Text as Data

Justin Grimmer

Associate Professor Department of Political Science University of Chicago

August 16th, 2017

Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2017)

- 1) Discovery: a hypothesis or view of the world
- 2) Measurement according to some organization
- 3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Text as Data Methods for Discovery

Text as Data Methods for Discovery Goal: Automatically Discover Organization (Similar Groups)

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Suppose documents live in a space

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Suppose documents live in a space \rightsquigarrow rich set of results from linear algebra

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Suppose documents live in a space \rightsquigarrow rich set of results from linear algebra

- Provides a geometry

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Suppose documents live in a space \rightsquigarrow rich set of results from linear algebra

- Provides a geometry → modify with word weighting

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Suppose documents live in a space \rightsquigarrow rich set of results from linear algebra

- Provides a geometry → modify with word weighting
- Natural notions of distance

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

Suppose documents live in a space \leadsto rich set of results from linear algebra

- Provides a geometry → modify with word weighting
- Natural notions of distance
- Building block for clustering, supervised learning, and scaling

$$Doc1 = (1, 1, 3, ..., 5)$$

Doc1 =
$$(1, 1, 3, ..., 5)$$

Doc2 = $(2, 0, 0, ..., 1)$

$$\begin{array}{rcl} \mathsf{Doc1} & = & (1,1,3,\dots,5) \\ \mathsf{Doc2} & = & (2,0,0,\dots,1) \\ \mathsf{Doc1}, \mathsf{Doc2} & \in & \Re^J \end{array}$$

$$\begin{array}{rcl} \mathsf{Doc1} & = & (1,1,3,\dots,5) \\ \mathsf{Doc2} & = & (2,0,0,\dots,1) \\ \mathsf{Doc1}, \mathsf{Doc2} & \in & \Re^J \end{array}$$

$$\begin{array}{rcl} \mathsf{Doc1} & = & (1,1,3,\dots,5) \\ \mathsf{Doc2} & = & (2,0,0,\dots,1) \\ \mathsf{Doc1}, \mathsf{Doc2} & \in & \Re^J \end{array}$$

$$Doc1 \cdot Doc2 = (1, 1, 3, ..., 5)'(2, 0, 0, ..., 1)$$

$$\begin{array}{rcl} \mathsf{Doc1} & = & (1,1,3,\ldots,5) \\ \mathsf{Doc2} & = & (2,0,0,\ldots,1) \\ \mathsf{Doc1}, \mathsf{Doc2} & \in & \Re^J \end{array}$$

Doc1 · **Doc2** =
$$(1, 1, 3, ..., 5)'(2, 0, 0, ..., 1)$$

= $1 \times 2 + 1 \times 0 + 3 \times 0 + ... + 5 \times 1$

$$\begin{array}{rcl} \mathsf{Doc1} & = & (1,1,3,\dots,5) \\ \mathsf{Doc2} & = & (2,0,0,\dots,1) \\ \mathsf{Doc1}, \mathsf{Doc2} & \in & \Re^J \end{array}$$

Doc1 · **Doc2** =
$$(1, 1, 3, ..., 5)'(2, 0, 0, ..., 1)$$

= $1 \times 2 + 1 \times 0 + 3 \times 0 + ... + 5 \times 1$
= 7

Length1.pdf



- Pythogorean Theorem: Side with length *a*

Length3.pdf

- Pythogorean Theorem: Side with length *a*
- Side with length *b* and right triangle

Length4.pdf

- Pythogorean Theorem: Side with length *a*
- Side with length *b* and right triangle
- $c = \sqrt{a^2 + b^2}$

Length4.pdf

- Pythogorean Theorem: Side with length *a*
- Side with length *b* and right triangle
- $c = \sqrt{a^2 + b^2}$
- This is generally true

Vector (Euclidean) Length

Definition

Suppose $\mathbf{v} \in \Re^J$. Then, we will define its length as

$$||\mathbf{v}|| = (\mathbf{v} \cdot \mathbf{v})^{1/2}$$

= $(v_1^2 + v_2^2 + v_3^2 + \dots + v_J^2)^{1/2}$

Initial guess \leadsto Distance metrics Properties of a metric: (distance function) $d(\cdot,\cdot)$. Consider arbitrary documents \boldsymbol{X}_i , \boldsymbol{X}_j , \boldsymbol{X}_k

Initial guess → Distance metrics

1)
$$d(\boldsymbol{X}_i, \boldsymbol{X}_j) \geq 0$$

Initial guess → Distance metrics

- 1) $d(X_i, X_j) \ge 0$
- 2) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$ if and only if $\boldsymbol{X}_i = \boldsymbol{X}_j$

Initial guess → Distance metrics

- 1) $d(X_i, X_j) \ge 0$
- 2) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$ if and only if $\boldsymbol{X}_i = \boldsymbol{X}_j$
- 3) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = d(\boldsymbol{X}_j, \boldsymbol{X}_i)$

Initial guess → Distance metrics

- 1) $d(X_i, X_j) \ge 0$
- 2) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$ if and only if $\boldsymbol{X}_i = \boldsymbol{X}_j$
- 3) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = d(\boldsymbol{X}_j, \boldsymbol{X}_i)$
- 4) $d(\boldsymbol{X}_i, \boldsymbol{X}_k) \leq d(\boldsymbol{X}_i, \boldsymbol{X}_j) + d(\boldsymbol{X}_j, \boldsymbol{X}_k)$

Initial guess → Distance metrics

Properties of a metric: (distance function) $d(\cdot, \cdot)$. Consider arbitrary documents X_i , X_j , X_k

- 1) $d(X_i, X_j) \ge 0$
- 2) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$ if and only if $\boldsymbol{X}_i = \boldsymbol{X}_j$
- 3) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = d(\boldsymbol{X}_j, \boldsymbol{X}_i)$
- 4) $d(\boldsymbol{X}_i, \boldsymbol{X}_k) \leq d(\boldsymbol{X}_i, \boldsymbol{X}_j) + d(\boldsymbol{X}_j, \boldsymbol{X}_k)$

Explore distance functions to compare documents ---

Initial guess → Distance metrics

Properties of a metric: (distance function) $d(\cdot, \cdot)$. Consider arbitrary documents X_i , X_j , X_k

- 1) $d(X_i, X_j) \ge 0$
- 2) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$ if and only if $\boldsymbol{X}_i = \boldsymbol{X}_j$
- 3) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = d(\boldsymbol{X}_j, \boldsymbol{X}_i)$
- 4) $d(\boldsymbol{X}_i, \boldsymbol{X}_k) \leq d(\boldsymbol{X}_i, \boldsymbol{X}_j) + d(\boldsymbol{X}_j, \boldsymbol{X}_k)$

Explore distance functions to compare documents \leadsto Do we want additional assumptions/properties?

Euclidean Distance

Doc1.pdf

Euclidean Distance

Doc2.pdf

Euclidean Distance

Doc3.pdf

Definition

The Euclidean distance between documents X_i and X_j as

$$||X_i - X_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Measuring the Distance Between Documents

Definition

The Euclidean distance between documents X_i and X_j as

$$||X_i - X_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Suppose $X_i = (1,4)$ and $X_j = (2,1)$. The distance between the documents is:

$$||(1,4) - (2,1)|| = \sqrt{(1-2)^2 + (4-1)^2}$$

= $\sqrt{10}$

What properties should similarity measure have?

- Maximum: document with itself

- Maximum: document with itself
- Minimum: documents have no words in common (orthogonal)

- Maximum: document with itself
- Minimum: documents have no words in common (orthogonal)
- Increasing when more of same words used

- Maximum: document with itself
- Minimum: documents have no words in common (orthogonal)
- Increasing when more of same words used
- ? s(a,b) = s(b,a).

What properties should similarity measure have?

- Maximum: document with itself
- Minimum: documents have no words in common (orthogonal)
- Increasing when more of same words used
- ? s(a,b) = s(b,a).

How should additional words be treated?

Measuring Similarity



Measure 1: Inner product

Measuring Similarity

Fig1.pdf

Measure 1: Inner product

$$(2,1)^{'} \cdot (1,4) = 6$$

Fig2.pdf

Fig2.pdf

Problem(?): length dependent

Fig2.pdf

Problem(?): length dependent

$$(4,2)^{'}(1,4) = 12$$

Fig3.pdf

Problem(?): length dependent

$$(4,2)'(1,4) = 12$$

 $a \cdot b = ||a|| \times ||b|| \times \cos \theta$

$$\cos \theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right)$$

$$\cos\theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right)$$
$$\frac{(4,2)}{||(4,2)||} = (0.89, 0.45)$$

$$\cos\theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right)$$

$$\frac{(4,2)}{||(4,2)||} = (0.89, 0.45)$$

$$\frac{(2,1)}{||(2,1)||} = (0.89, 0.45)$$

$$\cos \theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right) \\
\frac{(4,2)}{||(4,2)||} = (0.89, 0.45) \\
\frac{(2,1)}{||(2,1)||} = (0.89, 0.45) \\
\frac{(1,4)}{||(1,4)||} = (0.24, 0.97)$$

$$\cos \theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right) \\
\frac{(4,2)}{||(4,2)||} = (0.89, 0.45) \\
\frac{(2,1)}{||(2,1)||} = (0.89, 0.45) \\
\frac{(1,4)}{||(1,4)||} = (0.24, 0.97) \\
(0.89, 0.45)'(0.24, 0.97) = 0.65$$



 $\cos\theta \colon$ removes document length from similarity measure



 $\cos\theta$: removes document length from similarity measure Projects texts to unit length representation \leadsto onto sphere



 $\cos\theta$: removes document length from similarity measure Projects texts to unit length representation \leadsto onto sphere

Are all words created equal?

- Treat all words equally

- Treat all words equally
- Lots of noise

- Treat all words equally
- Lots of noise
- Reweight words

- Treat all words equally
- Lots of noise
- Reweight words
 - Accentuate words that are likely to be informative

- Treat all words equally
- Lots of noise
- Reweight words
 - Accentuate words that are likely to be informative
 - Make specific assumptions about characteristics of informative words

Are all words created equal?

