5.1 Binary trees

Binary tree basics

In a list, each node has up to one successor. In a ***binary tree***, each node has up to two children, known as a *left child* and a *right child*. "Binary" means two, referring to the two children. Some more definitions related to a binary tree:

- ***Leaf***: A tree node with no children.
- ***Internal node***: A node with at least one child.
- ***Parent***: A node with a child is said to be that child's parent. A node's ***ancestors*** include the node's parent, the parent's parent, etc., up to the tree's root.
- ***Root***: The one tree node with no parent (the "top" node).

Another section discusses binary tree usefulness; this section introduces definitions.

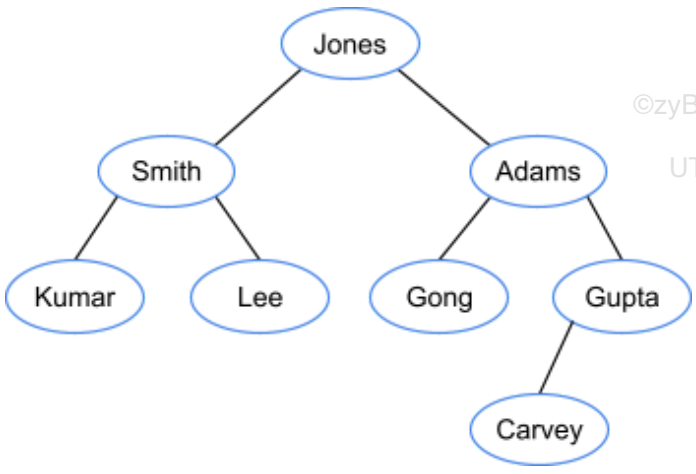
Below, each node is represented by just the node's key, as in B, although the node may have other data.

PARTICIPATION ACTIVITY 5.1.1: Binary tree basics.

Animation captions:

1. In a list, each node has up to one successor.
2. In a binary tree, each node has up to two children.
3. A tree is normally drawn vertically. Edge arrows are optional.
4. A node can have a left child and right child. A node with a child is called the child's parent.
5. First node: Root node. Node without child: Leaf node. Node with child: Internal nodes.

PARTICIPATION ACTIVITY 5.1.2: Binary tree basics.



1) Root node: _____.

Check Show answer

2) Smith's left child: ____

Check Show answer

3) The tree has four leaf nodes: Kumar, Lee, Gong, and _____.

Check Show answer

4) How many internal nodes does the tree have?

Check Show answer

5) The tree has an internal node, Gupta, with only one child rather than two. Is the tree still a binary tree? Type yes or no.

Check Show answer

Depth, level, and height

A few additional terms:

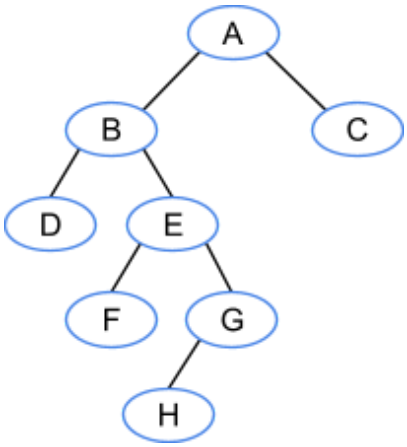
- The link from a node to a child is called an ***edge***.
- A node's ***depth*** is the number of edges on the path from the root to the node. The root node thus has depth 0.
- All nodes with the same depth form a tree ***level***.
- A tree's ***height*** is the largest depth of any node. A tree with just one node has height 0.

PARTICIPATION ACTIVITY 5.1.3: Binary tree terminology: height, depth, and level.

Animation captions:

1. The binary tree has edges A to B, A to C, and C to D.
2. A node's depth is the number of edges from the root to the node.
3. Nodes with the same depth form a level.
4. A tree's height is the largest depth of any node.

PARTICIPATION ACTIVITY 5.1.4: Binary tree height, depth, and level.



1) A's depth is 1.
   - True
   - False

2) E's depth is 2.
   - True
   - False

3) B and C form level 1.
   - True
   - False

4) D, F, and H form level 2.
   - True
   - False

5) The tree's height is 4.

- True
- False

6) A tree with just one node has height 0.

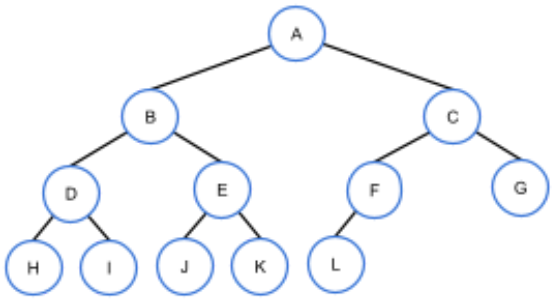
- True
- False

Special types of binary trees

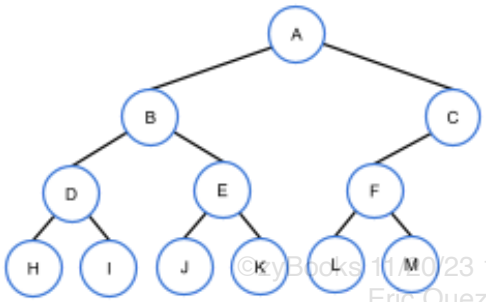
Certain binary tree structures can affect the speed of operations on the tree. The following describe special types of binary trees:

- A binary tree is ***full*** if every node contains 0 or 2 children.
- A binary tree is ***complete*** if all levels, except possibly the last level, contain all possible nodes and all nodes in the last level are as far left as possible.
- A binary tree is ***perfect***, if all internal nodes have 2 children and all leaf nodes are at the same level.

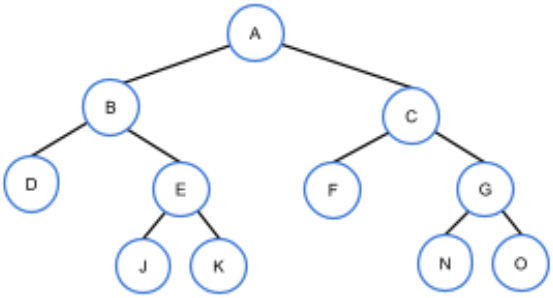
Figure 5.1.1: Special types of binary trees: full, complete, perfect.



