Vehicle Turning and Its Simulation

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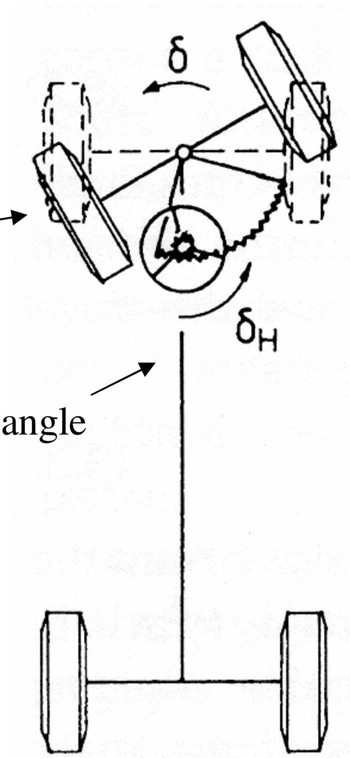
Overview

- Steering mechanisms
- Differentially-steered vehicle – kinematic
  - Animation of turning vehicle using Matlab
  - Animation of turning vehicle using LabVIEW
- Ackerman steered vehicle – kinematic
- Dynamic ‘bicycle’ model and its simulation

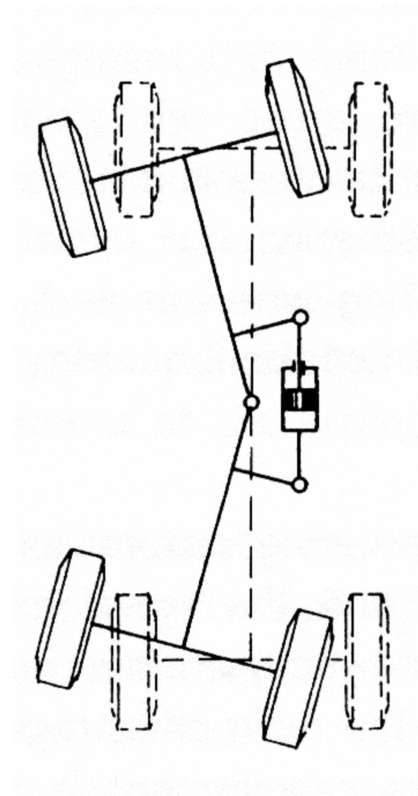
Classical Steering Mechanisms

5th wheel steering



δ_H = 'hand wheel' angle

'turntable steering'

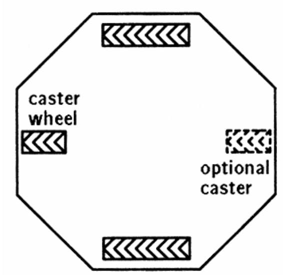


- Likely developed by the Romans, and preceded only by a 2 wheel cart.
- Consumes space
- Poor performance – unstable
- Longitudinal disturbance forces have large moment arms

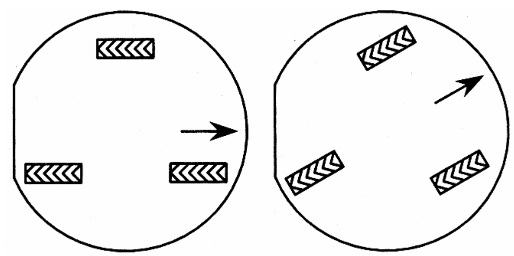
- Articulated-vehicle steering
- Tractors, heavy industrial vehicles

Common Steering Mechanisms

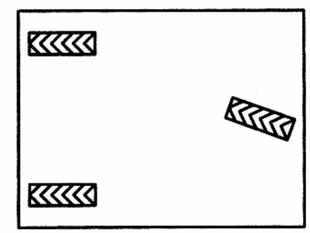
Differential steer

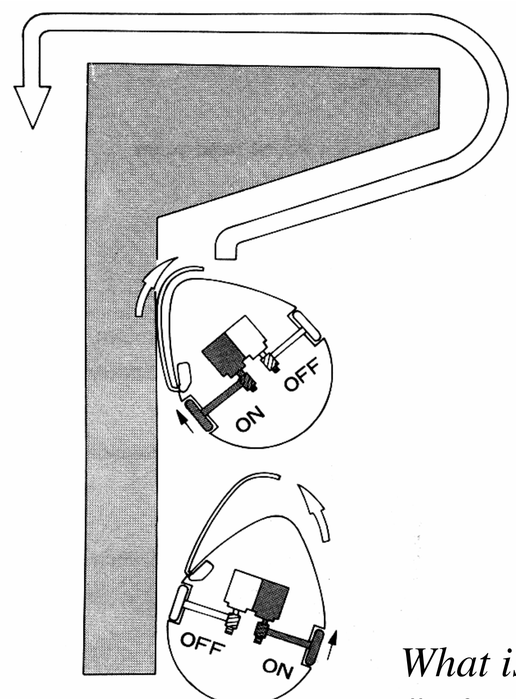


Synchro-drive

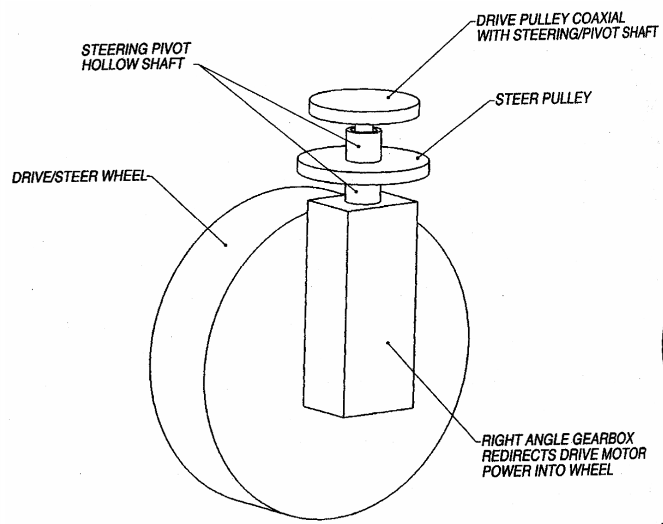


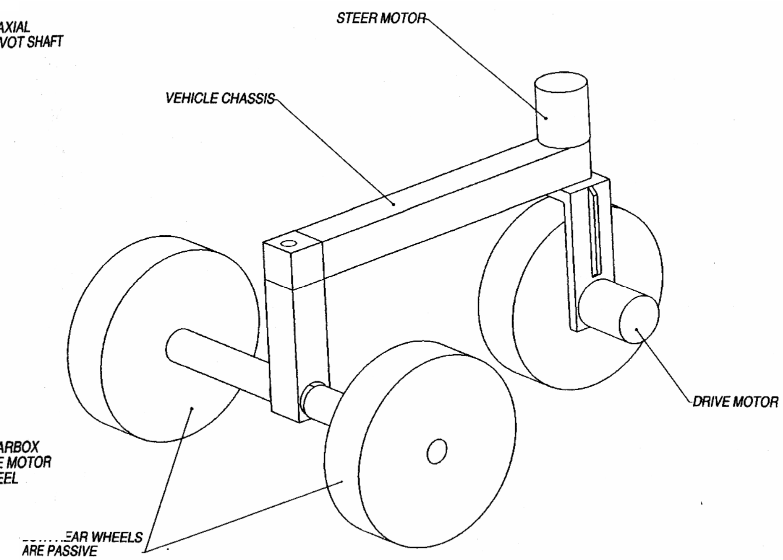
Tricycle





What is minimum # of actuators?

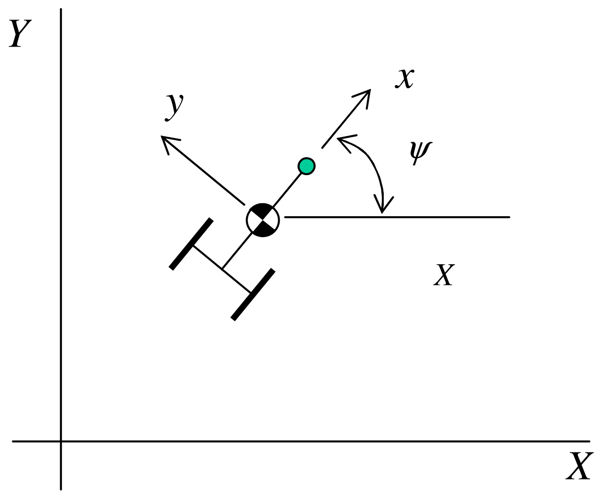




…and some systems also employ **Ackerman-type.**

Single-axle turning vehicles

Consider a simple 2D turning vehicle with kinematic state quantified by, $\mathbf{q}_I = \begin{bmatrix} X & Y & \psi \end{bmatrix}^\dagger$



The velocities in the local (body-fixed) reference frame are transformed into a global frame by the rotation matrix (see Appendix A),

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the velocities are then, $\dot{\mathbf{q}} = \mathbf{R}(\psi) \cdot \dot{\mathbf{q}}_I$

Inverting, we arrive at the velocities in the global reference frame, $\dot{\mathbf{q}}_I = \mathbf{\Psi}(\psi) \cdot \dot{\mathbf{q}}$

where,

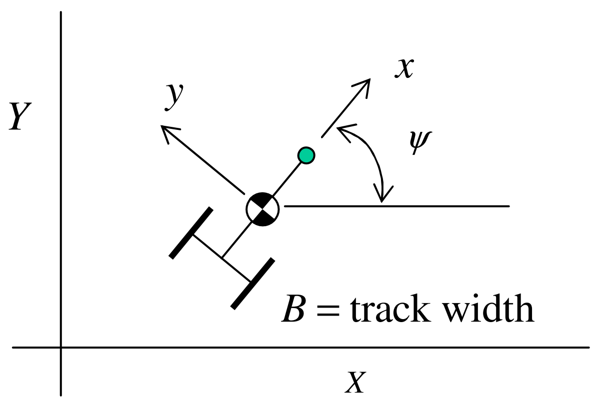
$$\mathbf{\Psi}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let's apply these relations to the case of a single-axis vehicle that has two wheels differentially driven with controlled speed.

Differentially-driven basic vehicle

For a *kinematic* model of a differentially-driven vehicle, we assume there is **no slip**, and that the wheels have controllable speeds, ω_1 and ω_2. The velocity of the CG in the local reference frame has a net effect from each wheel, composed as,

$$\left.\begin{aligned}\dot{x}_1 &= R_w \omega_1 \\ \dot{x}_2 &= R_w \omega_2\end{aligned}\right\}$$ Note these are the velocities at the wheels.



$$\therefore\ \dot{x} = \tfrac{1}{2}\left(\dot{x}_1 + \dot{x}_2\right) = \tfrac{1}{2} R_w (\omega_1 + \omega_2)$$

The lateral motion is constrained, so, $\dot{y} = 0$

The yaw rate is also composed by the net (constrained) motion of the two wheels, and you can show that:

$$\dot{\psi} = \frac{R_w}{B}\left(\omega_1 - \omega_2\right)$$

So the velocities in the global reference frame are,

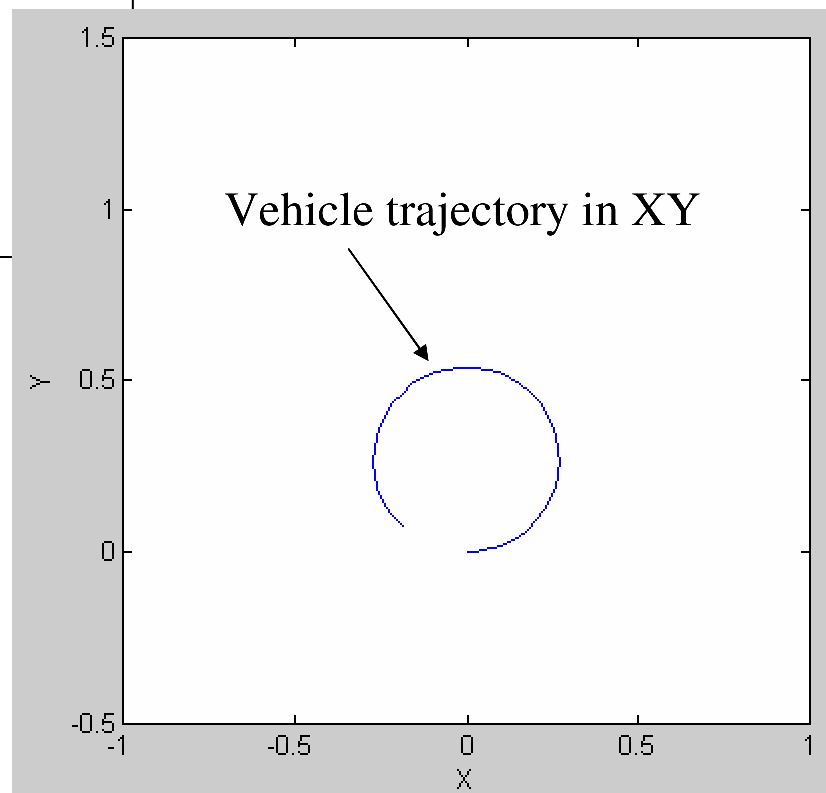
$$\dot{\mathbf{q}}_I = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \mathbf{\Psi}(\psi)\cdot\dot{\mathbf{q}} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{R_w}{2}\cos\psi(\omega_1+\omega_2) \\ \frac{R_w}{2}\sin\psi(\omega_1+\omega_2) \\ \frac{R_w}{B}(\omega_1-\omega_2) \end{bmatrix}$$

Basic vehicle trajectory simulation

A simple code in Matlab to plot out the vehicle trajectory is given below.

