

12.1 Huffman compression

Basic compression idea

Given data represented as some quantity of bits, **compression** transforms the data to use fewer bits. Compressed data uses less storage and can be communicated faster than uncompressed data.

The basic idea of compression is to encode frequently-occurring items using fewer bits. Ex: Uncompressed ASCII characters use 8 bits each, but compression uses fewer than 8 bits for more frequently occurring characters.

PARTICIPATION
ACTIVITY

12.1.1: The basic idea of compression is to use fewer bits for frequent items (and more bits for less-frequent items).



Animation content:

undefined

Animation captions:

1. The text "AAA Go" as ASCII would use $6 * 8 = 48$ bits. Such data is uncompressed.
2. Compression uses a dictionary of codes specific to the data. Frequent items get shorter codes. Here, A (which is most frequent) is 0, space 10, G 110, and o 111.
3. Thus, "AAA Go" is compressed as 0 0 0 10 110 111. The compressed data uses only eleven bits, much fewer than the 48 bits uncompressed.

PARTICIPATION
ACTIVITY

12.1.2: Basic compression.



Given the following dictionary:

00000000: 00

11111111: 01

00000010: 10

00000011: 110

00000100: 111

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023



- 1) Compress the following: 00000000
00000000 11111111 00000100

Show answer

- 2) Compress the following: 00000011
00000010

©zyBooks 11/20/23 11:11 104847/
Eric Quezada
UTEPACS2302ValeraFall2023

Show answer

- 3) Decompress the following: 00 01 00



Show answer

- 4) Is any code in the dictionary a prefix
of another code? Type yes or no.



Show answer

- 5) Decompress the following, in which
the spaces that were inserted above
for reading convenience are absent:
0011000.



Show answer

Building a character frequency table

©zyBooks 11/20/23 11:11 104847/
Eric Quezada
UTEPACS2302ValeraFall2023

Prior to compression, a character frequency table must be built for an input string. Such a table contains each distinct character from the input string and each character's number of occurrences.

Programming languages commonly provide a dictionary or map object to store the character frequency table.

**PARTICIPATION
ACTIVITY**

12.1.3: Building a character frequency table.

**Animation content:**

Static figure: A code block and a character frequency table.

Begin pseudocode:

```
BuildCharacterFrequencyTable(inputString) {  
    table = new Dictionary()  
    for (i = 0; i < inputString.length; i++) {  
        currentCharacter = inputString[i]  
        if (table has key for currentCharacter) {  
            table[currentCharacter] = table[currentCharacter] + 1  
        }  
        else {  
            table[currentCharacter] = 1  
        }  
    }  
    return table  
}
```

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302 Valera Fall 2023

BuildCharacterFrequencyTable("APPLES AND BANANAS")

End pseudocode.

Step 1: A new dictionary is created for the character frequency table and iteration through the input string's characters begins.

The code BuildCharacterFrequencyTable("APPLES AND BANANAS") is highlighted. Then, the code BuildCharacterFrequencyTable(inputString) { is highlighted, followed by the code table = new Dictionary(). The text table: (empty) appears below the code block. The code for (i = 0; i < inputString.length; i++) { is highlighted. The text i: 0 appears below the code block. The code currentCharacter = inputString[i] is highlighted. The text currentCharacter: A appears under the text i: 0.

Step 2: The current character, A, is not in the dictionary. So A is added to the dictionary with a frequency of 1.

The code if (table has key for currentCharacter) { is highlighted. The text not in table appears next to the text currentCharacter: A. The code table[currentCharacter] = 1 is highlighted. The text (empty) disappears after the word table under the code block. A new entry, A 1, appears in the table.

UTEP-CS2302 Valera Fall 2023

Step 3: The next character, P, is also not in the dictionary and is added with a frequency of 1.

The code for (i = 0; i < inputString.length; i++) { is highlighted, followed by the code currentCharacter = inputString[i]. The letter A is replaced with a P after the text currentCharacter. The code if (table has key for currentCharacter) { is highlighted. The text not in table appears next to the text currentCharacter: P. The code table[currentCharacter] = 1 is highlighted. A new entry, P 1, appears in the table.

Step 4: The next character, P, is already in the table, so the existing frequency is incremented from 1 to 2.

The code `currentCharacter = inputString[i]` is highlighted, and the P after the text `currentCharacter` is highlighted. The code `if (table has key for currentCharacter) {` is highlighted. The text in table appears next to the text `currentCharacter: P`. The code `table[currentCharacter] = table[currentCharacter] + 1` is highlighted. The P entry in the table is updated from 1 to 2.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

Step 5: For remaining characters, first occurrences set the frequency to 1 and subsequent occurrences increment the existing frequency.

The entire for loop is highlighted. The frequency table is filled in: A 5, P 2, L 1, E 1, S 2, (space) 2, N 3, D 1, B 1.

Animation captions:

1. A new dictionary is created for the character frequency table and iteration through the input string's characters begins.
2. The current character, A, is not in the dictionary. So A is added to the dictionary with a frequency of 1.
3. The next character, P, is also not in the dictionary and is added with a frequency of 1.
4. The next character, P, is already in the table, so the existing frequency is incremented from 1 to 2.
5. For remaining characters, first occurrences set the frequency to 1 and subsequent occurrences increment the existing frequency.

PARTICIPATION ACTIVITY

12.1.4: Character frequency counts.



Given the text "seems he fled", indicate the frequency counts.

1) s



- 1
- 2
- 3

2) e

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

- 2
- 5
- 6



3) Each m, h, f, l, and d

- 1
- 2
- 3

4) (space)

- 1
- 2
- 3

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023



Huffman coding

Huffman coding is a common compression technique that assigns fewer bits to frequent items, using a binary tree.

PARTICIPATION ACTIVITY

12.1.5: A binary tree can be used to determine the Huffman coding.



Animation captions:

1. Huffman coding first determines the frequencies of each item. Here, a occurs 4 times, b 3, c 2, and d 1. (Total is 10).
2. Each item is a "leaf node" in a tree. The pair of nodes yielding the lowest sum is found, and merged into a new node formed with that sum. Here, c and d yield $2 + 1 = 3$.
3. The merging continues. The lowest sum is b's 3 plus the new node's 3, yielding 6. (Note that c and d are no longer eligible nodes). The merging ends when only 1 node exists.
4. Each leaf node's encoding is obtained by traversing from the top node to the leaf. Each left branch appends a 0, and each right branch appends a 1, to the code.

PARTICIPATION ACTIVITY

12.1.6: Huffman coding example: Merging nodes.



A 100-character text has these character frequencies:

- A: 50
- C: 40
- B: 4
- D: 3
- E: 3

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023



1) What is the first merge?

- D and E: 6
- B and D: 7
- B and D and E: 10

2) What is the second merge?

- B and D: 7
- DE and B: 10
- C and A: 90

3) What is the third merge?

- DEB and C: 40
- DEB and C: 50

4) What is the fourth merge?

- None
- DEBC and A: 100

5) What is the fifth merge?

- None
- DEBCA and F

6) What is the code for A?

- 0
- 1

7) What is the code for C?

- 1
- 01
- 10

8) What is the code for B?

- 001
- 110

9) What is the code for D?

- 1110
- 1111

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPACS2302ValeraFall2023



10) What is the code for E?

- 1110
- 1111

11) 5 unique characters (A, B, C, D, E) can each be uniquely encoded in 3 bits (like 000, 001, 010, 011, and 100). With such a fixed-length code, how many bits are needed for the 100-character text?

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

- 100
- 300

12) For the Huffman code determined in the above questions, the number of bits per character is A: 1, C: 2, B: 3, D: 4, and E: 4. Recalling the frequencies in the instructions, how many bits are needed for the 100-character text?

- 14
- 166
- 300

Note: For Huffman encoded data, the dictionary must be included along with the compressed data, to enable decompression. That dictionary adds to the total bits used. However, typically only large data files get compressed, so the dictionary is usually a tiny fraction of the total size.

Building a Huffman tree

The data members in a Huffman tree node depend on the node type.