- Treat all words equally
- Lots of noise
- Reweight words
 - Accentuate words that are likely to be informative
 - Make specific assumptions about characteristics of informative words

How to generate weights?

Are all words created equal?

- Treat all words equally
- Lots of noise
- Reweight words
 - Accentuate words that are likely to be informative
 - Make specific assumptions about characteristics of informative words

How to generate weights?

- Assumptions about separating words

Are all words created equal?

- Treat all words equally
- Lots of noise
- Reweight words
 - Accentuate words that are likely to be informative
 - Make specific assumptions about characteristics of informative words

How to generate weights?

- Assumptions about separating words
- Use training set to identify separating words (Monroe, Ideology measurement)

What properties do words need to separate concepts?

What properties do words need to separate concepts?

- Used frequently

What properties do words need to separate concepts?

- Used frequently
- But not too frequently

What properties do words need to separate concepts?

- Used frequently
- But not too frequently

Ex. If all statements about OBL contain Bin Laden than this contributes nothing to similarity/dissimilarity measures

What properties do words need to separate concepts?

- Used frequently
- But not too frequently

Ex. If all statements about OBL contain Bin Laden than this contributes nothing to similarity/dissimilarity measures

Inverse document frequency:

What properties do words need to separate concepts?

- Used frequently
- But not too frequently

Ex. If all statements about OBL contain Bin Laden than this contributes nothing to similarity/dissimilarity measures

Inverse document frequency:

 $n_i = No.$ documents in which word j occurs

What properties do words need to separate concepts?

- Used frequently
- But not too frequently

Ex. If all statements about OBL contain Bin Laden than this contributes nothing to similarity/dissimilarity measures

Inverse document frequency:

$$n_j = No.$$
 documents in which word j occurs $idf_j = log \frac{N}{n_j}$

What properties do words need to separate concepts?

- Used frequently
- But not too frequently

Ex. If all statements about OBL contain Bin Laden than this contributes nothing to similarity/dissimilarity measures

Inverse document frequency:

$$n_j = No.$$
 documents in which word j occurs $idf_j = log \frac{N}{n_j}$ $idf = (idf_1, idf_2, ..., idf_J)$

Why log?

Why log?

- Maximum at $n_j=1$

Why log?

- Maximum at $n_i = 1$
- Decreases at rate $\frac{1}{n_i} \Rightarrow$ diminishing "penalty" for more common use

Why log?

- Maximum at $n_j = 1$
- Decreases at rate $\frac{1}{n_i} \Rightarrow$ diminishing "penalty" for more common use
- Other functional forms are fine, embed assumptions about penalization of common use

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{sf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J})$$

$$\begin{aligned} \mathbf{X}_{i,\mathrm{idf}} &\equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{tf}} \times \mathrm{idf} &= (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J}) \\ \mathbf{X}_{j,\mathrm{idf}} &\equiv \mathbf{X}_{j} \times \mathrm{idf} &= (X_{j1} \times \mathrm{idf}_{1}, X_{j2} \times \mathrm{idf}_{2}, \dots, X_{jJ} \times \mathrm{idf}_{J}) \end{aligned}$$

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{tf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J})$$

$$\mathbf{X}_{j,\mathrm{idf}} \equiv \mathbf{X}_{j} \times \mathrm{idf} = (X_{j1} \times \mathrm{idf}_{1}, X_{j2} \times \mathrm{idf}_{2}, \dots, X_{jJ} \times \mathrm{idf}_{J})$$

How Does This Matter For Measuring Similarity/Dissimilarity?

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{tf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J})$$

$$\mathbf{X}_{j,\mathrm{idf}} \equiv \mathbf{X}_{j} \times \mathrm{idf} = (X_{j1} \times \mathrm{idf}_{1}, X_{j2} \times \mathrm{idf}_{2}, \dots, X_{jJ} \times \mathrm{idf}_{J})$$

How Does This Matter For Measuring Similarity/Dissimilarity? Inner Product

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_i}_{\mathrm{tf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_1, X_{i2} \times \mathrm{idf}_2, \dots, X_{iJ} \times \mathrm{idf}_J)$$

$$\mathbf{X}_{j,\mathrm{idf}} \equiv \mathbf{X}_j \times \mathrm{idf} = (X_{j1} \times \mathrm{idf}_1, X_{j2} \times \mathrm{idf}_2, \dots, X_{jJ} \times \mathrm{idf}_J)$$

How Does This Matter For Measuring Similarity/Dissimilarity?

Inner Product

$$\mathbf{X}_{i,\mathrm{idf}} \cdot \mathbf{X}_{j,\mathrm{idf}} = (\mathbf{X}_i \times \mathrm{idf})' (\mathbf{X}_j \times \mathrm{idf})$$

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{tf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J})$$

$$\mathbf{X}_{j,\mathrm{idf}} \equiv \mathbf{X}_{j} \times \mathrm{idf} = (X_{j1} \times \mathrm{idf}_{1}, X_{j2} \times \mathrm{idf}_{2}, \dots, X_{jJ} \times \mathrm{idf}_{J})$$

How Does This Matter For Measuring Similarity/Dissimilarity?

Inner Product

$$\mathbf{X}_{i,\mathrm{idf}} \cdot \mathbf{X}_{j,\mathrm{idf}} = (\mathbf{X}_i \times \mathbf{idf})'(\mathbf{X}_j \times \mathbf{idf})$$

$$= (\mathrm{idf}_1^2 \times X_{i1} \times X_{j1}) + (\mathrm{idf}_2^2 \times X_{i2} \times X_{j2}) + \dots + (\mathrm{idf}_J^2 \times X_{iJ} \times X_{jJ})$$

Define:

Define:

$$\mathbf{\Sigma} = \begin{pmatrix} \mathsf{idf}_1^2 & 0 & 0 & \dots & 0 \\ 0 & \mathsf{idf}_2^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \mathsf{idf}_J^2 \end{pmatrix}$$

Define:

$$\mathbf{\Sigma} = \begin{pmatrix} idf_1^2 & 0 & 0 & \dots & 0 \\ 0 & idf_2^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & idf_J^2 \end{pmatrix}$$

If we use tf-idf for our documents, then

Define:

$$\mathbf{\Sigma} = \begin{pmatrix} idf_1^2 & 0 & 0 & \dots & 0 \\ 0 & idf_2^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & idf_J^2 \end{pmatrix}$$

If we use tf-idf for our documents, then

$$d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) = \sqrt{\sum_{m=1}^{J} (x_{im,idf} - x_{jm,idf})^2}$$
$$= \sqrt{(\boldsymbol{X}_i - \boldsymbol{X}_j)' \boldsymbol{\Sigma} (\boldsymbol{X}_i - \boldsymbol{X}_j)}$$

Final Product

Applying some measure of distance, similarity (if symmetric) yields:

$$\mathbf{D} = \begin{pmatrix} 0 & d(1,2) & d(1,3) & \dots & d(1,N) \\ d(2,1) & 0 & d(2,3) & \dots & d(2,N) \\ d(3,1) & d(3,2) & 0 & \dots & d(3,N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(N,1) & d(N,2) & d(N,3) & \dots & 0 \end{pmatrix}$$

Lower Triangle contains unique information N(N-1)/2

Clustering

Fully Automated Clustering

- 1) Distance metric when are documents close?
- 2) Objective function → how do we summarize distances?
- 3) Optimization method \leadsto how do we find optimal clustering?

THERE IS NO A PRIORI OPTIMAL METHOD Computer Assisted Clustering (Grimmer and King, 2011)

- crucial to combine human and computer insights

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized)

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \rightsquigarrow Partition documents into *K* clusters.

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters. Two parameters to estimate

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters. Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ .

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters. Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters.

Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

$$\boldsymbol{\theta}_k = (\theta_{1k}, \theta_{2k}, \dots, \theta_{Jk})$$

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters. Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

$$\theta_k = (\theta_{1k}, \theta_{2k}, \dots, \theta_{Jk})$$

 $\theta_k = exemplar for cluster k$

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters.

Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

$$\theta_k = (\theta_{1k}, \theta_{2k}, \dots, \theta_{Jk})$$

 $\theta_k = exemplar for cluster k$

2) T is an $N \times J$ matrix. Each row is an indicator vector.

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters. Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

$$\theta_k = (\theta_{1k}, \theta_{2k}, \dots, \theta_{Jk})$$

 $\theta_k = exemplar for cluster k$

2) T is an $N \times J$ matrix. Each row is an indicator vector. If observation i is from cluster k, then

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \rightsquigarrow Partition documents into K clusters.

Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

$$\theta_k = (\theta_{1k}, \theta_{2k}, \dots, \theta_{Jk})$$

 $\theta_k = \frac{\mathsf{exemplar}}{\mathsf{exemplar}}$ for cluster k

2) T is an $N \times J$ matrix. Each row is an indicator vector. If observation i is from cluster k, then

$$\boldsymbol{\tau}_{i} = (0,0,\ldots,0,\underbrace{1}_{k^{th}},0,\ldots,0)$$

N documents $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iJ})$ (normalized) Goal \leadsto Partition documents into K clusters. Two parameters to estimate

1) $K \times J$ matrix of cluster centers Θ . Cluster k has center

$$\boldsymbol{\theta}_k = (\theta_{1k}, \theta_{2k}, \dots, \theta_{Jk})$$

 $\theta_k = exemplar for cluster k$

2) T is an $N \times J$ matrix. Each row is an indicator vector. If observation i is from cluster k, then

$$\boldsymbol{\tau}_{i} = (0,0,\ldots,0,\underbrace{1}_{k^{th}},0,\ldots,0)$$

Hard Assignment



Assume squared euclidean distance

Assume squared euclidean distance

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \underbrace{\tau_{ik}}^{\text{cluster indicator}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

Assume squared euclidean distance

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \underbrace{\tau_{ik}}^{\text{cluster indicator}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

Calculate squared euclidean distance from center

Assume squared euclidean distance

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \overset{\text{cluster indicator}}{\underbrace{\tau_{ik}}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \overset{\text{cluster indicator}}{\underbrace{\tau_{ik}}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \overset{\text{cluster indicator}}{\underbrace{\tau_{ik}}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
 - If K = N then $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = 0$ (Minimum)

