# Information Retrieval and Text Mining

Modelling

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#### Session outline

- 1. Information Retrieval modeling
- 2. Boolean Retrieval Model
  - Boolean algebra in IR
  - Disjunctive Normal Form (DNF) queries
  - Extensions of the Boolean Model

# Learning outcomes

#### At the end of this session we will be able to:

- Explain what is an Information Retrieval (IR) model and describe its components
- Explain the Boolean Model for IR and describe its pros and cons
- Use Boolean Algebra in the context of IR
- Synthesize IR queries to Disjunctive Normal Form
- Discuss the applicability of Boolean IR for a given problem

# 1. IR Modeling

A general overview of document models for IR

Modeling in IR is a complex process aimed at producing a ranking of documents based on their relevance to a given query.

An IR model involves two core elements:

- The conception of a logical framework for representing documents and queries
- The definition of a ranking function that computes a rank for each document regarding a given query

- Modeling and Ranking
  - IR systems usually adopt index terms to index and retrieve documents
  - An index term is any word or expression that appears in the text of a document in the collection
  - Strengths
    - it can be implemented efficiently and it is simple to refer in a query. Simplicity reduces the
      effort of query formulation on the part of the user
  - Weaknesses
    - restricts the semantic of what can be expressed
    - abstracts document structure which can be of relevance
    - users have no training in properly forming their query

- Modeling and Ranking
  - Central problem of an IR system: predicting which documents the user will find relevant and which ones are irrelevant
  - Solution: IR systems implement a predictive algorithm.

- Characterization of an IR Model
  - Definition: An information retrieval model is a quadruple [D, Q, F, R(q<sub>i</sub>,d<sub>j</sub>)]
     where:
    - **D** is a set composed of logical views (or representations) of the documents in the corpus
    - **Q** is a set composed of logical views (or representations) of the users needs
    - **F** is a framework for modeling/representing documents, queries, and their relationships, such as:
      - sets and boolean relations
      - vectors and linear algebra operations
      - sample spaces and probability distributions
      - **—** ...
    - R(q<sub>i</sub>,d<sub>j</sub>) is a ranking function that associates a real number with a query representation q<sub>i</sub>
       ∈ Q and a document representation d<sub>i</sub> ∈ D.

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       ∈ Q and a document representation d<sub>i</sub> ∈ D.

# 2. Boolean Model in IR

A general overview of the Boolean Retrieval Model: boolean algebra and basic operators

Boolean retrieval process

## Boolean Retrieval Model

- The Boolean Retrieval Model is one of the earliest and simplest models used in information retrieval
- It is based on the Set Theory and Boolean Algebra (George Boole in the mid-19ths)
- Considers that index terms are present or absent in a document
- A query is essentially a conventional Boolean expression on index terms.

- In the context of IR, the Boolean Model:
  - represents documents and queries as sets of terms or keywords, and
  - retrieval is performed using logical operations: AND, OR, and NOT.

# **Boolean Operators**

• AND operator (Λ):

Α	В	АлВ
1	1	1
1	0	0
0	1	0
0	0	0

# **Boolean Operators**

• OR operator (v):

Α	В	AVB
1	1	1
1	0	1
0	1	1
0	0	0

# **Boolean Operators**

• NOT operator (¬):

A	¬ A
1	0
0	1

- Key characteristics and components of the Boolean model:
  - Documents: Each document in the collection is represented as a set of terms or keywords
  - Queries: rely on sets of terms or keywords
  - Boolean Operators:
    - AND Operator (Λ): Retrieves documents that contain all the specified terms.
    - OR Operator (V): Retrieves documents that contain at least one of the specified terms
    - NOT Operator (¬): Negates a term and retrieves documents not containing the specified term
  - Boolean Expressions: Queries are constructed by combining terms and operators into Boolean expressions
  - Retrieval Process: When a user submits a Boolean query, the retrieval system matches the query against the document collection using Boolean logic.

 How does an IR system grounded on the Boolean framework ranks retrieved documents?

#### It does not!

- In the Boolean Model of information retrieval, document ranking is not inherently performed. Instead, documents are retrieved in a binary manner – they are either relevant (match the query) or not relevant (do not match the query)
- Some extensions of the Boolean Model introduce ranking mechanisms.

- Basic Boolean Model (No Ranking)
- Query Representation: The query is represented using Boolean operators (AND, OR, NOT)
- Document Matching: Documents are retrieved based on exact matches to the query conditions
- Binary Relevance: A document is either retrieved (1) or not (0); there is no ranking
- Query: "machine AND learning"
  - Document 1: "Introduction to Machine Learning" → 

    (retrieved)
  - Document 2: "Machine Vision and Deep Learning" → <a> ✓</a> (retrieved)
  - Document 3: "Artificial Intelligence and Neural Networks" → X (not retrieved)
- Retrieved documents are not ranked; they are just returned as a set.