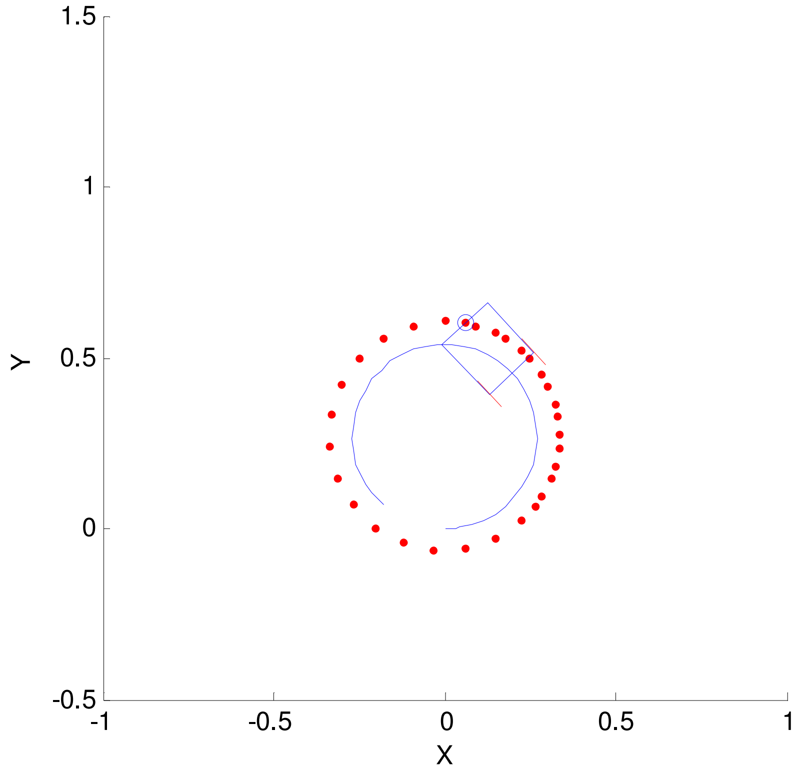
```
% Basic 2D single-axis vehicle kinematic model
% Requires right (#1) and left (#2) wheel velocities, omegaw1 and omegaw2,
% which are controlled inputs, to be passed as global parameters
% Wheel radius, R_w, and axle track width, B, are also required
function Xidot = basic_2D_vehicle(t,Xi)
global R_w B omegaw1 omegaw2
X = Xi(1); Y = Xi(2); psi = Xi(3);
% NOTE: these are global coordinates
Xdot = 0.5*cos(psi)*R_w*(omegaw1+omegaw2);
Ydot = 0.5*sin(psi)*R_w*(omegaw1+omegaw2);
psidot = R_w*(omegaw1-omegaw2)/B;

Xidot=[Xdot;Ydot;psidot];
```

```
% Test simulation script for basic_robot
clear all
global R_w B omegaw1 omegaw2
% Rw = wheel radius, B = track width
% omegaw1 = right wheel speed
R_w = 0.05; B = 0.18;
omegaw1 = 4; omegaw2 = 2;
Xi0=[0,0,0];
[t,Xi] = ode45(@basic_2D_vehicle,[0 10],Xi0);
N = length(t);
figure(1)
plot(Xi(:,1),Xi(:,2)), axis([-1.0 1.0 -0.5 1.5]), axis('square')
xlabel('X'), ylabel('Y')
```



Vehicle simulation with animation

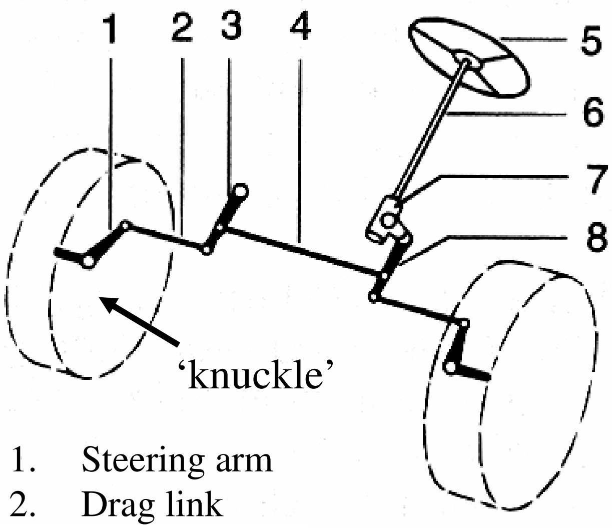
A code in Matlab to plot out the vehicle trajectory including a simple graphing animation of the vehicle body/orientation. This provides visual feedback on the model results.



The key elements of this code are:

1. Specify and plot initial location and orientation of the vehicle CG.
2. Initiate some 'handle graphics' functions for defining the 'body'.
3. Perform a fixed wheel speed simulation loop to find state, $\boldsymbol{q}$.
4. The state of the robot is used to define the position and orientation of the vehicle over time.
5. A simple routine is used to animate 2D motion of the vehicle by progressive plotting of the body/wheel positions.

Lankensperger/'Ackerman'-Type



1. Steering arm
2. Drag link
3. Idler arm
4. Tie rod/rack
5. Steering wheel
6. Steering shaft
7. Steering box
8. Pitman arm

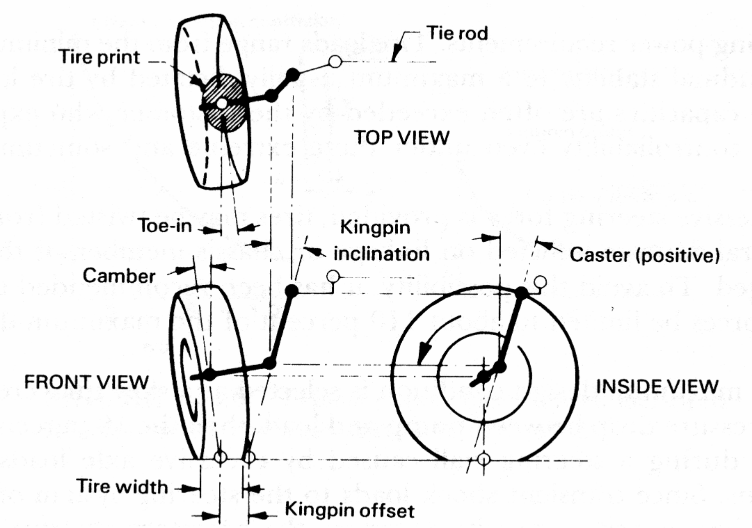
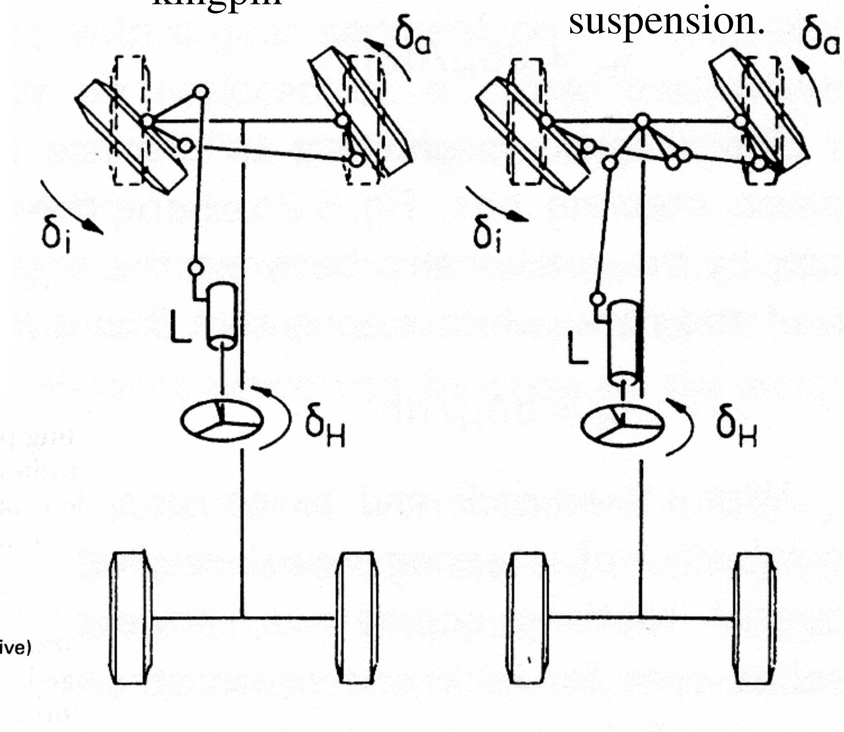


FIGURE 12-25 Tractor steering geometry. (From Wittren 1975.)

Rigid axle with kingpin

Divided track rods for independent suspension.



Turning at low (Ackermann) speed

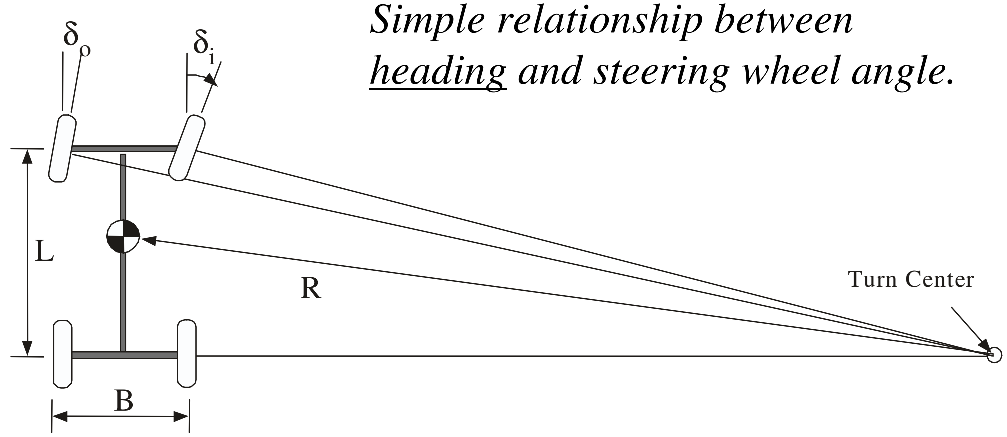
- What is 'low-speed'?
  - Negligible centrifugal forces
  - Tires need not develop lateral forces
- Pure rolling, no lateral sliding (minimum tire scrub).
- For proper geometry in the turn, the steer angles, δ, are given by:

$$\delta_o \cong \frac{L}{R + B/2} < \delta_i \cong \frac{L}{R - B/2}$$

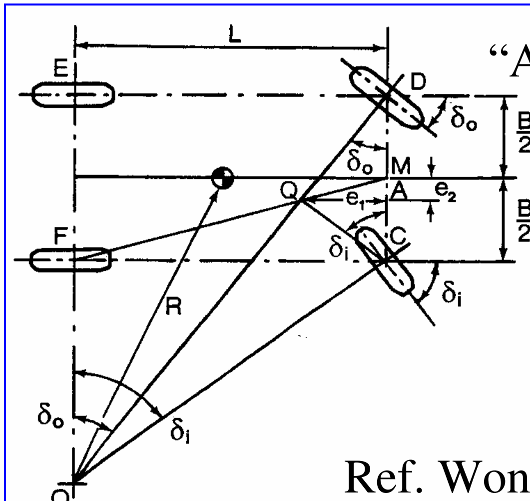
- The average value (small angles) is the **<u>Ackerman angle</u>**,

$$\delta_{\text{Ackermann}} = \frac{L}{R}$$

Simple relationship between <u>heading</u> and steering wheel angle.



"Ackermann steering geometry"



$$\cot\delta_o - \cot\delta_i = B/L$$

δ_o = steer angle of outside wheel

δ_i = steer angle of inside wheel

B = track

L = wheelbase

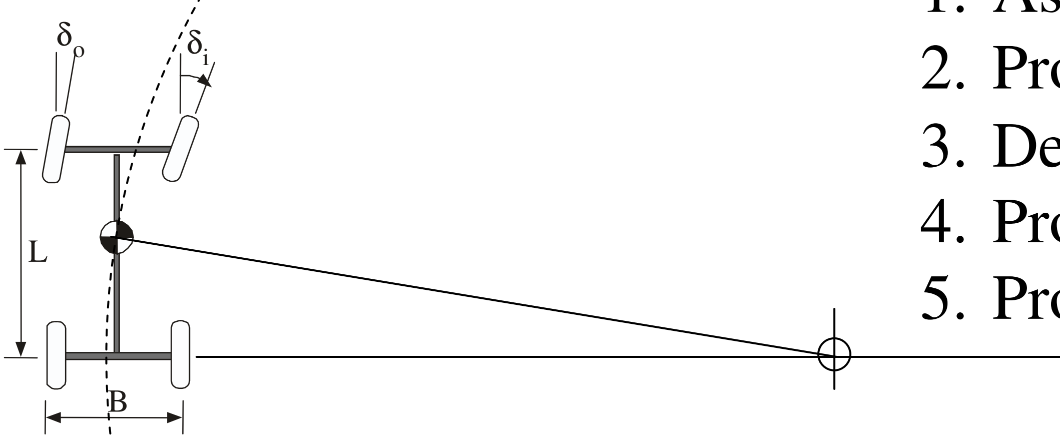
Ref. Wong, Ch. 5

Notes on Ackerman steering

- At low speed the wheels will roll without slip angle.
- If the rear wheels have no slip angle, the center of the turn lies on the projection of the rear axle. Each front steered wheel has a normal to the wheel plane that passes through the same center of the turn. This is what Ackerman geometry dictates.
- Correct Ackerman reduces tire wear (and is easy on terrain).
- Ackerman steering geometry leads to steering torques that increase with steer angle. The driver gets feedback about the extent to which wheels are turned. With parallel steer, the trend is different, becoming negative (not desirable in a steering system – positive feedback).
- Off-tracking of the rear wheels, Δ, is related to this geometry. The 'Δ' is $R[1-\cos(L/R)]$, or approximately $L^2/(2R)$.

What can you do with this?

Can you pass the vehicle through a given position?



1. Assume low-speed turning
2. Project along rear-axle
3. Define $R = L/\delta_{max}$
4. Project from CG
5. Project ideal turning path

2D vehicle with front-steered wheel

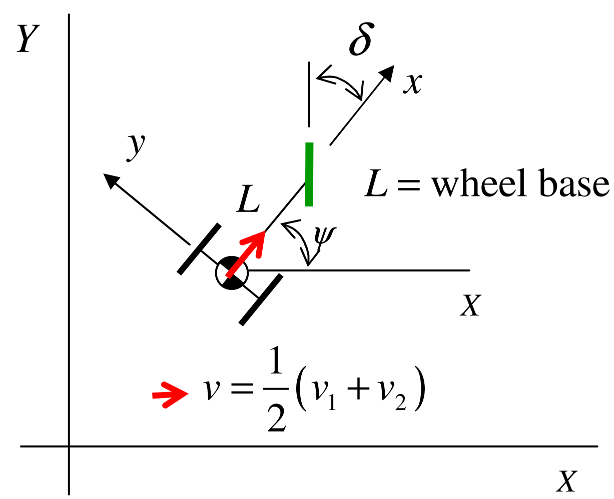
A wheeled vehicle is said to have *kinematic* (or Ackerman) steering when a wheel is actually given a steer angle, δ, as shown. A kinematic model for the steered basic vehicle in the inertial frame is given by the equations,

$$\dot{x} = v\cos\psi = R_w \omega \cos\psi$$

$$\dot{y} = v\sin\psi = R_w \omega \sin\psi$$

$$\dot{\psi} = \frac{v}{L}\tan\delta$$

‘tricycle’



where it is assumed that the wheels do not slip, so we can control the rotational speed and thus velocity at each wheel-ground contact.

So, the input ‘control variables’ are velocity, $v=R_w\omega$, and steer angle, δ.

In this example, the CG is located at the rear axle.

These kinematic equations can be easily simulated.