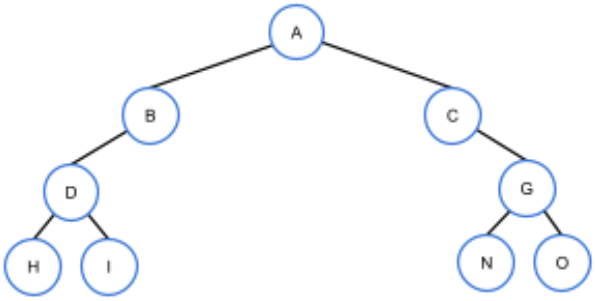
Not full, complete, not perfect



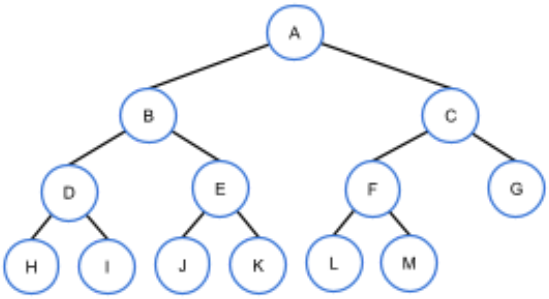
Not full, not complete, not perfect



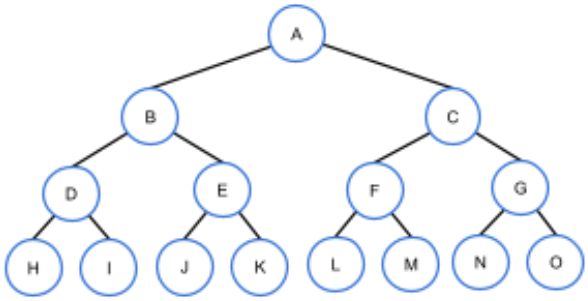
Full, not complete, not perfect



Not full, not complete, not perfect

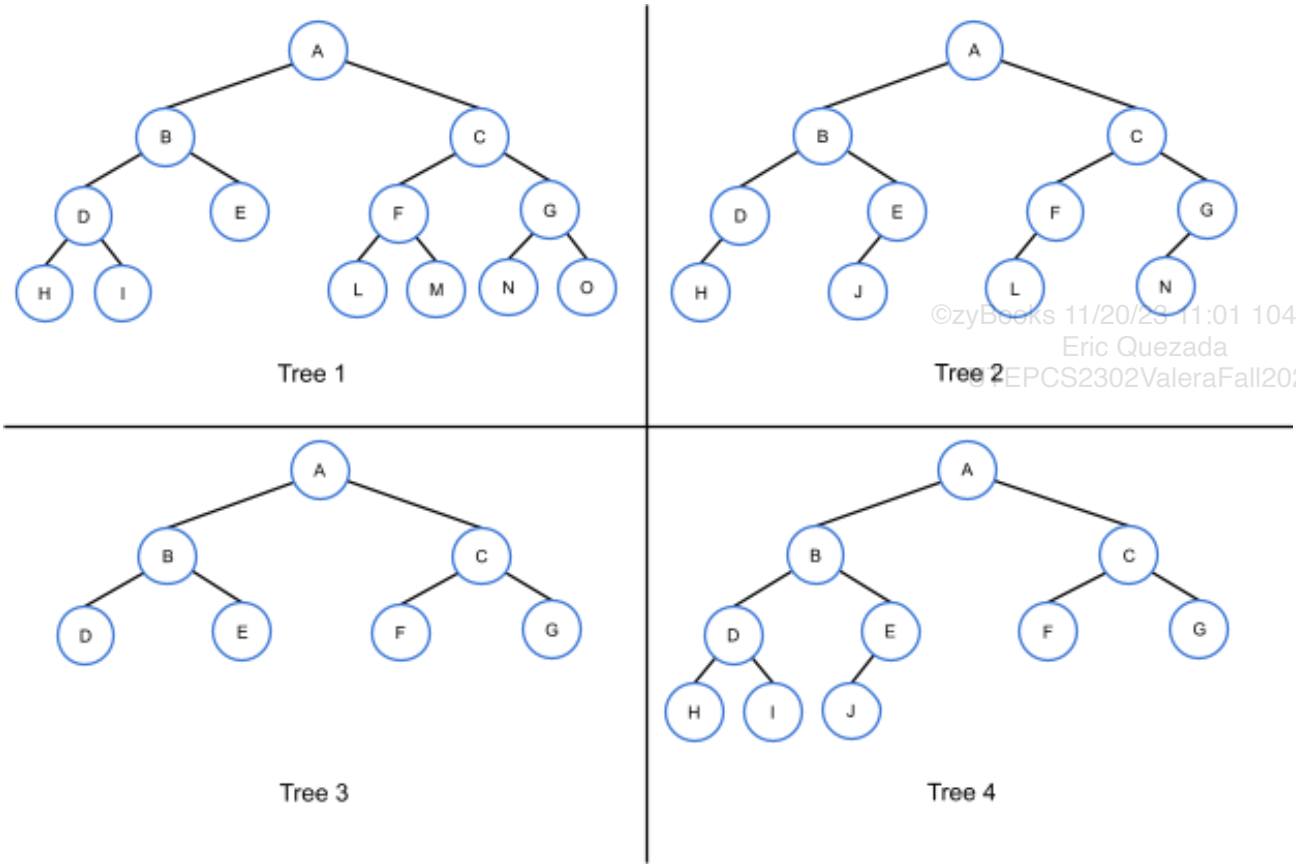


Full, complete, not perfect



Full, complete, perfect

PARTICIPATION ACTIVITY 5.1.5: Identifying special types of binary trees.



If unable to drag and drop, refresh the page.

Not full, complete, not perfect | **Full, complete, perfect**

Not full, not complete, not perfect | **Full, not complete, not perfect**

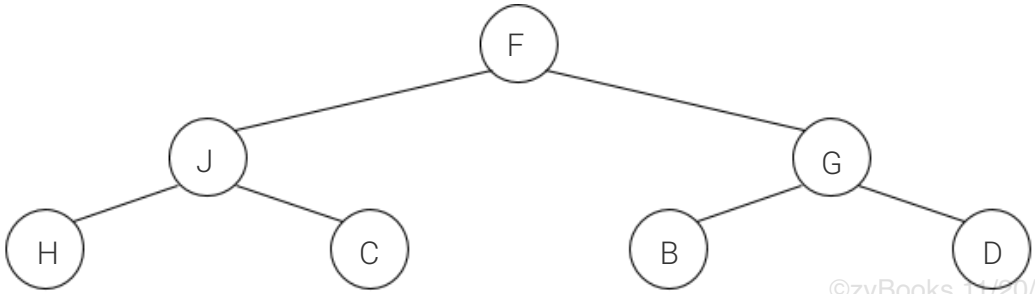
	Tree 1
	Tree 2
	Tree 3
	Tree 4

Reset

CHALLENGE ACTIVITY 5.1.1: Binary trees.

502696.2096894.qx3zqy7

Start



What is the root node? Ex: A

What is the parent of node D? Ex: A

What are the ancestors of node D? Ex: A, B, C

What are the leaf nodes? Ex: A, B, C

How many internal nodes does the tree have? Ex: 9

1 2 3

Check Next

5.2 Applications of trees

File systems

Trees are commonly used to represent hierarchical data. A tree can represent files and directories in a file system, since a file system is a hierarchy.

PARTICIPATION ACTIVITY 5.2.1: A file system is a hierarchy that can be represented by a tree.

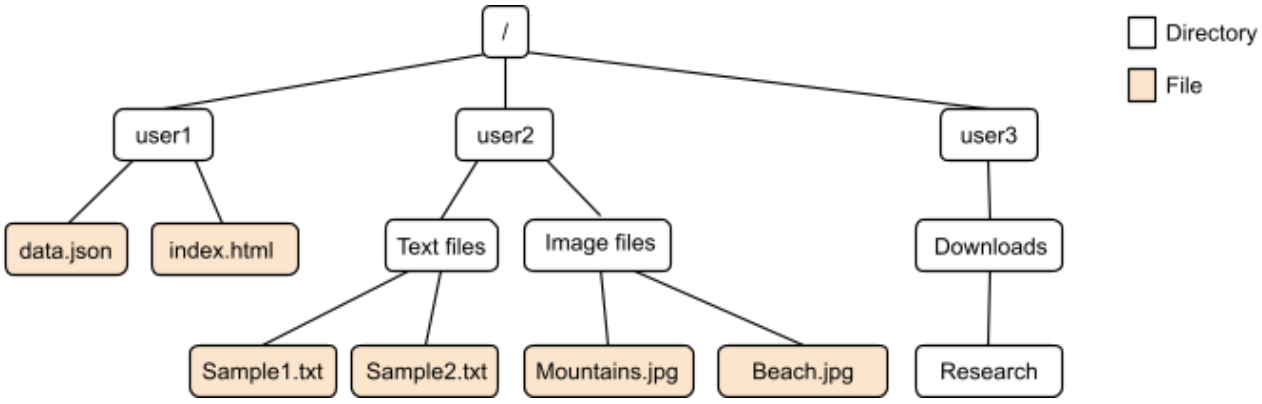
Animation content:

undefined

Animation captions:

1. A tree representing a file system has the filesystem's root directory ("/"), represented by the root node.
2. The root contains 2 configuration text files and 2 additional directories: user1 and user2.
3. Directories contain additional entries. Only empty directories will be leaf nodes. All files are leaf nodes.

PARTICIPATION ACTIVITY 5.2.2: Analyzing a file system tree.



1) What is the depth of the "Mountains.jpg" file node?

- 3
- 4

2) What is the tree's height?

- 3
- 4
- 14

3) What is the parent of the "Text files" node?

- The "user2" directory node
- The "Image files" directory node
- The "Sample1.txt" file node.

4) Which operation would increase the height of the tree?

- Adding a new file into the user1 directory
- Adding a new directory into the "Image files" directory
- Adding a new directory into the "Research" directory

PARTICIPATION ACTIVITY 5.2.3: File system trees.

1) A file in a file system tree is always a leaf node.

- True
- False

2) A directory in a file system tree is always an internal node.

- True
- False

3) Using a tree data structure to implement a file system requires that each directory node support a variable number of children.

- True
- False

Binary space partitioning

Binary space partitioning (***BSP***) is a technique of repeatedly separating a region of space into 2 parts and cataloging objects contained within the regions. A ***BSP tree*** is a binary tree used to store information for binary space partitioning. Each node in a BSP tree contains information about a region of space and which objects are contained in the region.

In graphics applications, a BSP tree can be used to store all objects in a multidimensional world. The BSP tree can then be used to efficiently determine which objects must be rendered on screen. The viewer's position in space is used to perform a lookup within the BSP tree. The lookup quickly eliminates a large number of objects that are not visible and therefore should not be rendered.

PARTICIPATION ACTIVITY 5.2.4: A BSP tree is used to quickly determine which objects do not need to be rendered.

Animation content:

undefined

Animation captions:

1. Data for a large, open 2-D world contains many objects. Only a few objects are visible on screen at any given moment.
2. Avoiding rendering off-screen objects is crucial for realtime graphics. But checking the intersection of all objects with the screen's rectangle is too time consuming.
3. A BSP tree represents partitioned space. The root represents the entire world and stores a list of all objects in the world, as well as the world's geometric boundary.
4. The root's left child represents the world's left half. The node stores information about the left half's geometric boundary, and a list of all objects contained within.
5. The root's right child contains similar information for the right half.
6. Using the screen's position within the world as a lookup into the BSP tree quickly yields the right node's list of objects. A large number of objects are quickly eliminated from the list of potential objects on screen.
7. Further partitioning makes the tree even more useful.

PARTICIPATION ACTIVITY 5.2.5: Binary space partitioning.

1) When traversing down a BSP tree, half the objects are eliminated each level.

   - ○ True
   - ○ False

2) A BSP implementation could choose to split regions in arbitrary locations, instead of right down the middle.

   - ○ True
   - ○ False

3) In the animation, if the parts of the screen were in 2 different regions, then all objects from the 2 regions would have to be analyzed when rendering.

- True
- False

4) BSP can be used in 3-D graphics as well as 2-D.

- True
- False

Using trees to store collections

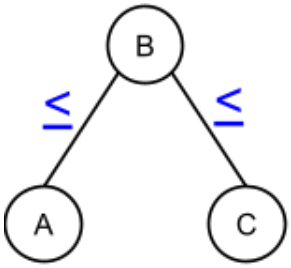
Most of the tree data structures discussed in this book serve to store a collection of values. Numerous tree types exist to store data collections in a structured way that allows for fast searching, inserting, and removing of values.

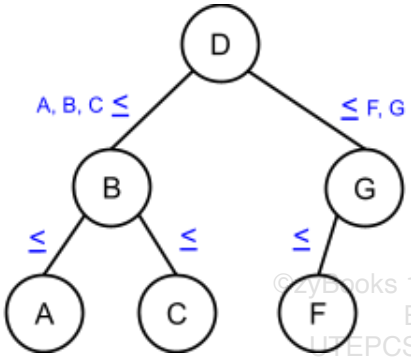
5.3 Binary search trees

Binary search trees

An especially useful form of binary tree is a ***binary search tree*** (BST), which has an ordering property that any node's left subtree keys ≤ the node's key, and the right subtree's keys ≥ the node's key. That property enables fast searching for an item, as will be shown later.

Figure 5.3.1: BST ordering property: For three nodes, left child is less-than-or-equal-to parent, parent is less-than-or-equal-to right child. For more nodes, all keys in subtrees must satisfy the property, for every node.





PARTICIPATION ACTIVITY 5.3.1: BST ordering properties.