 How does an IR system grounded on the Boolean framework ranks retrieved documents?

#### Ranking Extensions of the Boolean Model

• To introduce ranking in a Boolean retrieval system, heuristic methods can be applied.

#### **Term Frequency-Based Ranking**

- Count the number of query terms present in each document.
- Documents containing more of the query terms are ranked higher.

#### For the query "machine AND learning", if:

- Document 1 contains "machine" (5 times) and "learning" (3 times) → Score: 8
- Document 2 contains "machine" (2 times) and "learning" (1 time) → Score:
   3
- Ranking:
- Document 1 (score 8) appears before Document 2 (score 3).

#### Weighted Boolean Model (Term Importance)

- Assign different weights to query terms based on their importance (e.g., **TF-IDF** weighting).
- A document gets a higher score if it contains more important query terms.

- "AI" (is a common term, give it a low weight)
- "Quantum Computing" (is a rare term, give it a high weight).

#### **Proximity-Based Ranking**

• If the query terms appear closer together in a document, it is ranked higher.

#### For the query "machine AND learning"

- Document A: "Machine learning is a subset of AI."
- Document B: "Machine tools are useful, and learning is a continuous process."
- Since "machine" and "learning" appear closer in Document A, it gets a higher rank.

#### **Document Length Normalization**

 Shorter documents with exact query matches may be given higher relevance than long documents containing scattered matches

#### For the query "machine AND learning"

- Document A: "Machine learning is a subset of AI."
- Document B: "Machine tools are useful, and learning is a continuous process."
- Document A gets a higher rank.

The **pure Boolean Model does not rank documents**, ranking can be introduced using several heuristics, such as:

- Term Frequency-Based Ranking
- Weighted Boolean Model
- Proximity-Based Ranking
- Document Length Normalization

•

Disjunctive Normal Form (DNF) is a sum of products, an OR of AND terms.

- All elements of the term-document matrix are either 1 (one), to indicate presence of the term in the document, or 0 (zero), to indicate absence of the term in the document
- A query q is a Boolean expression on the index terms such as, for instance,  $[q=ka \land (kb \lor \neg kc)]$
- Given a query q, a term conjunctive component that satisfies its conditions is called a query conjunctive component c(q)
- By compiling all query conjunctive components, we can rewrite the query as a disjunction of those components. This is called the query **Disjunctive Normal Form**, which we refer to as  $q_{DNF}$

- In the Boolean model, a query is a conventional Boolean expression on index terms
  - Definition: Let c(q) be any of the query conjunctive components. Given a document d<sub>j</sub>, let c(d<sub>j</sub>) be the corresponding document conjunctive component. Then, the similarity of the document d<sub>j</sub> to the query q is defined as
  - $sim(d_j,q) = \begin{cases} 1 & if \exists c(q) \mid c(dj) \\ 0 & otherwise \end{cases}$

- Term-document matrix
  - Is a fundamental data structure used to represent the relationship between terms and documents in a document collection where:
    - rows represent terms (or words)
    - columns represent documents
    - each cell of the matrix contains a numerical value that indicates the frequency or some other measure of the presence of the corresponding term in the respective document.
    - In the case of the Boolean model the value is true (1) or false (0)

#### Term-document matrix:

- Let us consider transportation vehicles and the following document collection:
  - 1. Car: "A car is a common mode of transportation."
  - 2. Train: "Trains are used for long-distance travel."
  - 3. Plane: "Planes are fast and efficient for air travel."
  - 4. Boat: "Boats are essential for water transportation."
- Also consider the following terms:
  - car, train, plane, boat, transportation, travel, mode, fast, efficient, water, air, longdistance

• Term-document matrix:

	Doc1	Doc2	Doc3	Doc4
Car	1	0	0	0
Train	0	1	0	0
Plane	0	0	1	0
Boat	0	0	0	1
Transportation	1	0	0	1
Travel	0	1	1	0
Mode	1	0	0	0
Fast	0	0	1	0

• Term-document matrix:

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Transportation	1	0	0	1
Travel	0	1	1	0
Mode	1	0	0	0
Fast	0	0	1	0

**{Doc1, Doc4}** 

• Term-document matrix:

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Car	<u>1</u>	0	0	0
Train	0	1	0	0
Plane	0	0	1	0
Boat	0	0	0	1
Transportation	1	0	0	1
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{**Doc1**, **Doc4**}

