

Numération

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1 Introduction

Commençons par la base 10 :

Les chiffres de la base 10 sont $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$.

$$4735_{10} = 4 \times 10^3 + 7 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$$

2 Base binaire

Les chiffres de la base binaire sont 0 et 1.

Exemple 1 :

$$\begin{aligned} 101011_2 &= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 32 + 8 + 2 + 1 \\ &= 43_{10} \end{aligned}$$

2.1 Conversion de la base décimale à la base binaire

Exemple 2 :

Conversion les nombres suivants en base binaire :

- $a = 63$

$$\begin{aligned}
 63_{10} &= 111111_2 \\
 &= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 &= 32 + 16 + 8 + 4 + 2 + 1 \\
 &= 63
 \end{aligned}$$

- $b = 72$

$$\begin{array}{r}
 72 \mid 2 \\
 \textcolor{red}{0} \mid 36 \mid 2 \\
 \textcolor{red}{0} \mid 18 \mid 2 \\
 \textcolor{red}{0} \mid 9 \mid 2 \\
 \textcolor{red}{1} \mid 4 \mid 2 \\
 \textcolor{red}{0} \mid 2 \mid 2 \\
 \textcolor{red}{0} \mid 1 \mid 2 \\
 \textcolor{red}{1} \mid 0
 \end{array}$$

$$72_{10} = 1001000_2$$

2.2 Addition en base 2

Commençons par un exemple en base 10 :

$$S = 1469_{10} + 375_{10}$$

$$\begin{array}{r}
 1469_{10} \\
 + 375_{10} \\
 \hline
 1844_{10}
 \end{array}$$

- $1_2 + 1_2 = 4 \text{ (} 10_2 \text{)}$

$$\begin{array}{r}
 2 \mid 2 \\
 \textcolor{red}{0} \mid 1 \mid 2 \\
 \textcolor{red}{1} \mid 0
 \end{array} = 2_{10} = 10_2$$

- $1_2 + 0_2 = 1_2$
- $1_2 + 1_2 + 1_2 = 11_2$
- $T = 1111_2 + 101_2$

$$\begin{array}{r}
 1111_2 \\
 + 101_2 \\
 \hline
 10100_2
 \end{array}$$

Exercice 1 :

Convertir en base 10 :

$$a = 10111_2$$

$$\begin{aligned} & 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ & = 16 + 0 + 4 + 2 + 1 \\ & = 23_{10} \end{aligned}$$

$$b = 10011_2$$

$$\begin{aligned} & 1 \times 2^2 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ & = 16 + 0 + 0 + 2 + 1 \\ & = 19_{10} \end{aligned}$$

Exercice 2 :

Convertir en base 2 :

$$c = 131_{10}$$

$$\begin{array}{r} 131 \mid 2 \\ \textcolor{red}{1} \mid 65 \mid 2 \\ \textcolor{red}{1} \mid 32 \mid 2 \\ \textcolor{red}{0} \mid 16 \mid 2 \\ \textcolor{red}{0} \mid 8 \mid 2 \\ \textcolor{red}{0} \mid 4 \mid 2 \\ \textcolor{red}{0} \mid 2 \mid 2 \\ \textcolor{red}{0} \mid 1 \mid 2 \\ \textcolor{red}{1} \mid 0 \end{array}$$

$$131_{10} = 10000011_2$$

$$d = 75_{10}$$

$$\begin{array}{r} 75 \mid 2 \\ \textcolor{red}{1} \mid 37 \mid 2 \\ \textcolor{red}{1} \mid 18 \mid 2 \\ \textcolor{red}{0} \mid 9 \mid 2 \\ \textcolor{red}{1} \mid 4 \mid 2 \\ \textcolor{red}{0} \mid 2 \mid 2 \\ \textcolor{red}{0} \mid 1 \mid 2 \\ \textcolor{red}{1} \mid 0 \end{array}$$

$$75_{10} = 1001011_2$$

Exercice 3 :

Calculer les sommes suivantes :

$$e = 1111_2 + 111_2$$

$$\begin{array}{r} 1111_2 \\ + 111_2 \\ \hline 10110_2 \end{array}$$

$$f = 10011_2 + 1111_2$$

$$\begin{array}{r} 10011_2 \\ + 1111_2 \\ \hline 100010_2 \end{array}$$

Remarque :

$$\begin{aligned} 111,01_2 &= 111_2 + 1 \times 2_{10}^{-2} \\ &= 7 + 0,25 \\ &= 7,25_{10} \end{aligned}$$

$$\begin{aligned} 1011,11_2 &= 1011_2 + 11_2 \times 2^{-2} \\ &= (8 + 2 + 1) + 3 \times 2^{-2} \\ &= 11 + \frac{3}{4} \\ &= 11,75_{10} \end{aligned}$$