- Leaf nodes have two data members: a character from the input and an integer frequency for that character.
- Internal nodes have left and right child nodes, along with an integer frequency value that represents the sum of the left and right child frequencies.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

A Huffman tree can be built from a character frequency table.

PARTICIPATION
ACTIVITY

12.1.7: Building a Huffman tree.



Animation content:

undefined

Animation captions:

1. The character frequency table is built for the input string, BANANAS.
2. A leaf node is built for each table entry and enqueue in a priority queue. Lower frequencies have higher priority.
3. Leaf nodes for S and B are removed from the queue. A parent is built with the sum of the frequencies and is enqueue into the priority queue.
4. The two nodes with frequency 2 are dequeued and the parent with frequency $2 + 2 = 4$ is built.
5. The remaining 2 nodes are dequeued and given a parent.
6. When the priority queue has 1 node remaining, that node is the tree's root. The root is dequeued and returned.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302ValeraFall2023

PARTICIPATION ACTIVITY

12.1.8: HuffmanBuildTree function.



Assume `HuffmanBuildTree("zyBooks")` is called.

- 1) The character frequency table has _____ entries.

- 5
- 6
- 7



- 2) The leaf node at the back of the priority queue, `nodes`, before the while loop begins is _____.

- z | 1
- B | 1
- o | 2



- 3) The parent node for nodes `B` and `k` has a frequency of _____.

- 1
- 2
- 4

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302ValeraFall2023



Implementing with 1 node structure

Implementations commonly use the same node structure for leaf and internal nodes, instead of two distinct structures. Each node has a frequency, character, and 2 child pointers. The child pointers are set to null for leaves and the character is set to 0 for internal nodes.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPCS2302ValeraFall2023

Getting Huffman codes

Huffman codes for each character are built from a Huffman tree. Each character corresponds to a leaf node. The Huffman code for a character is built by tracing a path from the root to that character's leaf node, appending 0 when branching left or 1 when branching right.

PARTICIPATION
ACTIVITY

12.1.9: Getting Huffman codes.



Animation content:

undefined

Animation captions:

1. Huffman codes are built from a Huffman tree. Left branches add a 0 to the code, right branches add a 1.
2. HuffmanGetCodes starts at the root node with an empty prefix.
3. A recursive call is made on the root's left child. The node is a leaf and A's code is set to the current prefix, 0.
4. The first recursive call completes. The next recursive call is made for node 4 and a prefix of "1".
5. Node 4 and node 2 are not leaves, so additional recursive calls are made.
6. B's code is set to 100.
7. The remaining recursive calls set codes for S and N.
8. Each distinct character has a code. Characters B and S have lower frequencies than A and N and thus have longer codes.

PARTICIPATION
ACTIVITY

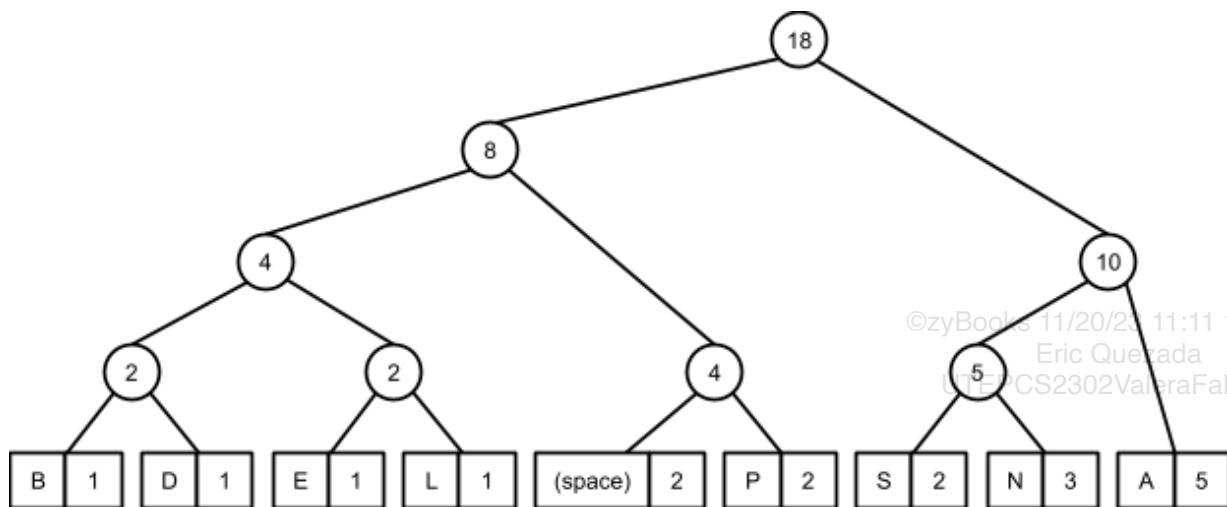
12.1.10: Huffman codes.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPCS2302ValeraFall2023

Below is the Huffman tree for "APPLES AND BANANAS"



©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

1) What is the Huffman code for A?

- 10
- 11
- 101



2) What is the Huffman code for P?

- 01
- 0000
- 011



3) What is the length of the longest
Huffman code?

- 3
- 4
- 5



Compressing data

To compress an input string, the Huffman codes are first obtained for each character. Then each character of the input string is processed, and corresponding bit codes are concatenated to produce the compressed result.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

Figure 12.1.1: HuffmanCompress function.

```
HuffmanCompress(inputString) {  
    // Build the Huffman tree  
    root = HuffmanBuildTree(inputString)  
  
    // Get the compression codes from the tree  
    codes = HuffmanGetCodes(root, "", new  
Dictionary())  
  
    // Build the compressed result  
    result = ""  
    for c in inputString {  
        result += codes[c]  
    }  
    return result  
}
```

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

**PARTICIPATION
ACTIVITY****12.1.11: Compressing data.**

Match each compression call to the result. Spaces are added between character codes for clarity, but would not exist in the actual compressed data.

If unable to drag and drop, refresh the page.

HuffmanCompress("zyBooks")**HuffmanCompress("aabbbac")****HuffmanCompress("BANANAS")**

100 0 11 0 11 0 101

00 101 010 11 11 011 100

11 11 0 0 0 11 10

Reset

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

Decompressing Huffman coded data

To decompress Huffman code data, one can use a Huffman tree and trace the branches for each bit, starting at the root. When the final node of the branch is reached, the result has been found. The process continues until the entire item is decompressed.

**Animation captions:**

1. The Huffman code is decompressed by first starting at the root. The branches are followed for each bit.
2. When the final node of the branch is reached, the result has been found. 11/20/23 11:11 1048447 Eric Quezada UTEPCS2302ValeraFall2023
3. Once the final node is reached decoding restarts at the root node.
4. The process continues until the entire item is decompressed.

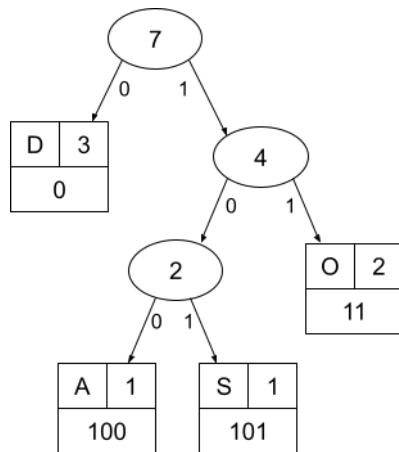
Figure 12.1.2: HuffmanDecompress function.

```
HuffmanDecompress(compressedString, treeRoot) {  
    node = treeRoot  
    result = ""  
    for (bit in compressedString) {  
        // Go to left or right child based on bit value  
        if (bit == 0)  
            node = node->left  
        else  
            node = node->right  
  
        // If the node is a leaf, add the character to  
        // the result and go back to the root  
        if (node is a leaf) {  
            result += node->character  
            node = treeRoot  
        }  
    }  
    return result  
}
```



Use the tree below to decompress 0111101000101.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023



©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

- 1) What is the first decoded character?

