Uncertainty and Hypothesis Tests

Review

Why do we care about *uncertainty*?

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Why do we care about *uncertainty*?

- to quantify how certain we can be of our empirical findings
- in other words, to quantify how likely is it that our result was just by chance

This is why we need probability theory.

Review

In your own words, what's the main insight from each of these probability concepts? Why are they important to political science research?

- LLN
- CLT

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In your own words, what's the main insight from each of these probability concepts? Why are they important to political science research?

- LLN
  - as sample size increases, sample mean converges to population mean
- CLT
  - as sample size increases, sampling distribution of sample means is approximately normal

Today

- Testing hypotheses three ways:
  1. one sample test for mean
  2. two sample test for difference in means
  3. linear regression coefficient test
- Note the logic and framework is the same as we go through these tests

Set up

Our research question: do female politicians promote different policies than men?

- Why might observation data not allow us to estimate a causal effect?

Set up

- Luckily for us, there was a randomized policy experiment in India since the mid 1990's in which 1/3 of village council heads have been *randomly* reserved for women.
- Why is this important? How does it allow us to estimate a causal effect of women as leaders on policies?

Source: Raghabendra Chattopadhyay and Esther Duflo. (2004). "Women as Policy Makers: Evidence from a Randomized Policy Experiment in India. *Econometrica*, Vol. 72, No. 5, pp. 1409-1443.

Set up

West Bengal data:

- `water` measures number of new or repaired drinking water facilities in village since the reserve policy started
- `irrigation` measures the number of new or repaired irrigation facilities in the village since the reserve policy started
- `reserved` indicator for whether the Gram Panchayat (level of government of interest) was reserved for women leaders or not

```
link <- "https://raw.githubusercontent.com/kosukeimai/qss/master/PREDIC
df <- read.csv(url(link), header = TRUE)
df[1:3, c("water", "irrigation", "reserved")]
```

```
##   water irrigation reserved
## 1    10          0        1
## 2     0          5        1
## 3     2          2        1
```

Hypothesis testing

Recall the steps for any hypothesis test (from the book, analagous to lecture):

1. State the null and alternative hypothesis.
2. Choose the appropriate test statistic and significance level α.
3. Determine the sampling distribution of the test statistic *given the null is true*.
4. Compute the p-value.
5. Reject the null if p-value $\leq \alpha$, otherwise "retain" (fail to reject) the null.

One sample test: Your task

An expert in this field thinks the average number of irrigation facilities across all the villages is 2. Test the hypothesis the average is greater than 2.

- Relevant info:

```
samp <- df$irrigation
mean(samp)
```

```
## [1] 3.263975
```

```
sd(samp)/sqrt(length(samp))
```

```
## [1] 0.5289967
```

One sample test: Answer

Step 1:

$$H_0 : \mu = 2$$

$$H_A : \mu > 2$$

One sample test: Answer

Step 2:

- We will use conventional level of significance of $\alpha = .05$
- We will use a Z-test statistic of the form:

$$Z = \frac{\bar{X} - \mu_0}{std.err.of\,\bar{X}}$$

- Remember standard error of mean is estimated by $sd\hat{(}X)/\sqrt{n}$

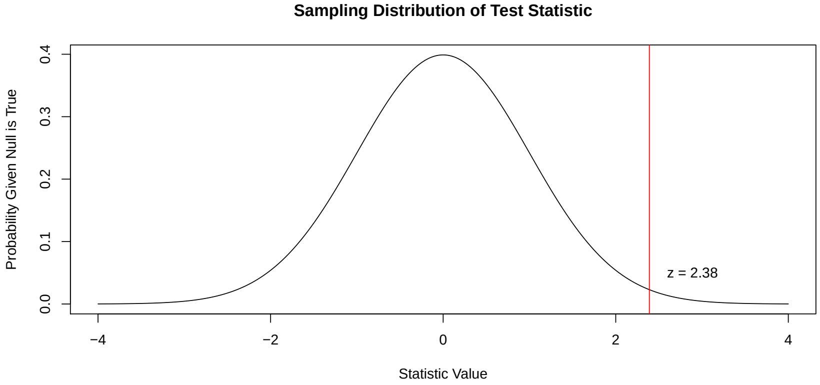
```
n <- length(df$irrigation)
std_err <- sd(df$irrigation)/sqrt(n)
z <- (mean(df$irrigation) - 2)/std_err
z #tip: z test-stat is unlikely to be bigger or smaller than, say, 4!
```

```
## [1] 2.389382
```

One sample test: Answer

Step 3:

- By CLT we know $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$
- and via "standardizing both sides"... $Z \sim N(0, 1)$
- So given the null is true (given the mean number of irrigation facilities is no different than 4) Z is distributed $N(0, 1)$



One sample test: Answer

Step 4:

- p-value is probability we'd observe this test statistic *or more extreme*, given the null is true
- Visually, does it seem unlikely we'd observe this result if the null were true? Maybe... but probably not. Let's calculate p-value.

```
pnorm(q = z, lower.tail = FALSE)
```

```
## [1] 0.008438371
```

One sample test: Answer

Step 5:

- $0.008 < .05$ so we reject the null (in favor of the alternative)

Note: important to know the steps, but R will do it for you

```
t.test(df$irrigation, mu = 2, alternative = "greater")
```

```
##
##  One Sample t-test
##
## data:  df$irrigation
## t = 2.3894, df = 321, p-value = 0.008727
## alternative hypothesis: true mean is greater than 2
## 95 percent confidence interval:
##  2.391335      Inf
## sample estimates:
## mean of x
##  3.263975
```

Two sample test: Your task

We hypothesize that female politicians support different policies than men. Test the hypothesis that reserving spots for female leaders influences the number of new/repaired drinking water facilities.