Exercice 4 :

Convertir en base 10 les nombres suivantes :

$$a = 11,11_2$$

$$\begin{aligned} 11_2 + 11_2 \times 2^{-2} \\ &= 3 + 3^{-2} \times \frac{1}{4} \\ &= 3,75_{10} \end{aligned}$$

$$b = 101,101_2$$

$$\begin{aligned} & 101_2 + 101_2 \times 2^{-2} \\ &= 5_{10} + 5 \times \frac{1}{8} \\ &= 5_{10} + \frac{5}{8} \\ &= 5,625_{10} \end{aligned}$$

$$c = 11,011_2$$

$$\begin{aligned} & 3_{10} + 11 \times 2^{-3} \\ &= 3_{10} + 3 \times \frac{1}{8} \\ &= 3 + \frac{3}{8} \\ &= 3,375_{10} \end{aligned}$$

3 Base octale

Les chiffres de la base 8 sont (0, 1, 2, 3, 4, 5, 6, 7).

$$8_{10} = 10_8 \quad 64_{10} = 100_8$$

3.1 De la base octale à la base décimale

$$A = 34_8$$

$$\begin{aligned} & 3 \times 8^1 + 4 \times 8^0 \\ &= 24 + 4 \\ &= 28_{10} \end{aligned}$$

$$B = 10_8$$

$$\begin{aligned} & 1 \times 8^1 + 0 \times 8^0 \\ &= 8 + 0 \\ &= 8_{10} \end{aligned}$$

$$C = 100_8$$

$$\begin{aligned} & 1 \times 8^2 + 0 \times 8^1 + 0 \times 8^0 \\ &= 64 + 0 + 0 \\ &= 64_{10} \end{aligned}$$

$$D = 101_8$$

$$\begin{aligned} & 1 \times 8^2 + 0 \times 8^1 + 1 \times 8^0 \\ &= 64 + 0 + 1 \\ &= 65_{10} \end{aligned}$$

3.2 De la base décimale à la base octale

$$E = 76_{10}$$

$$\begin{array}{r|l} 76 & 8 \\ \hline 4 & 9 \\ & \hline & 1 \\ & \hline & 1 \\ & \hline & 0 \end{array} \quad 76_{10} = 114_8$$

$$F = 242_{10}$$

$$\begin{array}{r|l} 242 & 8 \\ \hline 2 & 30 \\ & \hline & 6 \\ & \hline & 3 \\ & \hline & 0 \end{array} \quad 242_{10} = 362_8$$

$$G = 1004_{10}$$

$$\begin{array}{r|l} 1004 & 8 \\ \hline 4 & 125 \\ & \hline & 5 \\ & \hline & 15 \\ & \hline & 7 \\ & \hline & 1 \\ & \hline & 1 \\ & \hline & 0 \end{array} \quad 1004_{10} = 1754_8$$

3.3 Addition

Additionnons les nombres suivants en base octale :

a) $7_8 + 1_8$

$$7_8 + 1_8 = 10_8 = 1 \times 8^1 + 0 \times 8^0 = 8_{10}$$

b) $17_8 + 1_8$

$$\begin{array}{r} 17_8 \\ + 1_8 \\ \hline 20_8 \end{array}$$

c) $166_8 + 12_8$

$$\begin{array}{r} 166_8 \\ + 12_8 \\ \hline 200_8 \end{array}$$

d) $3721_8 + 4727_8$

$$\begin{array}{r} 3721_8 \\ + 4727_8 \\ \hline 10650_8 \end{array}$$

4 Base Hexadécimale

Les chiffres de la base 16 sont (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $A^{(10)}$, $B^{(11)}$, $C^{(12)}$, $D^{(13)}$, $E^{(14)}$, $F^{(15)}$).

