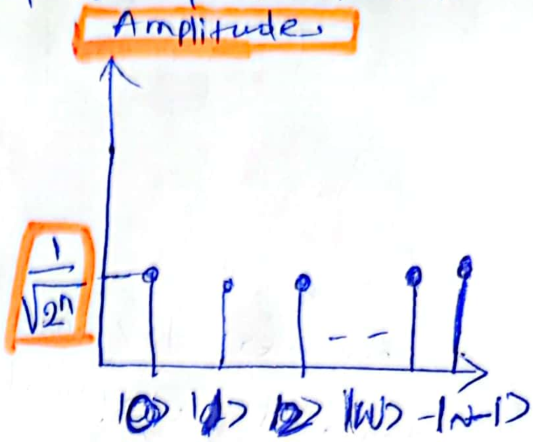
Grovers Algorithm.

Step 1:- which ever we want to find is present in $|s\rangle$ state



(superpositioned) states $\quad H^{\otimes n}|0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle = |s\rangle$

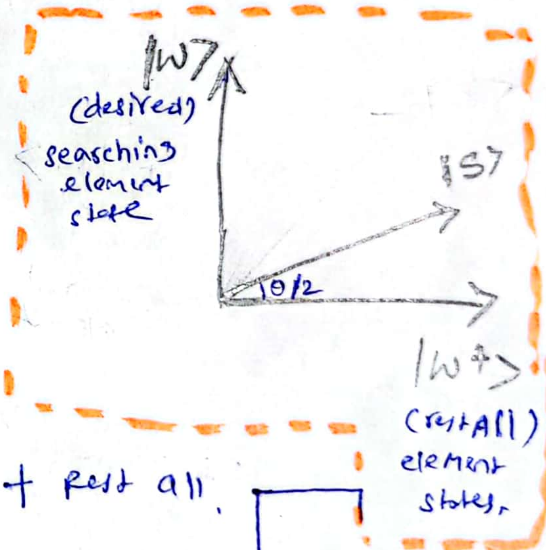
Σ, $|s\rangle$ $|w\rangle$

$\langle w^{\perp}|w\rangle = 0$

↓ represents transitional probability.

NOTE:- of $w^{\perp}$ & w are orthogonal thats why inner-product $= 0$

$$|w^{\perp}\rangle = \frac{\sqrt{2^n - 1}}{\sqrt{2^n}} \sum_{x \neq w} |x\rangle$$



$$|s\rangle = \frac{\sqrt{2^n - 1}}{\sqrt{2^n}} |w^{\perp}\rangle + \frac{1}{\sqrt{2^n}} |w\rangle$$

Here $|s\rangle$ consists of both desired search element state + rest all.

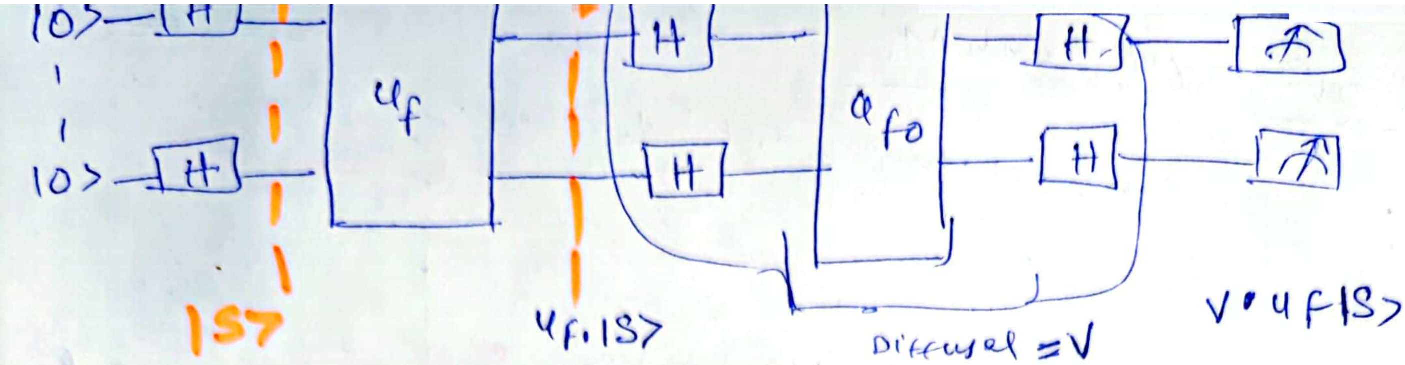
$$= \cos\frac{\theta}{2}|w^{\perp}\rangle + \sin\frac{\theta}{2}|w\rangle$$