Check

Show answer



- 2) What is the second decoded character?

Check

Show answer



- 3) 11 yields the third character O.
0 yields the fourth character D.



What is the next decoded character?

Check

Show answer



- 4) What is the decoded text?

Check

Show answer

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023



CHALLENGE ACTIVITY

12.1.1: Huffman compression.

502696.2096894.qx3zqy7

Start

Complete the character frequency table for the string "COFFEE AND TEA".

Character	Frequency
(space)	2
A	Ex: 4
C	
D	
E	
F	
N	
O	
T	

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

1

2

3

Check**Next**

12.2 Heuristics

Heuristics

In practice, solving a problem in the optimal or most accurate way may require more computational resources than are available or feasible. Algorithms implemented for such problems often use a **heuristic**: A technique that willingly accepts a non-optimal or less accurate solution in order to improve execution speed.

PARTICIPATION ACTIVITY

12.2.1: Introduction to the knapsack problem.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

Animation content:

undefined

Animation captions:

1. A knapsack is a container for items, much like a backpack or bag. Suppose a particular knapsack can carry at most 30 pounds worth of items.
2. Each item has a weight and value. The goal is to put items in the knapsack such that the weight ≤ 30 pounds and the value is maximized.
3. Taking a 20 pound item with an 8 pound item is an option, worth \$142.
4. If more than 1 of each item can be taken, 2 of item 1 and 1 of item 4 provide a better option, worth \$145.
5. Trying all combinations will give an optimal answer, but is time consuming. A heuristic algorithm may choose a simpler, but non-optimal approach.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

PARTICIPATION ACTIVITY

12.2.2: Heuristics.



- 1) A heuristic is a way of producing an optimal solution to a problem.
 True
 False
- 2) A heuristic technique used for numerical computation may sacrifice accuracy to gain speed.
 True
 False

**PARTICIPATION ACTIVITY**

12.2.3: The knapsack problem.



- Refer to the example in the animation above.
- 1) Which of the following options provides the best value?
 5 6-pound items
 2 6-pound items and 1 18-pound item
 3 8-pound items and 1 6-pound item.
 - 2) The optimal solution has a value of \$162 and has one of each item: 6-pound, 8-pound, and 18-pound.
 True
 False

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023



3) Which approach would guarantee finding an optimal solution?

- Taking the largest item that fits in
- the knapsack repeatedly until no more items will fit.
- Taking the smallest item
- repeatedly until no more items will fit in the knapsack.
- Trying all combinations of items
- and picking the one with maximum value.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

Heuristic optimization

A **heuristic algorithm** is an algorithm that quickly determines a near optimal or approximate solution. Such an algorithm can be designed to solve the **0-1 knapsack problem**: The knapsack problem with the quantity of each item limited to 1.

A heuristic algorithm to solve the 0-1 knapsack problem can choose to always take the most valuable item that fits in the knapsack's remaining space. Such an algorithm uses the heuristic of choosing the highest value item, without considering the impact on the remaining choices. While the algorithm's simple choices are aimed at optimizing the total value, the final result may not be optimal.

PARTICIPATION ACTIVITY

12.2.4: Non-optimal, heuristic algorithm to solve the 0-1 knapsack.



Animation content:

undefined

Animation captions:

1. The item list is sorted and the most valuable item is put into the knapsack first.
2. No remaining items will fit in the knapsack.
3. The resulting value of \$95 is inferior to taking the 12 and 8 pound items, collectively worth \$102.
4. The heuristic algorithm sacrifices optimality for efficiency and simplicity.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

PARTICIPATION ACTIVITY

12.2.5: Heuristic algorithm and the 0-1 knapsack problem.





- 1) Which is not commonly sacrificed by a heuristic algorithm?
- speed
 - optimality
 - accuracy

- 2) What restriction does the 0-1 knapsack problem have, in comparison with the regular knapsack problem?

©zyBooks 11/20/23 11:11 104847
Eric Quezada
UTEPCS2302ValeraFall2023

- The knapsack's weight limit cannot be exceeded.
- At most 1 of each item can be taken.
- The value of each item must be less than the item's weight.

- 3) Under what circumstance would the Knapsack01 function not put the most valuable item into the knapsack?

- The item list contains only 1 item.
- The weight of the most valuable item is greater than the knapsack's limit.



Self-adjusting heuristic

A **self-adjusting heuristic** is an algorithm that modifies a data structure based on how that data structure is used. Ex: Many self-adjusting data structures, such as red-black trees and AVL trees, use a self-adjusting heuristic to keep the tree balanced. Tree balancing organizes data to allow for faster access.

Ex: A self-adjusting heuristic can be used to speed up searches for frequently-searched-for list items by moving a list item to the front of the list when that item is searched for. This heuristic is self-adjusting because the list items are adjusted when a search is performed.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPCS2302ValeraFall2023

PARTICIPATION ACTIVITY

12.2.6: Move-to-front self-adjusting heuristic.



Animation content:

undefined

Animation captions:

1. 42 is at the end of a list with 8 items. A linear search for 42 compares against 8 items.
2. The move-to-front heuristic moves 42 to the front after the search.
3. Another search for 42 now only requires 1 comparison. 42 is left at the front of the list.
4. A search for 64 compares against 8 items and moves 64 to the front of the list.
5. 42 is no longer at the list's front, but a search for 42 need only compare against 2 items.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

**PARTICIPATION ACTIVITY**

12.2.7: Move-to-front self-adjusting heuristic.

Suppose a move-to-front heuristic is used on a list that starts as (56, 11, 92, 81, 68, 44).

- 1) A first search for 81 compares against how many list items?

- 0
- 3
- 4

- 2) A subsequent search for 81 compares against how many list items?

- 1
- 2
- 4

- 3) Which scenario results in faster searches?

- Back-to-back searches for the same key.
- Every search is for a key different than the previous search.



12.3 Python: Heuristics

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

0-1 Knapsack problem heuristic

The general knapsack problem seeks to maximize the total value of items placed into a knapsack such that the total weight of items in the knapsack doesn't exceed a predetermined weight. The 0-1 knapsack problem imposes the restriction that each item can be added at most once. A heuristic algorithm to

solve the knapsack problem first sorts items in descending order by value, and then iteratively places the most valuable items that fit within the remaining space into the knapsack until no more items can be added.

To construct a Python program to solve the knapsack problem using a heuristic, two classes are defined:

- An Item class to store each item's weight and value
- A Knapsack class to store the knapsack's maximum predetermined weight and the list of items the knapsack will hold

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302 Valera Fall 2023

The heuristic is defined in the knapsack_01() function. To sort the original list of items by value, the operator module's attrgetter() function (attribute getter) is imported and used in list's sort() method to identify the 'value' attribute as the sorting key. The **reverse = True** argument passed to list's sort method() sorts the items in descending order. Then the first item of the sorted list is added to the knapsack, changing the knapsack's remaining weight. The algorithm continues until the knapsack is full or until the next item in the list weighs more than the remaining weight in the knapsack.

Figure 12.3.1: 0-1 Knapsack problem heuristic.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302 Valera Fall 2023

```
from operator import attrgetter

class Item:
    def __init__(self, item_weight, item_value):
        self.weight = item_weight
        self.value = item_value

class Knapsack:
    def __init__(self, weight, items):
        self.max_weight = weight
        self.item_list = items

def knapsack_01(knapsack, item_list):
    # Sort the items in descending order based on value
    item_list.sort(key = attrgetter('value'), reverse = True)

    remaining = knapsack.max_weight
    for item in item_list:
        if item.weight <= remaining:
            knapsack.item_list.append(item)
            remaining = remaining - item.weight

# Main program
item_1 = Item(6, 25)
item_2 = Item(8, 42)
item_3 = Item(12, 60)
item_4 = Item(18, 95)
item_list = [item_1, item_2, item_3, item_4]
initial_knapsack_list = []

max_weight = int(input('Enter maximum weight the knapsack can hold: '))

knapsack = Knapsack(max_weight, initial_knapsack_list)
knapsack_01(knapsack, item_list)

print ('Objects in knapsack')
i = 1
sum_weight = 0
sum_value = 0

for item in knapsack.item_list:
    sum_weight += item.weight
    sum_value += item.value
    print ('%d: weight %d, value %d' % (i, item.weight, item.value))
    i += 1
print()

print('Total weight of items in knapsack: %d' % sum_weight)
print('Total value of items in knapsack: %d' % sum_value)
```

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

```
Enter maximum weight the knapsack can hold: 40
Objects in knapsack
1: weight 18, value 95
2: weight 12, value 60
3: weight 8, value 42

Total weight of items in knapsack: 38
Total value of items in knapsack: 197
```

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPACS2302ValeraFall2023

**PARTICIPATION ACTIVITY**

12.3.1: 0-1 Knapsack heuristic.