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \underbrace{\tau_{ik}}^{\text{cluster indicator}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
 - If K = N then $f(X, T, \Theta) = 0$ (Minimum)
 - Each observation in its own cluster

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \overset{\text{cluster indicator}}{\tau_{ik}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
 - If K = N then $f(X, T, \Theta) = 0$ (Minimum)
 - Each observation in its own cluster
 - $\theta_i = x_i$

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \underbrace{\tau_{ik}}^{\text{cluster indicator}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
 - If K = N then $f(X, T, \Theta) = 0$ (Minimum)
 - Each observation in its own cluster
 - $\boldsymbol{\theta}_i = \boldsymbol{x}_i$
 - If K = 1, $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = N \times \sigma^2$

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \overset{\text{cluster indicator}}{\tau_{ik}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
 - If K = N then $f(X, T, \Theta) = 0$ (Minimum)
 - Each observation in its own cluster
 - $\boldsymbol{\theta}_i = \boldsymbol{x}_i$
 - If K = 1, $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = N \times \sigma^2$
 - Each observation in same cluster



$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \underbrace{\tau_{ik}}^{\text{cluster indicator}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^{2}\right)}_{\text{Squared Euclidean Distance}}$$

- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
 - If K = N then $f(X, T, \Theta) = 0$ (Minimum)
 - Each observation in its own cluster
 - $\boldsymbol{\theta}_i = \boldsymbol{x}_i$
 - If K = 1, $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = N \times \sigma^2$
 - Each observation in same cluster
 - $oldsymbol{ heta}_1 = \mathsf{Average}$ across documents



Coordinate descent

Coordinate descent → iterate between labels and centers.

Coordinate descent → iterate between labels and centers.

Iterative algorithm: each iteration t

Coordinate descent → iterate between labels and centers.

Iterative algorithm: each iteration t

- Conditional on $\mathbf{\Theta}^{t-1}$ (from previous iteration), choose $\mathbf{\mathcal{T}}^t$

Coordinate descent → iterate between labels and centers.

Iterative algorithm: each iteration t

- Conditional on $\mathbf{\Theta}^{t-1}$ (from previous iteration), choose \mathbf{T}^t
- Conditional on T^t , choose Θ^t

Coordinate descent → iterate between labels and centers.

Iterative algorithm: each iteration t

- Conditional on $\mathbf{\Theta}^{t-1}$ (from previous iteration), choose $\mathbf{\mathcal{T}}^t$
- Conditional on T^t , choose Θ^t

Repeat until convergence \leadsto as measured as change in f dropping below threshold ϵ

Coordinate descent → iterate between labels and centers.

Iterative algorithm: each iteration t

- Conditional on $\mathbf{\Theta}^{t-1}$ (from previous iteration), choose \mathbf{T}^t
- Conditional on T^t , choose Θ^t

Repeat until convergence \leadsto as measured as change in f dropping below threshold ϵ

Change =
$$f(\mathbf{X}, \mathbf{T}^t, \mathbf{\Theta}^t) - f(\mathbf{X}, \mathbf{T}^{t-1}, \mathbf{\Theta}^{t-1})$$

1) initialize K cluster centers $\theta_1^t, \theta_2^t, \dots, \theta_K^t$.

- 1) initialize K cluster centers $\theta_1^t, \theta_2^t, \dots, \theta_K^t$.
- 2) Choose T^t

- 1) initialize K cluster centers $\theta_1^t, \theta_2^t, \dots, \theta_K^t$.
- 2) Choose T^t

$$\tau_{im}^t = \begin{cases} 1 \text{ if } m = \arg\min_k \sum_{j=1}^J (x_{ij} - \theta_{kj}^t)^2 \\ 0 \text{ otherwise }, \end{cases}.$$

- 1) initialize K cluster centers $\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \dots, \boldsymbol{\theta}_K^t$.
- 2) Choose T^t

$$\tau_{im}^t = \left\{ \begin{array}{l} 1 \text{ if } m = \arg\min_k \sum_{j=1}^J (x_{ij} - \theta_{kj}^t)^2 \\ 0 \text{ otherwise }, \end{array} \right.$$

In words: Assign each document x_i to the closest center θ_m^t

$$f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k = \sum_{i=1}^N \tau_{ik}^t \left(\sum_{j=1}^J (x_{ij} - \theta_{jk})^2 \right)$$

$$f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k} = \sum_{i=1}^{N} \tau_{ik}^{t} \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^{2} \right)$$
$$\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k}}{\partial \theta_{kj}} = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk})$$

$$f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k} = \sum_{i=1}^{N} \tau_{ik}^{t} \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^{2} \right)$$

$$\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k}}{\partial \theta_{kj}} = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk})$$

$$0 = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk}^{*})$$

$$f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k} = \sum_{i=1}^{N} \tau_{ik}^{t} \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^{2} \right)$$

$$\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k}}{\partial \theta_{kj}} = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk})$$

$$0 = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk}^{*})$$

$$= \sum_{i=1}^{N} \tau_{ij}^{t} x_{ij} - \theta_{jk}^{*} \sum_{i=1}^{N} \tau_{ij}^{t}$$

$$f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k} = \sum_{i=1}^{N} \tau_{ik}^{t} \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^{2} \right)$$

$$\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^{t}, \boldsymbol{\Theta})_{k}}{\partial \theta_{kj}} = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk})$$

$$0 = -2 \sum_{i=1}^{N} \tau_{ij}^{t} (x_{ij} - \theta_{jk}^{*})$$

$$= \sum_{i=1}^{N} \tau_{ij}^{t} x_{ij} - \theta_{jk}^{*} \sum_{i=1}^{N} \tau_{ij}^{t}$$

$$\frac{\sum_{i=1}^{N} \tau_{ik}^{t} x_{ij}}{\sum_{i=1}^{N} \tau_{ik}^{t}} = \theta_{jk}^{*}$$

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}}$$

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

In words: θ^{t+1} is the average of the documents assigned to k.

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

In words: θ^{t+1} is the average of the documents assigned to k. Optimization algorithm:

■ Initialize centers

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

- Initialize centers
- Do until converged:

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

- Initialize centers
- Do until converged:
 - lacksquare For each document, find closest center $\leadsto oldsymbol{ au}_i^t$

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

- Initialize centers
- Do until converged:
 - lacksquare For each document, find closest center $\leadsto oldsymbol{ au}_i^t$
 - lacksquare For each center, take average of assigned documents $\leadsto oldsymbol{ heta}_k^t$

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_{i}$$

- Initialize centers
- Do until converged:
 - lacktriangleright For each document, find closest center $\leadsto oldsymbol{ au}_i^t$
 - lacktriangle For each center, take average of assigned documents $\leadsto oldsymbol{ heta}_k^t$
 - Update change $f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta}^t) f(\boldsymbol{X}, \boldsymbol{T}^{t-1}, \boldsymbol{\Theta}^{t-1})$

Visual Example



KMeans1.pdf

KMeans2.pdf

KMeans3.pdf

KMeans4.pdf

KMeans5.pdf

KMeans6.pdf

KMeans7.pdf

KMeans8.pdf

KMeans9.pdf

KMeans10.pdf

KMeans11.pdf

KMeansFinal.pdf

An Example: Jeff Flake

To the R Code!

Unsupervised methods

Unsupervised methods→ low startup costs, high post-model costs

- Apply clustering methods, we have groups of documents

- Apply clustering methods, we have groups of documents
- How to interpret the groups?

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words
 - Use these to help infer differences across clusters

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words
 - Use these to help infer differences across clusters
- Transparency

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words
 - Use these to help infer differences across clusters
- Transparency
 - Debate what clusters are

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words
 - Use these to help infer differences across clusters
- Transparency
 - Debate what clusters are
 - Debate what they mean

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words
 - Use these to help infer differences across clusters
- Transparency
 - Debate what clusters are
 - Debate what they mean
 - Provide documents + organizations

- Apply clustering methods, we have groups of documents
- How to interpret the groups?
- Two (broad) methods:
 - Manual identification (Quinn et al 2010)
 - Sample set of documents from same cluster
 - Read documents
 - Assign cluster label
 - Automatic identification
 - Know label classes
 - Use methods to identify separating words
 - Use these to help infer differences across clusters
- Transparency
 - Debate what clusters are
 - Debate what they mean
 - Provide documents + organizations

- Previous Analysis Assumed We Know Number of Clusters

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

Think!

- No one statistic captures how you want to use your data

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

- No one statistic captures how you want to use your data
- But, can help guide your selection

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

- No one statistic captures how you want to use your data
- But, can help guide your selection
- Combination statistic + manual search

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

- No one statistic captures how you want to use your data
- But, can help guide your selection
- Combination statistic + manual search → discuss statistical methods/experimental methods on Thursday
- Humans should be the final judge

- Previous Analysis Assumed We Know Number of Clusters
- How Do We Choose Cluster Number?
- Cannot Compare f across clusters
 - Sum squared errors decreases as K increases
 - Trivial answer: each document in own cluster (useless)
 - Modeling problem: Fit often increases with features
- How do we choose number of clusters?

- No one statistic captures how you want to use your data
- But, can help guide your selection
- Combination statistic + manual search → discuss statistical methods / experimental methods on Thursday
- Humans should be the final judge
 - Compare insights across clusterings

- Notion of similarity and "good" partition → clustering

- Notion of similarity and "good" partition → clustering
- Many clustering methods:

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**
- How do we know we have something useful?

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**
- How do we know we have something useful?
 - Validation: read the documents

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - ...
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday
 - Validation: model based fit statistics

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday
 - Validation: model based fit statistics
- How do we know we have the "right" model?

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - ..
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday
 - Validation: model based fit statistics
- How do we know we have the "right" model?

YOU DON'T!

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - ..
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday
 - Validation: model based fit statistics
- How do we know we have the "right" model?

YOU DON'T! → And never will

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday
 - Validation: model based fit statistics
- How do we know we have the "right" model?