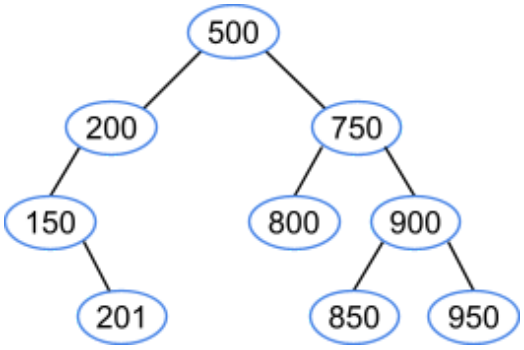
Animation content:

undefined

Animation captions:

1. BST ordering property: Left subtree's keys ≤ node's key, right subtree's keys ≥ node's key.
2. All keys in subtree must obey the ordering property. Not a BST.
3. All keys in subtree must obey the ordering property. Not a BST.
4. All keys in subtree must obey the ordering property. Valid BST.

PARTICIPATION ACTIVITY 5.3.2: Binary search tree: Basic ordering property.



1) Does node 900 and the node's subtrees obey the BST ordering property?

- ○ Yes
- ○ No

2) Does node 750 and the node's subtrees obey the BST ordering property?
   - Yes
   - No

3) Does node 150 and the node's subtrees obey the BST ordering property?
   - Yes
   - No

4) Does node 200 and the node's subtrees obey the BST ordering property?
   - Yes
   - No

5) Is the tree a binary search tree?
   - Yes
   - No

6) Is the tree a binary tree?
   - Yes
   - No

7) Would inserting 300 as the right child of 200 obey the BST ordering property (considering only nodes 300, 200, and 500)?
   - Yes
   - No

Searching

To ***search*** nodes means to find a node with a desired key, if such a node exists. A BST may yield faster searches than a list. Searching a BST starts by visiting the root node (which is the first currentNode below):

Figure 5.3.2: Searching a BST.

```
if (currentNode→key == desiredKey) {
   return currentNode; // The desired node was
found
}
else if (desiredKey < currentNode→key) {
   // Visit left child, repeat
}
else if (desiredKey > currentNode→key) {
   // Visit right child, repeat
}
```

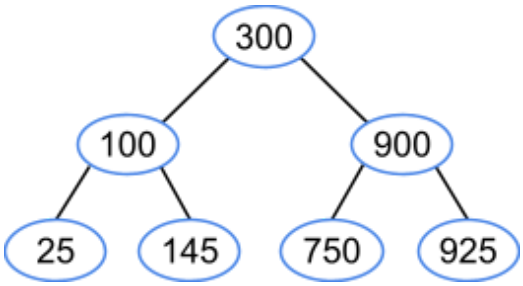

If a child to be visited doesn't exist, the desired node does not exist. With this approach, only a small fraction of nodes need be compared.

PARTICIPATION ACTIVITY 5.3.3: A BST may yield faster searches than a list.

Animation captions:

1. Searching a 7-node list may require up to 7 comparisons.
2. In a BST, if desired key equals current node's key, return found. If less, descend to left child. If greater, descend to right child.
3. Searching a BST may require fewer comparisons, in this case 3 vs. 7.

PARTICIPATION ACTIVITY 5.3.4: Searching a BST.



1) In searching for 145, what node is visited first?

Check Show answer

2) In searching for 145, what node is visited second?

Check Show answer

3) In searching for 145, what node is visited third?

Check Show answer

4) Which nodes would be visited when searching for 900? Write nodes in order visited, as: 5, 10

Check Show answer

5) Which nodes would be visited when searching for 800? Write nodes in order visited, as: 5, 10, 15

Check Show answer

6) What is the worst case (largest) number of nodes visited when searching for a key?

Check Show answer

BST search runtime

Searching a BST in the worst case requires H + 1 comparisons, meaning O(H) comparisons, where H is the tree height. Ex: A tree with a root node and one child has height 1; the worst case visits the root and the child: 1 + 1 = 2. A major BST benefit is that an N-node binary tree's height may be as small as O( ), yielding extremely fast searches. Ex: A 10,000 node list may require 10,000 comparisons, but a 10,000 node BST may require only 14 comparisons.

A binary tree's height can be minimized by keeping all levels full, except possibly the last level. Such an "all-but-last-level-full" binary tree's height is .

Table 5.3.1: Minimum binary tree heights for N nodes are equivalent to .

Nodes N	Height H			Nodes per level
1	0	0	0	1
2	1	1	1	1/1
3	1	1.6	1	1/2
4	2	2	2	1/2/1
5	2	2.3	2	1/2/2
6	2	2.6	2	1/2/3
7	2	2.8	2	1/2/4
8	3	3	3	1/2/4/1
9	3	3.2	3	1/2/4/2
...				
15	3	3.9	3	1/2/4/8
16	4	4	4	1/2/4/8/1

PARTICIPATION ACTIVITY 5.3.5: Searching a perfect BST with N nodes requires only O( ) comparisons.

Animation captions:

1. A perfect binary tree has height .
2. A perfect binary tree search is O( ), so O( ).
3. Searching a BST may be faster than searching a list.

PARTICIPATION ACTIVITY 5.3.6: Searching BSTs with N nodes.

What is the worst case (largest) number of comparisons given a BST with N nodes?

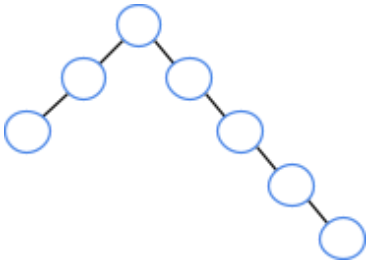
1) Perfect BST with N = 7

- ○
- ○
- ○

2) Perfect BST with N = 31

- ○ 31
- ○ 4
- ○ 5

3) Given the following tree.



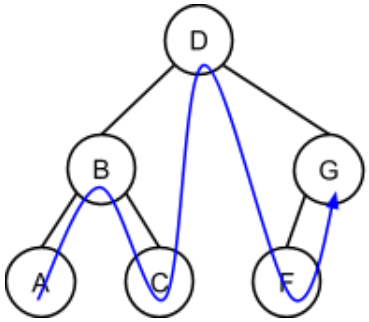
- ○ 3
- ○ 5

Successors and predecessors

A BST defines an ordering among nodes, from smallest to largest. A BST node's ***successor*** is the node that comes after in the BST ordering, so in A B C, A's successor is B, and B's successor is C. A BST node's ***predecessor*** is the node that comes before in the BST ordering.

If a node has a right subtree, the node's successor is that right subtree's leftmost child: Starting from the right subtree's root, follow left children until reaching a node with no left child (may be that subtree's root itself). If a node doesn't have a right subtree, the node's successor is the first ancestor having this node in a left subtree. Another section provides an algorithm for printing a BST's nodes in order.

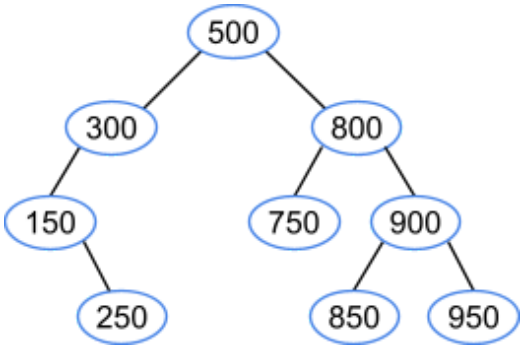
Figure 5.3.3: A BST defines an ordering among nodes.



BST ordering:
A B C D F G

Successor follows in ordering.
Ex: D's successor is F.

PARTICIPATION ACTIVITY 5.3.7: Binary search tree: Defined ordering.



1) The first node in the BST ordering is 150.
   - True
   - False

2) 150's successor is 250.
   - True
   - False

3) 250's successor is 300.
   - True
   - False

4) 500's successor is 850.
   - True
   - False

5) 950's successor is 150.

- True
- False

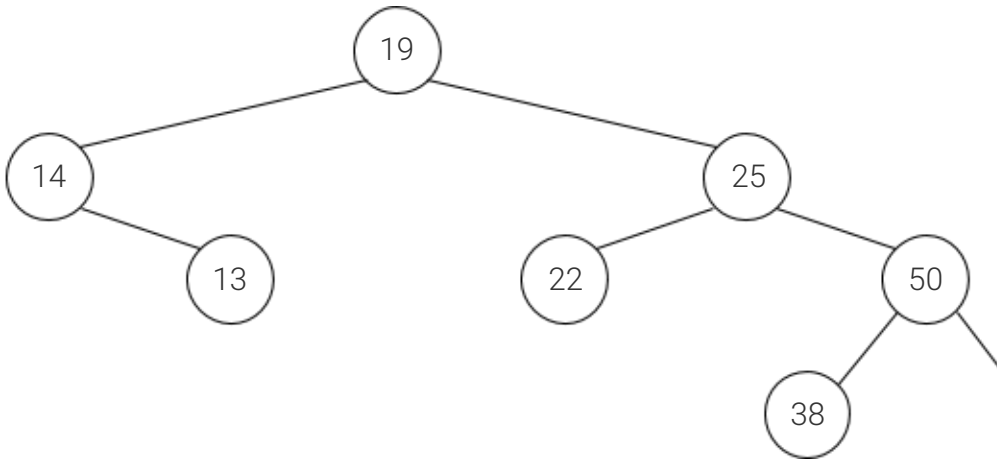
6) 950's predecessor is 900.

- True
- False

CHALLENGE ACTIVITY 5.3.1: Binary search trees.

502696.2096894.qx3zqy7

Start



Does node 14 and the node's subtrees obey the BST ordering property?

Does node 25 and the node's subtrees obey the BST ordering property?

Is the tree a BST?

1 2 3 4 5

Check Next

5.4 BST search algorithm

Given a key, a ***search*** algorithm returns the first node found matching that key, or returns null if a matching node is not found. A simple BST search algorithm checks the current node (initially the tree's root), returning that node as a match, else assigning the current node with the left (if key is less) or right (if key is greater) child and repeating. If such a child is null, the algorithm returns null (matching node not found).

PARTICIPATION ACTIVITY 5.4.1: BST search algorithm.