For questions 1 and 2, refer to the code above and assume `max_weight = 30`.

- 1) How many items are put into the knapsack using the heuristic?

- 1
- 2
- 3

- 2) Does the heuristic find the optimal solution to the 0-1 knapsack problem?

- Yes
- No

- 3) Using the heuristic, which items are added to the knapsack assuming:

```
item_1 = Item(6, 25)
item_2 = Item(8, 42)
item_3 = Item(12, 60)
item_4 = Item(18, 95)
```

`max_weight= 28`

- item_2 and item_1
- item_4, item_3, and item_2
- item_4 and item_2

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPACS2302ValeraFall2023



- 4) Using the heuristic, what is the weight sum of the items added to the knapsack assuming:

```
item_1 = Item(6, 25)
item_2 = Item(8, 42)
item_3 = Item(12, 60)
item_4 = Item(18, 95)
```

`max_weight= 32`

- 30
- 32

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

zyDE 12.3.1: 0-1 Knapsack problem heuristic.

The program below employs the 0-1 knapsack heuristic to add three items (item_4, item_2) to the knapsack; the sum of those items (38) is less than knapsack's max_weight. You can try running the program with different maximum weights to see how many items are added.

main.py

[Load default template](#)

```
1 from operator import attrgetter
2
3 class Item:
4     def __init__(self, item_weight, item_value):
5         self.weight = item_weight
6         self.value = item_value
7
8
9 class Knapsack:
10    def __init__(self, weight, items):
11        self.max_weight = weight
12        self.item_list = items
13
14
15 def knapsack_01(knapsack, item_list):
16     """
17     This function takes a knapsack object and a list of items and returns the
18     maximum value that can be obtained by adding items to the knapsack.
19     """
20
21     # If there are no items or the knapsack is full, return 0
22     if len(item_list) == 0 or knapsack.max_weight == 0:
23         return 0
24
25     # If there is only one item left, add it if it fits
26     if len(item_list) == 1:
27         item = item_list[0]
28         if item.weight <= knapsack.max_weight:
29             return item.value + knapsack_01(knapsack, item_list[1:])
30         else:
31             return knapsack_01(knapsack, item_list[1:])
32
33     # If there are two items left, consider both possibilities
34     if len(item_list) == 2:
35         item1 = item_list[0]
36         item2 = item_list[1]
37
38         if item1.weight <= knapsack.max_weight:
39             option1 = item1.value + knapsack_01(knapsack, item_list[1:])
40         else:
41             option1 = knapsack_01(knapsack, item_list[1:])
42
43         if item2.weight <= knapsack.max_weight - item1.weight:
44             option2 = item2.value + knapsack_01(knapsack, item_list[1:])
45         else:
46             option2 = knapsack_01(knapsack, item_list[1:])
47
48         return max(option1, option2)
49
50     # If there are more than two items left, consider the first item and
51     # the remaining items separately
52     else:
53         item1 = item_list[0]
54
55         if item1.weight <= knapsack.max_weight:
56             option1 = item1.value + knapsack_01(knapsack, item_list[1:])
57         else:
58             option1 = knapsack_01(knapsack, item_list[1:])
59
60         remaining_items = item_list[1:]
61         remaining_knapsack = Knapsack(knapsack.max_weight - item1.weight, remaining_items)
62
63         option2 = knapsack_01(remaining_knapsack, remaining_items)
64
65         return max(option1, option2)
```

40

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

Run

12.4 Greedy algorithms

Greedy algorithm

A **greedy algorithm** is an algorithm that, when presented with a list of options, chooses the option that is optimal at that point in time. The choice of option does not consider additional subsequent options, and may or may not lead to an optimal solution.

PARTICIPATION ACTIVITY

12.4.1: MakeChange greedy algorithm.



Animation content:

undefined

Animation captions:

1. The change making algorithm uses quarters, dimes, nickels, and pennies to make change equaling the specified amount.
2. The algorithm chooses quarters as the optimal coins, as long as the remaining amount is ≥ 25 .
3. Dimes offer the next largest amount per coin, and are chosen while the amount is ≥ 10 .
4. Nickels are chosen next. The algorithm is greedy because the largest coin \leq the amount is always chosen.
5. Adding one penny makes 91 cents.
6. This greedy algorithm is optimal and minimizes the total number of coins, although not all greedy algorithms are optimal.

PARTICIPATION ACTIVITY

12.4.2: Greedy algorithms.



- 1) If the MakeChange function were to make change for 101, what would be the result?

- 101 pennies
- 4 quarters and 1 penny
- 3 quarters, 2 dimes, 1 nickel, and 1 penny

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023



2) A greedy algorithm is attempting to minimize costs and has a choice between two items with equivalent functionality: the first costing \$5 and the second costing \$7. Which will be chosen?

- The \$5 item
- The \$7 item
- The algorithm needs more information to choose

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

3) A greedy algorithm always finds an optimal solution.

- True
- False



Fractional knapsack problem

The **fractional knapsack problem** is the knapsack problem with the potential to take each item a fractional number of times, provided the fraction is in the range [0.0, 1.0]. Ex: A 4 pound, \$10 item could be taken 0.5 times to fill a knapsack with a 2 pound weight limit. The resulting knapsack would be worth \$5.

While a greedy solution to the 0-1 knapsack problem is not necessarily optimal, a greedy solution to the fractional knapsack problem is optimal. First, items are sorted in descending order based on the value-to-weight ratio. Next, one of each item is taken from the item list, in order, until taking 1 of the next item would exceed the weight limit. Then a fraction of the next item in the list is taken to fill the remaining weight.

Figure 12.4.1: FractionalKnapsack algorithm.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

```
FractionalKnapsack(knapsack, itemList, itemListSize) {  
    Sort itemList descending by item's (value / weight)  
    ratio  
    remaining = knapsack->maximumWeight  
    for each item in itemList {  
        if (item->weight <= remaining) {  
            Put item in knapsack  
            remaining = remaining - item->weight  
        }  
        else if (remaining > 0) {  
            fraction = remaining / item->weight  
            Put (fraction * item) in knapsack  
            break  
        }  
    }  
}
```

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302 Valera Fall 2023

PARTICIPATION ACTIVITY**12.4.3: Fractional knapsack problem.**

Suppose the following items are available: 40 pounds worth \$80, 12 pounds worth \$18, and 8 pounds worth \$8.

- 1) Which item has the highest value-to-weight ratio?

- 40 pounds worth \$80
- 12 pounds worth \$18
- 8 pounds worth \$8



- 2) What would FractionalKnapsack put in a 20-pound knapsack?

- One 12-pound item and one 8-pound item
- One 40-pound item
- Half of a 40-pound item



- 3) What would FractionalKnapsack put in a 48-pound knapsack?