YOU DON'T! → And never will → but still useful for discovery (and measurement)

200

- Notion of similarity and "good" partition → clustering
- Many clustering methods:
 - Spectral clustering
 - Affinity Propagation
 - Non-parametric statistical models
 - Hierarchical clustering
 - Biclustering
 - **...**
- How do we know we have something useful?
 - Validation: read the documents
 - Validation: experiments to assess cluster quality → Thursday
 - Validation: model based fit statistics
- How do we know we have the "right" model?

YOU DON'T! → And never will → but still useful for discovery (and measurement)

200

J element long unit-length vector

J element long unit-length vector

$$x_i^* = \frac{x_i}{\sqrt{x_i'x_i}}$$

J element long unit-length vector

$$\mathbf{x}_{i}^{*} = \frac{\mathbf{x}_{i}}{\sqrt{\mathbf{x}_{i}'\mathbf{x}_{i}}}$$

Mixture of von Mises-Fisher (vMF) distributions:

J element long unit-length vector

$$\mathbf{x}_{i}^{*} = \frac{\mathbf{x}_{i}}{\sqrt{\mathbf{x}_{i}'\mathbf{x}_{i}}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$au_i \sim \overbrace{\mathsf{Multinomial}(1,\pi)}^{\mathsf{Mixture component}}$$

J element long unit-length vector

$$\mathbf{x}_{i}^{*} = \frac{\mathbf{x}_{i}}{\sqrt{\mathbf{x}_{i}'\mathbf{x}_{i}}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$oldsymbol{ au_i} au_i \sim egin{array}{ccc} ext{Mixture component} \ au_i & \sim & ext{Multinomial}(1,\pi) \ au_i^* | au_{ik} = 1, oldsymbol{\mu}_k & \sim & ext{vMF}(\kappa, oldsymbol{\mu}_k) \ ext{Language model} \end{array}$$

J element long unit-length vector

$$\mathbf{x}_{i}^{*} = \frac{\mathbf{x}_{i}}{\sqrt{\mathbf{x}_{i}'\mathbf{x}_{i}}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$oldsymbol{ au_i} au_i^* \sim egin{array}{c} ext{Mixture component} \ au_i^* | au_{ik} = 1, oldsymbol{\mu}_k & \sim \ ext{VMF}(\kappa, oldsymbol{\mu}_k) \ ext{Language model} \end{array}$$

Provides:

J element long unit-length vector

$$x_i^* = \frac{x_i}{\sqrt{x_i'x_i}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$oldsymbol{ au_i} au_i \sim egin{array}{ccc} ext{Mixture component} \ au_i & \sim & ext{Multinomial}(1,\pi) \ au_i^* | au_{ik} = 1, oldsymbol{\mu}_k & \sim & ext{vMF}(\kappa, oldsymbol{\mu}_k) \ ext{Language model} \end{array}$$

Provides:

 \blacksquare $\tau_i \leadsto$ Each document's cluster assignment

J element long unit-length vector

$$x_i^* = \frac{x_i}{\sqrt{x_i'x_i}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$oldsymbol{ au_i} au_i^* \sim egin{array}{c} ext{Mixture component} \ au_i^* | au_{ik} = 1, oldsymbol{\mu}_k & \sim ext{VMF}(\kappa, oldsymbol{\mu}_k) \ ext{Language model} \ \end{array}$$

Provides:

- $\blacksquare \tau_i \leadsto \mathsf{Each} \; \mathsf{document's} \; \mathsf{cluster} \; \mathsf{assignment}$
- $\pi = (\pi_1, \pi_2, \dots, \pi_K) \rightsquigarrow \text{Proportion of documents in each component}$

J element long unit-length vector

$$x_i^* = \frac{x_i}{\sqrt{x_i'x_i}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$oldsymbol{ au_i} au_i \sim egin{array}{c} ext{Mixture component} \ au_i & \sim & ext{Multinomial}(1,\pi) \ au_i^* | au_{ik} = 1, oldsymbol{\mu}_k & \sim & ext{vMF}(\kappa, oldsymbol{\mu}_k) \ ext{Language model} \end{array}$$

Provides:

- $\blacksquare \tau_i \leadsto \mathsf{Each} \; \mathsf{document's} \; \mathsf{cluster} \; \mathsf{assignment}$
- $\pi = (\pi_1, \pi_2, \dots, \pi_K) \rightsquigarrow \text{Proportion of documents in each component}$
- $\mu_k \rightsquigarrow$ Exemplar document for cluster k

A Motivating Clustering Model → Mixture of von Mises Fisher Distributions

J element long unit-length vector

$$x_i^* = \frac{x_i}{\sqrt{x_i'x_i}}$$

Mixture of von Mises-Fisher (vMF) distributions:

$$oldsymbol{ au_i} au_i \sim egin{array}{ccc} ext{Mixture component} \ au_i & \sim & ext{Multinomial}(1,\pi) \ au_i^* | au_{ik} = 1, oldsymbol{\mu}_k & \sim & ext{vMF}(\kappa, oldsymbol{\mu}_k) \ ext{Language model} \end{array}$$

Provides:

- $\tau_i \rightsquigarrow$ Each document's cluster assignment
- $\blacksquare \pi = (\pi_1, \pi_2, \dots, \pi_K) \leadsto \text{Proportion of documents in each component}$
- $\blacksquare \mu_k \leadsto \mathsf{Exemplar}$ document for cluster k

EM algorithm in slides appendix of Class 10 for my text as data course

Justin Grimmer (University of Chicago)

Text as Data

August 16th, 2017

33 / 1

How well does our model perform?

How well does our model perform?→ predict new documents?

How well does our model perform? → predict new documents? Problem

How well does our model perform? → predict new documents? Problem → in sample evaluation leads to overfit.

How well does our model perform? → predict new documents? Problem → in sample evaluation leads to overfit. Solution → evaluate performance on held out data

How well does our model perform? \leadsto predict new documents? Problem \leadsto in sample evaluation leads to overfit. Solution \leadsto evaluate performance on held out data For held out document $\boldsymbol{x}^*_{\text{out}}$

How well does our model perform? \leadsto predict new documents? Problem \leadsto in sample evaluation leads to overfit. Solution \leadsto evaluate performance on held out data For held out document $\boldsymbol{x}_{\text{out}}^*$

$$\log p(\boldsymbol{x}_{\text{out}}^*|\boldsymbol{\mu},\boldsymbol{\pi},\boldsymbol{X}) = \log \sum_{k=1}^K p(\boldsymbol{x}_{\text{out}}^*,\tau_{ik}|\boldsymbol{\mu}_k,\boldsymbol{\pi},\boldsymbol{X})$$

How well does our model perform? \leadsto predict new documents? Problem \leadsto in sample evaluation leads to overfit. Solution \leadsto evaluate performance on held out data For held out document $\boldsymbol{x}^*_{\text{out}}$

$$\begin{aligned} \log p(\boldsymbol{x}_{\text{out}}^*|\boldsymbol{\mu}, \boldsymbol{\pi}, \boldsymbol{X}) &= \log \sum_{k=1}^K p(\boldsymbol{x}_{\text{out}}^*, \tau_{ik} | \boldsymbol{\mu}_k, \boldsymbol{\pi}, \boldsymbol{X}) \\ &= \log \sum_{k=1}^K \left[\pi_k \exp(\kappa \boldsymbol{\mu}_k' \boldsymbol{x}_{\text{out}}^*) \right] \end{aligned}$$

How well does our model perform? \leadsto predict new documents? Problem \leadsto in sample evaluation leads to overfit. Solution \leadsto evaluate performance on held out data For held out document $\boldsymbol{x}^*_{\text{out}}$

$$\begin{split} \log p(\pmb{x}_{\text{out}}^*|\pmb{\mu},\pmb{\pi},\pmb{X}) &= \log \sum_{k=1}^K p(\pmb{x}_{\text{out}}^*,\tau_{ik}|\pmb{\mu}_k,\pmb{\pi},\pmb{X}) \\ &= \log \sum_{k=1}^K \left[\pi_k \exp(\kappa \pmb{\mu}_k^{'} \pmb{x}_{\text{out}}^*)\right] \\ \text{Perplexity}_{\text{word}} &= \exp\left(-\log p(\pmb{x}_{\text{out}}^*|\pmb{\mu},\pmb{\pi})\right) \end{split}$$

Perplex1.pdf

35 / 1

- Prediction → One Task

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it?

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

Chang et al 2009 ("Reading the Tea Leaves"):

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

Chang et al 2009 ("Reading the Tea Leaves"):

- Compare perplexity with human based evaluations

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

Chang et al 2009 ("Reading the Tea Leaves"):

- Compare perplexity with human based evaluations
- NEGATIVE relationship between perplexity and human based evaluations

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

Chang et al 2009 ("Reading the Tea Leaves"):

- Compare perplexity with human based evaluations
- NEGATIVE relationship between perplexity and human based evaluations

Different strategy → measure quality in topics and clusters

(Roberts, et al 2017

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

Chang et al 2009 ("Reading the Tea Leaves"):

- Compare perplexity with human based evaluations
- NEGATIVE relationship between perplexity and human based evaluations

Different strategy → measure quality in topics and clusters

- Statistics: measure cohesiveness and exclusivity (Roberts, et al 2017 Forthcoming)

- Prediction → One Task
- Do we care about it? → Social science application where we're predicting new texts?
- Does it correspond to how we might use the model?

Chang et al 2009 ("Reading the Tea Leaves"):

- Compare perplexity with human based evaluations
- NEGATIVE relationship between perplexity and human based evaluations

Different strategy → measure quality in topics and clusters

- Statistics: measure cohesiveness and exclusivity (Roberts, et al 2017 Forthcoming)
- Experiments: measure topic and cluster quality

Mathematical approaches

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

Humans → read texts

Mathematical approaches→ suppose we can capture quality with numbers assumes we're in the model→ including text representation Humans→ read texts Humans→ use cluster output

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

Humans → read texts

Humans → use cluster output

Do humans think the model is performing well?

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

Humans → read texts

Humans → use cluster output

Do humans think the model is performing well?

1) Topic Quality

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

Humans → read texts

Humans → use cluster output

Do humans think the model is performing well?