Animation content:

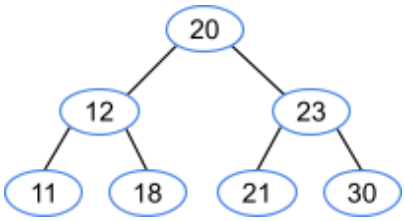
undefined

Animation captions:

1. BST search algorithm checks current node, returning a match if found. Otherwise, assigns current node with left (if key is less) or right (if key is greater) child and continues search.
2. If the child to be visited does not exist, the algorithm returns null indicating no match found.

PARTICIPATION ACTIVITY 5.4.2: BST search algorithm.

Consider the following tree.



1) When searching for key 21, what node is visited first?
   - 20
   - 21

2) When searching for key 21, what node is visited second?
   - 12
   - 23

3) When searching for key 21, what node is visited third?
   - 21
   - 30

4) If the current node matches the key, when does the algorithm return the node?

- Immediately
- Upon exiting the loop

5) If the child to be visited is null, when does the algorithm return null?

- Immediately
- Upon exiting the loop

6) What is the maximum loop iterations for a perfect binary tree with 7 nodes, if a node matches?

- 3
- 7

7) What is the maximum loop iterations for a perfect binary tree with 7 nodes, if no node matches?

- 3
- 7

8) What is the maximum loop iterations for a perfect binary tree with 255 nodes?

- 8
- 255

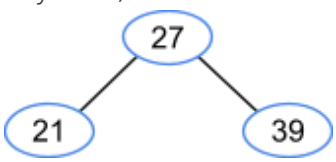
9) Suppose node 23 was instead 21, meaning two 21 nodes exist (which is allowed in a BST). When searching for 21, which node will be returned?

- Leaf
- Internal

PARTICIPATION ACTIVITY 5.4.3: BST search algorithm decisions.

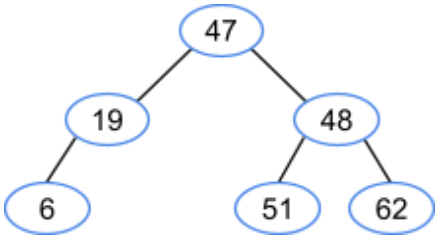
Determine cur's next assignment given the key and current node.

1) key = 40, cur = 27



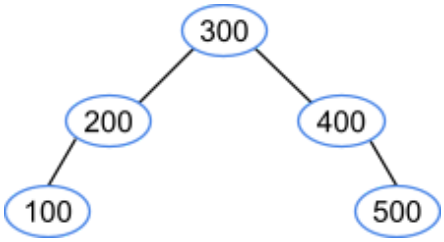
- 27
- 21
- 39

2) key = 6, cur = 47



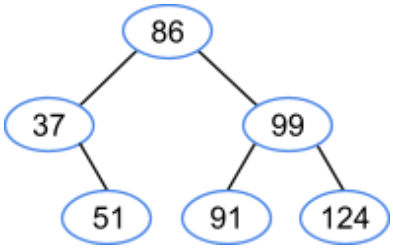
- 6
- 19
- 48

3) key 350, cur = 400



- Search terminates and returns null.
- 400
- 500

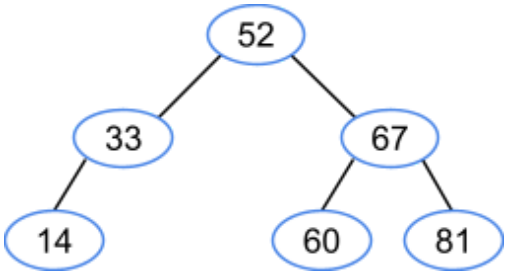
4) key 91, cur = 99



- 86
- 91
- 124

PARTICIPATION ACTIVITY 5.4.4: Tracing a BST search.

Consider the following tree. If node does not exist, enter null.



1) When searching for key 45, what node is visited first?

Check Show answer

2) When searching for key 45, what node is visited second?

Check Show answer

3) When searching for key 45, what node is visited third?

Check Show answer

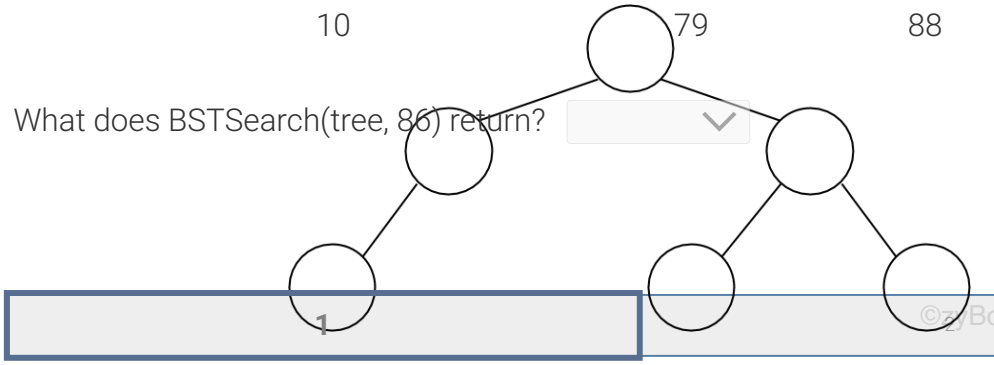
CHALLENGE ACTIVITY 5.4.1: BST search algorithm.

502696.2096894.qx3zqy7

Start

24

17 87



What does BSTSearch(tree, 86) return?

Check Next

5.5 BST insert algorithm

Given a new node, a BST ***insert*** operation inserts the new node in a proper location obeying the BST ordering property. A simple BST insert algorithm compares the new node with the current node (initially the root).

- *Insert as left child*: If the new node's key is less than the current node, and the current node's left child is null, the algorithm assigns that node's left child with the new node.
- *Insert as right child*: If the new node's key is greater than or equal to the current node, and the current node's right child is null, the algorithm assigns the node's right child with the new node.
- *Search for insert location*: If the left (or right) child is not null, the algorithm assigns the current node with that child and continues searching for a proper insert location.

PARTICIPATION ACTIVITY 5.5.1: Binary search tree insertions.

Animation content:

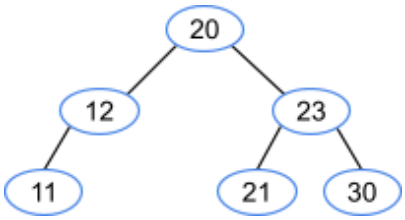
undefined

Animation captions:

1. A node inserted into an empty tree will become the tree's root.
2. The BST is searched to find a suitable location to insert the new node as a leaf node.

PARTICIPATION ACTIVITY 5.5.2: BST insert algorithm.

Consider the following tree.



1) Where will a new node 18 be inserted?

- 12's right child
- 11's right child

2) Where will a new node 11 be inserted? (So two nodes of 11 will exist).

- 11's left child
- 11's right child

3) Assume a perfect 7-node BST. How many algorithm loop iterations will occur for an insert?

- 3
- 7

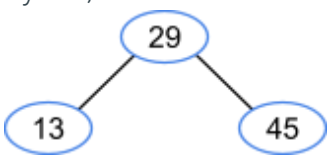
4) Assume a perfect 255-node BST. How many algorithm loop iterations will occur for an insert?

- 8
- 255

PARTICIPATION ACTIVITY 5.5.3: BST insert algorithm decisions.

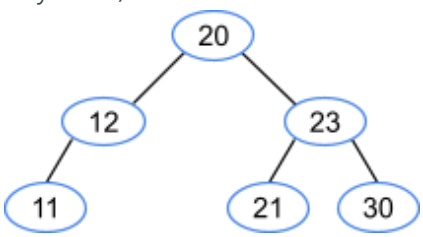
Determine the insertion algorithm's next step given the new node's key and the current node.

1) key = 7, currentNode = 29



- currentNode⋯→left = node
- currentNode = currentNode⋯→right
- currentNode = currentNode⋯→left

2) key = 18, currentNode = 12

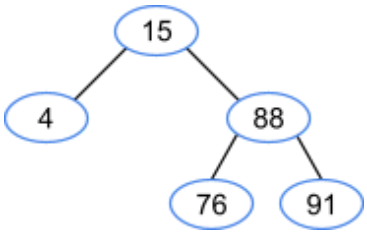


- currentNode⋯→left = node
- currentNode⋯→right = node
- currentNode = currentNode⋯→right

3) key = 87, currentNode = null, tree⋯→root = null (empty tree)

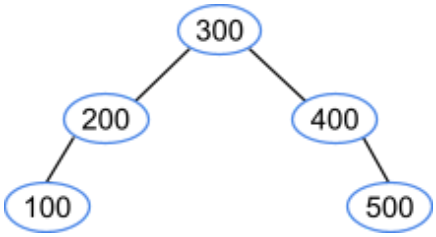
- tree⋯→root = node
- currentNode⋯→right = node
- currentNode⋯→left = node

4) key = 53, currentNode = 76



- currentNode⋯→left = node
- currentNode⋯→right = node
- tree⋯→root = node

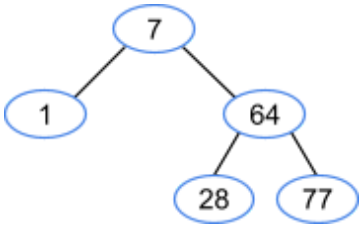
5) key = 600, currentNode = 400



- currentNode⋯→left = node
- currentNode = currentNode⋯→right
- currentNode⋯→right = node

PARTICIPATION ACTIVITY 5.5.4: Tracing BST insertions.

Consider the following tree.



1) When inserting a new node with key 35, what node is visited first?

Check Show answer

2) When inserting a new node with key 35, what node is visited second?

Check Show answer

3) When inserting a new node with key 35, what node is visited third?