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Travel	0	1	1	0
Mode	1	0	0	0
Fast	0	0	1	0

Q1 = Car OR Boat

Q2 = (Travel AND Fast) OR (Train)

{Doc2, Doc3}

Term-document matrix:

	Doc1	Doc2	Doc3	Doc4
Car	1	0	0	0
Train	0	<u>1</u>	0	0
Plane	0	0	1	0
Boat	0	0	0	1
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Q1 = Car OR Boat Q2 = (Travel AND Fast) OR (Train) **DNF** 

## Disjunctive Normal Form

In the **Boolean Model** of Information Retrieval, the **Disjunctive Normal Form (DNF)** of a query is obtained by rewriting the Boolean expression as a **disjunction (OR) of conjunctions (ANDs)** of terms.

#### **Steps to Obtain Disjunctive Normal Form (DNF)**

#### 1. Expand the Query Using Boolean Laws

- Apply distribution of AND over OR to rewrite the expression.
- Use De Morgan's Laws to simplify NOT operations when necessary.

#### 2. Express the Query as a Disjunction of Conjunctions

- Each **conjunctive clause** (AND clause) represents a set of terms that must be present together in a document.
- The OR operator combines multiple conjunctive clauses.

# Disjunctive Normal Form an OR of AND terms

**Disjunctive Normal Form (DNF)** is a **sum of products**, an **OR of AND terms** 

```
In the Boo
query is o 1st Law (Negation of AND):
conjunctic \neg(A \land B) = \neg A \lor \neg B
               •The negation of a conjunction is the disjunction of the negations.
Steps to O "Not (A and B)" is the same as "Not A or Not B."
1. Expand 2nd Law (Negation of OR):
              \neg(A \lor B) = \neg A \land \neg B
     • App •The negation of a disjunction is the conjunction of the negations.
     • "Not (A or B)" is the same as "Not A and Not B."

If A = "It is raining" and B = "It is cold"
2. Expres: Then "It is not raining and it is not cold" is the same as "It is not (raining or cold)."
```

• Each conjunctive clause (AND clause Why a together in a document.

The OR operator combines multiple

Disjunctive Normal Form (OR of AND terms)

... and not a

**Conjunctive Normal Form (AND of OR terms)?** 

## **Boolean Laws**

#### 1. Identity Laws

- A \( \) 1 = A
- A V 0 = A

(AND with 1 or OR with 0 does not change A)

#### 2. Null Laws (Dominance Laws)

- $A \wedge 0 = 0$
- A V 1 = 1

(AND with 0 results in 0, OR with 1 results in 1)

#### 3. Idempotent Laws

- A \( \text{A} = A \)
- A V A = A

(AND or OR with itself does nothing)

#### 4. Complement Laws

- $A \wedge \neg A = 0$
- A V ¬A = 1

(A AND NOT A is always false, A OR NOT A is always true)

#### 5. Double Negation Law

•  $\neg(\neg A) = A$ 

(Negation cancels out)

#### 6. Commutative Laws

- A Λ B = B Λ A
- A V B = B V A

(Order doesn't matter for AND/OR)

#### 7. Associative Laws

- $(A \wedge B) \wedge C = A \wedge (B \wedge C)$
- (A V B) V C = A V (B V C)

Grouping doesn't matter for AND/OR)

#### 8. Distributive Laws

- $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$
- $A \lor (B \land C) = (A \lor B) \land (A \lor C)$

(AND distributes over OR, OR distributes over AND)

#### 9. Absorption Laws

- A ∨ (A ∧ B) = A
- A ∧ (A ∨ B)=A

(Simplifies complex expressions)

#### 10. De Morgan's Laws

- $\neg(A \land B) = \neg A \lor \neg B$
- ¬(A ∨ B)=¬A ∧ ¬B

(Negation flips AND/OR)

# Disjunctive Normal Form

#### Why Use DNF in Information Retrieval?

- It simplifies Boolean queries for retrieval engines.
- Each conjunctive clause (AND clause) represents a minimal document retrieval condition.
- Allows efficient document filtering by breaking down queries into simpler conditions.

# Disjunctive Normal Form

**Example 1: Basic Boolean Query** 

**Given query:** 

 $(A \wedge (B \vee C))$ 

#### **Step-by-step transformation:**

1. Distribute **AND** over **OR**:

 $(A \wedge B) \vee (A \wedge C)$ 

2. The result is now in **DNF**:

 $(A \wedge B) \vee (A \wedge C)$ 

• This means a document must contain (A and B) OR (A and C) to be retrieved.