- One 40-pound item and one 8-pound item
- One 40-pound item and 2/3 of a 12-pound item

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302 Valera Fall 2023

Activity selection problem

The **activity selection problem** is a problem where 1 or more activities are available, each with a start and finish time, and the goal is to build the largest possible set of activities without time conflicts. Ex: When on vacation, various activities such as museum tours or mountain hikes may be available. Since vacation time is limited, the desire is often to engage in the maximum possible number of activities per day.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada
UTEP-CS2302-ValeraFall2023

A greedy algorithm provides the optimal solution to the activity selection problem. First, an empty set of chosen activities is allocated. Activities are then sorted in ascending order by finish time. The first activity in the sorted list is marked as the current activity and added to the set of chosen activities. The algorithm then iterates through all activities after the first, looking for a next activity that starts after the current activity ends. When such a next activity is found, the next activity is added to the set of chosen activities, and the next activity is reassigned as the current. After iterating through all activities, the chosen set of activities contains the maximum possible number of non-conflicting activities from the activities list.

PARTICIPATION ACTIVITY

12.4.4: Activity selection problem algorithm.



Animation content:

undefined

Animation captions:

1. Activities are first sorted in ascending order by finish time. The set of chosen activities initially has the activity that finishes first.
2. The morning mountain hike does not start after the history museum tour finishes and is not added to the chosen set of activities.
3. The boat tour is the first activity to start after the history museum tour finishes, and is the "greedy" choice.
4. Hang gliding and the fireworks show are chosen as 2 additional activities.
5. The maximum possible number of non-conflicting activities is 4, and 4 have been chosen.

PARTICIPATION ACTIVITY

12.4.5: ActivitySelection algorithm.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023



1) The fireworks show and the night movie both finish at 9 PM, so the sorting algorithm could have swapped the order of the 2. If the 2 were swapped, the number of chosen activities would not be affected.

- True
- False

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

2) Changing snorkeling's _____ would cause snorkeling to be added to the chosen activities.

- start time from 3 PM to 4 PM
- finish time from 5 PM to 4 PM



3) Regardless of any changes to the activity list, the activity with the _____ will always be in the result.

- earliest start time
- earliest finish time
- longest length



12.5 Python: Greedy algorithms

Greedy algorithm

A greedy algorithm solves a problem by assuming that the optimal choice at a given moment during the algorithm will also be the optimal choice overall. Greedy algorithms tend to be efficient, but certain types of problems exist where greedy algorithms don't find the *best* or *optimal* solution. However, greedy algorithms produce both efficient and optimal solutions for many problems, including the fractional knapsack problem and the activity selection problem.

Fractional knapsack problem

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023

The fractional knapsack problem is similar to the regular knapsack problem, except that fractional pieces of any item can be selected. The optimal solution can be achieved by first sorting the items by their value-to-weight ratio, in descending order. The items are then added to the knapsack in that order, taking whole units of items until only a fraction of the next item is possible. The highest fraction of that item that will fit in the knapsack is taken, and the algorithm ends.

The list's sort() method is used with a custom key function, value_to_weight_ratio(). The key function tells the sort() method what value to use to sort each item. For solving fractional knapsack problem, sort() will use the item's value divided by the item's weight. The items are sorted in descending order, so the call to sort() passes the `reverse = True` parameter.

Item objects have a fraction data member that is assigned with the fraction of the item (in the range [0.0, 1.0]) in the Knapsack. Items are assumed to be added fully, so fraction is assigned with 1.0 when the Item object is constructed; the value of the fraction data member is modified in the `fractional_knapsack()` function when only a part of the item fits in the knapsack.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP PCS2302 Valera Fall 2023

Figure 12.5.1: Fractional knapsack greedy algorithm.

```
# An individual item with a weight and value
class Item:
    def __init__(self, item_weight, item_value):
        self.weight = item_weight
        self.value = item_value
        self.fraction = 1.0

# The knapsack that contains a list of items and a maximum
# total weight
class Knapsack:
    def __init__(self, weight, items):
        self.max_weight = weight
        self.item_list = items

# A key function to be used to sort the items
def value_to_weight_ratio(item):
    return item.value / item.weight

# The Fractional Knapsack algorithm.
def fractional_knapsack(knapsack, item_list):
    # Sort the items in descending order based on value/weight
    item_list.sort(key = value_to_weight_ratio, reverse = True)

    remaining = knapsack.max_weight
    for item in item_list:
        # Check if the full item can fit into the knapsack or
        # only a fraction
        if item.weight <= remaining:
            # The full item will fit.
            knapsack.item_list.append(item)
            remaining = remaining - item.weight
        else:
            # Only a fractional part of the item will fit.
            item.fraction = remaining / item.weight
            knapsack.item_list.append(item)
            break
```

To test the algorithm, a Knapsack object is created with a user-defined capacity, along with a list of available items. The `fractional_knapsack()` method is executed. Then, the items in the knapsack are displayed with the items' weight, cost, and fraction.

Figure 12.5.2: A program to test the fractional knapsack greedy algorithm.

The knapsack is given a maximum weight of 35.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPoS2302ValeraFall2023

```
# Main program to test the fractional knapsack algorithm.
item_1 = Item(6, 25)
item_2 = Item(8, 42)
item_3 = Item(12, 60)
item_4 = Item(18, 95)
item_list = [item_1, item_2, item_3, item_4]
initial_knapsack_list = []

# Ask the user for the knapsack's maximum capacity.
max_weight = int(input('Enter maximum weight the knapsack can hold: '))

# Construct the knapsack object, then run the fractional_knapsack
# algorithm on it.
knapsack = Knapsack(max_weight, initial_knapsack_list)
fractional_knapsack(knapsack, item_list)

# Output the information about the knapsack. Show the contents
# of the knapsack, and the fraction of each item.
print ('Objects in knapsack')
i = 1
sum_weight = 0
sum_value = 0
for item in knapsack.item_list:
    sum_weight += item.weight * item.fraction
    sum_value += item.value * item.fraction
    print ('%d: %0.1f of weight %0.1f, value %0.1f' %
           (i, item.fraction, item.weight, item.value * item.fraction))
    i += 1
print()

# Display the total weight of the items as well as the total value.
print('Total weight of items in knapsack: %d' % sum_weight)
print('Total value of items in knapsack: %d' % sum_value)
```

```
Enter maximum weight the knapsack can hold: 35
Objects in knapsack
1: 1.0 of weight 18.0, value 95.0
2: 1.0 of weight 8.0, value 42.0
3: 0.8 of weight 12.0, value 45.0

Total weight of items in knapsack: 35.0
Total value of items in knapsack: 182.0
```

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPoS2302ValeraFall2023





- 1) The items are first sorted in descending order by weight.

True
 False



- 2) The algorithm finds the largest number of items that can fit into the knapsack.

True
 False



- 3) The algorithm always fills the knapsack to the maximum weight (so long as the list has enough items to do so).

True
 False

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

zyDE 12.5.1: Fractional knapsack greedy algorithm.

Try giving the knapsack different maximum weights. Note that all items in the knapsack have their full value (fraction = 1.0), but the final item has a fraction between 0.0 and 1.0 (inclusive). A maximum weight of 38 is a special case. What is unusual about the results? Are the results still technically correct? How could you modify the program to make the results more natural looking?

main.py

[Load default template](#)

```

1 # An individual item with a weight and value
2 class Item:
3     def __init__(self, item_weight, item_value):
4         self.weight = item_weight
5         self.value = item_value
6         self.fraction = 1.0
7
8 # The knapsack that contains a list of items and a maximum
9 # total weight
10 class Knapsack:
11     def __init__(self, weight, items):
12         self.max_weight = weight
13         self.item_list = items
14
15 # A key function to be used to sort the items
16

```

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

35

Run

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

Activity selection problem

The activity selection problem defines a set of "activities" (Ex: Tourist activities on a vacation), as well as when these activities start and finish (Ex: Time of day). The optimal solution will schedule the *most* activities possible without having any time conflicts.

The greedy algorithm starts by sorting the activities, using the activity finish times as the sorting key, from earliest to latest. The first activity in the list is selected and added to the set of chosen activities. The second activity in the sorted list is selected only if the activity does not conflict with the first activity. If the second activity *does* conflict with the first, then the activity is not selected. The third activity in the sorted list is selected only if the activity does not conflict with the second activity. The process continues until the entire sorted list of activities is processed.