Check Show answer

4) Where is the new node inserted? Type: left or right

Check Show answer

BST insert algorithm complexity

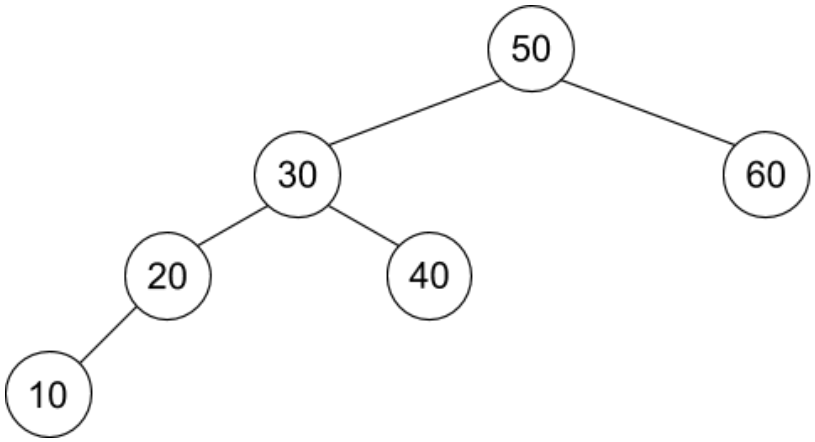
The BST insert algorithm traverses the tree from the root to a leaf node to find the insertion location. One node is visited per level. A BST with N nodes has at least levels and at most levels. Therefore, the runtime complexity of insertion is best case and worst case .

The space complexity of insertion is because only a single pointer is used to traverse the tree to find the insertion location.

CHALLENGE ACTIVITY 5.5.1: BST insert algorithm.

502696.2096894.qx3zqy7

Start



Where will a new node 25 be inserted?

child of node

1 2 3

Check Next

Exploring further:

- Binary search tree visualization

5.6 BST remove algorithm

Given a key, a BST ***remove*** operation removes the first-found matching node, restructuring the tree to preserve the BST ordering property. The algorithm first searches for a matching node just like the search algorithm. If found (call this node X), the algorithm performs one of the following sub-algorithms:

- *Remove a leaf node:* If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with null. Else, if X was the root, the root pointer is assigned with null, and the BST is now empty.
- *Remove an internal node with single child:* If X has a parent (so X is not the root), the parent's left or right child (whichever points to X) is assigned with X's single child. Else, if X was the root, the root pointer is assigned with X's single child.
- *Remove an internal node with two children:* This case is the hardest. First, the algorithm locates X's successor (the leftmost child of X's right subtree), and copies the successor to X. Then, the algorithm recursively removes the successor from the right subtree.

PARTICIPATION ACTIVITY 5.6.1: BST remove: Removing a leaf, or an internal node with a single child.

Animation captions:

1. Removing a leaf node: The parent's right child is assigned with null.
2. Remove an internal node with a single child: The parent's right child is assigned with node's single child.

PARTICIPATION ACTIVITY 5.6.2: BST remove: Removing internal node with two children.

Animation captions:

1. Find successor: Leftmost child in node 25's right subtree is node 27.
2. Copy successor to current node.
3. Remove successor from right subtree.

Figure 5.6.1: BST remove algorithm.

```
BSTRemove(tree, key) {
   par = null
   cur = tree→root
   while (cur is not null) { // Search for node
      if (cur→key == key) { // Node found
         if (cur→left is null && cur→right is null) { // Remove leaf
            if (par is null) // Node is root
               tree→root = null
            else if (par→left == cur)
               par→left = null
            else
               par→right = null
         }
         else if (cur→right is null) {                // Remove node with
only left child
            if (par is null) // Node is root
               tree→root = cur→left
            else if (par→left == cur)
               par→left = cur→left
            else
               par→right = cur→left
         }
         else if (cur→left is null) {                // Remove node with
only right child
            if (par is null) // Node is root
               tree→root = cur→right
            else if (par→left == cur)
               par→left = cur→right
            else
               par→right = cur→right
         }
         else {                                      // Remove node with two
children
            // Find successor (leftmost child of right subtree)
            suc = cur→right
            while (suc→left is not null)
               suc = suc→left
            successorData = Create copy of suc's data
            BSTRemove(tree, suc→key)     // Remove successor
            Assign cur's data with successorData
         }
         return // Node found and removed
      }
      else if (cur→key < key) { // Search right
         par = cur
         cur = cur→right
      }
      else {                     // Search left
         par = cur
         cur = cur→left
      }
   }
   return // Node not found
}
```

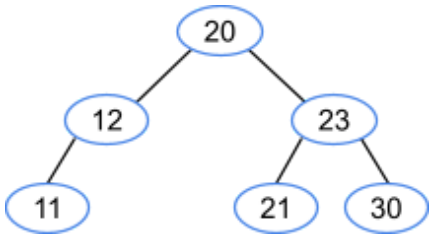
BST remove algorithm complexity

The BST remove algorithm traverses the tree from the root to find the node to remove. When the node being removed has two children, the node's successor is found and a recursive call is made. One node is visited per level, and in the worst case scenario the tree is traversed twice from the root to a leaf. A BST with nodes has at least levels and at most levels. So removal's worst case time complexity is for a BST with levels and worst case for a tree with levels.

Two pointers are used to traverse the tree during removal. When the node being removed has two children, a third pointer and a copy of one node's data are also used, and one recursive call is made. So removal's space complexity is always .

PARTICIPATION ACTIVITY 5.6.3: BST remove algorithm.

Consider the following tree. Each question starts from the original tree. Use this text notation for the tree: (20 (12 (11, -), 23 (21, 30))). The - means the child does not exist.



1) What is the tree after removing 21?

- (20 (12 (11, -), 23 (-, 30)))
- (20 (12 (11, -), 23))

2) What is the tree after removing 12?

- (20 (- (11, -), 23 (21, 30)))
- (20 (11, 23 (21, 30)))

3) What is the tree after removing 20?

- (21 (12 (11, -), 23 (-, 30)))
- (23 (12 (11, -), 30 (21, -)))

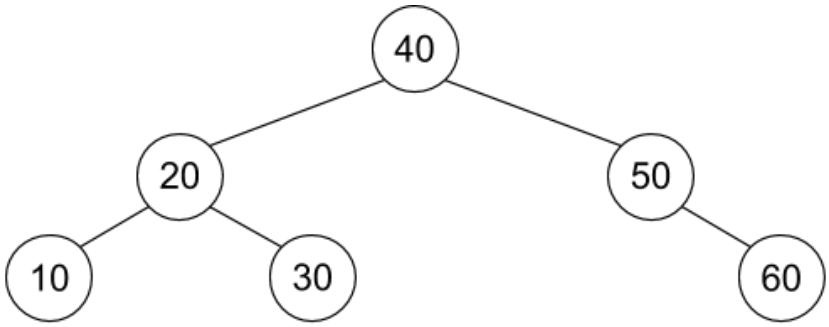
4) Removing a node from an N-node nearly-full BST has what computational complexity?

- O( )
- O( )

CHALLENGE ACTIVITY 5.6.1: BST remove algorithm.

502696.2096894.qx3zqy7

Start



BSTRemove(tree, 30) executes.

What is the left child of 20?

What is the right child of 20?

1 2 3

Check Next

5.7 BST inorder traversal

A ***tree traversal*** algorithm visits all nodes in the tree once and performs an operation on each node. An ***inorder traversal*** visits all nodes in a BST from smallest to largest, which is useful for example to print

the tree's nodes in sorted order. Starting from the root, the algorithm recursively prints the left subtree, the current node, and the right subtree.

Figure 5.7.1: BST inorder traversal algorithm.

```
BSTPrintInorder(node) {
  if (node is null)
      return

  BSTPrintInorder(node→left)
  Print node
  BSTPrintInorder(node→right)
}
```

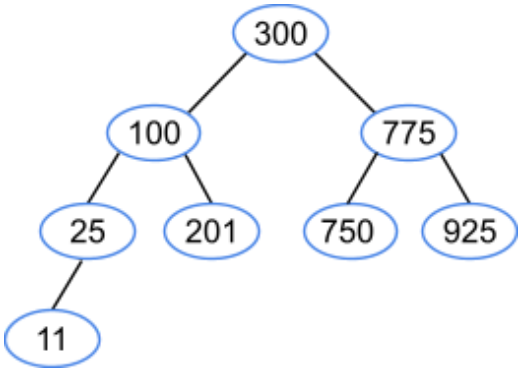

PARTICIPATION ACTIVITY 5.7.1: BST inorder print algorithm.

Animation captions:

1. An inorder traversal starts at the root. Recursive call descends into left subtree.
2. When left done, current is printed, then recursively descend into right subtree.
3. Return from recursive call causes ascending back up the tree; left is done, so do current and right.
4. Continues similarly.

PARTICIPATION ACTIVITY 5.7.2: Inorder traversal of a BST.

Consider the following tree.



1) What node is printed first?

Check Show answer

2) Complete the tree traversal after node 300's left subtree has been printed.

```
11 25 100 201
```

Check Show answer

3) How many nodes are visited?

Check Show answer

4) Using left, current, and right, what ordering will print the BST from largest to smallest? Ex: An inorder traversal uses left current right.

Check Show answer

5.8 BST height and insertion order

BST height and insertion order

Recall that a tree's ***height*** is the maximum edges from the root to any leaf. (Thus, a one-node tree has height 0.)

The *minimum* N-node binary tree height is , achieved when each level is full except possibly the last. The *maximum* N-node binary tree height is N - 1 (the - 1 is because the root is at height 0).

Searching a BST is fast if the tree's height is near the minimum. Inserting items in random order naturally keeps a BST's height near the minimum. In contrast, inserting items in nearly-sorted order

leads to a nearly-maximum tree height.

PARTICIPATION ACTIVITY 5.8.1: Inserting in random order keeps tree height near the minimum. Inserting in sorted order yields the maximum.

Animation captions:

1. Inserting in random order naturally keeps tree height near the minimum, in this case 3 (minimum: 2)
2. Inserting in sorted order yields the maximum height, in this case 6.
3. If nodes are given beforehand, randomizing the ordering before inserting keeps tree height near minimum.

PARTICIPATION ACTIVITY 5.8.2: BST height.

Draw a BST by hand, inserting nodes one at a time, to determine a BST's height.

1) A new BST is built by inserting nodes in this order:
`6 2 8`

What is the tree height? (Remember, the root is at height 0)

Check Show answer

2) A new BST is built by inserting nodes in this order:
`20 12 23 18 30`

What is the tree height?

Check Show answer

3) A new BST is built by inserting nodes in this order:
`30 23 21 20 18`

What is the tree height?

Check Show answer

4) A new BST is built by inserting nodes in this order:
`30 11 23 21 20`

What is the tree height?