Note:

$$\omega_{CG} = \dot{\psi} = \frac{v_t}{L}$$

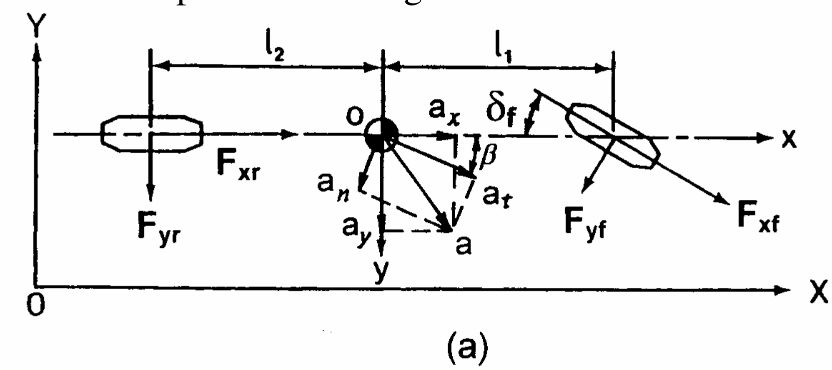
$$\tan\delta = \frac{v_t}{v}$$

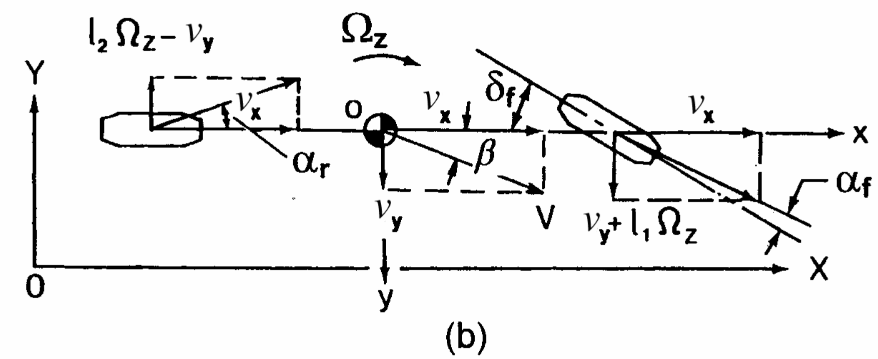
Summary of vehicle turning

- The models introduced here provide additional review of fundamental dynamics principles and how they are applied to vehicle systems.
- The kinematic model is commonly used in mobile robot applications.

The ‘Bicycle’ or Single-Track Model

Adapted from Wong





- Assume: symmetric vehicle, no roll or pitch
- Represent the two wheels on the front and rear axles by a single equivalent wheel.
- The bicycle model will have at least three states:
  - –forward translational momentum or velocity of the CG
  - –lateral translational momentum or velocity of the CG
  - –yaw angular momentum or velocity about the CG

Refer to Wong, Chapter 5, Eqs. 5.25 – 5.27:

$$m\left(\dot{v}_x - V_y\Omega_z\right) = \underbrace{F_{xf}\cos(\delta_f)}_{\text{front drive}} + \underbrace{F_{xr}}_{\text{rear drive}} - \underbrace{F_{yf}\sin(\delta_f)}_{\text{lateral force effect}}$$

$$m\left(\dot{v}_y + V_x\Omega_z\right) = F_{yr} + F_{yf}\cos(\delta_f) + F_{xf}\sin(\delta_f)$$

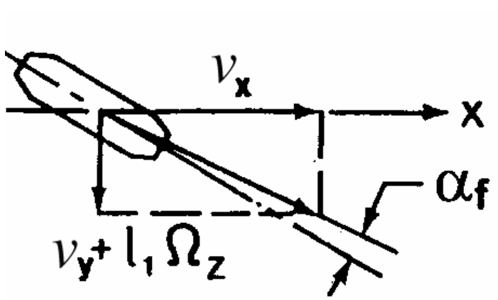
$$I_z\dot{\Omega}_z = l_1 F_{yf}\cos(\delta_f) - l_2 F_{yr} + l_1 F_{xf}\sin(\delta_f)$$

How would you add a disturbance? Need to determine the ‘external forces’.

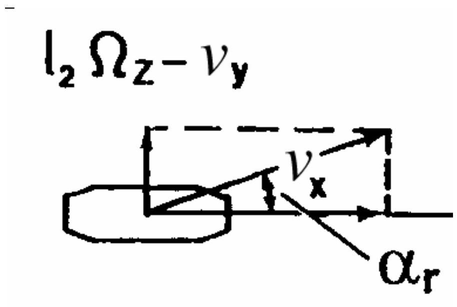
Lateral forces are related to slip angles

The slip angle, α, is derived using body-fixed variables.

Wong (2001)

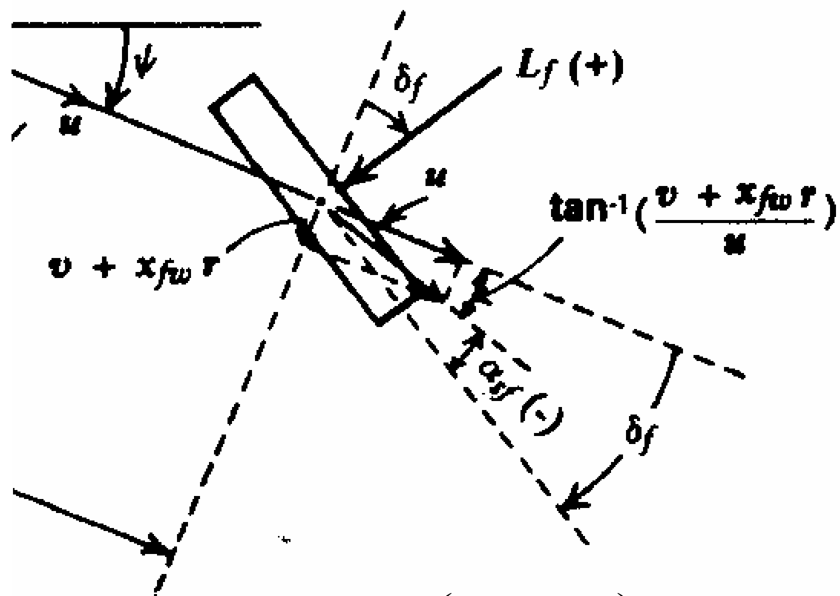


$$\alpha_f = \delta_f - \tan^{-1}\left(\frac{v_y + l_1\Omega_z}{v_x}\right)$$



$$\alpha_r = \tan^{-1}\left(\frac{l_2\Omega_z - v_y}{v_x}\right)$$

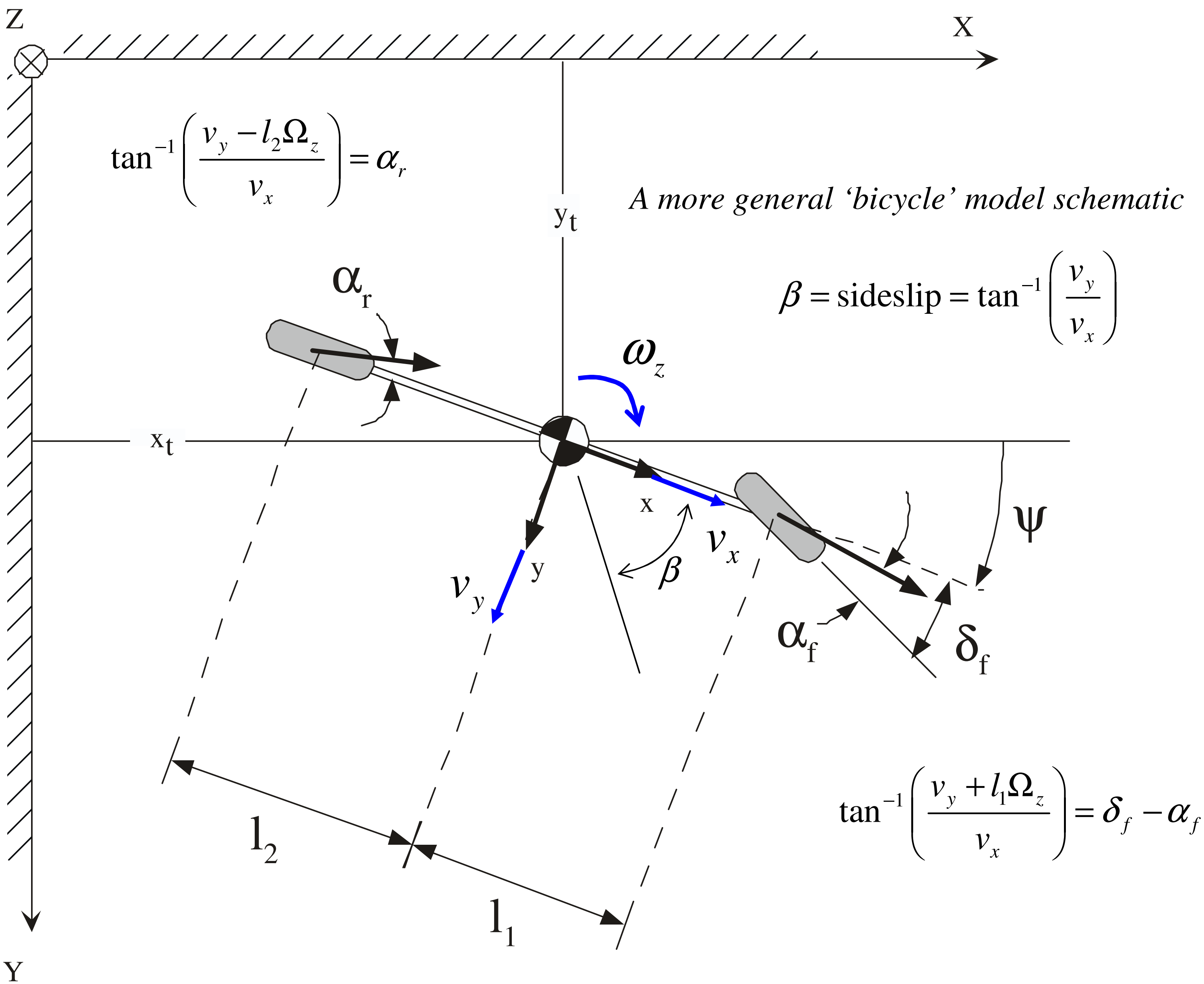
From Liljedahl, et al (1996)



$$-\alpha_f = \tan^{-1}\left(\frac{v_y + l_1\Omega_z}{v_x}\right) - \delta_f$$

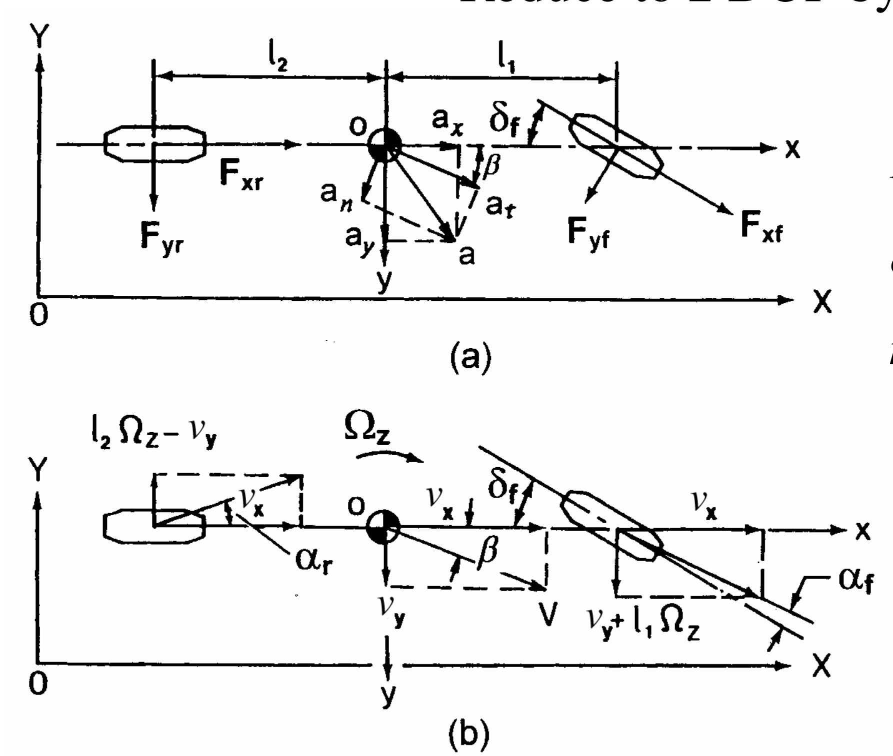
$$-\alpha_r = \tan^{-1}\left(\frac{v_y - l_2\Omega_z}{v_x}\right) - \underbrace{\delta_r}_{\text{rear steer}}$$

A more general 'bicycle' model schematic



Reduced bicycle model

Reduce to 2 DOF by taking V_x = constant = V



$$v_x = V$$

$$\delta_f = \delta = \text{small steering angle}$$

$$m\left(\cancel{\dot{v}}_x - V_y\Omega_z\right) = \underbrace{F_{xf}\cos(\delta_f)}_{\text{front drive}} + \underbrace{F_{xr}}_{\text{rear drive}} - \underbrace{F_{yf}\sin(\delta_f)}_{\text{lateral force effect}}$$

$$m\dot{v}_y = F_{yr} + F_{yf} + \underbrace{F_{xf}\delta_f}_{\sim 0} - m v_x \Omega_z$$

$$I_z\dot{\Omega}_z = l_1 F_{yf} - l_2 F_{yr} + \underbrace{l_1 F_{xf}\delta_f}_{\sim 0}$$

Note how δ falls out, but it enters again in the lateral forces, since,

$$\alpha_f = \delta_f - \tan^{-1}\left(\frac{v_y + l_1\Omega_z}{v_x}\right) \approx \delta_f - \left(\frac{v_y + l_1\Omega_z}{v_x}\right) \qquad \alpha_r = \tan^{-1}\left(\frac{l_2\Omega_z - v_y}{v_x}\right) \approx \left(\frac{l_2\Omega_z - v_y}{v_x}\right)$$

Vehicle trajectory calculations

Recall the models solve for forward and lateral velocity and yaw velocity, resulting from input steer angles, δ, relative to the body-fixed axes.

To find the trajectory of the CG in the Earth-based coordinates, we must use the transformation equations, which requires we add the following equations for basic 2-D trajectory simulations.

$$\dot{X} = v_x \cos(\psi) - v_y \sin(\psi)$$

$$\dot{Y} = v_x \sin(\psi) + v_y \cos(\psi)$$

$$\dot{\psi} = \Omega_z$$

Case Study with Baseline Model

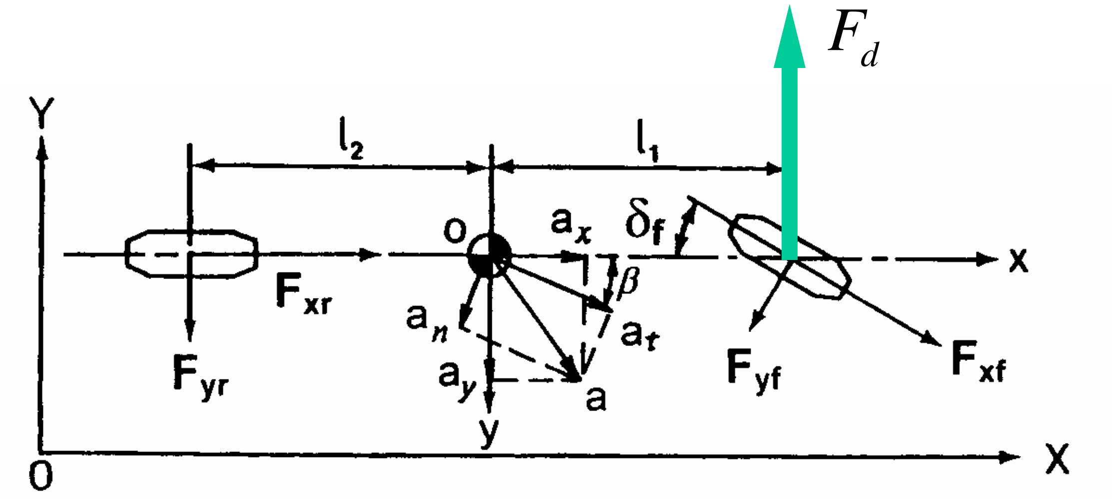
- We examine directional stability, reviewing Steeds use of Rocard's (linearized) model (see class handout), which is essentially the basic bicycle model with steer angle, $\delta = 0$.
- Determine if the vehicle described by the baseline parameter data given (next slide) is stable, and compare with results from a simulation of the bicycle model subjected to a force 'perturbation' at the front wheel applied in the +y direction.
- Use the Matlab code (available online) as a starting point (making changes/corrections as needed) or build a Simulink or LabVIEW simulation model.
- Additional 'car specifications' data is provided as Appendix B.

