Recursion

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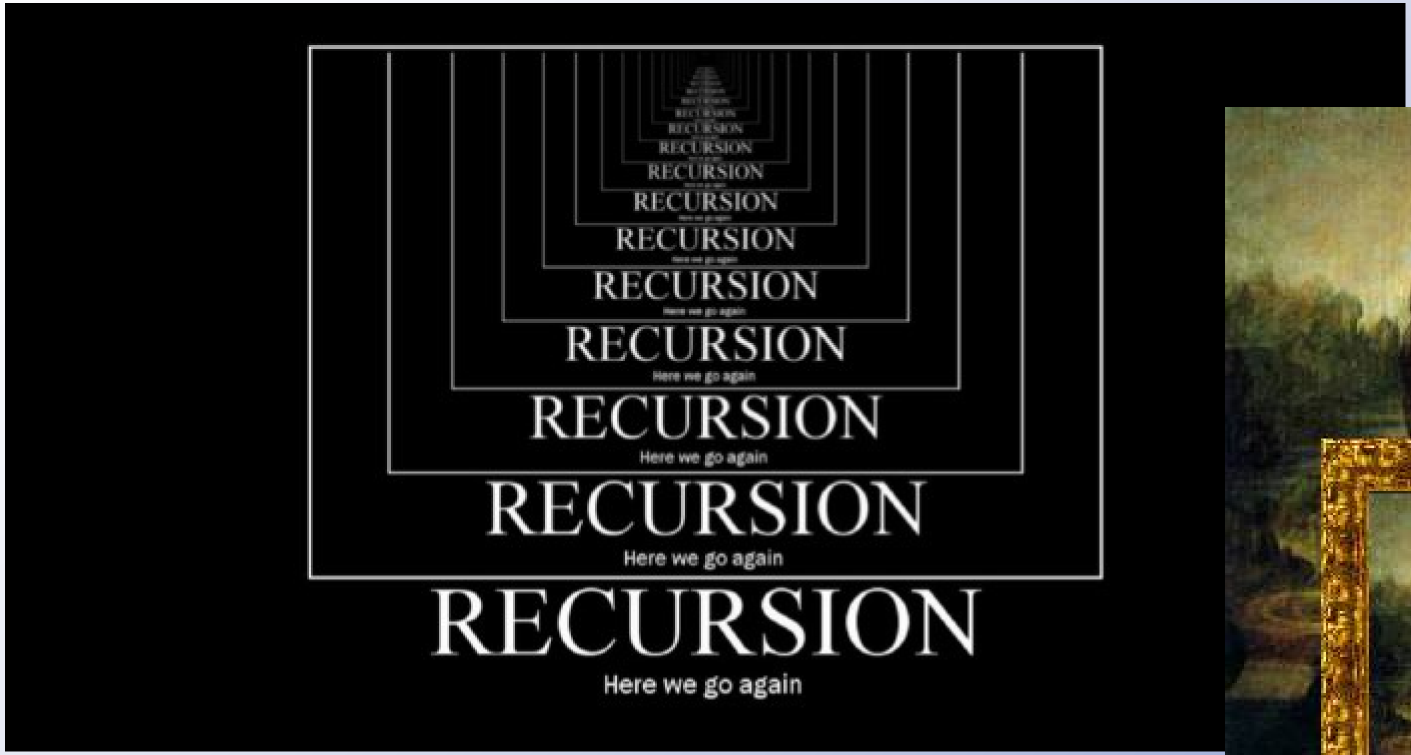
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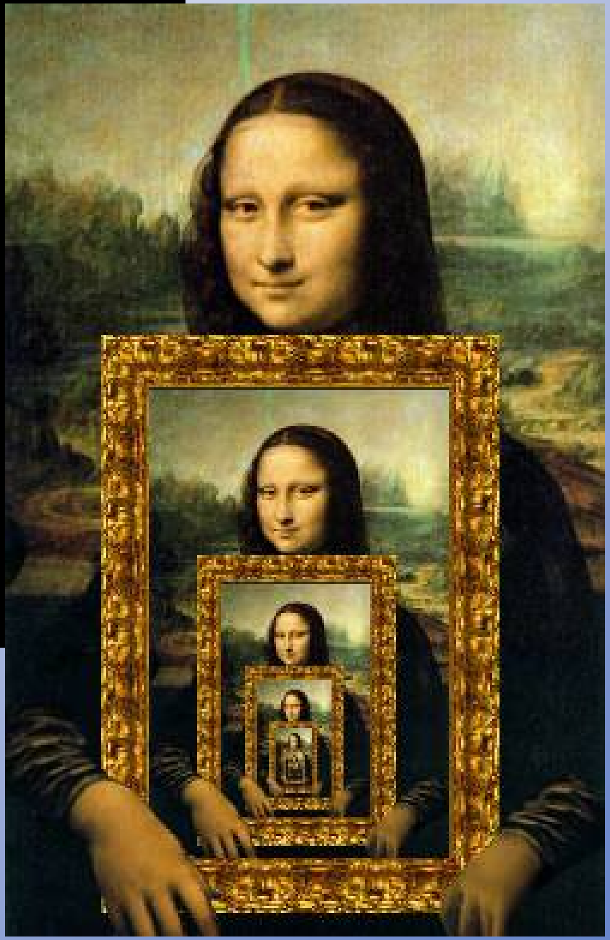