Check Show answer

5) A new BST is built by inserting 255 nodes in sorted order. What is the tree height?

Check Show answer

6) A new BST is built by inserting 255 nodes in random order. What is the minimum possible tree height?

Check Show answer

BSTGetHeight algorithm

Given a node representing a BST subtree, the height can be computed as follows:

- If the node is null, return -1.
- Otherwise recursively compute the left and right child subtree heights, and return 1 plus the greater of the 2 child subtrees' heights.

PARTICIPATION ACTIVITY 5.8.3: BSTGetHeight algorithm.

Animation content:

undefined

Animation captions:

1. BSTGetHeight(tree⇢root) is called to get the height of the tree. The height of the root's left child is determined first using a recursive call.
2. BSTGetHeight for node 18 makes a recursive call on node 12. BSTGetHeight on node 12 makes a recursive call on the null left child, which returns -1.
3. Returning to the BSTGetHeight(node 12) call, a recursive call is now made on the right child. Node 14 is a leaf, so both recursive calls return -1.
4. BSTGetHeight(node 14) returns 1 + max(-1, -1) = 1 + -1 = 0.
5. BSTGetHeight(node 12) has completed 2 recursive calls and returns 1 + max(-1, 0) = 1. BSTGetHeight(node 18) makes the recursive call on the null right child, which returns -1.
6. A recursive call is made for each node in the tree. BSTGetHeight(tree⇢root) returns 1 + max(2, 1) = 3, which is the tree's height.

PARTICIPATION ACTIVITY 5.8.4: BSTGetHeight algorithm.

1) BSTGetHeight returns 0 for a tree with a single node.
   - ○ True
   - ○ False

2) The base case for BSTGetHeight is when the node argument is null.
   - ○ True
   - ○ False

3) The worst-case time complexity for BSTGetHeight is O(log N), where N is the number of nodes in the tree.
   - ○ True
   - ○ False

4) BSTGetHeight would also work if the recursive call on the right child was made before the recursive call on the left child.

- True
- False

5.9 BST parent node pointers

A BST implementation often includes a parent pointer inside each node. A balanced BST, such as an AVL tree or red-black tree, may utilize the parent pointer to traverse up the tree from a particular node to find a node's parent, grandparent, or siblings. The BST insertion and removal algorithms below insert or remove nodes in a BST with nodes containing parent pointers.

Figure 5.9.1: BSTInsert algorithm for BSTs with nodes containing parent pointers.

```
BSTInsert(tree, node) {
   if (tree→root == null) {
      tree→root = node
      node→parent = null
      return
   }

   cur = tree→root
   while (cur != null) {
      if (node→key < cur→key) {
         if (cur→left == null)
{
            cur→left = node
            node→parent = cur
            cur = null
         }
         else
            cur = cur→left
      }
      else {
         if (cur→right == null)
{
            cur→right = node
            node→parent = cur
            cur = null
         }
         else
            cur = cur→right
      }
   }
}
```


Figure 5.9.2: BSTReplaceChild algorithm.

```
BSTReplaceChild(parent, currentChild,
newChild) {
   if (parent→left != currentChild &&
       parent→right != currentChild)
      return false

   if (parent→left == currentChild)
      parent→left = newChild
   else
      parent→right = newChild

   if (newChild != null)
      newChild→parent = parent
   return true
}
```

Figure 5.9.3: BSTRemoveKey and BSTRemoveNode algorithms for BSTs with nodes containing parent pointers.

```
BSTRemoveKey(tree, key) {
   node = BSTSearch(tree, key)
   BSTRemoveNode(tree, node)
}

BSTRemoveNode(tree, node) {
   if (node == null)
      return

   // Case 1: Internal node with 2 children
   if (node→left != null && node→right != null) {
      // Find successor
      succNode = node→right
      while (succNode→left)
         succNode = succNode→left

      // Copy value/data from succNode to node
      node = Copy succNode

      // Recursively remove succNode
      BSTRemoveNode(tree, succNode)
   }

   // Case 2: Root node (with 1 or 0 children)
   else if (node == tree→root) {
      if (node→left != null)
         tree→root = node→left
      else
         tree→root = node→right

      // Make sure the new root, if non-null, has a null
parent
      if (tree→root != null)
         tree→root→parent = null
   }

   // Case 3: Internal with left child only
   else if (node→left != null)
      BSTReplaceChild(node→parent, node, node→left)

   // Case 4: Internal with right child only OR leaf
   else
      BSTReplaceChild(node→parent, node, node→right)
}
```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Eric Quezada
UTEPCS2302ValeraFall2023

PARTICIPATION ACTIVITY 5.9.1: BST parent node pointers.

1) `BSTInsert` will not work if the tree's root is null.

- True
- False

2) `BSTReplaceChild` will not work if the parent pointer is null.

- True
- False

3) `BSTRemoveKey` will not work if the key is not in the tree.

- True
- False

4) `BSTRemoveNode` will not work to remove the last node in a tree.

- True
- False

5) `BSTRemoveKey` uses `BSTRemoveNode`.

- True
- False

6) `BSTRemoveNode` uses `BSTRemoveKey`.

- True
- False

7) `BSTRemoveNode` may use recursion.

- True
- False

8) `BSTRemoveKey` will not properly update parent pointers when a non-root node is being removed.

- True
- False

9) All calls to `BSTRemoveNode` to remove a non-root node will result in a call to `BSTReplaceChild`.

- True
- False

5.10 BST: Recursion

BST recursive search algorithm

BST search can be implemented using recursion. A single node and search key are passed as arguments to the recursive search function. Two base cases exist. The first base case is when the node is null, in which case null is returned. If the node is non-null, then the search key is compared to the node's key. The second base case is when the search key equals the node's key, in which case the node is returned. If the search key is less than the node's key, a recursive call is made on the node's left child. If the search key is greater than the node's key, a recursive call is made on the node's right child.

PARTICIPATION ACTIVITY 5.10.1: BST recursive search algorithm.

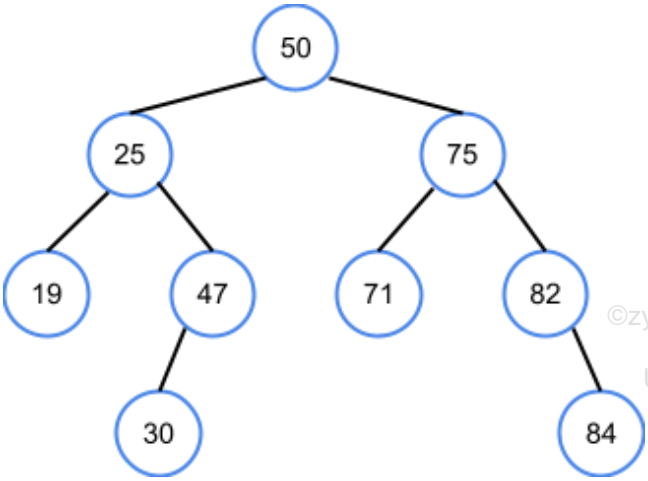
Animation content:

undefined

Animation captions:

1. A call to BSTSearch(tree, 40) calls the BSTSearchRecursive function with the tree's root as the node argument.
2. The search key 40 is less than 64, so a recursive call is made on the root node's left child.
3. An additional recursive call searches node 32's right child. The key 40 is found and node 40 is returned.
4. Each function returns the result of a recursive call, so BSTSearch(tree, 40) returns node 40.

PARTICIPATION ACTIVITY 5.10.2: BST recursive search algorithm.



1) How many calls to BSTSearchRecursive are made by calling BSTSearch(tree, 71)?

- 2
- 3
- 4

2) How many calls to BSTSearchRecursive are made by calling BSTSearch(tree, 49)?

- 3
- 4
- 5

3) What is the maximum possible number of calls to BSTSearchRecursive when searching the tree?

- 4
- 5

BST get parent algorithm

A recursive BST get-parent algorithm searches for a parent in a way similar to the normal BST search algorithm. But instead of comparing the search key with a candidate node's key, the search key is compared with the keys of the candidate node's children.

Figure 5.10.1: BST get parent algorithm.

```
BSTGetParent(tree, node) {
   return BSTGetParentRecursive(tree→root, node)
}

BSTGetParentRecursive(subtreeRoot, node) {
   if (subtreeRoot is null)
      return null

   if (subtreeRoot→left == node or
       subtreeRoot→right == node) {
      return subtreeRoot
   }

   if (node→key < subtreeRoot→key) {
      return BSTGetParentRecursive(subtreeRoot→left,
node)
   }
   return BSTGetParentRecursive(subtreeRoot→right,
node)
}
```


PARTICIPATION ACTIVITY 5.10.3: BST get parent algorithm.

1) BSTGetParent() returns null when the node parameter is the tree's root.
   - True
   - False

2) BSTGetParent() returns a leaf node when the node parameter is null.
   - True
   - False

3) The base case for BSTGetParentRecursive() is when subtreeRoot is null or is node's parent.
   - True
   - False

Recursive BST insertion and removal

BST insertion and removal can also be implemented using recursion. The insertion algorithm uses recursion to traverse down the tree until the insertion location is found. The removal algorithm uses the

recursive search functions to find the node and the node's parent, then removes the node from the tree. If the node to remove is an internal node with 2 children, the node's successor is recursively removed.

Figure 5.10.2: Recursive BST insertion and removal.