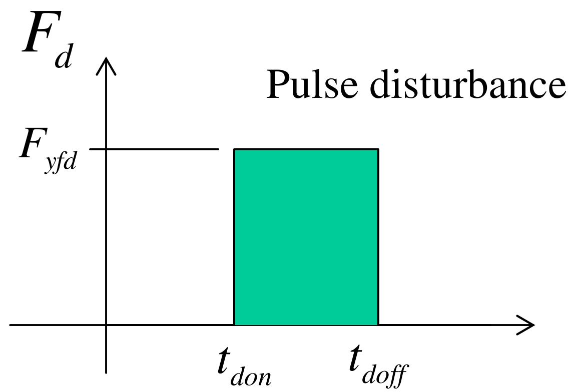
Model Simulation Data

(In Matlab script form)

```
g = 9.81;                 % gravitational acceleration, m/s^2

L = 3.075;                % wheelbase, m
a = 1.568;                % G behind front axle
b = L-L1;                 % rear axle

m = 1945;                 % total mass, kg
W = m*g;                  % weight, N

iyaw = 0.992;             % yaw dynamic index
% the following is a defined relation between yaw dynamic index and Iz
Iz = iyaw*m*a*b;          % moment of inertia about z, kg-m^2

Wf = b*W/L;               % static weight on front axle
Wr = a*W/L;               % static weight on rear axle
% Refer to Wong, Section 1.4 for guide to the following parameters
CCf = 0.171*180/pi;       % front corning stiffness coefficient, /rad
CCr = 0.181*180/pi;       % rear corning stiffness coefficient, /rad

Cf = CCf*Wf/2;            % corning stiffness per tire, N/rad (front)
Cr = CCr*Wr/2;            % rear cornering stiffness per tire, N/rad (rear)
```

Disturbance Force Application






```
if (t>=tdon & t<=tdoff)
   Fd = Fyfd;
else
   Fd = 0;
end
```

Add disturbance as a lateral force and related moment.

```
Vy_dot = (-m*Vx*omegaz + Fyr + Fyf*cos(deltaf) + Fxf*sin(deltaf) + Fd)/m;

omegaz_dot = (L1*Fyf*cos(deltaf) - L2*Fyr + L1*Fxf*sin(deltaf) + Fd*L1)/Iz;
```

For Nominal Case, Vehicle Stabilizes

Given case:

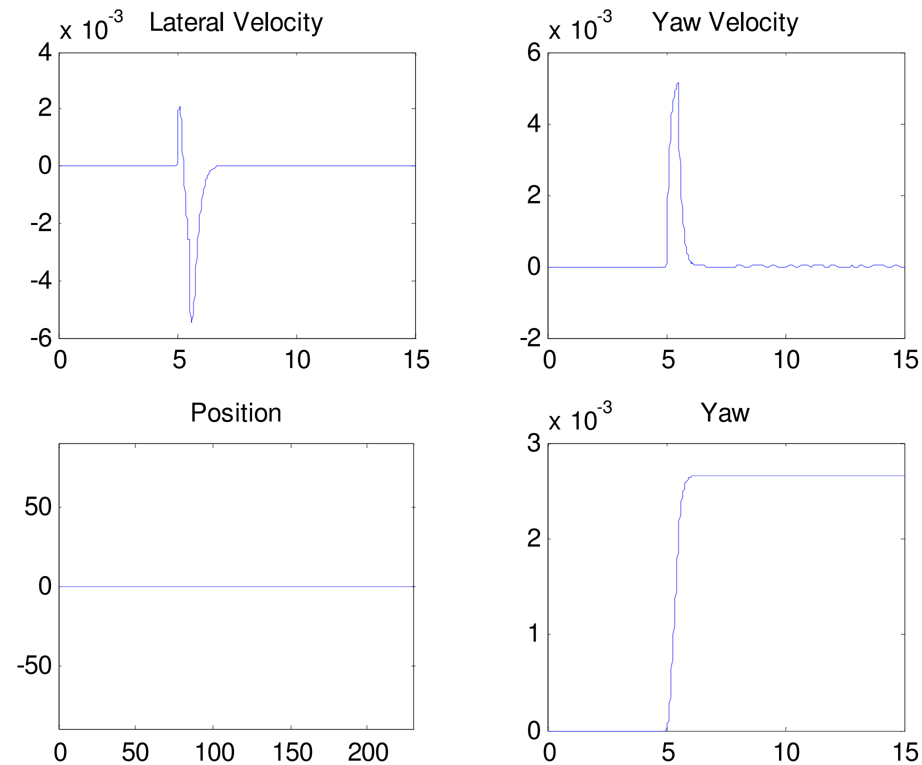
$R_{factor} = 4200.4$

V_c = non-existent

$v_x = V = 5L/\text{sec}$

A lateral pulse disturbance is applied at front wheel.

Afterwards, the vehicle stabilizes (lateral and yaw velocities go to zero).



Used: simple_bicycle.m

Change Lateral Stiffness

In this case, the rear lateral stiffness was cut in *half*.

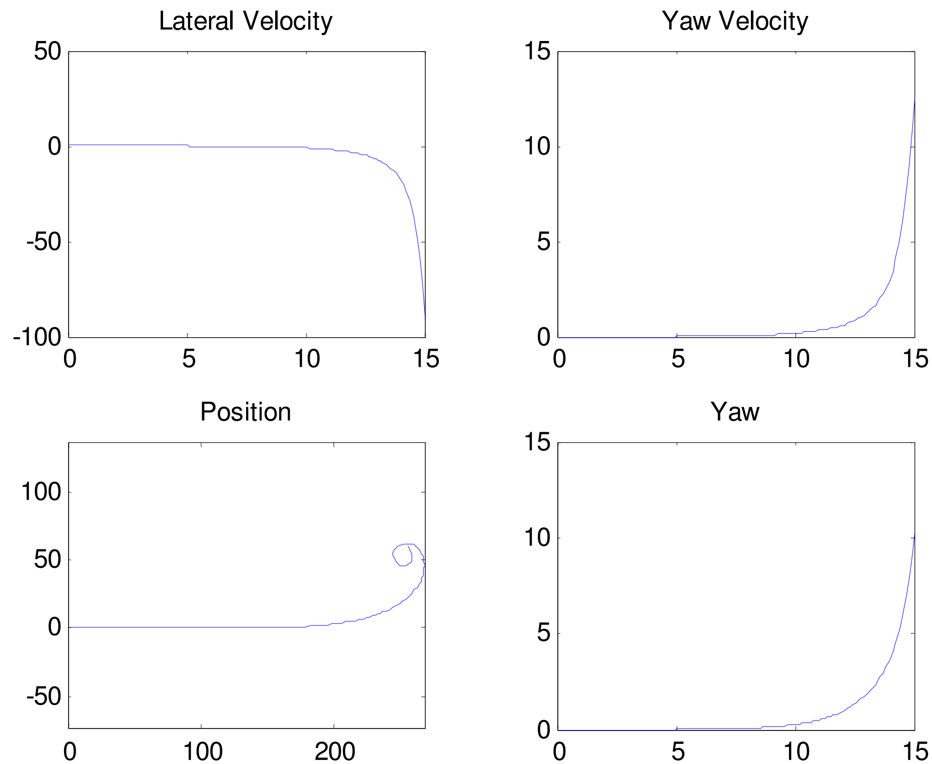
$R_{factor} = -33814.$

$V_c = 18.2282$

Let:

$V = 1.2 * V_c = 21.8$

After the pulse disturbance, the vehicle does not stabilize.

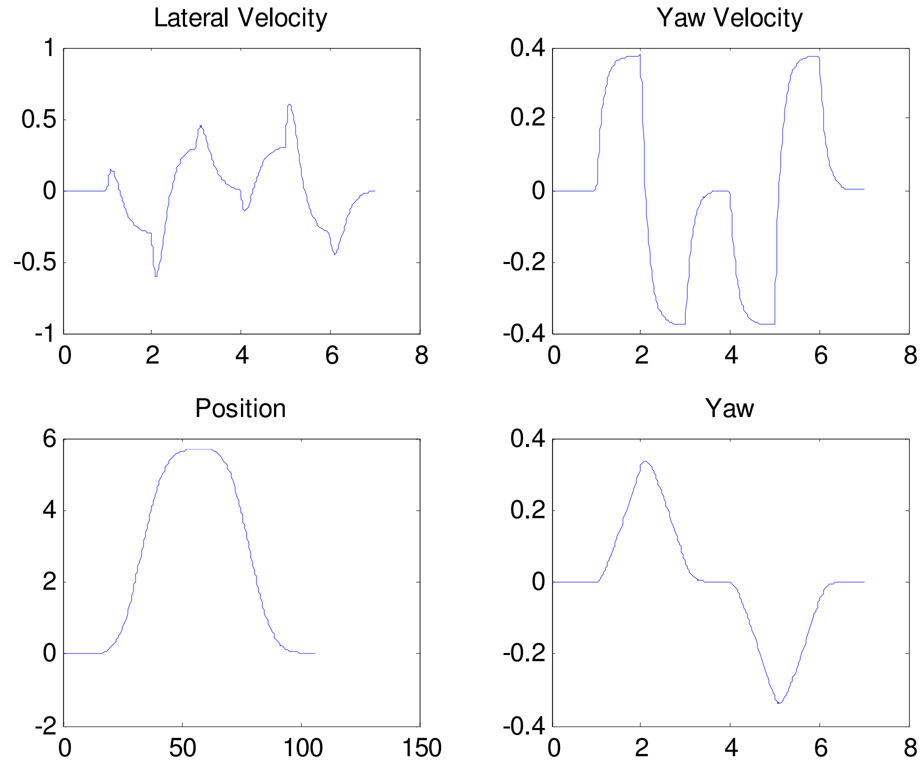


Used: simple_bicycle.m

Example of a Double Lane Change

Using the ***stable*** case for the 'simple_bicycle' model, an 'open loop' lane change is achieved by:

```
% double lane change
if (t<1) deltaf = 0; end;
if (t>=1 & t<2) deltaf = steer_angle; end;
if (t>=2 & t<3) deltaf = -steer_angle; end;
if (t>=3 & t<4) deltaf = 0; end;
if (t>=4 & t<5) deltaf = -steer_angle; end;
if (t>=5 & t<6) deltaf = steer_angle; end;
if (t>=6) deltaf = 0; end;
```

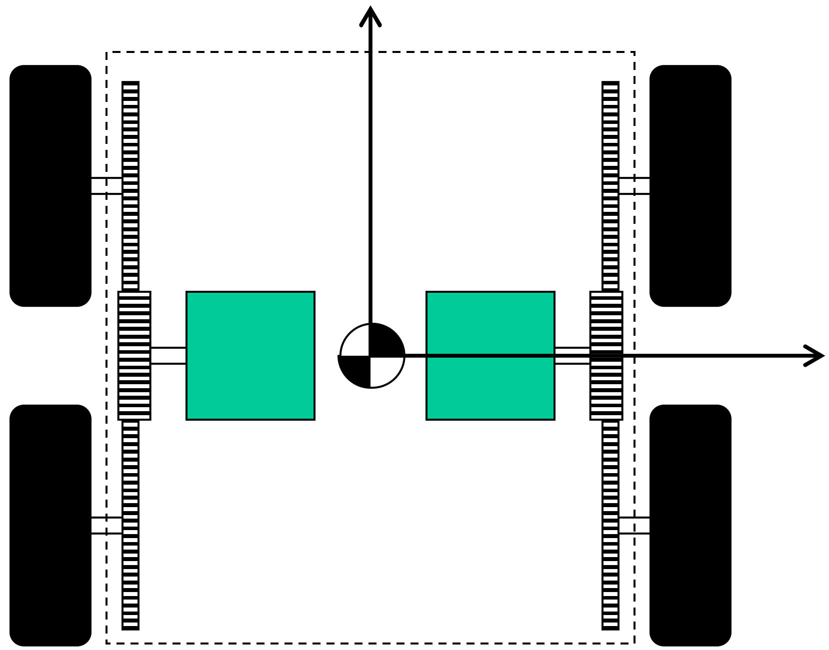


Used: simple_bicycle.m

Bicycle Model Summary

- The classical bicycle (or single-track) model forms the basis for understanding steering and steering control.
- Lateral forces induced by tires depend on wheel slip angle and play a key role in lateral stability. See also the Rocard handout.
- We saw earlier that longitudinal traction (driving or braking) influences lateral forces, so driven bicycle models (where there are traction/braking forces) require us to model the coupling between longitudinal and lateral tire forces (friction ellipse).
- In extreme maneuvers, for example, it would be possible to predict yaw instability if lateral forces were significantly reduced.

