NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF INFORMATION TECHNOLOGY, MATHEMATICS AND ELECTRICAL ENGINEERING
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Deadline: 02.02.2011

TTT4115 Kommunikasjonsteori
Exercise 2

Problem 1

A linear filter with impulse response $h(t)$ is real, stable, and time invariant. The input process, $X(t)$, is real and wide sense stationary with zero mean, $\mu_X = 0$. It follows that the output process, $Y(t)$, is also wide sense stationary with zero mean.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

a) Show that the following relation exists between the autocorrelation functions of the output and input signals:

$$R_Y(\tau) = R_X(\tau) * \left(\int_{-\infty}^{\infty} h(t)h(t+\tau)dt \right).$$

b) The power spectral density, $S(f)$, and the autocorrelation function, $R(\tau)$, constitute a Fourier transform pair:

$$S(f) = \mathcal{F}\left\{R(\tau)\right\}.$$

How would you more or less directly argue based on the result in a) and the Fourier relationship that the following equation is correct:

$$S_Y(f) = \left|H(f)\right|^2 S_X(f).$$

c) Find the variance of $Y(t)$, σ_Y^2, expressed by:

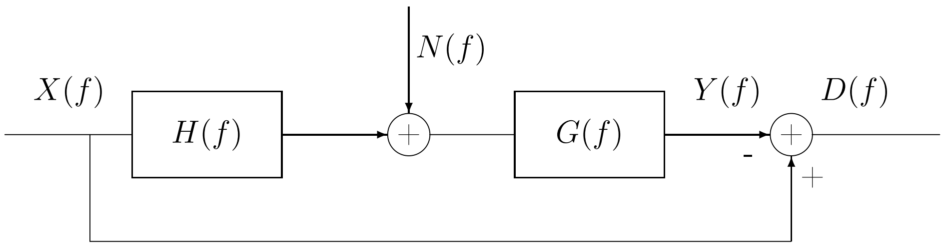
i. the autocorrelation $R_X(\tau)$ and
ii. the power spectral density $S_X(f)$.

d) Assume that $X(t)$ is unit variance, white noise process so that $\sigma_X^2 = 1$. Show that the following relation is then fulfilled:

$$\sigma_Y^2 = \int_{-\infty}^{\infty} \left|h(t)\right|^2 dt.$$

Problem 2

In this problem we study a linear communication system which transmits a time discrete signal given by $X(f)$ with sampling frequency f_s. The signal first experiences filtering by the transmit filter $H(f)$. Then an additive channel noise, $N(f)$, which is statistically independent of the signal, corrupts the signal. Finally, a receiver filter, $G(f)$, produces the output $Y(f)$.



a) To test the performance of the system, we compare the output signal to the input signal by forming the difference $D(f) = X(f) - Y(f)$. Find $D(f)$ expressed by the signal and noise Fourier transforms and the frequency responses of the filters.

b) Derive the power spectral density, $S_D(f)$, of the difference signal when the input signal and noise signal have power spectral densities $S_X(f)$ and $S_N(f)$, respectively.

c) The filter which maximizes the signal-to-noise ratio at the output is the Wiener filter given by

$$G(f) = \frac{H^*(f)S_X(f)}{|H(f)|^2 S_X(f) + S_N(f)}.$$

Find $S_D(f)$ expressed by $S_X(f)$, $S_N(f)$, and $|H(f)|^2$.

d) We study the following three cases:

- $H(f) = A$.
- $H(f) = B/\sqrt{S_X(f)}$.
- $H(f) = CS_X^{-1/4}(f)$.

Find the expressions for $S_D(f)$ in terms of the power spectral densities for the above cases when the channel noise is white and given by $N(f) = N_0/2$.

e) Assume that the power spectral density of the input signal is given by $S_X(f) = e^{-4|f/f_s|}$ defined in the range $f \in [-f_s/2, f_s/2]$ where $f_s = 1$. Compute the numerical values of A, B and C that would make the output power from the transmit filter equal to 1 for all three cases.

f) Assume that the noise power spectral density is given by $N_0/2 = 0.1$. Compute the distortion σ_D^2 for the three cases.

g) Discuss the optimality of the suggested systems.