An activity is represented with a Python class called `Activity`. Three data members are defined: `name`, `start_time`, and `finish_time`. A `conflicts_with()` method is also defined to determine whether or not a time conflict exists between two `Activity` objects.

Figure 12.5.3: Activity class for the activity selection problem.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

```

class Activity:
    def __init__(self, name, initial_start_time, initial_finish_time):
        self.name = name
        self.start_time = initial_start_time
        self.finish_time = initial_finish_time

    def conflicts_with(self, other_activity):
        # No conflict exists if this activity's finish_time comes
        # at or before the other activity's start_time
        if self.finish_time <= other_activity.start_time:
            return False
        # No conflict exists if the other activity's finish_time
        # comes at or before this activity's start_time
        elif other_activity.finish_time <= self.start_time:
            return False
        # In all other cases the two activity's conflict with each
        # other
        else:
            return True

# Main program to test Activity objects
activity_1 = Activity('History museum tour', 9, 10)
activity_2 = Activity('Morning mountain hike', 9, 12)
activity_3 = Activity('Boat tour', 11, 14)

print('History museum tour conflicts with Morning mountain hike:',
      activity_1.conflicts_with(activity_2))
print('History museum tour conflicts with Boat tour:',
      activity_1.conflicts_with(activity_3))
print('Morning mountain hike conflicts with Boat tour:',
      activity_2.conflicts_with(activity_3))

```

```

History museum tour conflicts with Morning mountain hike: True
History museum tour conflicts with Boat tour: False
Morning mountain hike conflicts with Boat tour: True

```

The `activity_selection()` function takes a list of `Activity` objects as a parameter and finds an optimal selection of activities using the greedy algorithm.

The algorithm uses the list's `sort()` method, using the key function `attrgetter` imported from the `operator` module. This key function allows the `sort()` method to sort the list based on the indicated item attribute, namely the `finish_time` attribute.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

Figure 12.5.4: `activity_selection()` function.

```

from operator import attrgetter

def activity_selection(activities):
    # Start with an empty list of selected activities
    chosen_activities = []

    # Sort the list of activities in increasing order of finish_time
    activities.sort(key = attrgetter("finish_time"))

    # Select the first activity, and add it to the chosen_activities list
    current = activities[0]
    chosen_activities.append(current)

    # Process all the remaining activities, in order
    for i in range(1, len(activities)):

        # If the next activity does not conflict with
        # the most recently selected activity, select the
        # next activity
        if not activities[i].conflicts_with(current):
            chosen_activities.append(activities[i])
            current = activities[i]

    # The chosen_activities list is an optimal list of
    # activities with no conflicts
    return chosen_activities

# Program to test Activity Selection greedy algorithm. The
# start_time and finish_time are represented with integers
# (ex. "20" is 20:00, or 8:00 PM).
activity_1 = Activity('Fireworks show', 20, 21)
activity_2 = Activity('Morning mountain hike', 9, 12)
activity_3 = Activity('History museum tour', 9, 10)
activity_4 = Activity('Day mountain hike', 13, 16)
activity_5 = Activity('Night movie', 19, 21)
activity_6 = Activity('Snorkeling', 15, 17)
activity_7 = Activity('Hang gliding', 14, 16)
activity_8 = Activity('Boat tour', 11, 14)

activities = [ activity_1, activity_2, activity_3, activity_4,
               activity_5, activity_6, activity_7, activity_8 ]

# Use the activity_selection() method to get a list of optimal
# activities with no conflicts.
itinerary = activity_selection(activities)
for activity in itinerary:
    # Output the activity's information.
    print('%s - start time: %d, finish time: %d' %
          (activity.name, activity.start_time, activity.finish_time))

```

History museum tour - start time: 9, finish time: 10
 Boat tour - start time: 11, finish time: 14
 Hang gliding - start time: 14, finish time: 16
 Fireworks show - start time: 20, finish time: 21

PARTICIPATION ACTIVITY

12.5.2: Activity selection algorithm.

- 1) The Activity class data members start_time and finish_time can be assigned with any data type that supports the \leq operator.

- True
- False

- 2) The optimal result for activity selection is the total amount of time spent in the activities.

- True
- False

- 3) The greedy algorithm for activity selection won't necessarily find the optimal result, but will find a result that is close to optimal.

- True
- False

zyDE 12.5.2: Testing the activity selection greedy algorithm.

Try running the activity selection greedy algorithm with different activity start and finish times. Ex: What happens if the start time for "Snorkeling" is changed to 16? Can you change the integer values for start and finish times to real `datetime.time` objects? Does the algorithm work as expected? (Hint: remember to import the `datetime` module, and then use `datetime.time(hour=9)` to represent 9AM or `datetime.time(hour=17)` to represent 5PM.)

main.py

Load default template

```
1 from operator import attrgetter
2
3 class Activity:
4     def __init__(self, name, initial_start_time, initial_finish_time)
5         self.name = name
6         self.start_time = initial_start_time
7         self.finish_time = initial_finish_time
8
9     def conflicts_with(self, other_activity):
10        # No conflict exists if this activity's finish_time comes
```

```

11      # at or before the other activity's start_time.
12      if self.finish_time <= other_activity.start_time:
13          return False
14
15      # No conflict exists if the other activity's finish_time
16      ...

```

Run

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

12.6 Dynamic programming

Dynamic programming overview

Dynamic programming is a problem solving technique that splits a problem into smaller subproblems, computes and stores solutions to subproblems in memory, and then uses the stored solutions to solve the larger problem. Ex: Fibonacci numbers can be computed with an iterative approach that stores the 2 previous terms, instead of making recursive calls that recompute the same term many times over.

PARTICIPATION ACTIVITY

12.6.1: FibonacciNumber algorithm: Recursion vs. dynamic programming.



Animation content:

undefined

Animation captions:

1. The recursive call hierarchy of FibonacciNumber(4) shows each call made to FibonacciNumber.
2. Several terms are computed more than once.
3. The iterative implementation uses dynamic programming and stores the previous 2 terms at a time.
4. For each iteration, the next term is computed by adding the previous 2 terms. The previous and current terms are also updated for the next iteration.
5. 3 loop iterations are needed to compute FibonacciNumber(4). Because the previous 2 terms are stored, no term is computed more than once.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023

PARTICIPATION ACTIVITY

12.6.2: FibonacciNumber implementation.





1) If the recursive version of FibonacciNumber(3) is called, how many times will FibonacciNumber(2) be called?

- 1
- 2
- 3

2) Which version of FibonacciNumber is faster for large term indices?

- Recursive version
- Iterative version
- Neither

3) Which version of FibonacciNumber is more accurate for large term indices?

- Recursive version
- Iterative version
- Neither

4) The recursive version of FibonacciNumber has a runtime complexity of $O(\quad)$. What is the runtime complexity of the iterative version?

- $O(\quad)$
- $O(\quad)$
- $O(\quad)$

PARTICIPATION ACTIVITY

12.6.3: Dynamic programming.



1) Dynamic programming avoids recomputing previously computed results by storing and reusing such results.

- True
- False

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023



2) Any algorithm that splits a problem into smaller subproblems is using dynamic programming.

- True
- False

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

Longest common substring

The **longest common substring** algorithm takes 2 strings as input and determines the longest substring that exists in both strings. The algorithm uses dynamic programming. An $N \times M$ integer matrix keeps track of matching substrings, where N is the length of the first string and M the length of the second. Each row represents a character from the first string, and each column represents a character from the second string.

An entry at i, j in the matrix indicates the length of the longest common substring that ends at character i in the first string and character j in the second. An entry will be 0 only if the 2 characters the entry corresponds to are not the same.

The matrix is built one row at a time, from the top row to the bottom row. Each row's entries are filled from left to right. An entry is set to 0 if the two characters do not match. Otherwise, the entry at i, j is set to 1 plus the entry in $i - 1, j - 1$.

PARTICIPATION ACTIVITY

12.6.4: Longest common substring algorithm.