DaNI: 4-wheeled, differentially driven



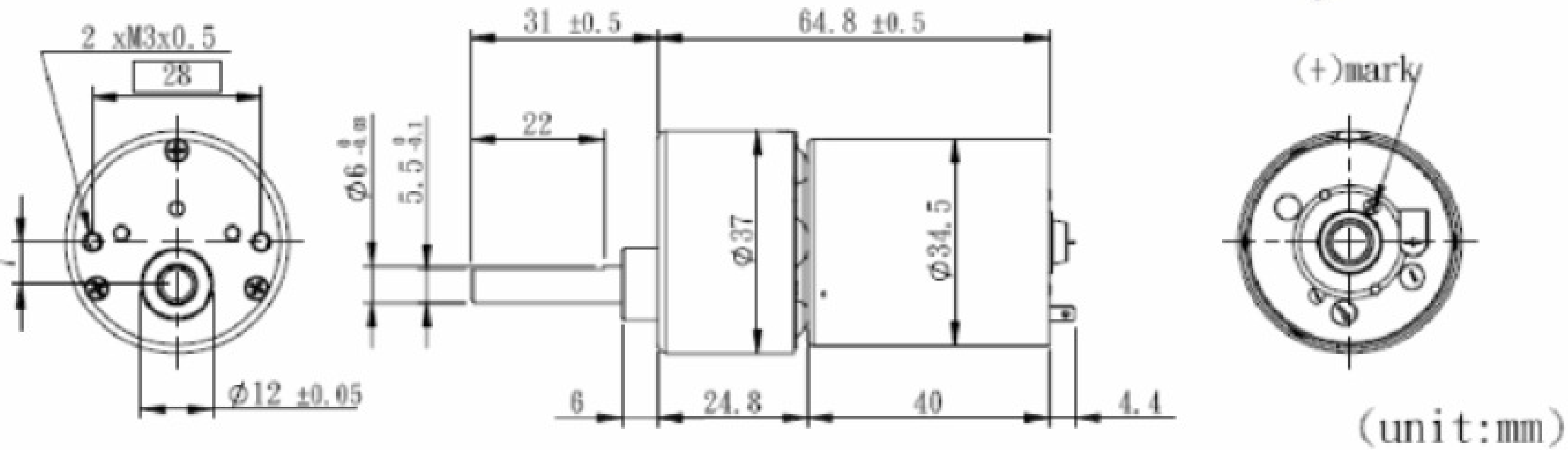
- Motor: Tetrix DC (handout)
- Gear Ratio: 2:1
- Shaft Diameter: 4.73 mm
- Wheel Diameter: 100 mm
- Wheel base: 133 mm.

Under what conditions would you need to use a dynamic model?

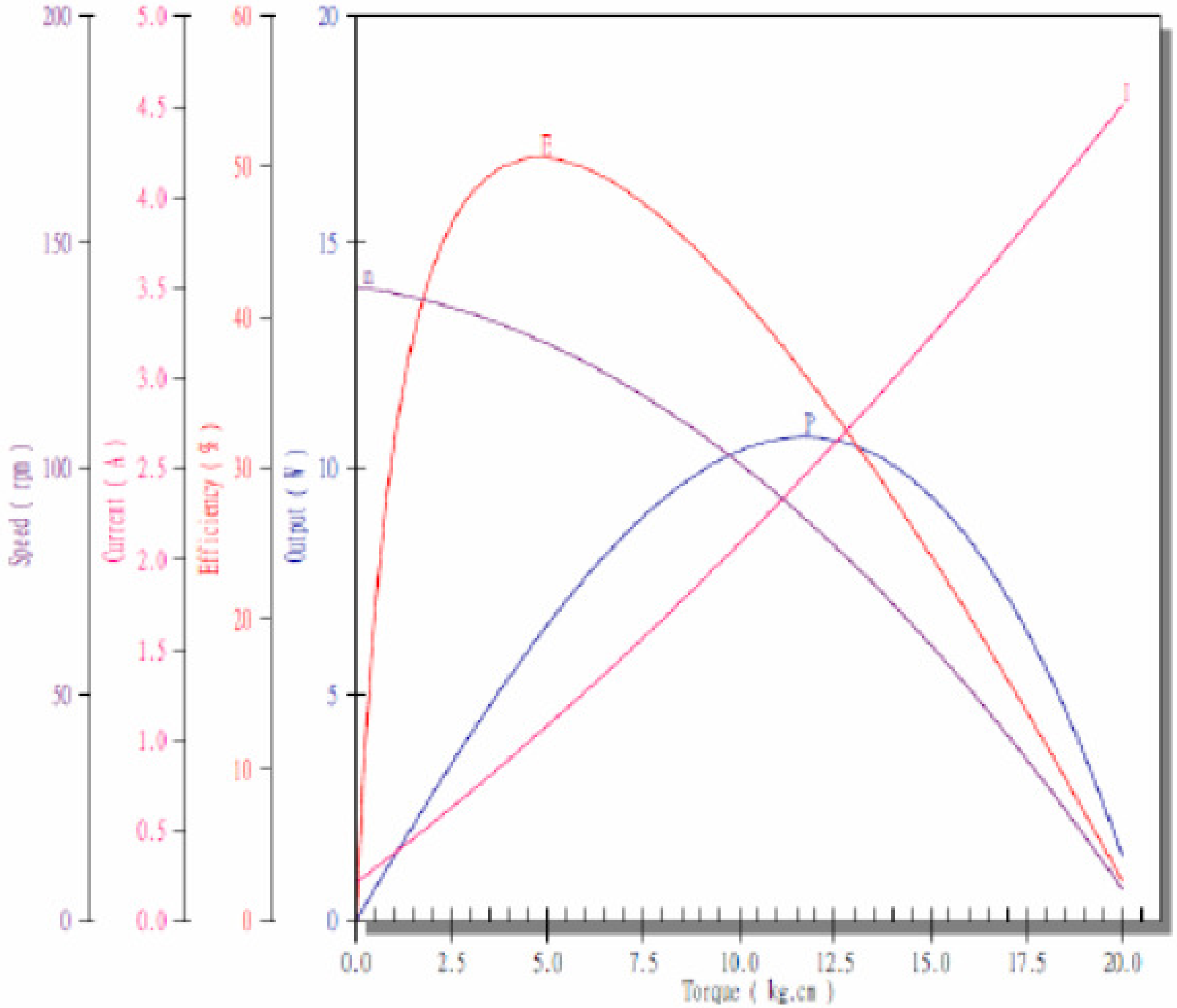
GENERAL SPECIFICATION FOR TETRIX DC DRIVE MOTOR

Test Voltage	: 12 Volts DC
No Load Speed	: 146 RPM (nominal) can deviate +/- due to manufacturing tolerances
No Load Current	: 0.17 Amps
Gear Ratio	: 1:52

(A) Dimensional Figures:



(B) MOTOR PERFORMANCE CURVES AND CHARACTERISTICS:



Supply Voltage = 12 VDC

AT NO LOAD

SPEED	=	146	RPM
CURRENT	=	0.17	AMP

AT STALL EXTRAPOLATION

TORQUE	=	20	kg-cm
CURRENT	=	4.55	AMP

AT MAXIMUM EFFICIENCY

EFFICIENCY	=	50	%
SPEED	=	128	RPM
TORQUE	=	4.85	kg-cm
CURRENT	=	1.05	AMP
OUTPUT	=	6.38	WATTS

AT MAXIMUM OUTPUT

SPEED	=	89	RPM
TORQUE	=	11.72	kg-cm
CURRENT	=	2.46	AMP
OUTPUT	=	10.7	WATTS
EFFICIENCY	=	36.22	%

Torque Conversions:
1.0 kgf-cm = 0.098 Nm = 98 mNm
= 13.887 oz-in = 0.867 lb-in

References

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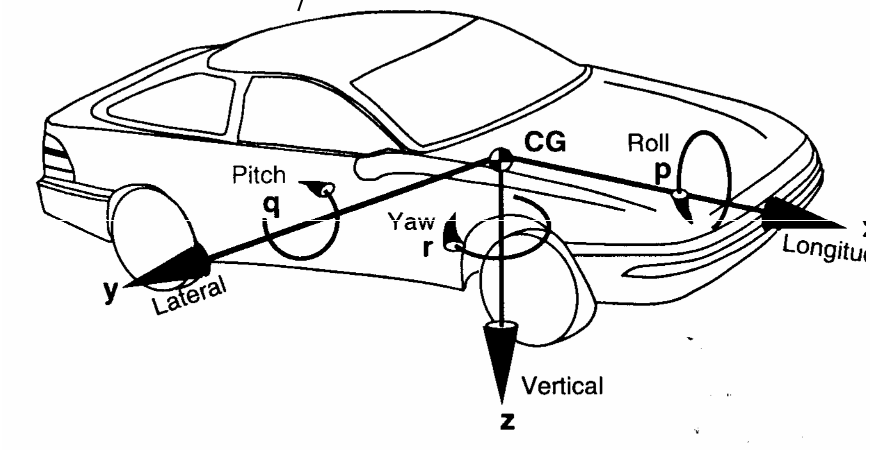
Appendix A

Coordinate Transformations for Basic Vehicle Trajectory Calculations

Coordinate Transformations – 1

Cardan angles:

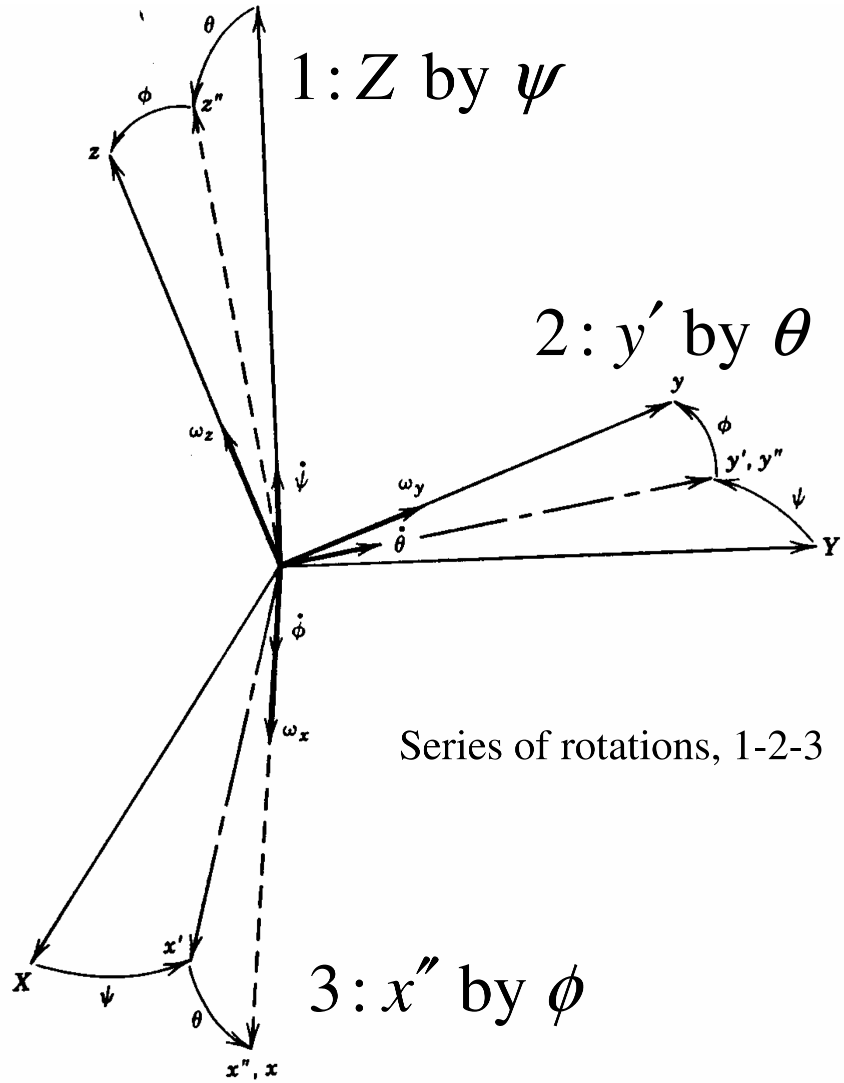
Yaw - ψ
Pitch - θ
Roll - ϕ



Body-fixed angular velocities:

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^{\dagger}$$

(also see the bond graph ‘gyrator ring’ model)



Transformations depend on angles

Need the 'Cardan angles' to determine these transformations, so you introduce additional* states, ϕ(roll), θ (pitch), and ψ (yaw).

Relate the rates of change of these angles to body-fixed angular velocities, $\omega_x, \omega_y, \omega_z$.

From the axes on the previous slide, show

$$\omega_x = \dot{\phi} - \dot{\psi}\sin\theta$$

$$\omega_y = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi$$

$$\omega_z = -\dot{\theta}\sin\phi + \dot{\psi}\cos\theta\sin\phi$$

Solve for $\dot{\theta}$ and $\dot{\psi}$

$$\dot{\theta} = \cos\phi \cdot \omega_y - \sin\phi \cdot \omega_z$$

$$\dot{\psi} = \frac{\sin\phi}{\cos\theta} \cdot \omega_y + \frac{\cos\phi}{\sin\theta}\omega_z$$

so,

$$\dot{\phi} = \omega_x + \sin\phi\frac{\sin\theta}{\cos\theta} \cdot \omega_y + \cos\phi\frac{\sin\theta}{\cos\theta}\omega_z$$

In a simulation, these three equations must be integrated to find the three Cardan angles.

For planar 2D:

$\phi = 0$ (no roll), $\omega_x = 0$

$\theta = 0$ (no pitch), $\omega_y = 0$

$\dot{\theta} = 1 \cdot 0 - 0 \cdot \omega_z = 0$

$\dot{\psi} = 0 \cdot 0 + 1 \cdot \omega_z = \omega_z$

$\dot{\phi} = 0 + 0 + 0 \cdot \omega_z = 0$

Global Trajectory Calculation

When the state equations a 2D vehicle model are in the form of body-fixed forward and lateral velocity and yaw velocity, the trajectory of the CG can be found in the Earth-based coordinates by using the transformation relations.

The 2-D trajectory would be found by integrating,

$$\dot{x}_t = v_x \cos(\psi) - v_y \sin(\psi)$$

$$\dot{y}_t = v_x \sin(\psi) + v_y \cos(\psi)$$

$$\dot{\psi} = \omega_z$$

APPENDIX B

Example Car Specifications*

Table C.1 Example Car Specifications

Parameter		Symbol	Units	A	B	C	D	E	F	G	H
1 Vehicle and type				Sedan	Sedan	Coupe	Sedan	Sedan	Sports	Racing Formula One	Racing C1 Sports Prototype
2 Wheelbase		l	m	2.040	2.398	2.580	2.770	3.075	2.500	2.718	2.910
3 Front track	f	T_f	m	1.206	1.362	1.440	1.480	1.560	1.540	1.804	1.600
4 Rear track	r	T_r	m	1.164	1.324	1.440	1.440	1.540	1.540	1.626	1.524
5 Total mass (part load)		m	kg	727	1045	1175	1435	1945	1600	625	1020
6 G behind front axle		a	m	0.775	1.031	1.161	1.302	1.568	1.250	1.631	1.717
7 G height		H	m	0.550	0.610	0.620	0.635	0.672	0.500	0.280	0.350
8 Yaw dynamic index		i_Y	-	0.860	1.060	0.980	1.030	0.992	0.920	0.650	0.700
Suspension											
1 Wheel rate (no ARB)	f	k_{Wf}	kN/m	16.00	12.40	12.80	12.95	13.40	20.00	300	230
(per wheel)	r	k_{Wr}	kN/m	10.00	10.60	13.80	14.72	17.00	20.00	200	400
2 Roll stiffness of ARB	f	k_{Bf}	Nm/deg	0	146	248	210	205	1000	10000	4000
	r	k_{Br}	Nm/deg	0	0	0	73	0	250	2000	1000

*Table C.1 from J.C. Dixon, "Tires, Suspension and Handling" (2nd ed.), SAE, Warrendale, PA, 1996.