- 1) Topic Quality
- 2) Cluster Quality

- 1) Take *M* top words for a topic
- 2) Randomly select a top word from another topic
 - 2a) Sample the topic number from I from K-1 (uniform probability)
 - 2b) Sample word j from the M top words in topic I
 - 2c) Permute the words and randomly insert the intruder:
 - List:

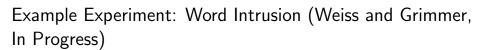
test =
$$(v_{k,3}, v_{k,1}, v_{l,j}, v_{k,2}, v_{k,4}, v_{k,5})$$

bowl, flooding, olympic, olympics, nfl, coach

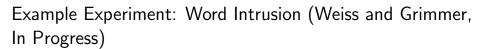
bowl, flooding, olympic, olympics, nfl, coach

stocks, investors, fed, guns, trading, earning

stocks, investors, fed, guns, trading, earning



Higher rate of intruder identification → more exclusive/cohesive topics



Higher rate of intruder identification → more exclusive/cohesive topics

Deploy on Mechanical Turk

Cluster Quality (Grimmer and King 2011)

Assessing Cluster Quality with experiments

Assessing Cluster Quality with experiments

- Goal: group together similar documents

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

Design to assess cluster quality

- Estimate clusterings

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

- Estimate clusterings
- Sample pairs of documents (hint: you only need to compare discrepant pairs)

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

- Estimate clusterings
- Sample pairs of documents (hint: you only need to compare discrepant pairs)
- Scale: (1) unrelated, (2) loosely related, (3) closely related (richer instructions, based on thing you want to cluster on)

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

- Estimate clusterings
- Sample pairs of documents (hint: you only need to compare discrepant pairs)
- Scale: (1) unrelated, (2) loosely related, (3) closely related (richer instructions, based on thing you want to cluster on)
- Cluster Quality = mean(within cluster) mean(between clusters)

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

- Estimate clusterings
- Sample pairs of documents (hint: you only need to compare discrepant pairs)
- Scale: (1) unrelated, (2) loosely related, (3) closely related (richer instructions, based on thing you want to cluster on)
- Cluster Quality = mean(within cluster) mean(between clusters)
- Select clustering with highest cluster quality

Assessing Cluster Quality with experiments

- Goal: group together similar documents
- Who knows if similarity measure corresponds with semantic similarity
- → Inject human judgement on pairs of documents

- Estimate clusterings
- Sample pairs of documents (hint: you only need to compare discrepant pairs)
- Scale: (1) unrelated, (2) loosely related, (3) closely related (richer instructions, based on thing you want to cluster on)
- Cluster Quality = mean(within cluster) mean(between clusters)
- Select clustering with highest cluster quality
- Can be used to compare any clusterings, regardless of source

How do we Choose K?

Generate many candidate models

- 1) Assess using numerical values
- 2) Use experiments
- 3) Read
- 4) Final decision --> combination

There are a lot of different clustering models (and many variations within each): k-means

There are a lot of different clustering models (and many variations within each):

k-means , Mixture of multinomials

There are a lot of different clustering models (and many variations within each):

k-means, Mixture of multinomials, k-medoids

There are a lot of different clustering models (and many variations within each):

k-means, Mixture of multinomials, k-medoids, affinity propagation

There are a lot of different clustering models (and many variations within each):

k-means , Mixture of multinomials , k-medoids , affinity propagation , agglomerative Hierarchical

There are a lot of different clustering models (and many variations within each):

k-means, Mixture of multinomials, k-medoids, affinity propagation, agglomerative Hierarchical fuzzy k-means, trimmed k-means, k-Harmonic means, fuzzy k-medoids, fuzzy k modes, maximum entropy clustering, model based hierarchical (agglomerative), proximus, ROCK, divisive hierarchical, DISMEA, Fuzzy, QTClust, self-organizing map, self-organizing tree, unnormalized spectral, MS spectral, NJW Spectral, SM Spectral, Dirichlet Process Multinomial, Dirichlet Process Normal, Dirichlet Process von-mises Fisher, Mixture of von mises-Fisher (EM), Mixture of von Mises Fisher (VA), Mixture of normals, co-clustering mutual information, co-clustering SVD, LLAhclust, CLUES, bclust, c-shell, qtClustering, LDA, Express Agenda Model, Hierarchical Dirichlet process prior, multinomial, uniform process mulitinomial, Chinese Restaurant Distance Dirichlet process multinomial, Pitmann-Yor Process multinomial, LSA, ...

- Large quantitative literature on cluster analysis

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method —

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method is mathematically impossible:

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:
 - Many choices: model-based, subspace, spectral, grid-based, graph-based, fuzzy k-modes, affinity propagation, self-organizing maps,...

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:
 - Many choices: model-based, subspace, spectral, grid-based, graph-based, fuzzy k-modes, affinity propagation, self-organizing maps,...
 - Well-defined statistical, data analytic, or machine learning foundations

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:
 - Many choices: model-based, subspace, spectral, grid-based, graph-based, fuzzy k-modes, affinity propagation, self-organizing maps,...
 - Well-defined statistical, data analytic, or machine learning foundations
 - How to add substantive knowledge: With few exceptions, unclear

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:
 - Many choices: model-based, subspace, spectral, grid-based, graph-based, fuzzy k-modes, affinity propagation, self-organizing maps,...
 - Well-defined statistical, data analytic, or machine learning foundations
 - How to add substantive knowledge: With few exceptions, unclear
 - The literature: little guidance on when methods apply

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:
 - Many choices: model-based, subspace, spectral, grid-based, graph-based, fuzzy k-modes, affinity propagation, self-organizing maps,...
 - Well-defined statistical, data analytic, or machine learning foundations
 - How to add substantive knowledge: With few exceptions, unclear
 - The literature: little guidance on when methods apply
 - Deriving such guidance: difficult or impossible

- Large quantitative literature on cluster analysis
- The Goal an optimal application-independent cluster analysis method — is mathematically impossible:
 - No free lunch theorem: every possible clustering method performs equally well on average over all possible substantive applications
- Existing methods:
 - Many choices: model-based, subspace, spectral, grid-based, graph-based, fuzzy k-modes, affinity propagation, self-organizing maps,...
 - Well-defined statistical, data analytic, or machine learning foundations
 - How to add substantive knowledge: With few exceptions, unclear
 - The literature: little guidance on when methods apply
 - Deriving such guidance: difficult or impossible

Deep problem in cluster analysis literature: full automation requires more information

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best
 - Impossible in Practice

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best
 - Impossible in Practice
 - Solution: Organized list

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best
 - Impossible in Practice
 - Solution: Organized list
 - Insight: Many clusterings are perceptually identical

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best
 - Impossible in Practice
 - Solution: Organized list
 - Insight: Many clusterings are perceptually identical
 - Consider two clusterings of 10,000 documents, we move one document from 5 to 6.

Fully Automated \rightarrow Computer Assisted (Grimmer and King 2011)

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best
 - Impossible in Practice
 - Solution: Organized list
 - Insight: Many clusterings are perceptually identical
 - Consider two clusterings of 10,000 documents, we move one document from 5 to 6.
- How to organize clusterings so humans can undestand?

Fully Automated \rightarrow Computer Assisted (Grimmer and King 2011)

- Fully Automated Clustering may succeed, fails in general. Too hard to know when to apply models
- An alternative: Computer Assisted Clustering
 - Easy (if you don't think about it): list all clustering, choose best
 - Impossible in Practice
 - Solution: Organized list
 - Insight: Many clusterings are perceptually identical
 - Consider two clusterings of 10,000 documents, we move one document from 5 to 6.
- How to organize clusterings so humans can undestand?
- Our answer: a geography of clusterings

1) Code text as numbers (in one or more of several ways)

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings
- 4) Create a metric space of clusterings, and a 2-D projection

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings
- 4) Create a metric space of clusterings, and a 2-D projection
- 5) Introduce the local cluster ensemble to summarize any point, including points with no existing clustering

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings
- 4) Create a metric space of clusterings, and a 2-D projection
- 5) Introduce the local cluster ensemble to summarize any point, including points with no existing clustering
 - New Clustering: weighted average of clusterings from methods

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings
- 4) Create a metric space of clusterings, and a 2-D projection
- 5) Introduce the local cluster ensemble to summarize any point, including points with no existing clustering
 - New Clustering: weighted average of clusterings from methods
- 6) Use animated visualization: use the local cluster ensemble to explore the space of clusterings (smoothly morphing from one into others)

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings
- 4) Create a metric space of clusterings, and a 2-D projection
- 5) Introduce the local cluster ensemble to summarize any point, including points with no existing clustering
 - New Clustering: weighted average of clusterings from methods
- 6) Use animated visualization: use the local cluster ensemble to explore the space of clusterings (smoothly morphing from one into others)
- 7) --> Millions of clusterings easily comprehended

- 1) Code text as numbers (in one or more of several ways)
- 2) Apply many different clustering methods to the data each representing different (unstated) substantive assumptions
 - Introduce sampling methods to extend search beyond existing methods
- 3) Develop a metric between clusterings
- 4) Create a metric space of clusterings, and a 2-D projection
- 5) Introduce the local cluster ensemble to summarize any point, including points with no existing clustering
 - New Clustering: weighted average of clusterings from methods
- 6) Use animated visualization: use the local cluster ensemble to explore the space of clusterings (smoothly morphing from one into others)
- 7) --- Millions of clusterings easily comprehended
- 8) (Or, our new strategy: represent entire Bell space directly; no need to examine document contents)

Crosas, Grimmer, King, and Stewart (2017) → Consilience

Consilience.com example (email me for assignment + access)

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)
- David Mayhew's (1974) famous typology

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)
- David Mayhew's (1974) famous typology
 - Advertising

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)
- David Mayhew's (1974) famous typology
 - Advertising
 - Credit Claiming

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)
- David Mayhew's (1974) famous typology
 - Advertising
 - Credit Claiming
 - Position Taking

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)
- David Mayhew's (1974) famous typology
 - Advertising
 - Credit Claiming
 - Position Taking
- Data: 200 press releases from Frank Lautenberg's office (D-NJ)

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)
- David Mayhew's (1974) famous typology
 - Advertising
 - Credit Claiming
 - Position Taking
- Data: 200 press releases from Frank Lautenberg's office (D-NJ)
- Apply our method (relying on many clustering algorithms)