Animation content:

undefined

Animation captions:

1. Comparing "Look" and "zyBooks" requires a 4×7 matrix.
2. In the first row, 0 is entered for each pair of mismatching characters.
3. In the next row, 'o' matches in 2 entries. In both cases the upper-left value is 0, and 1 is entered into the matrix.
4. Two matches for 'o' exist in the next row as well, with the second having a 1 in the upper-left entry.
5. The character 'k' matches once in the last row and an entry of $2 + 1 = 3$ is entered.
6. The maximum entry in the matrix is the longest common substring's length. The maximum entry's row index is the substring ending index in the first string.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

PARTICIPATION ACTIVITY

12.6.5: Longest common substring matrix.



Consider the matrix below for the two strings "Programming" and "Problem".

	P	r	o	g	r	a	m	m	i	n	g
P	1	0	0	0	0	0	0	0	0	0	0
r	0	2	0	0	?	0	0	0	0	0	0
o	0	0	?	0	0	0	0	0	0	0	0
b	0	0	0	0	0	0	0	0	0	0	0
l	0	0	0	0	0	0	0	0	0	0	0
e	0	0	0	0	0	0	0	0	0	0	0
m	0	0	0	0	0	0	1	?	0	0	0

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

1) What should be the value in the green cell?

- 0
- 1
- 2

2) What should be the value in the yellow cell?

- 1
- 2
- 3

3) What should be the value in the blue cell?

- 0
- 1
- 2

4) What is the longest common substring?

- Pr
- Pro
- mm

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

Longest common substring algorithm complexity

The longest common substring algorithm operates on two strings of length N and M. For each of the N characters in the first string, M matrix entries are computed, making

the runtime complexity $O(\quad)$. Since an $N \times M$ integer matrix is built, the space complexity is also $O(\quad)$.

Common substrings in DNA

A real-world application of the longest common substring algorithm is to find common substrings in DNA strings. Ex: Common substrings between 2 different DNA sequences may represent shared traits. DNA strings consist of characters C, T, A, and G.

PARTICIPATION ACTIVITY

12.6.6: Finding longest common substrings in DNA.



Animation content:

undefined

Animation captions:

1. Finding common substrings in DNA strings can be used for detecting things such as genetic disorders or for tracing evolutionary lineages.
2. DNA strings are very long, often billions of characters. Dynamic programming is crucial to obtaining a reasonable runtime.
3. Optimizations can lower memory usage by keeping only the following in memory: previous row data and largest matrix entry information.

PARTICIPATION ACTIVITY

12.6.7: Common substrings in DNA.



- 1) Which cannot be a character in a DNA string?

- A
- B
- C

- 2) If an animal's DNA string is available, a genetic disorder might be found by finding the longest common substring from the DNA of another animal ____ the disorder.

- with
- without



3) When computing row X in the matrix, what information is needed, besides the 2 strings?

- Row X - 1
- Row X + 1
- All rows before X

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPCS2302ValeraFall2023

PARTICIPATION ACTIVITY

12.6.8: Longest common substrings - critical thinking.



1) If the largest entry in the matrix were the only known value, what could be determined?

- The starting index of the longest
- common substring within either string
- The character contents of the common substring
- The length of the longest common substring

2) Suppose only the row and column indices for the largest entry in the matrix were known, and not the value of the largest or any other matrix entry.

What can be determined in O(1)?

- Only the longest common
- substring's ending index within either string
- The longest common substring's
- starting and ending indices within either string

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPCS2302ValeraFall2023

The longest common substring algorithm can be implemented such that only the previously computed row and the largest matrix entry's location and value are stored in memory. With this optimization, the space complexity is reduced to O(n^2). The runtime complexity remains O(n^2).

12.7 Python: Dynamic programming

©zyBooks 11/20/23 11:11 1048447

Dynamic programming for the longest common substring problem

Eric Quezada
UTEP-CS2302-ValeraFall2023

Dynamic programming is a technique where solutions to subproblems are stored in memory and used to quickly find the solution to the full problem.

The longest common substring problem is solved efficiently using dynamic programming. A matrix, a list of lists, stores the length of the longest common substring found in two strings as the strings' characters are examined. Matrix element (i, j) is assigned with zero if the first string's i^{th} character doesn't match the second string's j^{th} character. If the two characters *do* match, then matrix element (i, j) is assigned with the matrix element $(i-1, j-1) + 1$.

Note that string slices in Python use the syntax `str1[start_index : end_index + 1]` to get the substring from `start_index` up to and including `end_index`.

Figure 12.7.1: Dynamic programming algorithm for the longest common substring problem.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

```
def longest_common_substring(str1, str2):  
  
    # Create the matrix as a list of lists.  
    matrix = []  
    for i in range(len(str1)):  
        # Each row is started as a list of zeros.  
        matrix.append([0] * len(str2))  
  
    # Variables to remember the largest value, and the  
    # position it  
    # occurred at.  
    max_value = 0  
    max_value_row = 0  
    max_value_col = 0  
    for row in range(len(str1)):  
        for col in range(len(str2)):  
  
            # Check if the characters match  
            if str1[row] == str2[col]:  
                # Get the value in the cell that's up and to  
                # the  
                # left, or 0 if no such cell  
                up_left = 0  
                if row > 0 and col > 0:  
                    up_left = matrix[row - 1][col - 1]  
  
                # Set the value for this cell  
                matrix[row][col] = 1 + up_left  
                if matrix[row][col] > max_value:  
                    max_value = matrix[row][col]  
                    max_value_row = row  
                    max_value_col = col  
            else:  
                matrix[row][col] = 0  
  
    # The longest common substring is the substring  
    # in str1 from index max_value_row - max_value + 1,  
    # up to and including max_value_col.  
    start_index = max_value_row - max_value + 1  
    return str1[start_index : max_value_row + 1]
```

PARTICIPATION ACTIVITY

12.7.1: Longest common substring dynamic programming algorithm.



Refer to the code above.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023



- 1) Assuming str1 and str2 are both longer than 7 characters, what is up_left's value when row = 7 and col = 0?

- 0
- The value at matrix[7][0]
- The value at matrix[6][0]
- The value depends on what the two strings are.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEPACS2302ValeraFall2023



- 2) `str1 = "GCCGTATTGT"`

`max_value = 4`

`max_value_row = 6`

Without knowing what str2 is, what is the longest common substring between str1 and str2?

- GTAT
- TTGT
- Not enough information is given.



- 3) `str1 = "embrace"`

`str2 = "braille"`

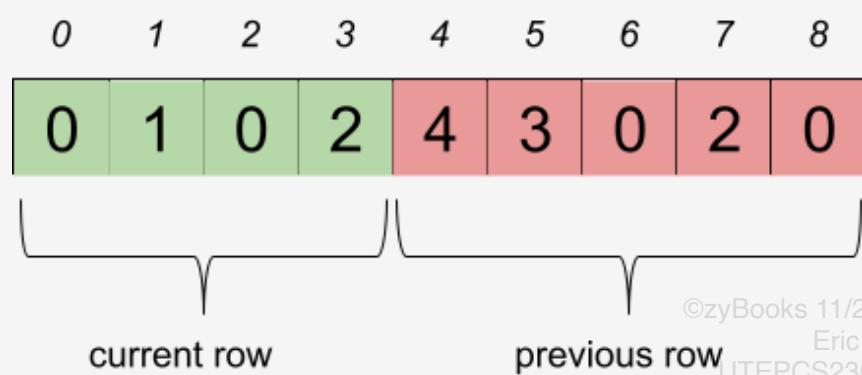
When the algorithm finishes, what is the value of matrix[3][1]?

- 0
- 1
- 2

A space-saving optimization

As the matrix fills in, only the previous row is needed to fill the current row. Thus, the only space required is two rows, plus the variables for the length and row of the longest substring found so far. In fact, once a value is used as an up_left value, the row element can be overwritten as part of the current row. Thus, only a single row is truly needed — the values before the current column are the current row's values, while values after the current column are the previous row's values. Ex: The current column is index 4 with the following row. The indices 0 - 3 (colored green) are from the current row, and indices 4 - 83 (colored red) are from the previous row.