APPENDIX C

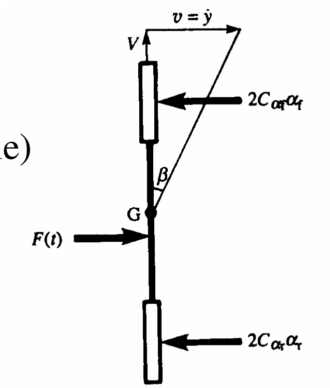
2D ‘handling’ models with slip – 1

1 DOF, constant forward velocity, lateral slip

1 DOF
pure sideslip
(no yaw)

$$\beta = \frac{\dot{y}}{V}$$

(small angle)



$$\dot{p}_y = m\dot{v}_y = m\ddot{y}$$
$$= \sum F_y = -C_{\alpha f}\alpha_f - C_{\alpha r}\alpha_r$$

2[nd] order system in y

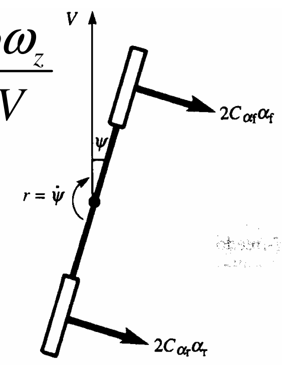
$$C_0 = C_{\alpha f} + C_{\alpha r}$$

zeroth moment cornering stiffness

1 DOF
pure yaw
(pin at CG)

$$\alpha_f = \psi - \frac{a\omega_z}{V}$$

$$\alpha_r = \psi + \frac{b\omega_z}{V}$$



$$\dot{h}_z = I_z\ddot{\psi}$$
$$= \sum T_z = aC_{\alpha f}\alpha_f - bC_{\alpha r}\alpha_r$$

2[nd] order system in yaw

$$C_1 = aC_{\alpha f} - bC_{\alpha r}$$

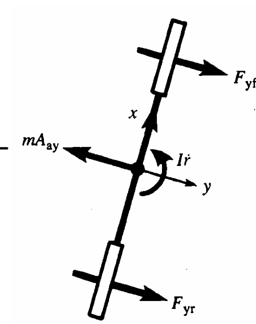
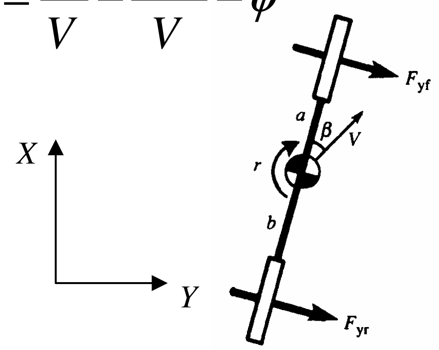
yaw stiffness

$$\frac{C_2}{V} = \frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{V}$$

yaw damping

2D ‘handling’ models with slip – 2

2 DOF – no steering angle, fixed *V*, lateral slip

2 DOF sideslip and yaw – vehicle fixed axes	$\beta = \frac{v_y}{V}$ $\alpha_f = \beta + \frac{a\omega_z}{V}$ $\alpha_r = \beta - \frac{b\omega_z}{V}$ 	$\dot{p}_y = m\dot{v}_y$ $= \sum F_y = C_{\alpha f}\alpha_f + C_{\alpha r}\alpha_r$ $\dot{h}_z = I_z\dot{\omega}_z = I_z\dot{r}$ $= \sum T_z = aC_{\alpha f}\alpha_f - bC_{\alpha r}\alpha_r$ Coupling occurs through the slip angle definitions	Terms will appear with: $C_0 = C_{\alpha f} + C_{\alpha r}$ $C_1 = aC_{\alpha f} - bC_{\alpha r}$ $\frac{C_2}{V} = \frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{V}$
2 DOF sideslip and yaw – earth fixed axes	$\alpha_f = \frac{v_Y}{V} + \frac{a\omega_z}{V} - \psi$ $\alpha_r = \frac{v_Y}{V} - \frac{b\omega_z}{V} - \psi$ 	Could use transformation, or re-write equations and add: $\psi =$ heading angle $\dot{\psi} = \omega_z$	

Appendix D:
More on Steering Mechanisms

Department of Mechanical Engineering
The University of Texas at Austin

General Steering System Requirements

- A steering system should be insensitive to disturbances from the ground/road while providing the driver/controller with essential ‘feedback’ as needed to maintain stability.
- The steering system should achieve the required turning geometry. For example, it may be required to satisfy the Ackerman condition.
- The vehicle should be responsive to steering corrections.
- The orientation of the steered wheels with respect to the vehicle should be maintained in a stable fashion. For example, passenger vehicles require that the steered wheels automatically return to a straight-ahead stable equilibrium position.
- It should be possible to achieve reasonable handling without excessive control input (e.g., a minimum of steering wheel turns from one locked position to the other).

(DRAFT)

Passenger Steering Requirements

- Driver should alter steering wheel angle to keep deviation from course low.
- Correlation between steering wheel and driving direction is not linear due to: a) turns of the steering wheel, b) steered wheel alterations, c) lateral tire loads, and d) alteration of driving direction.
- Driver must steer to account for compliance in steering system, chassis, etc., as well as need to change directions.
- Driver uses visual as well as 'haptic' feedback. For example, roll inclination of vehicle body, vibration, and feedback through the steering wheel (effect of self-centering torque on wheels).
- It is believed that the feedback from the steering torque coming back up through the steering system from the wheels is the most important information used by many drivers.

Impact of Steering Geometry

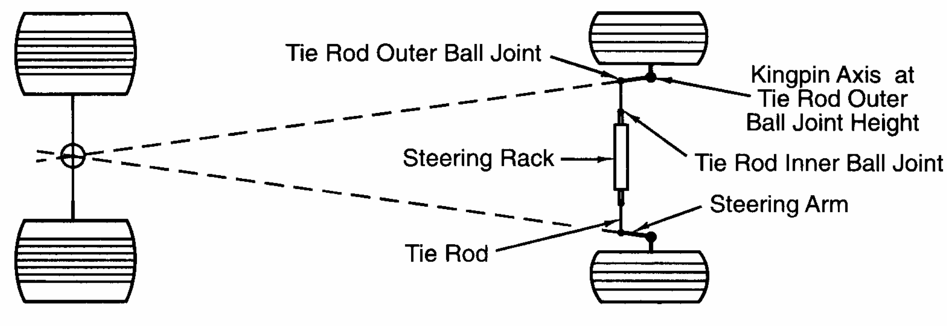
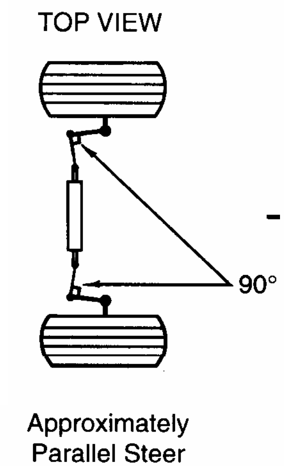
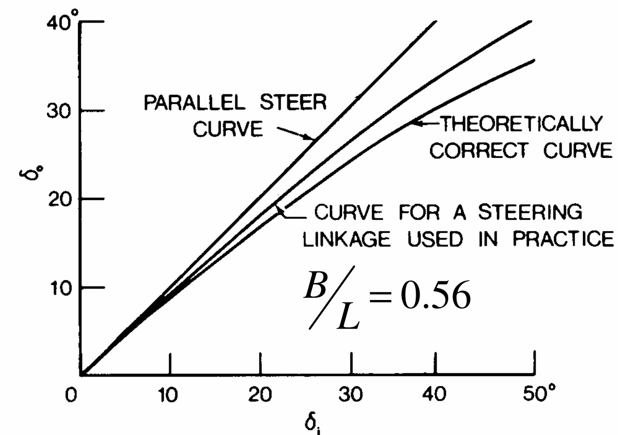
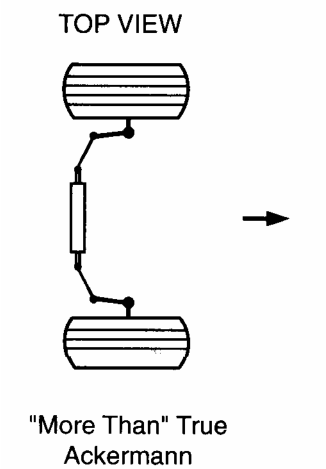


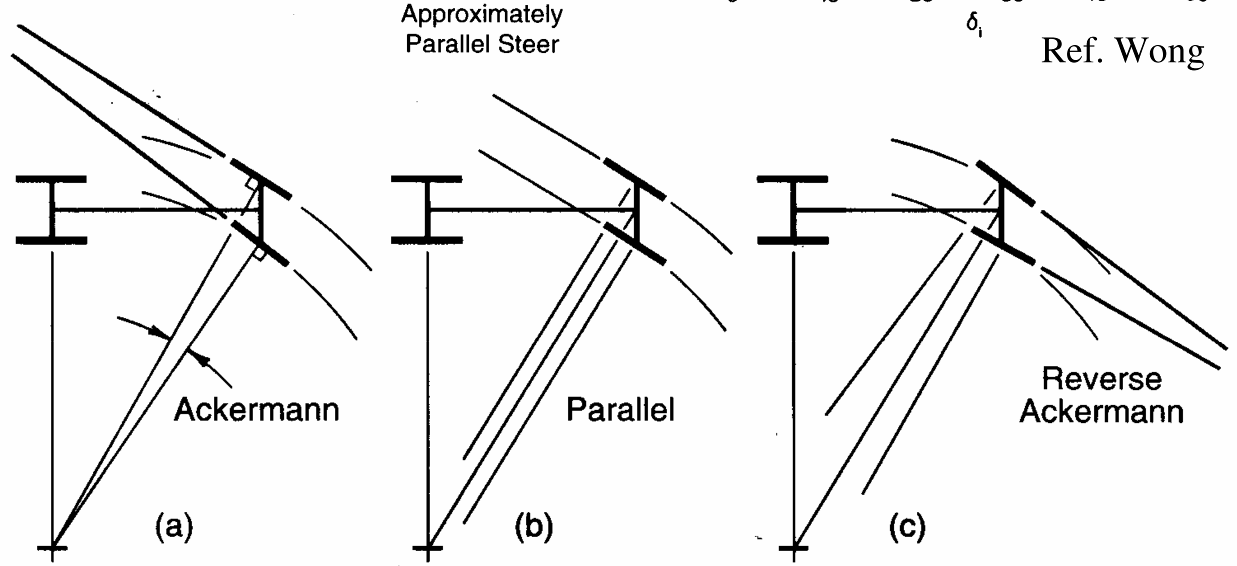
Figure 19.3 Ackermann geometry, with steering rack behind the axle line.





Ref. Wong

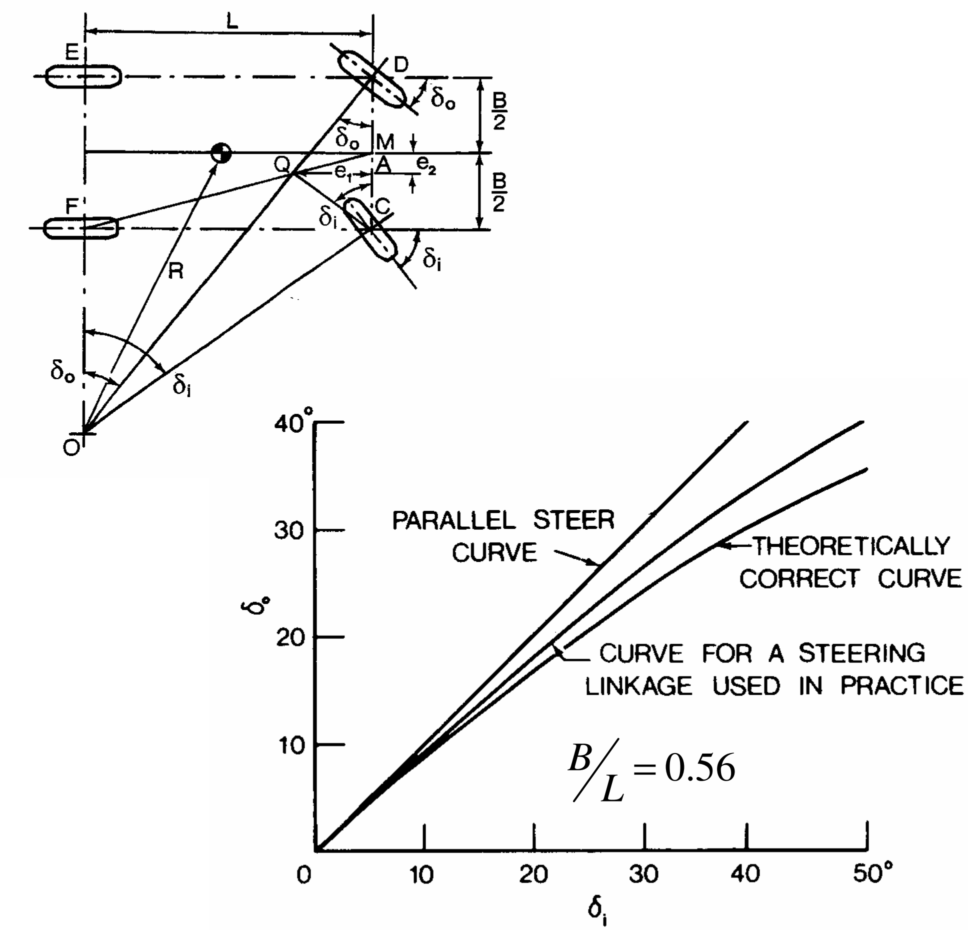




Ref. Milliken & Milliken

Figure 19.2 Ackermann steering geometry.

Steering Results and Error in Ackerman



Ref. Wong

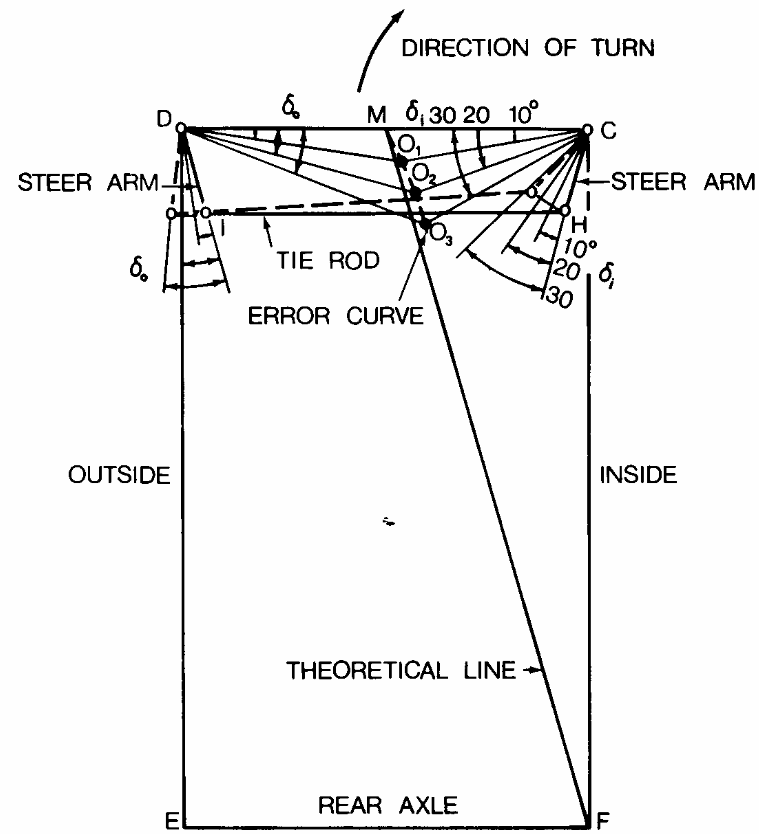


Fig. 5.4 Error curve of a steering linkage.