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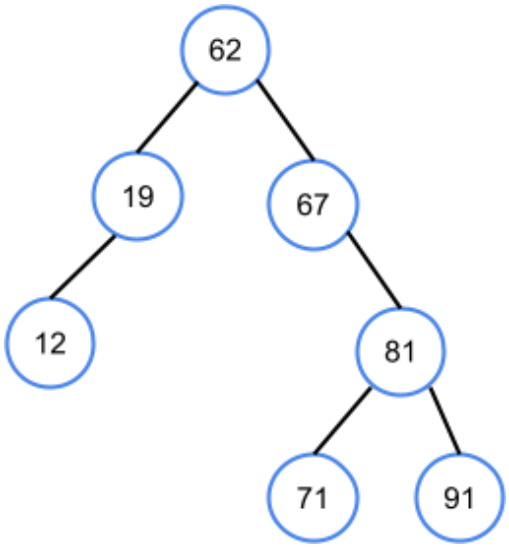
```
BSTInsert(tree, node) {
   if (tree→root is null)
      tree→root = node
   else
      BSTInsertRecursive(tree→root, node)
}

BSTInsertRecursive(parent, nodeToInsert) {
   if (nodeToInsert→key < parent→key) {
      if (parent→left is null)
         parent→left = nodeToInsert
      else
         BSTInsertRecursive(parent→left,
nodeToInsert)
   }
   else {
      if (parent→right is null)
         parent→right = nodeToInsert
      else
         BSTInsertRecursive(parent→right,
nodeToInsert)
   }
}

BSTRemove(tree, key) {
   node = BSTSearch(tree, key)
   parent = BSTGetParent(tree, node)
   BSTRemoveNode(tree, parent, node)
}

BSTRemoveNode(tree, parent, node) {
   if (node == null)
      return false

   // Case 1: Internal node with 2 children
   if (node→left != null && node→right != null) {
      // Find successor and successor's parent
      succNode = node→right
      successorParent = node
      while (succNode→left != null) {
         successorParent = succNode
         succNode = succNode→left
      }

      // Copy the value from the successor node
      node = Copy succNode

      // Recursively remove successor
      BSTRemoveNode(tree, successorParent, succNode)
   }

   // Case 2: Root node (with 1 or 0 children)
   else if (node == tree→root) {
      if (node→left != null)
         tree→root = node→left
      else
         tree→root = node→right
   }

   // Case 3: Internal with left child only
```

```
   else if (node→left != null) {
      // Replace node with node's left child
      if (parent→left == node)
         parent→left = node→left
      else
         parent→right = node→left
   }

   // Case 4: Internal with right child only OR leaf
   else {
      // Replace node with node's right child
      if (parent→left == node)
         parent→left = node→right
      else
         parent→right = node→right
   }

   return true
}
```


PARTICIPATION ACTIVITY 5.10.4: Recursive BST insertion and removal.



The following operations are executed on the above tree:
BSTInsert(tree, node 70)
BSTInsert(tree, node 56)
BSTRemove(tree, 67)

1) Where is node 70 inserted?

- Node 67's left child
- Node 71's left child
- Node 71's right child

2) How many times is BSTInsertRecursive called when inserting node 56?

- 2
- 3
- 5

3) How many times is BSTRemoveNode called when removing node 67?

- 1
- 2
- 3

4) What is the maximum number of calls to BSTRemoveNode when removing one of the tree's nodes?

- 2
- 4
- 5

5.11 Tries

Overview

A ***trie*** (or ***prefix tree***) is a tree representing a set of strings. Each non-root node represents a single character. Each node has at most one child per distinct alphabet character. A ***terminal node*** is a node that represents a terminating character, which is the end of a string in the trie.

Tries provide efficient storage and quick search for strings, and are often used to implement auto-complete and predictive text input.

PARTICIPATION ACTIVITY 5.11.1: Trie representing the set of strings: bat, cat, and cats.

Animation content:

undefined

Animation captions:

1. The following trie represents a set of two strings. Each string can be determined by traversing the path from the root to a leaf.
2. Suppose "cats" is added to the trie. Adding another child for 'c' would violate the trie requirements.
3. So the existing child for 'c' is reused.
4. Similarly, nodes for 'a' and 't' are reused. Node 't' has a new child added for 's'.
5. Exactly one terminal node exists for each string. Other nodes may be shared by multiple strings.

PARTICIPATION ACTIVITY 5.11.2: Trie representing the set of strings: bat, cat, and cats.

Refer to the trie above.

1) The terminal nodes can be removed, and instead the last character of a string can be a leaf.
   - True
   - False

2) Adding the string "balance" would create a new branch off the root node.
   - True
   - False

3) Inserting a string that doesn't already exist in the trie requires allocation of at least 1 new node.
   - True
   - False

Trie insert algorithm

Given a string, a ***trie insert*** operation creates a path from the root to a terminal node that visits all the string's characters in sequence.

A current node pointer initially points to the root. A loop then iterates through the string's characters. For each character C:

1. A new child node is added only if the current node does not have a child for C.
2. The current node pointer is assigned with the current node's child for C.

After all characters are processed, a terminal node is added and returned.

PARTICIPATION ACTIVITY 5.11.3: Trie insert algorithm.

Animation content:

undefined

Animation captions:

1. The trie starts with an empty root node. Adding "APPLE" first adds a new child for 'A' to the root node.
2. New nodes are built for each remaining character in "APPLE".
3. The terminal node is added, completing insertion of "APPLE".
4. When adding "APPLY", the first 4 character nodes are reused.
5. Node L has no child for 'Y', so a new node is added. The terminal node is also added.
6. When adding "APP", only 1 new node is needed, the terminal node.

PARTICIPATION ACTIVITY 5.11.4: Trie insert algorithm.

Assume a trie is built by executing the following code.

```
trieRoot = new TrieNode()
TrieInsert(trieRoot, "cat")
TrieInsert(trieRoot, "cow")
TrieInsert(trieRoot, "crow")
```

1) When inserting "cat", _____ new nodes are created.

- 1
- 3
- 4

2) When inserting "crow", _____ new nodes are created.

- 1
- 4
- 5

3) If `TrieInsert(trieRoot, "cow")` is called a second time, _____ new nodes are created.

- 0
- 1
- 4

Trie search algorithm

Given a string, a ***trie search*** operation returns the terminal node corresponding to that string, or null if the string is not in the trie.

PARTICIPATION ACTIVITY 5.11.5: Trie search algorithm.

Animation content:

undefined

Animation captions:

1. The search for "PAPAYA" starts at the root and iterates through one node per character. The terminal node is returned, indicating that the string was found.
2. The search for "GRAPE" ends quickly, since the root has no child for 'G'.
3. The search for "PEA" gets to the node for 'A'. No terminal child node exists after 'A', so null is returned.

PARTICIPATION ACTIVITY 5.11.6: Trie search algorithm.

Refer to the trie above. Assume that to "visit" a node means accessing the node's children in TrieSearch.

1) `TrieSearch(root, "PINEAPPLE")` returns _____.

- the trie's root node
- the terminal node for "APPLE"
- null

2) When searching for "PLUM", _____ visited.

- only the root node is
- the root and the root's 'P' child node are
- all nodes in the root's 'P' child subtree are

3) TrieSearch will visit at most _____ nodes in a single search.

- 7
- 9
- 41

Trie remove algorithm

Given a string, a ***trie remove*** operation removes the string's corresponding terminal node and all non-root ancestors with 0 children.

PARTICIPATION ACTIVITY 5.11.7: Trie remove algorithm.

Animation content:

undefined

Animation captions:

1. TrieRemove is called to remove "BANANA". TrieRemove then calls TrieRemoveRecursive, passing the trie's root, the string "BANANA", and a character index of 0.
2. The root has a child for 'B'. A recursive removal call is made for the child and the next character index.
3. Recursive calls continue until the terminal node's parent is reached.
4. The terminal node is removed from the node's children and true is returned.
5. After returning from each recursive call, child nodes with 0 children are also removed.
6. When removing "APPLE", 4 nodes are removed.
7. Removal operations only remove nodes that are exclusive to the string being removed.

PARTICIPATION ACTIVITY 5.11.8: Trie remove algorithm.

Refer to the trie above. Match each operation to the statement that is true when the operation executes. Assume that "BANANA" and "APPLE" have already been removed.

If unable to drag and drop, refresh the page.

`TrieRemove(root, "CHERRY")`
`TrieRemove(root, "PLUM")`

`TrieRemove(root, "PEAR")`

`TrieRemove(root, "AVOCADO")`
`TrieRemove(root, "APRICOT")`

`TrieRemove(root, "PAPAYA")`

	Remove's the root's child node for 'A'.
	`charIndex` is 6 at the moment a terminal node is removed.
	Has no effect.
	Removes a total of 2 nodes from the trie.

Reset

Trie time complexities

Implementations commonly use a lookup table for a trie node's children, allowing retrieval of a child node from a character in O(1) time. Therefore, to insert, remove, or search for a string of length M in a trie takes O(M) time. The trie's current size does not affect each operation's time complexity.

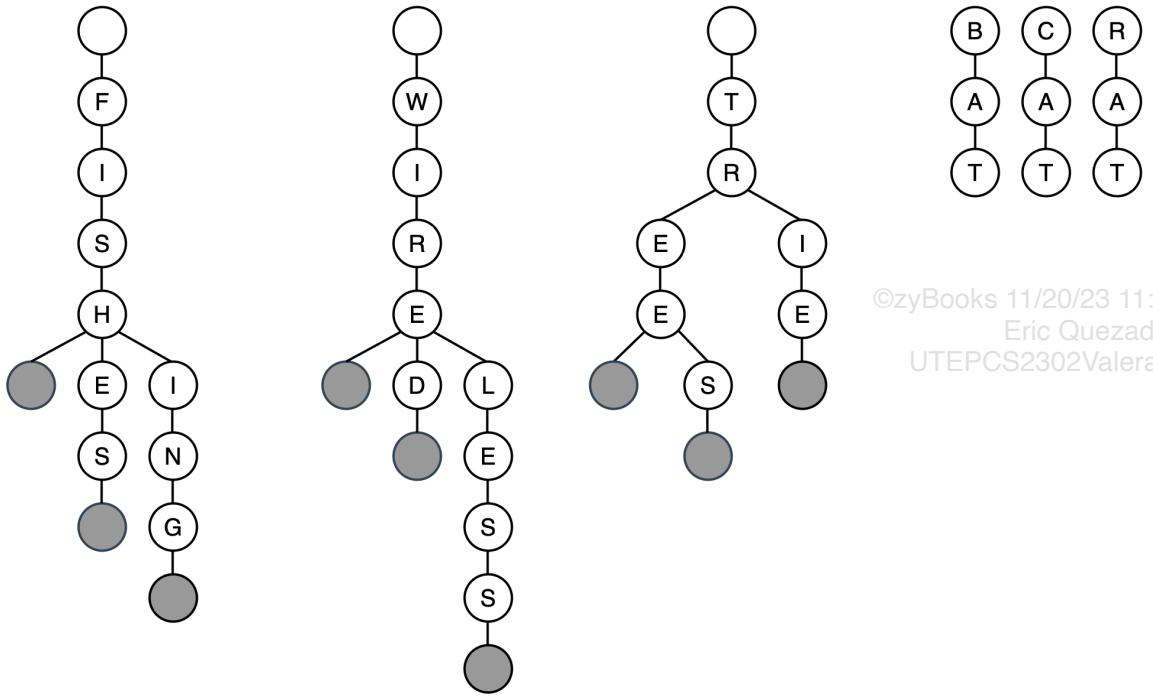
CHALLENGE ACTIVITY 5.11.1: Tries.