Each point is a clustering Affinity Propagation-Cosine (Dueck and Frey 2007)





Each point is a clustering Affinity Propagation-Cosine (Dueck and Frey 2007) Close to:

Mixture of von Mises-Fisher distributions (Banerjee et. al. 2005)

⇒ Similar clustering of documents



Space between methods:



Space between methods:



Space between methods: local cluster ensemble





Found a region with clusterings that all reveal the same important insight

Mixture:



Mixture:

0.39 Hclust-Canberra-McQuitty

0.13 Hclust-Correlation-Ward

0.09 Hclust-Pearson-Ward





Mixture:

- 0.39 Hclust-Canberra-McQuitty
- 0.30 Spectral clustering Random Walk (Metrics 1-6)
- 0.13 Hclust-Correlation-Ward
- 0.09 Hclust-Pearson-Ward
- 0.04 Spectral clustering Symmetric (Metrics 1-6)



Mixture:

- 0.39 Hclust-Canberra-McQuitty
- 0.30 Spectral clustering Random Walk (Metrics 1-6)
- 0.13 Hclust-Correlation-Ward
- 0.09 Hclust-Pearson-Ward
- 0.05 Kmediods-Cosine
- 0.04 Spectral clustering Symmetric (Metrics 1-6)





Credit Claiming, Pork:

"Sens. Frank R. Lautenberg (D-NJ) and Robert Menendez (D-NJ) announced that the U.S. Department of Commerce has awarded a \$100,000 grant to the South Jersey Economic Development District"



Credit Claiming, Legislation:

"As the Senate begins its recess, Senator Frank Lautenberg today pointed to a string of victories in Congress on his legislative agenda during this work period"



Advertising:

"Senate Adopts Lautenberg/Menendez Resolution Honoring Spelling Bee Champion from New Jersey"

Example Discovery: Partisan Taunting



Partisan Taunting:

"Republicans Selling Out Nation on Chemical Plant Security"

Important Concept Overlooked in Mayhew's (1974) typology

 "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]



Important Concept Overlooked in Mayhew's (1974) typology



- "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]
- "The scopes trial took place in 1925. Sadly, President Bush's veto today shows that we haven't progressed much since then" [Healthcare]

Important Concept Overlooked in Mayhew's (1974) typology



- "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]
- "The scopes trial took place in 1925. Sadly, President Bush's veto today shows that we haven't progressed much since then" [Healthcare]
- "Every day the House Republicans dragged this out was a day that made our communities less safe." [Homeland Security]

Important Concept Overlooked in Mayhew's (1974) typology

Definition: Explicit, public, and negative attacks on another political party or its members



- "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]
- "The scopes trial took place in 1925. Sadly, President Bush's veto today shows that we haven't progressed much since then" [Healthcare]
- "Every day the House Republicans dragged this out was a day that made our communities less safe." [Homeland Security]

Important Concept Overlooked in Mayhew's (1974) typology

Definition: Explicit, public, and negative attacks on another political party or its members

Consequences for representation: Deliberative, Polarization, Policy



- "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]
- "The scopes trial took place in 1925. Sadly, President Bush's veto today shows that we haven't progressed much since then" [Healthcare]
- "Every day the House Republicans dragged this out was a day that made our communities less safe." [Homeland Security]

- Discovered using 200 press releases; 1 senator.

- Discovered using 200 press releases; 1 senator.
- Demonstrate prevalence using senators' press releases.

- Discovered using 200 press releases; 1 senator.
- Demonstrate prevalence using senators' press releases.
- Apply supervised learning method: measure proportion of press releases a senator taunts other party

- Discovered using 200 press releases; 1 senator.
- Demonstrate prevalence using senators' press releases.
- Apply supervised learning method: measure proportion of press releases a senator taunts other party



- Discovered using 200 press releases; 1 senator.
- Demonstrate prevalence using senators' press releases.
- Apply supervised learning method: measure proportion of press releases a senator taunts other party



Over Time Tauting Rates in Speeches



1) FAC methods tuned to problem

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions
 - Topic Models (FAC)

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions
 - Topic Models (FAC)
 - Semi-supervised methods (CAC)

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions
 - Topic Models (FAC)
 - Semi-supervised methods (CAC)
 - Manual methods

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions
 - Topic Models (FAC)
 - Semi-supervised methods (CAC)
 - Manual methods
 - Observe group with most productivity 20-30 years later

- 1) FAC methods tuned to problem
 - Provides single answer, uncertainty estimates
 - Imposes many unstated assumptions, narrow set of conceptualizations considered
 - Difficult for political scientist to tune to their problem
- 2) CAC methods to explore a space of partitions
 - Varies assumptions, ensures many different conceptualizations considered
 - Burden on user to discover conceptualization
 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions
 - Topic Models (FAC)
 - Semi-supervised methods (CAC)
 - Manual methods
 - Observe group with most productivity 20-30 years later
 - To identify limits of methods, when to use which approach, need evaluations for the usefulness of conceptualizations

Stylometry Who Wrote Disputed Federalist Papers?

Federalist papers → Mosteller and Wallace (1963)

- Persuade citizens of New York State to adopt constitution
- Canonical texts in study of American politics
- 77 essays
 - Published from 1787-1788 in Newspapers
 - And under the name Publius, anonymously

Who Wrote the Federalist papers?

- Jay wrote essays 2, 3, 4,5, and 64
- Hamilton: wrote 43 papers
- Madison: wrote 12 papers

Disputed: Hamilton or Madison?

- Essays: 49-58, 62, and 63
- Joint Essays: 18-20

Task: identify authors of the disputed papers.

Task: Classify papers as Hamilton or Madison using dictionary methods

Setting up the Analysis

Training → papers Hamilton, Madison are known to have authored Test → unlabeled papers

Preprocessing:

- Hamilton/Madison both discuss similar issues
- Differ in extent they use stop words
- Focus analysis on the stop words

Setting up the Analysis

- $\mathbf{Y} = (Y_1, Y_2, ..., Y_N) = (Hamilton, Hamilton, Madison, ..., Hamilton)$ $N \times 1$ matrix with author labels
- Define the number of words in federalist paper i as num $_i$

$$\mathbf{X} = \begin{pmatrix} \frac{1}{\mathsf{num}_1} & \frac{2}{\mathsf{num}_1} & \frac{0}{\mathsf{num}_1} & \cdots & \frac{3}{\mathsf{num}_1} \\ \frac{0}{\mathsf{num}_2} & \frac{1}{\mathsf{num}_2} & \frac{0}{\mathsf{num}_2} & \cdots & \frac{0}{\mathsf{num}_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{0}{\mathsf{num}_N} & \frac{0}{\mathsf{num}_N} & \frac{1}{\mathsf{num}_N} & \cdots & \frac{0}{\mathsf{num}_N} \end{pmatrix}$$

 $N \times J$ counting stop word usage rate

-
$$\theta = (\theta_1, \theta_2, \dots, \theta_J)$$

Word weights.

Objective Function

Heuristically: find $\theta^* = (\theta_1^*, \theta_2^*, \dots, \theta_J^*)$ used to create score

$$p_i = \sum_{j=1}^J \theta_j^* X_{ij}$$

that maximally discriminates between categories

LDALine.pdf

Objective Function

Define:

$$oldsymbol{\mu}_{\mathsf{Madison}} = rac{1}{N_{\mathsf{Madison}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) oldsymbol{X}_i$$
 $oldsymbol{\mu}_{\mathsf{Hamilton}} = rac{1}{N_{\mathsf{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) oldsymbol{X}_i$

Objective Function

We can then define functions that describe the "projected" mean and variance for each author

$$g(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison}) = \frac{1}{N_{\mathsf{Madison}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) \boldsymbol{\theta}' \boldsymbol{X}_i = \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Madison}}$$

$$g(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) = \frac{1}{N_{\mathsf{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) \boldsymbol{\theta}' \boldsymbol{X}_i = \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Hamilton}}$$

$$s(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison}) = \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) (\boldsymbol{\theta}' \boldsymbol{X}_i - \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Madison}})^2$$

$$s(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) = \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) (\boldsymbol{\theta}' \boldsymbol{X}_i - \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Hamilton}})^2$$

Objective Function --> Optimization

$$f(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}) = \frac{(g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) - g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison}))^2}{s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) + s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison})}$$

$$= \frac{\left(\boldsymbol{\theta}'(\boldsymbol{\mu}_{\mathsf{Hamilton}} - \boldsymbol{\mu}_{\mathsf{Madison}})\right)^2}{\mathsf{Scatter}_{\mathsf{Hamilton}} + \mathsf{Scatter}_{\mathsf{Madison}}}$$

Optimization \rightsquigarrow find θ^* to maximize $f(\theta, X, Y)$, assuming independence across dimensions.

(Fisher's) Linear Discriminant Analysis

Optimization >>> Word Weights

For each word j, construct weight θ_j^* ,

$$\begin{array}{ll} \mu_{j,\mathsf{Hamilton}} & = & \frac{\sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) X_{ij}}{\sum_{j=1}^{J} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) X_{ij}} \\ \mu_{j,\mathsf{Madison}} & = & \frac{\sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) X_{ij}}{\sum_{j=1}^{J} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) X_{ij}} \\ \sigma_{j,\mathsf{Hamilton}}^2 & = & \mathsf{Var}(X_{i,j} | \mathsf{Hamilton}) \\ \sigma_{j,\mathsf{Madison}}^2 & = & \mathsf{Var}(X_{i,j} | \mathsf{Madison}) \end{array}$$

We can then generate weight θ_i^* as

$$\theta_{j}^{*} = \frac{\mu_{j, \text{Hamilton}} - \mu_{j, \text{Madison}}}{\sigma_{j, \text{Hamilton}}^{2} + \sigma_{j, \text{Madison}}^{2}}$$

Optimization \sim Trimming the Dictionary

- Trimming weights: Focus on discriminating words (very simple regularization)
- Cut off: For all $| heta_j^*| < 0.025$ set $heta_j^* = 0.$

Classification → Determining Authorship

For each disputed document i, compute discrimination statistic

$$p_i = \sum_{j=1}^J \theta_j^* X_{ij}$$

 $p_i \rightsquigarrow \text{classification (linear discriminator)}$

- Above midpoint in training set \rightarrow Hamilton text
- Below midpoint in training set \rightarrow Madison text

Findings: Madison is the author of the disputed federalist papers.