"Steering error curves" for a front beam axle

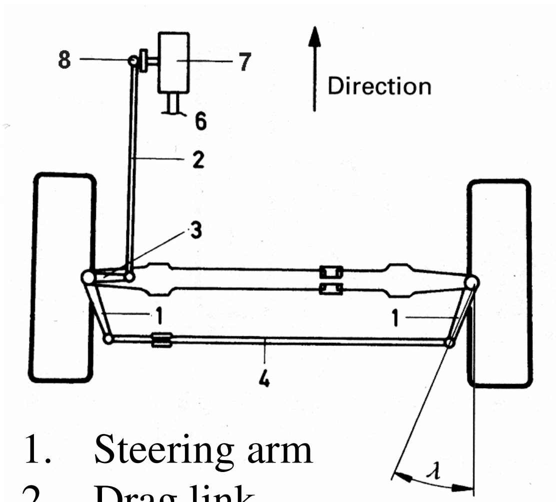
Impact on Steering Geometry
of using Ackermann in High Speed

A car with a steering geometry chosen for low-speed (Ackermann) will not be as effective at higher vehicle speeds.

In a high speed turn, the inside tire will have a lower normal force, which means it could achieve the same amount of lateral cornering force with a smaller slip angle.

Using Ackermann will result in more instances in which the inside tire is dragged along at *too high* a slip angle, unnecessarily raising the temperature and slowing the vehicle down with excessive slip induced drag.

Steering Systems – Rigid Axle

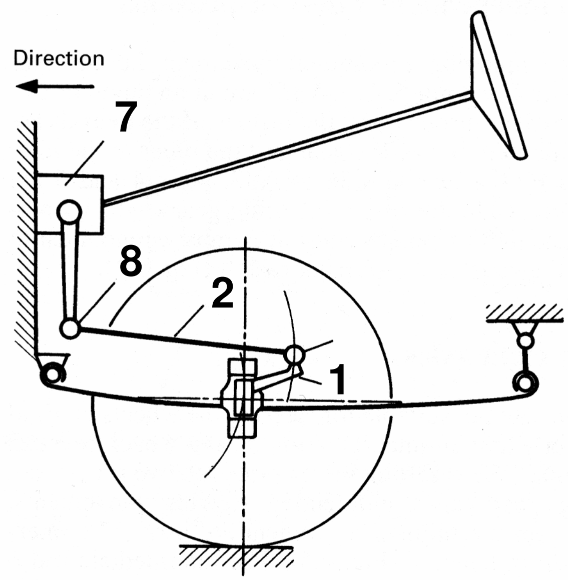


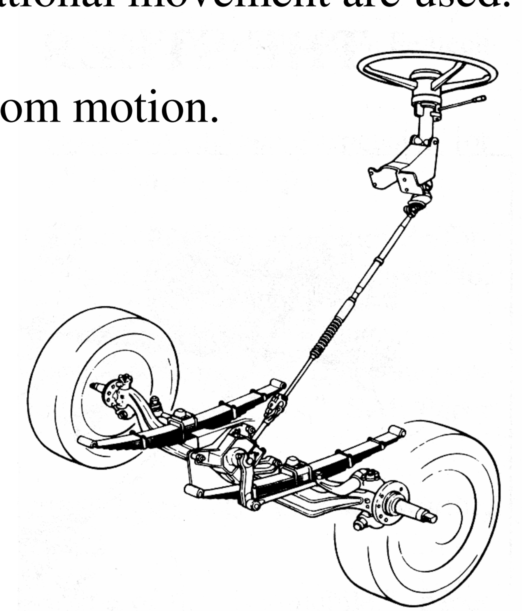
1. Steering arm
2. Drag link
3. Idler arm
4. Tie rod/rack
5. Steering wheel
6. Steering shaft
7. Steering box
8. Pitman arm

Rack and pinion is not suitable for steering wheels on rigid front axles, as the axles tend to move in the longitudinal direction. This movement between the wheels and steering system can induce unintended steering action.

Only steering gears (1) with rotational movement are used.

Design must minimize effects from motion.





You may see some older model Toyota Land Cruisers that use this type of steering design.

Axle-Beam Steering Linkage

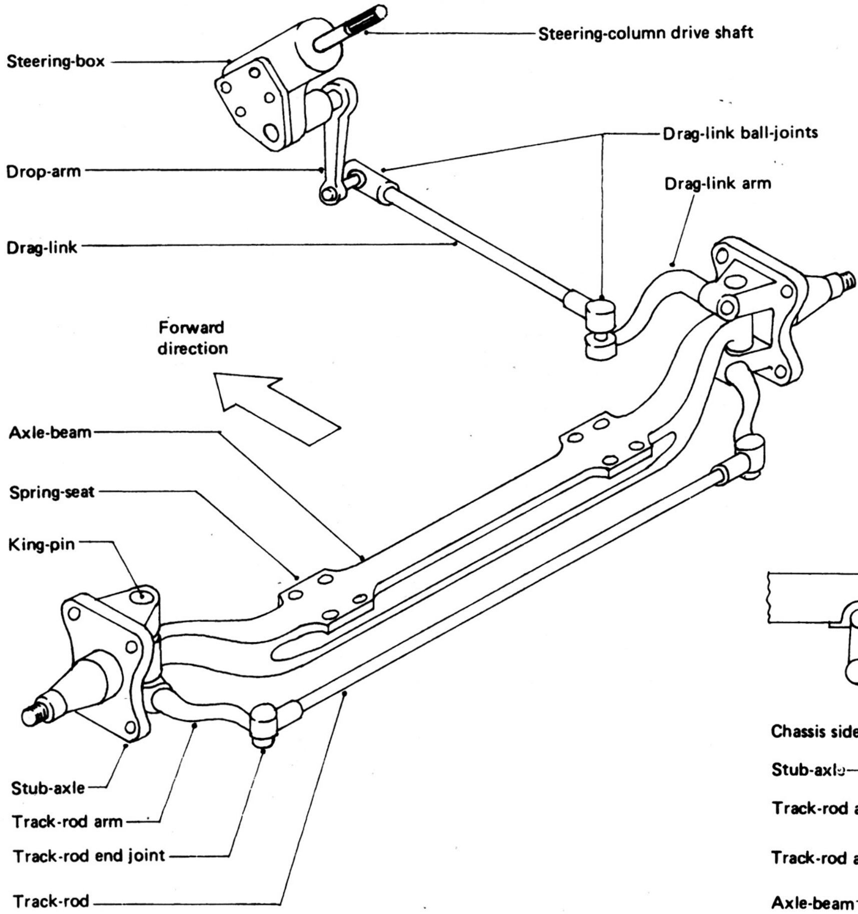
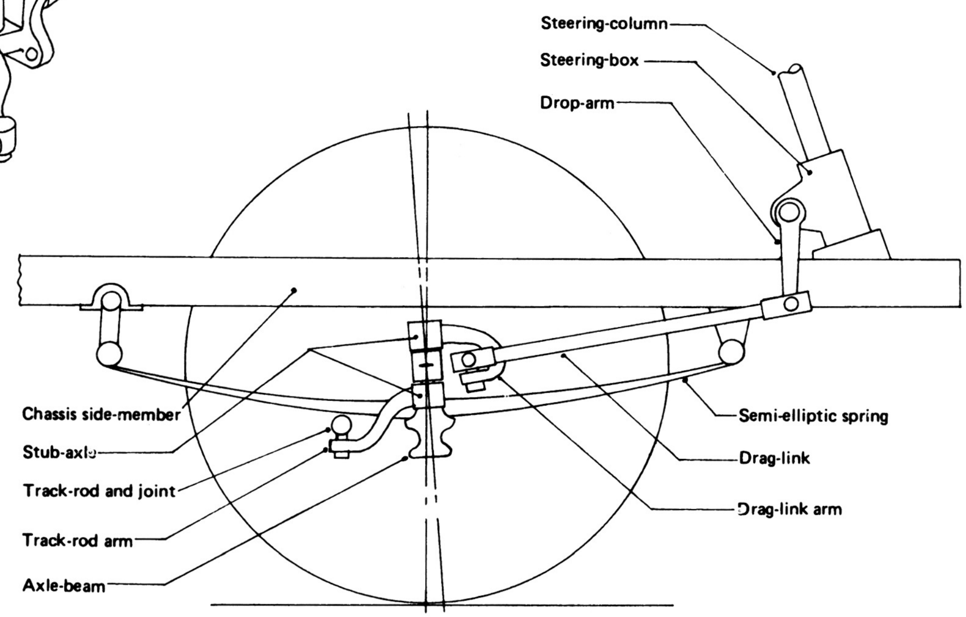


Fig. 8.1 Pictorial view of a typical axle-beam steering-linkage layout

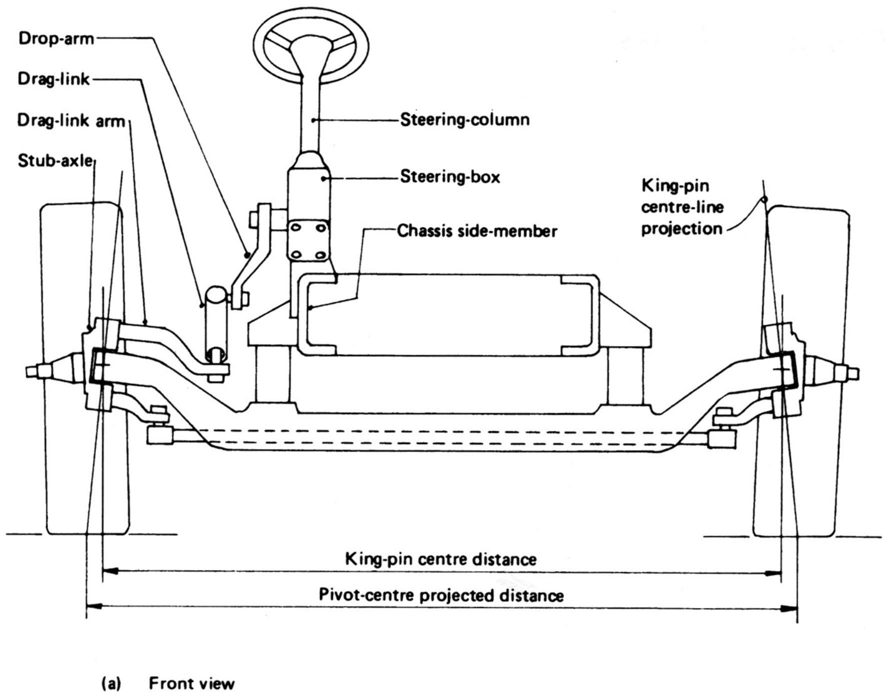
Heisler (1999)



(c) Side view

Fig. 8.2 (*continued*) Axle-beam steering linkage with longitudinal-located drag-link

Axle-Beam Steering Linkage



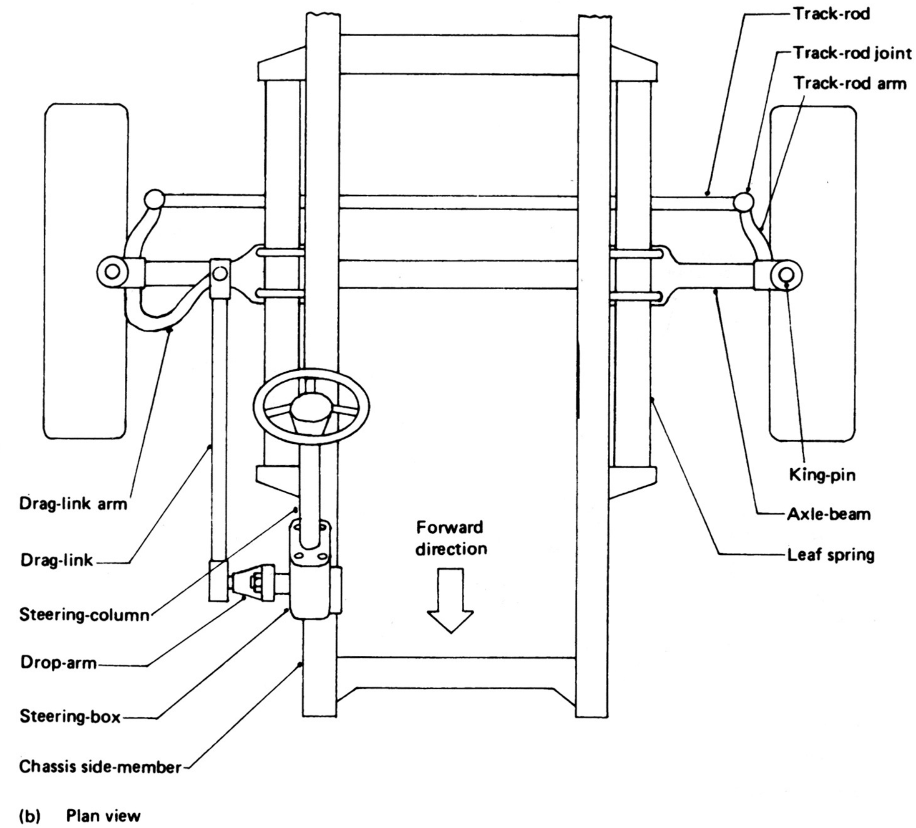


Fig. 8.2 Axle-beam steering linkage with longitudinal-located drag-link

Heisler (1999)

