NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF INFORMATION TECHNOLOGY, MATHEMATICS AND ELECTRICAL ENGINEERING
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Deadline: 26.01.2011

TTT4115 Kommunikasjonsteori
Exercise 1

Problem 1

A binary signal having the value of ± 1 is detected in the presence of additive white Gaussian noise of zero mean and variance σ^2. What is the probability density function of the signal observed at the input to the detector? Derive an expression for the probability that the observed signal is larger than a specified threshold α.

Problem 2

A random process $X(t)$ is defined by

$$X(t) = A\cos(2\pi f_c t)$$

where A is a Gaussian-distributed random variable of zero mean and variance σ_A^2. This random process is applied to an ideal integrator, producing the output

$$Y(t) = \int_0^t X(\tau)\, d\tau$$

(a) Determine the probability density function of the output $Y(t)$ at a particular time t_k.
(b) Determine whether or not $Y(t)$ is stationary.
(c) Determine whether or not $Y(t)$ is ergodic.

Problem 3

Let X(t) be a stationary, Gaussian process with autocorrelation function $R_X(\tau)$. This process is applied to a square-law device defined by the input-output relation

$$Y(t) = X^2(t),$$

where $Y(t)$ is the output.

a) Show that the mean of $Y(t)$ is $R_X(0)$.
b) Show that the aoutocovariance of $Y(t)$ is $2R_X(\tau)$.

Given: For two zero mean, Gaussian variables U and V the following relation applies:

$$E\{U^2V^2\} = E\{U^2\}E\{V^2\} + 2E^2\{UV\}.$$