502696.2096894.qx3zqy7

Start

Select all valid tries.

☐ A ☐ B ☐ C ☐ D



1 2 3 4

Check Next

5.12 Python: Binary search tree

Constructing the node and binary search tree class

A binary search tree can be implemented in Python using a Node class and a BinarySearchTree class. The Node class contains a key value and data members for the left and right children nodes. The BinarySearchTree class contains a data member for the tree's root (a Node object).

Figure 5.12.1: Node and BinarySearchTree class definitions.

The Node class	The BinarySearchTree class

```
class Node:
    def __init__(self, key):
        self.key = key
        self.left = None
        self.right = None
```

```
class BinarySearchTree:
    def __init__(self):
        self.root = None
```


PARTICIPATION ACTIVITY 5.12.1: Implementation of the BinarySearchTree class.

If unable to drag and drop, refresh the page.

Node **BinarySearchTree** **root**

	The class that represents an element in a tree, with data members `key`, `left` and `right`.
	The class that represents an entire tree with data members for the tree's root node.
	The Node instance that represents the single key value at the tree's top.

Reset

Binary search tree search() method

The binary search tree search algorithm first assigns current_node with the root. A loop is then entered, and one of three cases occurs:

- Case 1: The current node's key matches the desired key, so current_node is returned from the method.
- Case 2: The desired key is less than the current node's key, so current_node is assigned with current_node.left.
- Case 3: The desired key is greater than the current node's key, so current_node is assigned with current_node.right.

The loop terminates once current_node is None, meaning the desired key is not in the tree.

Figure 5.12.2: BinarySearchTree search() method.

```
def search(self, desired_key):
    current_node = self.root
    while current_node is not None:
        # Return the node if the key matches.
        if current_node.key == desired_key:
            return current_node

        # Navigate to the left if the search key
is
        # less than the node's key.
        elif desired_key < current_node.key:
            current_node = current_node.left

        # Navigate to the right if the search key
is
        # greater than the node's key.
        else:
            current_node = current_node.right

    # The key was not found in the tree.
    return None
```


PARTICIPATION ACTIVITY 5.12.2: BinarySearchTree search() method.

1) The search() method has a Node instance parameter.

   - True
   - False

2) The search() method returns True if the desired key is found in the tree, or False if the key is not found.

   - True
   - False

3) If a specific key is in a well-balanced tree, then the search() method will find the desired key within comparisons.

   - True
   - False

Binary search tree insert() method

The insert() method is used to insert a new node into the tree. If the tree is empty, then the root data member is assigned with the new node. If the tree is not empty, current_node is assigned with the root node, and a loop is entered. Inside the loop, if the new node's key is less than the current node's key, then the new node is inserted as the current node's left child (if the current node has no left child), or current_node is assigned with current_node.left. If the new node's key is greater than or equal to the current node's key, then the new node is inserted as the current node's right child (if the current node has no right child), or current_node is assigned with current_node.right.

Figure 5.12.3: Binary search tree insert() method.

The insert() method	A program to test the insert() method

```
def insert(self, node):

    # Check if the tree is empty
    if self.root is None:
        self.root = node
    else:
        current_node = self.root
        while current_node is not
None:
            if node.key <
current_node.key:
                # If there is no
left child, add the new
                # node here;
otherwise repeat from the
                # left child.
                if
current_node.left is None:

current_node.left = node
                    current_node =
None
                else:
                    current_node =
current_node.left
            else:
                # If there is no
right child, add the new
                # node here;
otherwise repeat from the
                # right child.
                if
current_node.right is None:

current_node.right = node
                    current_node =
None
                else:
                    current_node =
current_node.right
```

```
# Main program to test insert and
search methods.
tree = BinarySearchTree()
node_a = Node(17)
node_b = Node(32)
node_c = Node(10)
node_d = Node(3)
node_e = Node(21)

tree.insert(node_a)
tree.insert(node_b)
tree.insert(node_c)
tree.insert(node_d)
tree.insert(node_e)

# Search for key 3.
found_node = tree.search(3)
if found_node is not None:
    print('Found node with key =
%d' % found_node.key)
else:
    print('Key 3 not found.')

# Search for key 99.
found_node = tree.search(99)
if found_node is not None:
    print('Found node with key =
%d' % found_node.key)
else:
    print('Key 99 not found.')
```



```
Found node with key = 3.
Key 99 not found.
```

PARTICIPATION ACTIVITY 5.12.3: Binary search tree insert() method.

The following values are inserted into a binary search tree in order: 30, 50, 60, 20, 40.

1) What is the key in node 50's parent? Enter "None" if the 50 node has no parent.

Check Show answer

2) What is the key in the 50 node's right child? Enter "None" if the 50 node has no right child.

Check Show answer

3) Node 50's left child is assigned with what key? Enter "None" if the 50 node has no left child.

Check Show answer

4) In the *worst* case, inserting a new node into a tree with N nodes requires how many comparisons?

Check Show answer

Binary search tree remove() method

The remove() method is more complicated than search() and insert() because several cases exist. Different actions are taken depending on what the kind of node is being removed.

- Case 1: The node being removed is a leaf. The parent's left or right data member is assigned with None, depending on which side the node is.
- Case 2: The node being removed has one child. The parent's left or right data member is assigned with the removed node's single child.
- Case 3:The node being removed has two children. The node's key is replaced by the successor's key, and then the successor (which will fall under either Case 1 or Case 2) is removed. The successor is the next largest node in the tree, and is always the right child's leftmost child.

Figure 5.12.4: Binary search tree remove method in Python.

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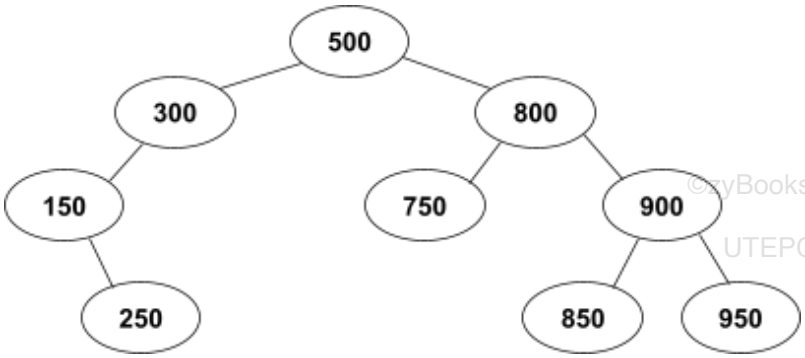
```
def remove(self, key):
    parent = None
    current_node = self.root

    # Search for the node.
    while current_node is not None:

        # Check if current_node has a matching key.
        if current_node.key == key:
            if current_node.left is None and current_node.right is None:  #
Case 1
                if parent is None: # Node is root
                    self.root = None
                elif parent.left is current_node:
                    parent.left = None
                else:
                    parent.right = None
                return  # Node found and removed
            elif current_node.left is not None and current_node.right is
None:  # Case 2
                if parent is None: # Node is root
                    self.root = current_node.left
                elif parent.left is current_node:
                    parent.left = current_node.left
                else:
                    parent.right = current_node.left
                return  # Node found and removed
            elif current_node.left is None and current_node.right is not
None:  # Case 2
                if parent is None: # Node is root
                    self.root = current_node.right
                elif parent.left is current_node:
                    parent.left = current_node.right
                else:
                    parent.right = current_node.right
                return  # Node found and removed
            else:                                       # Case 3
                # Find successor (leftmost child of right subtree)
                successor = current_node.right
                while successor.left is not None:
                    successor = successor.left
                current_node.key = successor.key      # Copy successor to
current node
                parent = current_node
                current_node = current_node.right     # Remove successor
from right subtree
                key = parent.key                      # Loop continues with
new key
        elif current_node.key < key: # Search right
            parent = current_node
            current_node = current_node.right
        else:                        # Search left
            parent = current_node
            current_node = current_node.left

    return # Node not found
```

PARTICIPATION ACTIVITY 5.12.4: Binary search tree remove() method.



1) If node 750 is removed, which case is applicable?
   - Case 1
   - Case 2
   - Case 3

2) If node 150 is removed, which case is applicable?
   - Case 1
   - Case 2
   - Case 3

3) If node 800 is removed, which case is applicable?
   - Case 1
   - Case 2
   - Case 3

4) Which node is node 800's successor?
   - 500
   - 900
   - 850

zyDE 5.12.1: Working with a binary search tree.

The following program inserts some user-defined values into a binary search tree, and the removes one of the values.

An overloaded __str__() method that uses functions in the included TreePrint.py has been included to create a way to display a tree.

Try inserting values in different orders:

- random: 3 10 7 2 8 4 9 5 1 6
- sorted: 1 2 3 4 5 6 7 8 9 10
- reverse sorted: 10 9 8 7 6 5 4 3 2 1

For each of the above, after inserting all the values try to remove the value 5. Try to predic what each tree will look like before and after removing the value 5.

Current file: main.py Load default templat

```
1  # Main program to test Binary search tree.
2  from Node import Node
3  from BinarySearchTree import BinarySearchTree
4
5  tree = BinarySearchTree()
6
7  user_values = input('Enter insert values with spaces between: ')
8  print()
9
10 for value in user_values.split():
11     new_node = Node(int(value))
12     tree.insert(new_node)
13
14 print('Initial tree:')
15 print(tree)
```

```
3 10 7 2 8 4 9 5 1 6
5
```

Run

5.13 LAB: BST validity checker

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