Front Axle and Tie Rod Assembly

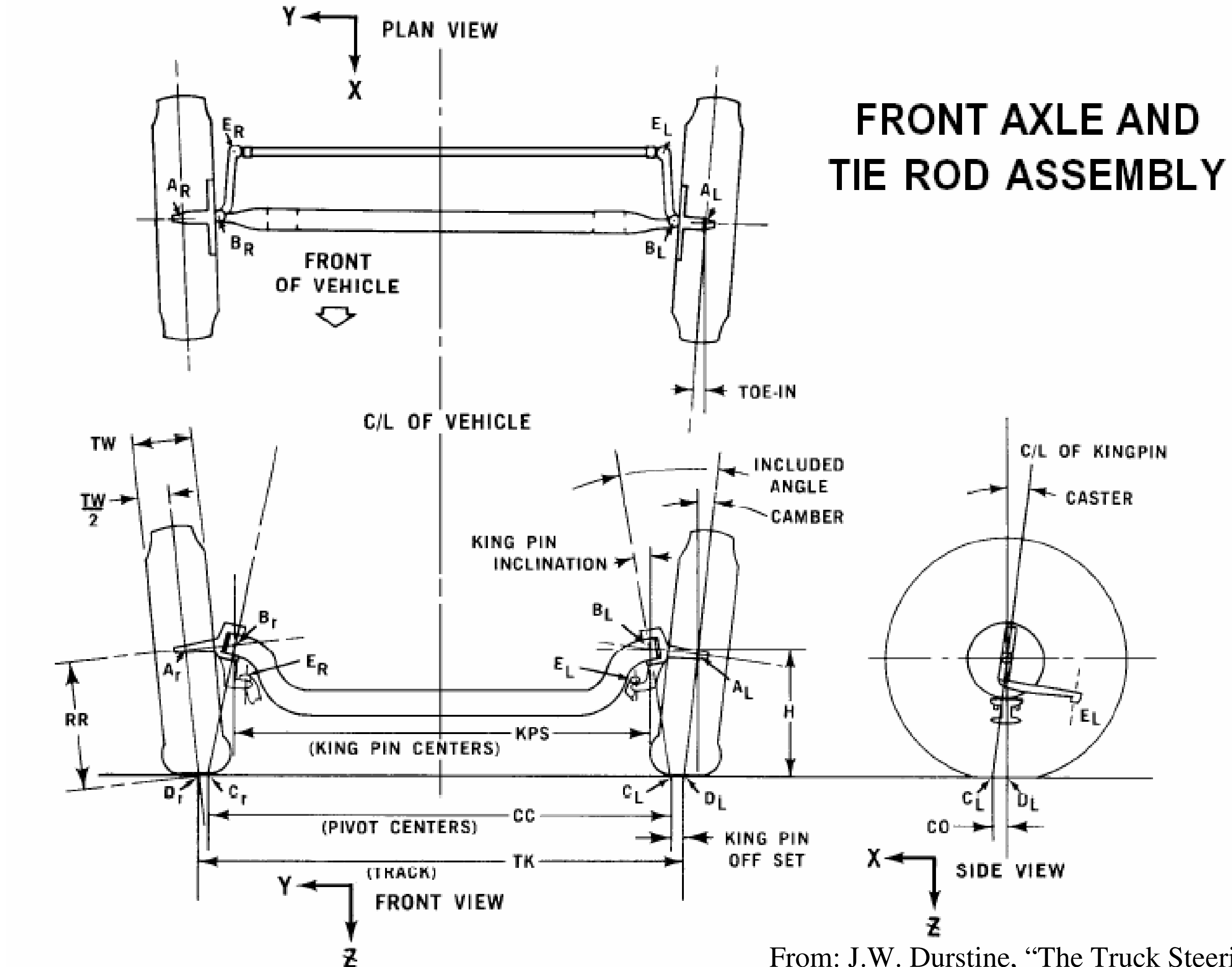


Figure 30

From: J.W. Durstine, "The Truck Steering System From Hand Wheel to Road Wheel", SAE, SP-374, 1973 (L. Ray Bukendale Lecture).

Steering Gearbox and Ratio

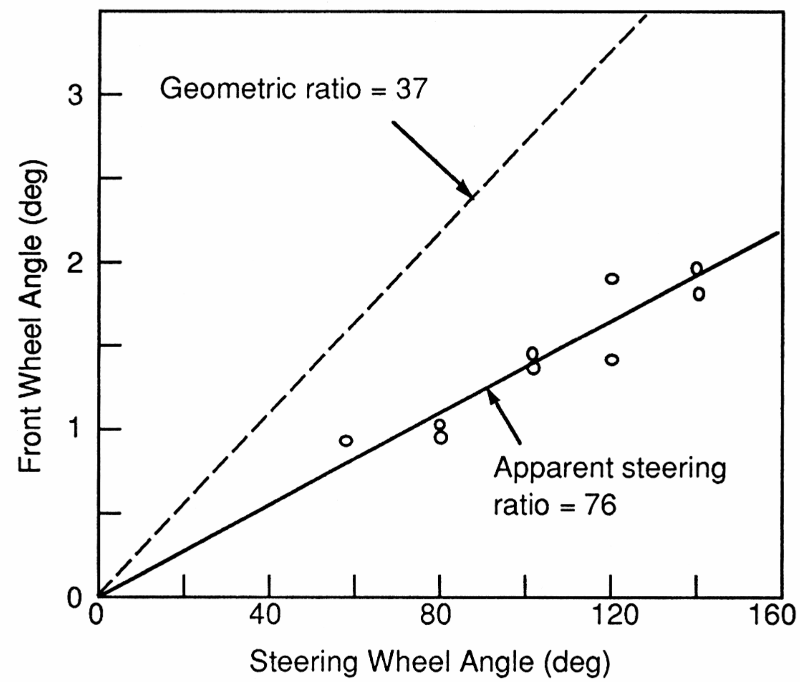
The gearbox provides the primary means for reducing the rotational input from the steering wheel and the steering axis.

Steering wheel to road wheel ratios may vary with angle, but have values of 15:1 in passenger cars and may go as high as 36:1 for heavy trucks.

Rack and pinions are commonly designed to have a variable gear ratio depending on steer angle.

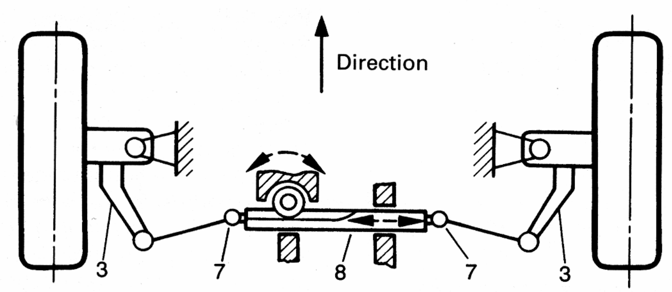
The actual steer ratio can be influenced by steering system effects, such as compliance. The plot here from Gillespie shows how much this can change.

Experimentally measured steering ratio on a truck.

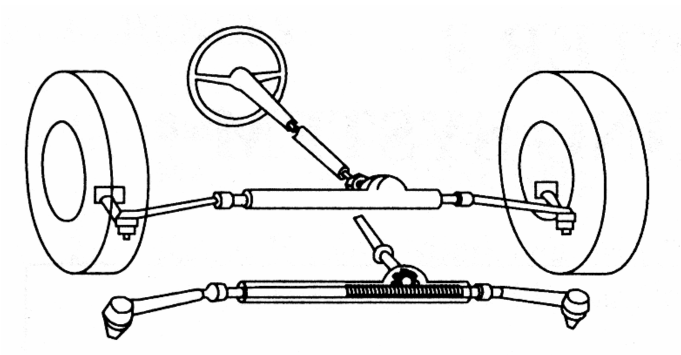


From Gillespie (Fig. 8.18).

Rack-and-Pinion – 1



3 Steering arms
7 Tie rod joints
8 Steering rack



Used on most passenger cars and some light trucks, as well as on some heavier and high speed vehicles.

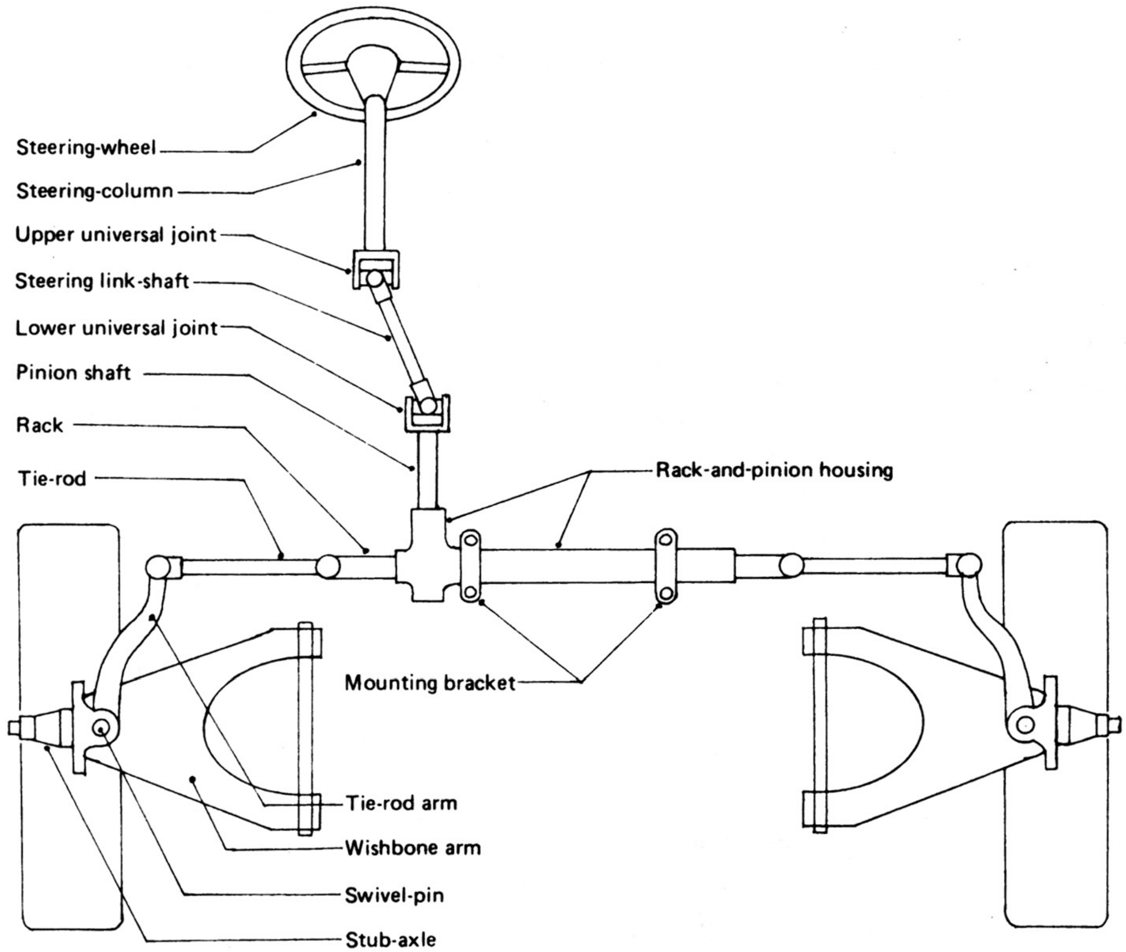
Used on vehicles with independent suspensions.

Some advantages: simple, manufacturing ease, efficient, minimal backlash, tie rods can be joined to rack, minimal compliance, compact, eliminate idler arm and intermediate rod

Some disadvantages: sensitive to impacts, greater stress in tie rod, since it is efficient you feel disturbances, size of steering angle depends on rack travel so you have short steering arms and higher forces throughout, cannot be used on rigid axles

Steering ratio is ratio of pinion revolutions to rack travel.

Rack-and-Pinion – 2



Heisler (1999)

Fig. 8.5 Rack-and-pinion steering-linkage layout

Split track-rod with relay-rod

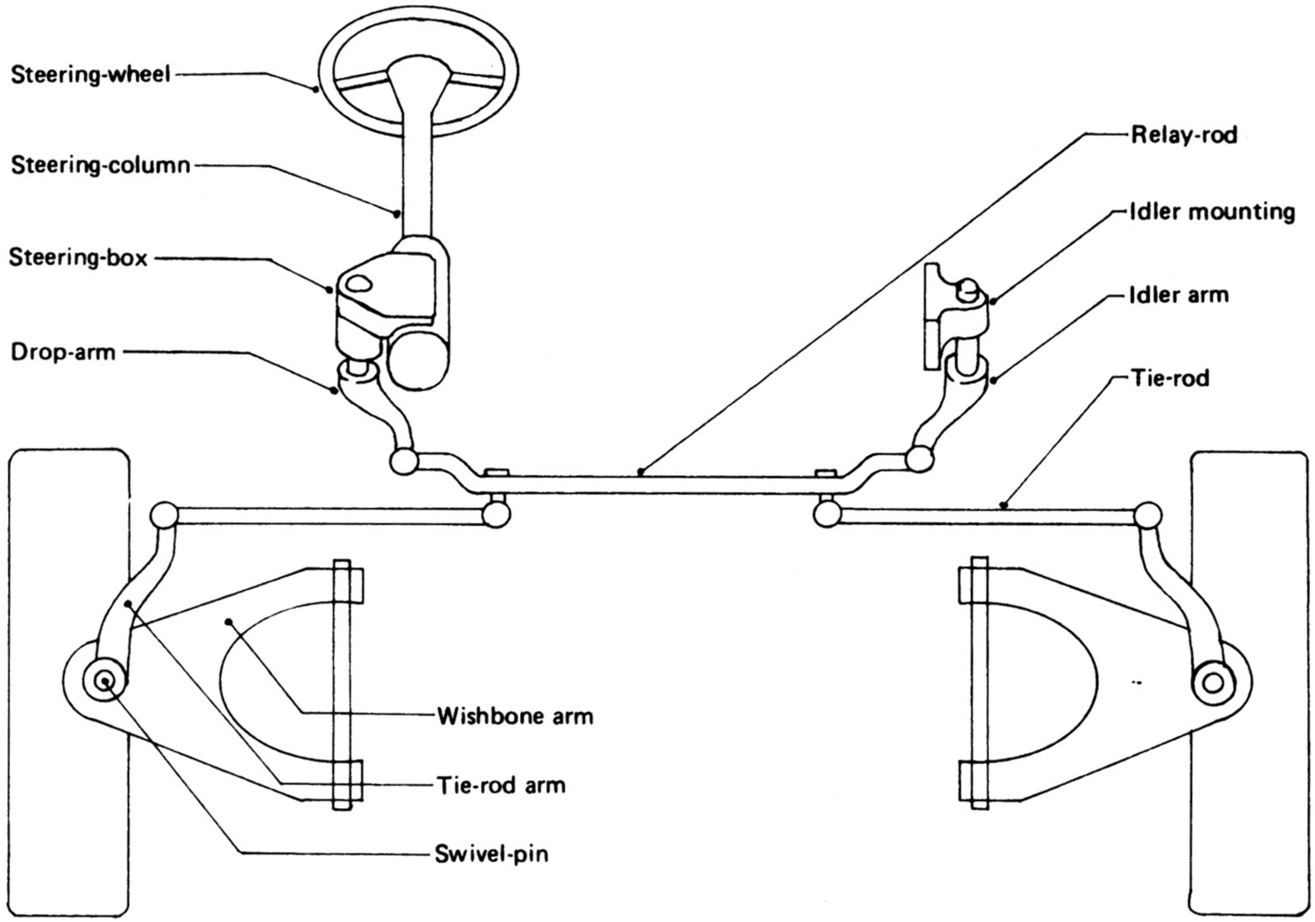


Fig. 8.4 Split track-rod with relay-rod and idler steering-linkage layout

Heisler (1999)

Summary

- Steering mechanisms were reviewed briefly
- Steering geometry – how do you select steering system?
- Steering control – how does it translate into steering mechanism design?
- Relationship to suspension? (Segel, Gillespie)

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