Inferring Separating Words Classification → Custom Dictionaries

Classification → Custom Dictionaries

- Stylometry → Classify Authors

- Stylometry → Classify Authors
- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant

- Stylometry → Classify Authors
- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting

- Stylometry → Classify Authors
- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group

- Stylometry → Classify Authors
- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group
 - Difference in Republican, Democratic language → Partisan words

- Stylometry → Classify Authors
- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group
 - Difference in Republican, Democratic language → Partisan words
 - Difference in Liberal, Conservative language → Ideological Language

- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group
 - Difference in Republican, Democratic language → Partisan words
 - Difference in Liberal, Conservative language → Ideological Language
 - Difference in Secret/Not Secret Language

 Secretive Language (Gill and Spirling 2014)

- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group
 - Difference in Republican, Democratic language → Partisan words
 - Difference in Liberal, Conservative language → Ideological Language
 - Difference in Secret/Not Secret Language

 Secretive Language (Gill and Spirling 2014)
 - Difference in Toy advertising



- Stylometry → Classify Authors
- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group
 - Difference in Republican, Democratic language → Partisan words
 - Difference in Liberal, Conservative language → Ideological Language
 - Difference in Secret/Not Secret Language

 Secretive Language (Gill and Spirling 2014)
 - Difference in Toy advertising
 - Difference in Language across groups → Labeling output from Clustering/Topic Models

Classification → Custom Dictionaries

- Dictionary based classification → Gentzkow and Shapiro (2010) and measures of media slant
- Dictionary based classification → Customized to particular setting Fictitious Prediction Problem → Infer words that are indicative of some class/group
 - Difference in Republican, Democratic language → Partisan words
 - Difference in Liberal, Conservative language \rightsquigarrow Ideological Language
 - Difference in Secret/Not Secret Language

 Secretive Language (Gill and Spirling 2014)
 - Difference in Toy advertising
 - Difference in Language across groups → Labeling output from Clustering/Topic Models

Vague and Difficult to derive before hand

Congressional Press Releases and Floor Speeches

- Collected 64,033 press releases

- Collected 64,033 press releases
- Problem: are they distinct from floor statements (approx. 52,000 during same time)?

- Collected 64,033 press releases
- Problem: are they distinct from floor statements (approx. 52,000 during same time)?
 - Yes: press releases have different purposes, targets, and need not relate to official business

- Collected 64,033 press releases
- Problem: are they distinct from floor statements (approx. 52,000 during same time)?
 - Yes: press releases have different purposes, targets, and need not relate to official business
 - No: press releases are just reactive to floor activity, will follow floor statements

- Collected 64,033 press releases
- Problem: are they distinct from floor statements (approx. 52,000 during same time)?
 - Yes: press releases have different purposes, targets, and need not relate to official business
 - No: press releases are just reactive to floor activity, will follow floor statements
- Deeper question: what does it mean for two text collections to be different?

- Collected 64,033 press releases
- Problem: are they distinct from floor statements (approx. 52,000 during same time)?
 - Yes: press releases have different purposes, targets, and need not relate to official business
 - No: press releases are just reactive to floor activity, will follow floor statements
- Deeper question: what does it mean for two text collections to be different?
- One Answer: texts used for different purposes

- Collected 64,033 press releases
- Problem: are they distinct from floor statements (approx. 52,000 during same time)?
 - Yes: press releases have different purposes, targets, and need not relate to official business
 - No: press releases are just reactive to floor activity, will follow floor statements
- Deeper question: what does it mean for two text collections to be different?
- One Answer: texts used for different purposes
- Partial answer: identify words that distinguish press releases and floor speeches

Mutual Information

- Unconditional uncertainty (entropy):

- Unconditional uncertainty (entropy):
 - Randomly sample a press release

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_j

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_j
 - Randomly sample a press release

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_j
 - Randomly sample a press release
 - Guess press release/floor statement

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_i
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_i
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty
 - Unrelated: Conditional uncertainty = uncertainty

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_i
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty
 - Unrelated: Conditional uncertainty = uncertainty
 - Perfect predictor: Conditional uncertainty = 0

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_i
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty
 - Unrelated: Conditional uncertainty = uncertainty
 - Perfect predictor: Conditional uncertainty = 0
- Mutual information(X_j): uncertainty conditional uncertainty (X_j)

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_j
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty
 - Unrelated: Conditional uncertainty = uncertainty
 - Perfect predictor: Conditional uncertainty = 0
- Mutual information(X_j): uncertainty conditional uncertainty (X_j)
 - Maximum: Uncertainty $o X_j$ is perfect predictor

- Unconditional uncertainty (entropy):
 - Randomly sample a press release
 - Guess press release/floor statement
 - Uncertainty about guess
 - Maximum: No. press releases = No. floor statements
 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_j
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty
 - Unrelated: Conditional uncertainty = uncertainty
 - Perfect predictor: Conditional uncertainty = 0
- Mutual information(X_j): uncertainty conditional uncertainty (X_j)
 - Maximum: Uncertainty $\rightarrow X_i$ is perfect predictor
 - Minimum: $0 \to X_i$ fails to separate speeches and floor statements

- $Pr(Press) \equiv Probability$ selected document press release

- $Pr(Press) \equiv Probability$ selected document press release
- $Pr(Speech) \equiv Probability selected document speech$

- $Pr(Press) \equiv Probability$ selected document press release
- Pr(Speech) ≡ Probability selected document speech
- Define entropy H(Doc)

- $Pr(Press) \equiv Probability$ selected document press release
- Pr(Speech) ≡ Probability selected document speech
- Define entropy H(Doc)

$$H(\mathsf{Doc}) = -\sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t) \log_2 \mathsf{Pr}(t)$$

- $Pr(Press) \equiv Probability$ selected document press release
- Pr(Speech) ≡ Probability selected document speech
- Define entropy H(Doc)

$$H(\mathsf{Doc}) = -\sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t) \log_2 \mathsf{Pr}(t)$$

- log₂? Encodes bits

- $Pr(Press) \equiv Probability$ selected document press release
- Pr(Speech) ≡ Probability selected document speech
- Define entropy H(Doc)

$$H(\mathsf{Doc}) = -\sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t) \log_2 \mathsf{Pr}(t)$$

- log₂? Encodes bits
- Maximum: Pr(Press) = Pr(Speech) = 0.5

- $Pr(Press) \equiv Probability$ selected document press release
- Pr(Speech) ≡ Probability selected document speech
- Define entropy H(Doc)

$$H(\mathsf{Doc}) = -\sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t) \log_2 \mathsf{Pr}(t)$$

- log₂? Encodes bits
- Maximum: Pr(Press) = Pr(Speech) = 0.5
- Minimum: $Pr(Press) \rightarrow 0 \text{ (or } Pr(Press) \rightarrow 1)$

- Consider presence/absence of word X_j

- Consider presence/absence of word X_j
- Define conditional entropy $H(Doc|X_j)$

- Consider presence/absence of word X_j
- Define conditional entropy $H(Doc|X_j)$

$$H(\mathsf{Doc}|X_j) = -\sum_{s=0}^{1} \sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t,X_j=s) \log_2 \mathsf{Pr}(t|X_j=s)$$

- Consider presence/absence of word X_j
- Define conditional entropy $H(Doc|X_j)$

$$H(\mathsf{Doc}|X_j) = -\sum_{s=0}^{1} \sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t,X_j=s) \log_2 \mathsf{Pr}(t|X_j=s)$$

- Maximum: X_j unrelated to Press Releases/Floor Speeches

- Consider presence/absence of word X_j
- Define conditional entropy $H(Doc|X_j)$

$$H(\mathsf{Doc}|X_j) = -\sum_{s=0}^{1} \sum_{t \in \{\mathsf{Pre},\mathsf{Spe}\}} \mathsf{Pr}(t,X_j=s) \log_2 \mathsf{Pr}(t|X_j=s)$$

- Maximum: X_j unrelated to Press Releases/Floor Speeches
- Minimum: X_j is a perfect predictor of press release/floor speech

- Define Mutual Information (X_j) as

- Define Mutual Information (X_j) as

Mutual Information
$$(X_j) = H(Doc) - H(Doc|X_j)$$

- Define Mutual Information (X_j) as

Mutual Information
$$(X_j) = H(Doc) - H(Doc|X_j)$$

- Maximum: entropy $\Rightarrow H(\mathsf{Doc}|X_j) = 0$

- Define Mutual Information (X_j) as

Mutual Information
$$(X_j) = H(Doc) - H(Doc|X_j)$$

- Maximum: entropy $\Rightarrow H(\mathsf{Doc}|X_j) = 0$
- Minimum: $0 \Rightarrow H(Doc|X_j) = H(Doc)$.

- Define Mutual Information (X_j) as

Mutual Information
$$(X_j) = H(Doc) - H(Doc|X_j)$$

- Maximum: entropy $\Rightarrow H(\mathsf{Doc}|X_j) = 0$
- Minimum: $0 \Rightarrow H(Doc|X_j) = H(Doc)$.

Bigger mutual information \Rightarrow better discrimination

- Define Mutual Information (X_j) as

Mutual Information
$$(X_j) = H(Doc) - H(Doc|X_j)$$

- Maximum: entropy $\Rightarrow H(\mathsf{Doc}|X_j) = 0$
- Minimum: $0 \Rightarrow H(Doc|X_j) = H(Doc)$.