©zyBooks 11/20/23 11:11 1048447
Eric Quezada



©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

The "2" at index 3 has just been finished, and the value previously at index 3 is stored in variable up_left. The value at index 4 is saved temporarily in the saved_current variable; after the new value at index 4 is put into the row, up_left is assigned with saved_current.

The rest of the algorithm is the same. The optimized function uses _____ space instead of the space required by the unoptimized function. Both algorithms require _____ time to execute.

Figure 12.7.2: A space-saving optimized version of the longest common substring algorithm.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

```

def longest_common_substring_optimized(str1, str2):
    # Create one row of the matrix.
    matrix_row = [0] * len(str2)

    # Variables to remember the largest value, and the row it
    # occurred at.
    max_value = 0
    max_value_row = 0
    for row in range(len(str1)):
        # Variable to hold the upper-left value from the
        # current matrix position.
        up_left = 0
        for col in range(len(str2)):
            # Save the current cell's value; this will be
            up_left
            # for the next iteration.
            saved_current = matrix_row[col]

            # Check if the characters match
            if str1[row] == str2[col]:
                matrix_row[col] = 1 + up_left

                # Update the saved maximum value and row,
                # if appropriate.
                if matrix_row[col] > max_value:
                    max_value = matrix_row[col]
                    max_value_row = row
            else:
                matrix_row[col] = 0

            # Update the up_left variable
            up_left = saved_current

        # The longest common substring is the substring
        # in str1 from index max_value_row - max_value + 1,
        # up to and including max_value_row.
        start_index = max_value_row - max_value + 1
    return str1[start_index : max_value_row + 1]

```

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302 Valera Fall 2023

PARTICIPATION ACTIVITY

12.7.2: Optimized longest common substring algorithm.



- 1) The optimized version of the longest common substring algorithm is faster than the original version.

- True
- False

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302 Valera Fall 2023



- 2) The optimized version of the longest common substring algorithm requires the length of str1 is the same as the length of str2.

- True
 - False

zyDE 12.7.1: Longest common substring algorithms.

The program below includes both original and optimized versions of the longest common substring algorithm. Try running the program with various inputs to confirm that both algorithms work as expected.

main.py

Load default template

```
1 def longest_common_substring(str1, str2):
2     # Create the matrix as a list of lists.
3     matrix = []
4     for i in range(len(str1)):
5         # Each row is started as a list of zeros.
6         matrix.append([0] * len(str2))
7
8     # Variables to remember the largest value, and the row it
9     # occurred at.
10    max_value = 0
11    max_value_row = 0
12    for row in range(len(str1)):
13        for col in range(len(str2)):
14
15            # Check if the characters match
```

Look
zyBooks

Ryn

12.8 LAB: Longest common subsequences

Overview

The longest common **substring** algorithm is presented elsewhere in this book. The longest common **subsequence** algorithm is similar. Unlike a substring, a subsequence need not be continuous. Ex: "ARTS" is a subsequence of **ALGORITHMS**, but is not a substring.

A dynamic programming matrix can be used to solve the longest common subsequence problem. Rules for populating the matrix differ slightly from the longest common substring algorithm. Both algorithms populate rows from top to bottom, and left to right across a row. Each entry `matrix[R][C]` is computed as follows:

- If characters match, both algorithms assign `matrix[R][C]` with `1 + matrix[R - 1][C - 1]`.
- If characters do not match, the longest common _____.
 - substring algorithm assigns `matrix[R][C]` with `0`
 - subsequence algorithm assigns `matrix[R][C]` with `max(matrix[R - 1][C], matrix[R][C - 1])`

Each algorithm uses 0 for out of bounds entries. Ex: When computing `matrix[0][0]` for a character match, instead of trying to access `matrix[-1][-1]`, 0 is used instead.

Sample matrix

The image below shows the longest common subsequence matrix for strings "ALASKAN" and "BANANAS". Entries corresponding to a character match are highlighted.

	B	A	N	A	N	A	S	
A	0	1	1	1	1	1	1	Matching character
L	0	1	1	1	1	1	1	
A	0	1	1	2	2	2	2	
S	0	1	1	2	2	2	3	
K	0	1	1	2	2	2	3	
A	0	1	1	2	2	3	3	
N	0	1	2	2	3	3	3	

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-ValeraFall2023

The largest number in the matrix indicates the length of the longest common subsequence. Ex: The largest entry in the matrix above is 3, so the longest common subsequence between "ALASKAN" and "BANANAS" is 3 characters long.

Multiple longest common subsequences

A pair of strings may have more than one longest common subsequence. Ex: "ALASKAN" and "BANANAS" have three longest common subsequences: "AAA", "AAN", and "AAS".

Step 1: Determine how to make the LCS set from the matrix

The matrix can be used to build a set of all longest common subsequences. Before writing any code, consider two options for building the LCS set:

1. Build the entire numerical matrix, then traverse the matrix to build the LCS set.
2. Build a matrix of structures that include the numerical entry plus the LCS set.

The image below illustrates approach #2.

Legend: Matching character (Yellow square)

	A	B	B	A
B	0 {}	1 { "B" }	1 { "B" }	1 { "B" }
A	1 { "A" }	1 { "A", "B" }	1 { "A", "B" }	2 { "BA" }
A	1 { "A" }	1 { "A", "B" }	1 { "A", "B" }	2 { "AA", "BA" }
B	1 { "A" }	2 { "AB" }	2 { "AB", "BB" }	2 { "AB", "BB", "AA", "BA" }

Step 2: Implement the **LCSMatrix** class

The LCSMatrix class is declared in the LCSMatrix.py file. Access LCSMatrix.py by clicking on the orange arrow next to main.py at the top of the coding window. The `__init__()`, `get_entry()`, and `get_longest_common_subsequences()` methods must be completed. An attribute must also be added for the matrix data. Each matrix entry may be an integer or a more complex object, depending on the choice made in step 1.

After adding an attribute for the matrix, complete the methods to satisfy the requirements below.

- **__init__()**: Two lines of code are given to assign the row_count and column_count attributes to string1's length and string2's length, respectively. The remainder of the method must be implemented to build the longest common subsequence matrix.
 - Use case sensitive character comparisons. Ex: 'a' and 'A' are **not** equal.
 - **get_entry()**: Returns the numerical entry at the given row and column indices, or 0 if either index is out of bounds.
 - **get_longest_common_subsequences()**: Returns a Python **set** object. The set contains strings indicating all longest common subsequences for the two strings passed to __init__().

Step 3: Test in develop mode, then submit

Code in main.py runs several test cases. Every test case tests the set returned from get_longest_common_subsequences(). Some test cases also test matrix entries returned from get_entry(). Unit tests are similar.

Ensure that all tests pass in develop mode before submitting code.

502696.2096894.7x32qy/

LAB
ACTIVITY

12.8.1: LAB: Longest common subsequences

5 / 10

Downloadable files

[main.py](#)

[LCSMatrix.py](#)

, and

[LCSTestCase.py](#)

[Download](#)

File is marked as read only

Current file: [main.py](#) ▾

```
1 from LCSTestCase import LCSTestCase
2
3 class PrintFeedback:
4     def write(self, string_to_write):
5         print(string_to_write, end="")
6 test_feedback = PrintFeedback()
7
8 test_cases = [
9     LCSTestCase(
10         "ALASKAN",
11         "BANANAS",
12         { "AAA", "AAS", "AAN" } )
```

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEPCS2302ValeraFall2023

13

14

15

16

[0, 1, 1, 1, 1, 1, 1],

[0, 1, 1, 1, 1, 1, 1],

Develop mode**Submit mode**

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

©zyBooks 11/20/23 11:11 1048447

Eric Quezada

UTEP-CS2302-Valera-Fall2023

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above)

**main.py**
(Your program)

Output

Program output displayed here

Coding trail of your work

[What is this?](#)

11/12 U-0, 5-5 min:3

©zyBooks 11/20/23 11:11 1048447
Eric Quezada
UTEP-CS2302-Valera-Fall2023