4.1 De la base 16 à la base 10

- $1F_{16} = 1 \times 16^1 + F \times 16^0$
 $= 1 \times 16^1 + 15 \times 16^0$
 $= 31_{10}$
- $6B_{16} = 6 \times 16^1 + 11 \times 16^0$
 $= 107_{10}$
- $23D_{16} = 2 \times 16^2 + 3 \times 16^1 + 13 \times 16^0$
 $= 2 \times 256 + 48 + 13$
 $= 512 + 48 + 13$
 $= 573_{10}$

4.2 De la base 10 à la base 16

a) 2022_{10}

$$\begin{array}{r|l}
 2022 & 16 \\
 \hline
 6 & 126 \\
 & \hline
 & 14 & 7 & 16 \\
 & & \hline
 & & 7 & 0
 \end{array}
 \quad 2022_{10} = 7E6_{16}$$

b) 256_{10}

$$\begin{array}{r|l}
 256 & 16 \\
 \hline
 0 & 16 \\
 & \hline
 & 0 & 1 & 16 \\
 & & \hline
 & & 1 & 0
 \end{array}
 \quad 256_{10} = 100_{16}$$

c) 1024_{10}

$$\begin{array}{r|l}
 1024 & 16 \\
 \hline
 0 & 64 \\
 & \hline
 & 0 \\
 & \hline
 & 4 \\
 & \hline
 & 4 \\
 & \hline
 & 0
 \end{array}
 \quad 1024_{10} = 400_{16}$$

d) 336_{10}

$$\begin{array}{r|l}
 336 & 16 \\
 \hline
 0 & 21 \\
 & \hline
 & 5 \\
 & \hline
 & 1 \\
 & \hline
 & 1 \\
 & \hline
 & 0
 \end{array}
 \quad 336_{10} = 150_{16}$$

4.3 De la base 10 à la base 16 et à la base 2

Base 10	Base 16	Base 2
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

4.4 De la base 16 à la base 2 et l'inverse

4.4.1 De la base 16 à la base 2

$$F4_{16} = 1111\textcolor{red}{0100}0010_2$$

4.4.2 De la base 2 à la base 16

$$\overbrace{\textcolor{red}{000}1}^{10}_{16} \overbrace{1011}^{11}_{16} \overbrace{1100}^{12}_{16} \overbrace{0111}^{13}_{16}_2 = 1BC7_{16}$$

4.5 Addition en base 16

a) $F_{16} + 1_{16}$

$$\begin{array}{r} 15_{10} \\ + 1_{10} \\ \hline 16_{10} \end{array} \quad 16_{10} = 1 \times 16^1 + 0 \times 16^0 = 10_{16}$$

$$\textcolor{red}{16_{10} = 10_{16}}$$

b) $12_{16} + 6_{16}$

$$\begin{array}{r} 12_{16} \\ + 6_{16} \\ \hline 18_{16} \end{array} \quad \begin{array}{r} 18_{10} \\ + 6_{10} \\ \hline 24_{10} \end{array} \quad 12_{16} = 1 \times 16^1 + 2 \times 16^0 = 18_{10}$$

$$\neq \textcolor{red}{C_{16} = 12_{10}}$$

c) $A_{16} + B_{16}$

$$10 + 11 = 21 = 1 \times 16^1 + 5 \times 16^0 = 15_{16}$$

d) $ABC_{16} + BAC_{16}$

$$\begin{array}{r} ABC_{16} \\ + BAC_{16} \\ \hline 1668_{16} \end{array} \quad \begin{array}{r} C_{16} = 12_{10} \\ + C_{16} = 12_{10} \\ \hline 24_{10} = 18_{16} \end{array} \quad B + B = 22_{10} = 16_{16}$$

e) $167_{16} + B_{16}$

$$\begin{array}{r} 167_{16} \\ + B_{16} \\ \hline 172_{16} \end{array} \quad 7_{16} + B_{16} = \begin{array}{r} 7_{10} \\ + 11_{10} \\ \hline 18_{10} \end{array} = 12_{16}$$

Exercice 5 :

Effectuer les additions suivants :

- $11101_2 + 1111_2$

$$\begin{array}{r} 11101_2 \\ + 1111_2 \\ \hline 101100_2 \end{array}$$

- $3716_8 + 5162_8$

$$\begin{array}{r} 3716_8 \\ + 5162_8 \\ \hline 11100_8 \end{array}$$

- $2AB_{16} + 16_{16}$

$$\begin{array}{r} 2AB_{16} \\ + 16_{16} \\ \hline 2C1_{16} \end{array} \quad B_{16} = 11_{10} \quad 17_{10} = 11_{16}$$

- $4C2_{16} + C7_{16}$

$$\begin{array}{r} 4C2_{16} \\ + C7_{16} \\ \hline 589_{16} \end{array}$$

- $16_{16} + 10_{16}$

$$\begin{array}{r} 16_{16} \\ + 10_{16} \\ \hline 26_{16} \end{array}$$