Bigger mutual information \Rightarrow better discrimination

Objective function and optimization \leadsto estimate probabilities that we then place in mutual information

Formula for mutual information (based on ML estimates of probabilities)

```
n_p = Number Press Releases
  n_s = Number of Speeches
   D = n_p + n_s
  n_j = \sum_{i=1}^D X_{i,j} (No. docs X_j appears)
 n_{-i} = No. docs X_i does not appear
 n_{i,p} = No. press and X_i
 n_{i,s} = No. speech and X_i
n_{-i,p} = No. press and not X_i
n_{-i,s} = No. speech and not X_i
```

Formula for Mutual Information

$$MI(X_{j}) = \frac{n_{j,p}}{D} \log_{2} \frac{n_{j,p}D}{n_{j}n_{p}} + \frac{n_{j,s}}{D} \log_{2} \frac{n_{j,s}D}{n_{j}n_{s}} + \frac{n_{-j,p}}{D} \log_{2} \frac{n_{-j,p}D}{n_{-j}n_{p}} + \frac{n_{-j,s}}{D} \log_{2} \frac{n_{-j,s}D}{n_{-j}n_{s}}.$$











What's Different?

- Press Releases: Credit Claiming



- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words



What's Different?

- Press Releases: Credit Claiming

- Floor Speeches: Procedural Words

- Validate: Manual Classification



- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words
- Validate: Manual Classification
- Sample 500 Press Releases, 500 Floor Speeches

What's Different About Press Releases



What's Different?

- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words
- Validate: Manual Classification
- Sample 500 Press Releases, 500 Floor Speeches
- Credit Claiming: 36% Press Releases, 4% Floor Speeches

What's Different About Press Releases



What's Different?

- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words
- Validate: Manual Classification
- Sample 500 Press Releases, 500 Floor Speeches
- Credit Claiming: 36% Press Releases, 4% Floor Speeches
- Procedural: 0% Press Releases, 44% Floor Speeches

What's Different About Press Releases

What's Different?

- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words
- Validate: Manual Classification
- Sample 500 Press Releases, 500 Floor Speeches
- Credit Claiming: 36% Press Releases, 4% Floor Speeches
- Procedural: 0% Press Releases, 44% Floor Speeches
- Validate: Topic Classification

Monroe, Colaresi, and Quinn (2009) → what makes a word partisan?

Monroe, Colaresi, and Quinn (2009) → what makes a word partisan? Argue for using Log Odds Ratio, weighted by variance

$$P(E) = 1 - P(E^c)$$

$$P(E) = 1 - P(E^{c})$$

$$Odds(E) = \frac{P(E)}{1 - P(E)}$$

$$P(E) = 1 - P(E^{c})$$

$$Odds(E) = \frac{P(E)}{1 - P(E)}$$

$$Odds Ratio(E, F) = \frac{\frac{P(E)}{(1 - P(E))}}{\frac{P(F)}{1 - P(F)}}$$

$$P(E) = 1 - P(E^c)$$

$$Odds(E) = \frac{P(E)}{1 - P(E)}$$

$$Odds \ \mathsf{Ratio}(E, F) = \frac{\frac{P(E))}{(1 - P(E))}}{\frac{P(F)}{1 - P(F)}}$$

$$\mathsf{Log} \ \mathsf{Odds} \ \mathsf{Ratio}(E, F) = \log\left(\frac{P(E)}{1 - P(E)}\right) - \log\left(\frac{P(F)}{1 - P(F)}\right)$$

Monroe, Colaresi, and Quinn (2009) \rightsquigarrow what makes a word partisan? Argue for using Log Odds Ratio, weighted by variance Recall: For some event E and F

$$P(E) = 1 - P(E^{c})$$

$$Odds(E) = \frac{P(E)}{1 - P(E)}$$

$$Odds \ Ratio(E, F) = \frac{\frac{P(E)}{(1 - P(E))}}{\frac{P(F)}{1 - P(F)}}$$

$$Log \ Odds \ Ratio(E, F) = \log\left(\frac{P(E)}{1 - P(E)}\right) - \log\left(\frac{P(F)}{1 - P(F)}\right)$$

Strategy Construct objective function on *proportions* (and then calculate log-odds)

イロト (個) (重) (重) (重) のQの

Monroe, Colaresi, and Quinn (2009) \rightsquigarrow what makes a word partisan? Argue for using Log Odds Ratio, weighted by variance Recall: For some event E and F

$$P(E) = 1 - P(E^{c})$$

$$Odds(E) = \frac{P(E)}{1 - P(E)}$$

$$Odds \ Ratio(E, F) = \frac{\frac{P(E)}{(1 - P(E))}}{\frac{P(F)}{1 - P(F)}}$$

$$Log \ Odds \ Ratio(E, F) = \log\left(\frac{P(E)}{1 - P(E)}\right) - \log\left(\frac{P(F)}{1 - P(F)}\right)$$

Strategy Construct objective function on *proportions* (and then calculate log-odds)

イロト (個) (重) (重) (重) のQの

Suppose we're interested in how a word separates partisan speech.

 $\mathbf{Y} = (Republican, Republican, Democrat, \dots, Republican)$

X =Unnormalized matrix of word counts $N \times J$ Define

$$\mathbf{x}_{\mathsf{Republican}} = (\sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{i1}, \sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{i2}, \dots, \sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{iJ})$$

with $N_{Republican} = Total$ number of Republican words

 $\pi_{\mathsf{Republican}} \ \sim \ \mathsf{Dirichlet}(lpha)$

```
m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha}) \ m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})
```

```
m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha}) \ m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})
```

This implies an objective function on π ,

$$m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha})$$
 $m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})$

This implies an objective function on π ,

$$p(\pmb{\pi}|\pmb{lpha},\pmb{X},\pmb{Y}) \; \propto \; p(\pmb{\pi}|\pmb{lpha})p(\pmb{x}_{\mathsf{Republican}}|\pmb{\pi}\pmb{lpha},\pmb{Y})$$

$$m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha}) \ m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})$$

This implies an objective function on π ,

$$p(\pi|\alpha, \mathbf{X}, \mathbf{Y}) \propto p(\pi|\alpha)p(\mathbf{x}_{\mathsf{Republican}}|\pi\alpha, \mathbf{Y})$$
 $\propto \frac{\Gamma(\sum_{j=1}^{J} lpha_j)}{\prod_{j} \Gamma(lpha_j)} \prod_{i=1}^{J} \pi_j^{lpha_j - 1} \pi_j^{\mathsf{x}_{\mathsf{Republican}, j}}$

$$m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha})$$
 $m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})$

This implies an objective function on π ,

$$p(\pi|\alpha, \mathbf{X}, \mathbf{Y}) \propto p(\pi|\alpha)p(\mathbf{x}_{\mathsf{Republican}}|\pi\alpha, \mathbf{Y})$$

$$\propto \frac{\Gamma(\sum_{j=1}^{J} \alpha_j)}{\prod_{j} \Gamma(\alpha_j)} \prod_{j=1}^{J} \pi_j^{\alpha_j - 1} \pi_j^{\mathsf{x}_{\mathsf{Republican}, j}}$$

 $p(\boldsymbol{\pi}|\boldsymbol{\alpha},\boldsymbol{X},\boldsymbol{Y})$ is a Dirichlet distribution:

$$m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha})$$
 $m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})$

This implies an objective function on π ,

$$egin{array}{ll}
ho(m{\pi}|m{lpha},m{X},m{Y}) & \propto &
ho(m{\pi}|m{lpha})
ho(m{x}_{\mathsf{Republican}}|m{\pi}m{lpha},m{Y}) \ & \propto & rac{\Gamma(\sum_{j=1}^Jlpha_j)}{\prod_j\Gamma(lpha_j)}\prod_{j=1}^J\pi_j^{lpha_j-1}\pi_j^{\mathsf{x}_{\mathsf{Republican},j}} \end{array}$$

 $p(\boldsymbol{\pi}|\boldsymbol{\alpha},\boldsymbol{X},\boldsymbol{Y})$ is a Dirichlet distribution:

$$\pi^*_{\mathsf{Republican},j} = \frac{\mathsf{x}_{\mathsf{Republican},j} + \alpha_j}{\mathsf{N}_{\mathsf{Republican}} + \sum_{j=1}^J \alpha_j}$$

◆ロト ◆昼 ト ◆ 差 ト ◆ 差 ・ か へ ()

$$m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha})$$
 $m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})$

This implies an objective function on π ,

$$egin{array}{ll}
ho(m{\pi}|m{lpha},m{X},m{Y}) & \propto &
ho(m{\pi}|m{lpha})
ho(m{x}_{\mathsf{Republican}}|m{\pi}m{lpha},m{Y}) \ & \propto & rac{\Gamma(\sum_{j=1}^Jlpha_j)}{\prod_j\Gamma(lpha_j)}\prod_{j=1}^J\pi_j^{lpha_j-1}\pi_j^{\mathsf{x}_{\mathsf{Republican},j}} \end{array}$$

 $p(\boldsymbol{\pi}|\boldsymbol{\alpha},\boldsymbol{X},\boldsymbol{Y})$ is a Dirichlet distribution:

$$\pi^*_{\mathsf{Republican},j} = \frac{\mathsf{x}_{\mathsf{Republican},j} + \alpha_j}{\mathsf{N}_{\mathsf{Republican}} + \sum_{j=1}^J \alpha_j}$$

◆ロト ◆昼 ト ◆ 差 ト ◆ 差 ・ か へ ()

Calculating Log Odds Ratio

Define log Odds Ratio; as

$$\log \mathsf{Odds} \; \mathsf{Ratio}_j \;\; = \;\; \log \left(\frac{\pi_{\mathsf{Republican},j}}{1 - \pi_{\mathsf{Republican},j}} \right) - \log \left(\frac{\pi_{\mathsf{Democratic},j}}{1 - \pi_{\mathsf{Democratic},j}} \right)$$

$$Var(\log Odds \ Ratio_j) \approx \frac{1}{x_{jD} + \alpha_j} + \frac{1}{x_{jR} + \alpha_j}$$

$$Std. \ Log \ Odds_j = \frac{\log Odds \ Ratio_j}{\sqrt{Var(\log Odds \ Ratio_j)}}$$

Applying the Model

https://gist.github.com/thiagomarzagao/5851207 How do Republicans and Democrats differ in debate? Condition on topic and examine word usage

- Press Releases (64,033)
- Topic Coded
- Given press release is about topic, what are the features that distinguish Republican and Democratic language?

Mutual Information, Standardized Log Odds



Mutual Information, Standardized Log Odds



Gentzkow, Shapiro, and Taddy (2017): Rhetorical Polarization