# Disjunctive Normal Form

**Example 2: More Complex Query** 

#### Given query:

 $(A \lor B) \land (C \lor D)$ 

#### **Step-by-step transformation:**

1. Apply distribution:

 $(A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$ 

2. This is now in **DNF** form:

 $(A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$ 

 This means a document must contain any of the AND-combinations to be retrieved.

**Scenario: Searching a Document Collection** 

Assume we have a **Boolean query** for retrieving documents from a collection of articles.

#### **User Query:**

(Machine V Deep) ∧ (Learning V AI)

This means the user wants documents containing either "Machine" or "Deep", and at the same time, containing either "Learning" or "Al".

### **Step 1: Convert to DNF**

Using distribution of AND over OR:

(Machine  $\land$  Learning)  $\lor$  (Machine  $\land$  AI)  $\lor$  (Deep  $\land$  Learning)  $\lor$  (Deep  $\land$  AI)

- Now, the query consists of four conjunctive (AND) clauses:
  - Machine AND Learning
  - Machine AND AI
  - Deep AND Learning
  - Deep AND AI

### Step 2: Apply to a Document Collection

- Assume we have the following documents in our system:
- D1 "Machine Learning is a key area of AI research."
- D2 "Deep Learning is revolutionizing Al applications."
- D3 "Machine intelligence is different from Large Language Models."
- D4 "Al and Machine Learning work together in modern computing."
- D5 "Deep Learning and AI are driving innovations."

### **Step 3: Retrieve Matching Documents**

Now, we check which documents satisfy **at least one** of the four DNF clauses:

- 1. (Machine AND Learning) → Matches D1, D4
- 2. (Machine AND AI) → Matches D1, D4
- 3. (Deep AND Learning) → ✓ Matches D2, D5
- 4. (Deep AND AI) → Matches D2, D5

Thus, the retrieved documents are:

**D1, D2, D4, D5** (D3 is excluded since it doesn't match any clause).

### **Corpus**

- D1 "Machine Learning is a key area of AI research."
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- D3 "Machine intelligence is different from Large Language Models."
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### Query

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### Query

#### **Step 4: Ranking (Optional)**

Since the **Boolean Model** doesn't inherently rank results, we can use:

- Term Frequency (TF): Rank documents higher if they contain the query terms more frequently.
- Proximity Ranking: Rank documents higher if the matched words appear closer together.

#### For example:

D1 could be ranked higher than D4, both have "Machine" and "Learning" close by but in D1 it appears first.

#### **Step 4: Ranking (Optional)**

#### Final Retrieved Results (Ranked)

- 1. D5 "Deep Learning and AI are driving innovations."
- 2. D2 "Deep Learning is revolutionizing AI applications."
- 3. D1 "Machine Learning is a key area of Al research."
- 4. D4 "Al and Machine Learning work together in modern computing."

### Highlights:

- DNF breaks complex queries into smaller, manageable conditions.
- Documents are retrieved based on logical AND conditions.
- Ranking can be added for better result ordering.

Boolean Retrieval helps in **efficiently filtering** documents before applying more advanced ranking techniques.

- Term-document matrix:
  - Let us consider an automobile and the following corpus:
    - 1. Doc1: "An automobile consists of various components, including wheels and a spark plug."
    - 2. Doc2: "Regular maintenance of automobiles involves checking the condition of wheels and replacing the spark plug."
    - 3. Doc3: "The invention of the spark plug revolutionized automobile engines, leading to improved performance."
    - 4. Doc4: "Wheels are an essential part of automobiles, providing mobility, while spark plugs ignite fuel in the engine."
  - Also consider the following terms / vocabulary / lexicon:
    - {automobile, wheels, spark plug}
  - And query q
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Term-document matrix:

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	Doc1	Doc2	Doc3	Doc4
automobile	1	1	1	1
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spark plug	1	1	1	1



Term-document matrix:

q = (automobile Λ wheels) v spark plug

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D1, D2, D4, D3

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Term-document matrix:

q = (automobile v wheels) A (wheels v spark plug)

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	Doc1	Doc2	Doc3	Doc4
automobile	1	1	1	1
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spark plug	1	1	1	1



• Term-document matrix:



	Doc1	Doc2	Doc3	Doc4
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spark plug	1	1	1	1

Term-document matrix:

q = (automobile Λ wheels) ν ¬(wheels Λ spark plug)

q = automobile V ¬wheels V ¬spark

	Doc1	Doc2	Doc3	Doc4
automobile	1	1	1	1
wheels	1	1	0	1
spark plug	1	1	1	1



#### • Limitations:

- Since the decision is binary, without any notion of grading scale, it limits retrieval quality
- It is not easy to translate information needs into boolean expressions

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