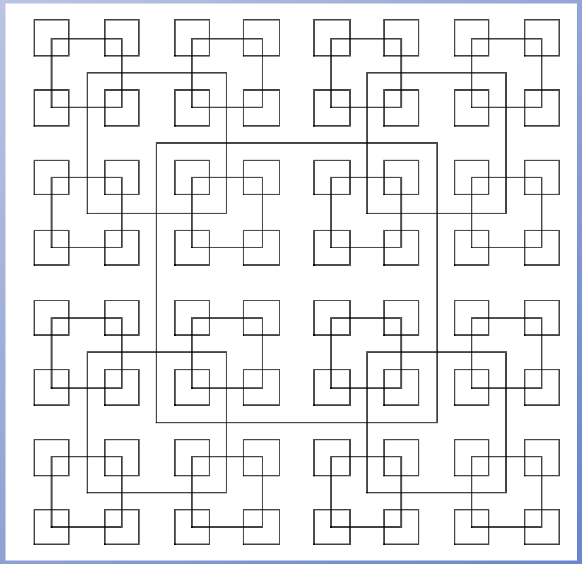

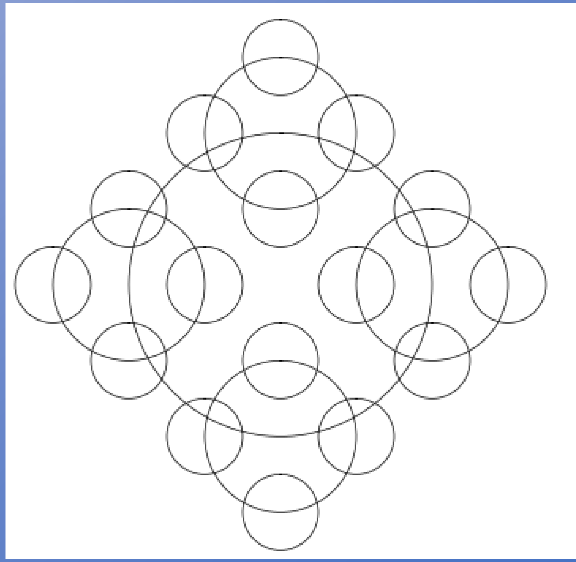

Recursion

- **Recursion** is a method of solving a problem where the solution depends on solutions to smaller instances of the same problem.
- Recursion solves such problems by using functions that call themselves from within their own code.
- The approach can be applied to many types of problems,
- Recursion is one of the central ideas of computer science.

Is recursion ever necessary?

The short answer is:

NO

Anything that can be achieved with recursion can be achieved with iteration

Iteration is usually more efficient than recursion

When *should* we use recursion (non-exhaustive list):

1. When the algorithm to implement is defined recursively
   - Mergesort
   - Quicksort
   - Binary search
2. To traverse a data structure that is defined recursively
   - Linked lists
   - Binary search trees
3. To explore all possibilities
   - Permutations – find all the permutations of the characters in a string
     - p('cat') = 'cat', 'cta', 'act', 'atc', 'tca', 'tac'
   - Combinations - find all the binary numbers that have n digits
     - b(3) = '000','001','010','011','100','101',110','111'
4. To search incrementally and exhaustively for solutions to a problem (a.k.a backtracking):
   - Subset sum: is there a subset of a set of integers S that adds up to g?
     - Subsetsum({2,5,8,12}, 4) = No
     - Subsetsum({2,5,8,12},17) = Yes (subset = {5,12})

Elements of recursion:

1. Bases case(s) – You must ALWAYS have some base cases which can be solved without recursion
2. Making progress-For the cases that are to be solved recursively, the recursive call must always be to a case that makes progress towards the base case
3. Design rule – Assume all recursive calls work, then show that the original call will work – **Ignore the fact that the function is recursive!**
4. Compound interest rule – Never duplicate work by solving the same instance of a problem in separate recursive calls

More on the design rule

What do we mean when we say: 'assume all recursive calls work'?