- Relevant info:

```
x_t <- df$water[df$reserved == 1] # outcome for treated units
x_c <- df$water[df$reserved == 0] # outcome for control units
mean(x_t)
```

```
## [1] 23.99074
```

```
mean(x_c)
```

```
## [1] 14.73832
```

```
std_error <- sqrt(var(x_t)/length(x_t) + var(x_c)/length(x_c))
std_error
```

```
## [1] 5.100282
```

Two Sample test: Answer

Step 1:

- $H_0 : \mu_T - \mu_C = 0$
- $H_A : \mu_T - \mu_C \neq 0$

In words:

- Null–having reserved seats for female politicians *does not* influence the number of drinking water facilities in the villages.
- Alternative–having reserved seats for female politicians *does* influence the number of drinking water facilities in the villages.

Two Sample test: Answer

Step 2:

- We will use conventional level of significance of $\alpha = .05$
- We will use a Z-test statistic of the form:

$$Z = \frac{(\bar{X_T} - \bar{X_C}) - 0}{\sqrt{\frac{1}{n_T}\hat{\sigma^2_T} + \frac{1}{n_C}\hat{\sigma^2_C}}}$$

- Remember standard error is different for different test stats

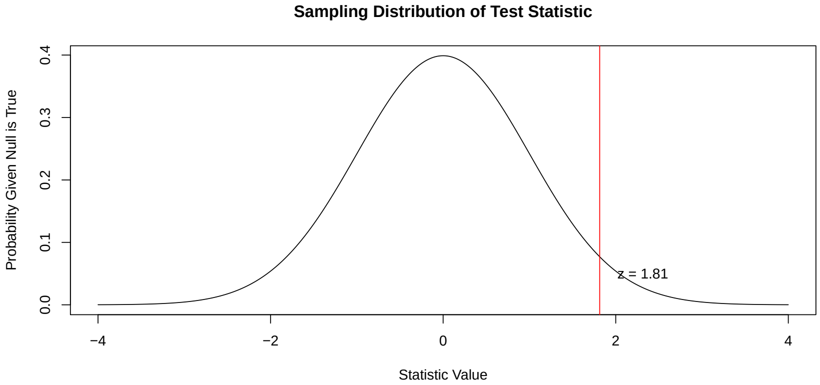
```
z <- (mean(x_t) - mean(x_c))/std_error
z
```

```
## [1] 1.8141
```

Two sample test: Answer

Step 3:

- By CLT we know $Z \sim N(0, 1)$
- So given the null is true (given the difference in means is 0) Z is distributed $N(0, 1)$



One sample test: Answer

Step 4:

- Recall, p-value is probability we'd observe this test statistic *or more extreme*, given the null is true
- Visually, does it seem unlikely we'd observe this result if the null were true? Maybe... maybe not? Let's calculate p-value.

```
pnorm(q = -abs(z), lower.tail = TRUE)*2
```

```
## [1] 0.06966231
```

One sample test: Answer

Step 5:

- $0.0696 > .05$ so “retain” or fail to reject the null

Regression

Forshadowing next week

- the steps for hypothesis testing for regression coefficients is no different
- by CLT,

$$z-\text{ score of }\beta = \frac{\hat{\beta}-\beta}{\text{std. err. of }\hat{\beta}} \sim N(0,1)$$

Let's conduct the same hypothesis test using a regression framework.

Regression: Answers

Step 1:

- $H_0 : \beta = 0$
- $H_A : \beta \neq 0$

Regression: Answer

Step 2: use conventional level of significance of $\alpha = .05$ and z-test statistic stated above

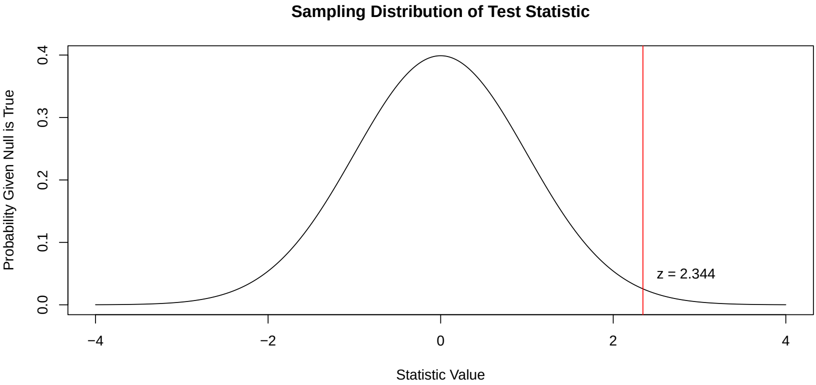
```r
mod <- lm(water ~ reserved, data = df)
summary(mod)
```

```
##
## Call:
## lm(formula = water ~ reserved, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -23.991 -14.738  -7.865   2.262 316.009
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   14.738      2.286   6.446 4.22e-10 ***
## reserved       9.252      3.948   2.344   0.0197 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.45 on 320 degrees of freedom
```

Regression: Answer

Step 3:

- By CLT we know $Z \sim N(0, 1)$
- So given the null is true (given the coefficient is 0) Z is distributed $N(0, 1)$



Regression: Answers

Step 4:

- Recall, p-value is probability we'd observe this test statistic *or more extreme*, given the null is true
- Visually, does it seem unlikely we'd observe this result if the null were true? Probably! Let's calculate p-value.

```
pnorm(q = -abs(2.344), lower.tail = TRUE)*2
```

```
## [1] 0.01907817
```

Regression: Answer

Step 5:

- $0.0197 < \alpha$ at .05 level, so we reject the null
- So our finding that $\beta = 9.252$ was unlikely to be by chance. The number of drinking facilities increased by about 9 due to the reservation of seats for women.