\mathcal{F}^{-1}\left\{\dfrac{7}{(w^2+6w+45)} \cdot \dfrac{7}{(8+jw)^2}\right\}$

$$x(t) = \mathcal{F}^{-1}\{A(w)B(w)\} = \frac{1}{2\pi}\, a(t) \cdot b(t)$$

$\rightarrow a(t) = \mathcal{F}^{-1}\{A(w)\}$ $b(t) = \mathcal{F}^{-1}\{B(w)\}$

- primer factor:

$w^2+6w+45 = (w+3)^2+36$

$$\mathcal{F}\{e^{-a|t|}e^{jw_0t}\} = \frac{2a}{a^2+(w-w_0)^2} \rightarrow a=6 \quad w_0=-3$$

$$\mathcal{F}\{e^{-6|t|}e^{-j3t}\} = \frac{12}{(w+3)^2+36}$$

$$\mathcal{F}^{-1}\left\{\frac{7}{(w^2+6w+45)}\right\} = \frac{7}{12}e^{-6|t|}e^{-j3t}$$

- Segundo factor

$\dfrac{7}{(8+jw)^2}$ $a>0$

$$\mathcal{F}\{t\,e^{-at}u(t)\} = \frac{7}{(a+jw)^2}$$

$$\mathcal{F}^{-1}\left\{\frac{7}{(8+jw)^2}\right\} = t\,e^{-8t}u(t)$$

$$x(t) = \frac{7}{2\pi}\left(\frac{7}{12}e^{-6|t|}e^{-j3t}\right) \cdot \left(t\,e^{-8t}u(t)\right)$$

d) $\mathcal{F}\{3t^2\}$

$$\mathcal{F}\{t^n x(t)\} = j^n \frac{d^n}{d\omega^n} X(\omega)$$

$$\rightarrow \mathcal{F}\{t^n\} = 2\pi j^n \delta^{(n)}(\omega)$$

$n=2$: $\rightarrow \mathcal{F}(t^2) = 2\pi j^2 \delta''(\omega) = -2\pi\delta''(\omega)$

$$\mathcal{F}\{3t^2\} = -6\pi\delta''(\omega)$$

e) $$\frac{B}{T}\sum_{n=-\infty}^{\infty}\left(\frac{1}{a^2+(\omega-n\omega_0)^2} + \frac{1}{a+j(\omega-n\omega_0)}\right)$$

inversa de $\dfrac{1}{a^2+(\omega-\omega_0)^2}$

Usando $\mathcal{F}\{e^{-a|t|}e^{j\omega_0 t}\} = \dfrac{2a}{a^2+(\omega-\omega_0)^2}$

despejamos:

$$\mathcal{F}^{-1}\left\{\frac{1}{a^2+(\omega-n\omega_0)^2}\right\} = \frac{1}{2a}e^{-a|t|}e^{jn\omega_0 t}$$

$$\sum_n \frac{1}{a^2+(\omega-\omega_0)^2} \rightarrow \frac{1}{2a}e^{-a|t|}\sum_n e^{jn\omega_0 t}$$

$$\mathcal{F}^{-1}\left\{\sum_n \frac{1}{a^2+(\omega-n\omega_0)^2}\right\} = \frac{1}{2a}\cdot\frac{2\pi}{\omega_0}\sum_k e^{-a|kT|}\delta(t-kT)$$

$$= \frac{\pi}{a\omega_0}\sum_k e^{-a|kT|}\delta(t-kT)$$

$\omega_0 = 2\pi/T \rightarrow \dfrac{T}{2a}\sum_k e^{-a|kT|}\delta(t-kT)$

- inversa de $\dfrac{1}{a+j(\omega-n\omega_0)}$

$$\rightarrow \mathcal{F}\{u(t)\, e^{-at}\, e^{jn\omega_0 t}\} = \frac{1}{a+j(\omega-n\omega_0)}$$

$$\sum_n \frac{1}{a+j(\omega-n\omega_0)} \longrightarrow u(t)e^{-at}\sum_n e^{jn\omega_0 t}$$

$$\mathcal{F}^{-1}\left\{\sum_n \frac{1}{a+j(\omega-n\omega_0)}\right\} = \frac{2\pi}{\omega_0}\sum_k u(kT)\, e^{-akT}\delta(t-kT)$$

con $\omega_0 = 2\pi/T$

$$\frac{2\pi}{\omega_0}\sum_{k=0}^{\infty} e^{-akT}\delta(t-kT) \longrightarrow T\sum_{k=0}^{\infty} e^{-akT}\delta(t-kT)$$

$$\boxed{\frac{B}{2a}\sum_{k=-\infty}^{\infty} e^{-a|kT|}\delta(t-kT) + B\sum_{k=0}^{\infty} e^{-akT}\delta(t-kT)}$$