1. They **return** the expected value(s), according to the definition of the problem

or

2. They produce the expected **output**, according to the definition of the problem

Remember:

Every call to a recursive function has its own local variables (even if the have the same name in all calls)

Recursive calls cannot change the values of local variables in the calling function - this is the most common error when writing recursive functions!

What does the **return** statement do?

Misunderstanding of **return** statements is another source of problems for beginning programmers

Do you understand exactly what **return** does?

A **return statement** **ends** the execution of a function and **returns** control to the calling function. Execution resumes in the calling function at the point immediately following the call. A **return statement** may also **return** a value to the calling function.

Elements of recursion:

1. Bases case(s) – You must ALWAYS have some base cases which can be solved without recursion
2. Making progress-For the cases that are to be solved recursively, the recursive call must always be to a case that makes progress towards the base case
3. Design rule – If all recursive calls work, then the original call will work (assume recursive calls work, show original call will work) – **Ignore the fact that the function is recursive!**
4. Compound interest rule – Never duplicate work by solving the same instance of a problem in separate recursive calls

Checking if all elements of recursion are satisfied

An example: factorial

```
1 def factorial(n):
2     if n<=0:
3         return 1          # (1) Base case – solved without recursion
4     else:
5         return n*factorial(n-1)   # (2) Making progress - recursive call is made to a simpler instance of the problem
```

(3) Design rule:
Assume factorial(n-1) returns the factorial of n-1 (DON'T TRACE!)
Then the function works correctly, since $n! = n(n-1)!$ (line 5)

(4) Compound interest rule:
There's no repeated computation

Checking if all elements of recursion are satisfied

An example: adding the digits of an integer

```
def sum_digits(n):
    if n<=0:
        return 0    # (1) Base case – solved without recursion
    else:
        return n%10 + sum_digits(n//10)    # (2) Making progress - recursive call is made to a simpler instance of the problem
```

(3) Design rule:
Assume sum_digits(n//10) returns the sum of all the digits except for the last one, then the function works correctly, since n%10 is the last digit

(4) Compound interest rule:
There’s no repeated computation

Checking if all elements of recursion are satisfied

An example: Fibonacci numbers

```
def fib(n):
    if n<2:            # (1) Base case – solved without recursion
        return n
    else:
        return fib(n-1) + fib(n-2)   # (2) Making progress - recursive calls are
                                     # made to simpler instances of the problem
```

(3) Design rule:
Assume fib(n- 1) and fib(n-2) return the correct values, then the function works correctly, since fib(n) = fib(n-1) + fib(n-2)

(4) Compound interest rule:
It breaks the compound interest rule. For example, fib(n-2) will be called by both fib(n) and fib(n-1). Thus the function will run in exponential time; it will never finish for most values of n.

Checking if all elements of recursion are satisfied

An example: adding the elements of a list

```
def sum_list(L):
    s = 0
    if len(L)==0:           # (1) Base case – solved without recursion
        return s
    s += L[0]
    sum_list(L[1:])         # (2) Making progress - recursive call is
    return s                #     made to a simpler instance of the problem
```

(3) Design rule:

Assume `sum_list(L[1:])` returns the sum of L from index 1 to the end. Then the function does not work, since it returns s, which is equal to L[0].

(4) Compound interest rule:

There's no repeated computation

Checking if all elements of recursion are satisfied

An example: adding the elements of a list

```
def sum_list(L):
    s = 0
    if len(L)==0:         # (1) Base case – solved without recursion
        return s
    s += L[0]
    s += sum_list(L[1:])  # (2) Making progress - recursive call is
                          # made to a simpler instance of the problem
    return s
```

(3) Design rule:
Assume `sum_list(L[1:])` returns the sum of L from index 1 to the end. Then the function works correctly, since the sum of L is L[0] plus the sum of L[1:].

(4) Compound interest rule:
There’s no repeated computation