1. $P\left(+ \,\middle|\, \begin{matrix} A_1=T \\ A_2=F \\ A_3=T \end{matrix}\right) = \dfrac{P\left(\begin{matrix} A_1=T \\ A_2=F \\ A_3=T \end{matrix} \,\middle|\, +\right) * P(+)}{P\left(\begin{matrix} A_1=T \\ A_2=F \\ A_3=T \end{matrix}\right)}$

Based on Bayes theorem only one may be known and the probabil of other ~~notes~~ may not be estimated mathematically - To address the scenario we creat a compliment of the above scenario or a negative - And compare which is larger based on a Naive Bayes learn which ever is bigger in value of the positive or negative is valid.

$P\left(+ \,\middle|\, \begin{matrix} A_1=T \\ A_2=F \\ A_3=T \end{matrix}\right), \quad P\left(- \,\middle|\, \begin{matrix} A_1=T \\ A_2=F \\ A_3=T \end{matrix}\right)$

Using Naive Bayes learning assumption all the descriptive are Indepedent

$P(\varepsilon_1 \varepsilon_2 \varepsilon_3) = P(\varepsilon_1) \times P(\varepsilon_2) \times P(\varepsilon_3) \ldots \times P(\varepsilon_n)$

$\alpha \; P(A_1=T|+) \times P(A_2=F|+) \times P(A_3=T|+)$

$0/2 \times 1/2 \times 1/2 \times 2/6$ Applying laplace smoothing

$\frac{0+1}{2+3} \times \frac{1+1}{2+3} \times \frac{1+1}{2+3} = 1/5 * \frac{2}{5} * \frac{2}{5} = \frac{2}{125} \times 2/6$

$\alpha \; (A_1=T|-) * P(A_2=F|-) \times P(A_3=T|-) \times \frac{n(-)}{n(S)}$

$2/4 * 2/4 \times 1/3 \times 4/6 = 1/2 \times 1/2 \times 1/3 \times 4/6 =$

Valid - $P\left(- \,\middle|\, \begin{matrix} A_1=T \\ A_2=F \\ A_3=T \end{matrix}\right) > P\left(+ \,\middle|\, \begin{matrix} A_1=F \\ A_2=F \\ A_3=T \end{matrix}\right)$

4(a) (2) To find the point for which the probability of being produce by both normal distributions is the same, we need to solve the equations

$$\frac{\exp\left(-\frac{(x-4)^2}{2}\right)}{\sqrt{1/2} * \exp\left(-\frac{(x-8)^2}{8}\right)} = 1$$

$$\ln\left(\exp\left(-\frac{(x-4)^2}{2}\right)\right) = \ln\left(\sqrt{1/2}\exp\left(-\frac{(x-8)^2}{8}\right)\right)$$

$$\frac{-(x-4)^2}{2} = \ln\left(\sqrt{1/2}\right) - \frac{(x-8)^2}{8}$$

$$\frac{-(x-4)^2}{2} = -\ln(\sqrt{2}) - \frac{(x-8)^2}{8}$$

$$(x-4)^2 = 2\ln(\sqrt{2}) + (x-8)^2/4$$

$$(x-4)^2 = 2\ln(2^{1/2}) + (x^2 - 16x - 64)/4$$

$$(x-4)^2 = 2\ln(2^{1/2}) + (x^2 - 16x + 64)/4$$

$$x^2 - 8x + 16 = 2\ln(2^{1/2}) + (x^2/4 - 4x + 16)$$

$$(x^2 - 8x + 16) - (x^2/4 - 4x + 16) = 2\ln(2^{1/2})$$

$$x^2 - 8x + 16 - (x^2/4 - 4x + 16) = 2\ln(2^{1/2})$$

$$x^2 - 8x + 16 - x^2/4 + 4x - 16 = 2\ln(2^{1/2})$$

$$\cancel{x^2 - 8x + 16}$$

$$3x - x^2/4 = 2\ln(2^{1/2})$$

$$x(3 - x/4) = 2\ln(2^{1/2})$$

$$x = \frac{8\ln(2^{1/2})}{12 - x}$$

$$x(12-x) = 8\ln(2^{1/2})$$

$$x^2 - 12x + 8\ln(2^{1/2}) = 0$$

Solve this with quadratic equations

$$x^2 - 12x + 8\ln(2^{1/2}) = 0$$

Using the quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 1$, $b = -12$, and $c = 8\ln(2^{1/2})$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 1 \cdot 8\ln(2^{1/2})}}{2 \cdot 1}$$

$$= \frac{12 \pm \sqrt{144 - 32\ln(2)}}{2}$$

$$= \frac{12 \pm \sqrt{144 - 32\ln(2)}}{2}$$

$$x = 6 \pm \frac{\sqrt{144 - 32\ln(2)}}{2}$$

$$x = 6 \pm \frac{\sqrt{144 - 22}}{2}$$

$$x = 6 \pm \frac{11.04}{2}$$

$$x = 6 \pm 5.5$$

$x = 6 + 5.5$ OR x_1 $6 - 5.5$

$x_1 = 11.5$ OR $x_2 = 1.1$

based on a independent and ... distributed

Find the MLE of the parameter λ ~~for a gamma distribution~~ given that the parameter α is Known, we need to maximize the likehood

(4b) $$f(x|\alpha,\lambda) = \frac{1}{\Gamma(\alpha)} \lambda^{\alpha} x^{\alpha-1} e^{-\lambda x} \quad \text{with } 0 \leq x < \infty$$

$$L(\lambda : x_1, x_2 \ldots x_n) = \prod_{i=1}^{k} \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-\lambda x}$$

Taking the natural logarithm of the likehood funct below

$$\ln L(\lambda : x_1, x_2, \ldots, x_n) = n \ln(\lambda^{\alpha}) - n \ln(\Gamma(\alpha)) + (\alpha-1) \sum_{i=1}^{k} \ln(x_i) - \lambda \sum_{i=1}^{n} x_i$$

Differential the log-likelihood functn with respect to λ, will set the derivative equal to zero.

$$\frac{d}{d\lambda} \ln L(\lambda; x_1, x_2, \ldots, x_n) = \frac{n\alpha}{\lambda} - \sum_{i=1}^{k} x_i$$

$$\frac{d}{d\lambda} = 0$$

$$\frac{n\alpha}{\lambda} - \sum_{i=1}^{n} x_i = 0$$

$$\lambda = \frac{n\alpha}{\sum_{i=1}^{k} x_i}$$ Maximum Likelihood Estimator