

Exercise 1

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Problem 1

$$Y(t) = X(t) + w(t) \quad (1)$$

$$f_Y(x) = f_X(x) * f_w(x) \quad (2)$$

$$f_X(x) = \frac{1}{2} (\delta(x+1) + \delta(x-1)) \quad (3)$$

$$f_w(x) = N(0, \sigma^2) \quad (4)$$

$$f_Y(x) = \frac{1}{2} (N(-1, \sigma^2) + N(1, \sigma^2)) \quad (5)$$

$$P(Y \geq \alpha) = 1 - F_Y(\alpha) \quad (6)$$

$$F_w(\alpha) = \Phi\left(\frac{\alpha}{\sigma}\right) \quad (7)$$

$$F_Y(\alpha) = \frac{1}{2} \left(\Phi\left(\frac{\alpha+1}{\sigma}\right) + \Phi\left(\frac{\alpha-1}{\sigma}\right) \right) \quad (8)$$

$$P(Y \geq \alpha) = 1 - \frac{1}{2} \left(\Phi\left(\frac{\alpha+1}{\sigma}\right) + \Phi\left(\frac{\alpha-1}{\sigma}\right) \right) \quad (9)$$

$$P(Y \geq \alpha) = 1 - \frac{1}{4} \left(2 + \operatorname{erf}\left(\frac{\alpha+1}{\sqrt{2}\sigma}\right) + \operatorname{erf}\left(\frac{\alpha-1}{\sqrt{2}\sigma}\right) \right) \quad (10)$$

$$(11)$$

Problem 2

Problem 3

a)

$$R_X(\tau) = E[X(t)X(t + \tau)] \quad (12)$$

$$= \frac{1}{T} \int_0^T X(t)X(t + \tau) dt \quad (13)$$

$$R_X(0) = \frac{1}{T} \int_0^T X(t)X(t) dt \quad (14)$$

$$= \frac{1}{T} \int_0^T X^2(t) dt \quad (15)$$

$$= \frac{1}{T} \int_0^T Y(t) dt = E[Y(t)] \quad (16)$$

b)

Autocovariance:

$$K_{YY}(\tau) = E[Y(t)Y(t + \tau)] \quad (17)$$

$$E[Y(t)Y(t + \tau)] = E[Y(t)]E[Y(t + \tau)] + 2E^2[\sqrt{Y(t)}\sqrt{Y(t + \tau)}] \quad (18)$$