$$\theta = 2 \arcsin\left(\frac{1}{\sqrt{2^n}}\right) \quad -(1.1)$$

NOTE:-

$|s\rangle$ is the projection onto $|w\rangle$ and $|w^{\perp}\rangle$

P.T.O



Remember |S> has both |w> and |w⁺> meaning that

x1 Desired element to search, remaining other elements also 0

x2 After passing through $\boxed{U_f}$ ϕ shift Happens.

Step-02 Phase inversion

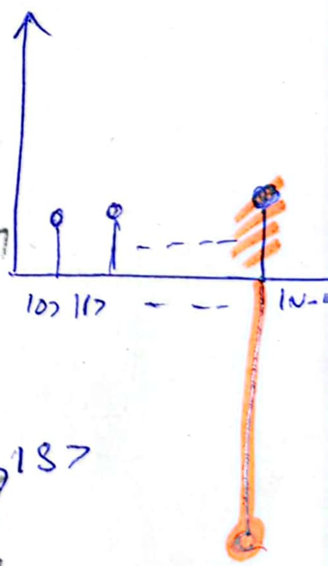
|w>, |S> — same
s≠w

↳ $\boxed{U_f|w\rangle = -|w\rangle}$

desired element

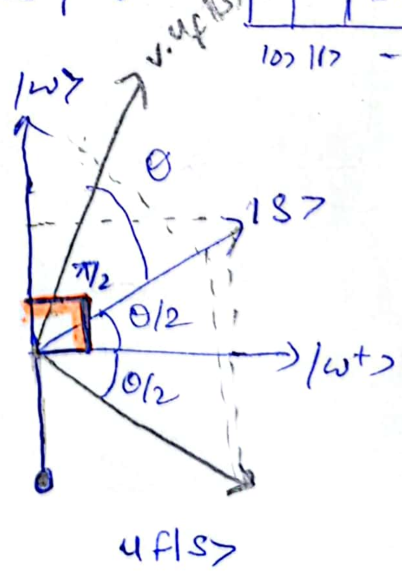
↳ all other no change $U_f|w^+\rangle = |w^+\rangle$ or

$U_f|x\rangle = |x\rangle$, when $x \neq w$



$U_f|s\rangle = (I - 2|w\rangle\langle w^+|)|s\rangle$

|S> is projected on w twice
ϕ change happend on |w> state not on the |w⁺>
(making it odd one out)



$\theta + \frac{\theta}{2}$
$\Rightarrow \frac{3\theta}{2}$

Step-3:- inversion About Mean.

$V = (2|S\rangle\langle S| - I)$

page #3

$(V.U_f)^{\gamma} |s\rangle$

we run rotation γ times every time it is at angle θ comming closer & closer to the $|w\rangle$ desired state

$|w\rangle$

for one rotation θ then for γ Number of rotations

it will be $\gamma\theta + \frac{\theta}{2} = \frac{\pi}{2}$

(defined in 1.1 page #1)

$\theta = 2 \arcsin\left(\frac{1}{\sqrt{2^n}}\right)$

$\gamma\theta = \frac{\pi}{2} - \frac{\theta}{2}$

if $N = 2^n$ if

n is large

$\gamma = \frac{\pi}{2\theta} - \frac{1}{2}$

if denominator is large whole quantity will become small

$2.2\ eq^n \longrightarrow = \frac{\pi}{2 \cdot 2\left(\frac{1}{\sqrt{2^n}}\right)} - \frac{1}{2}$

constant term

so, Approximating $\boxed{\sin\theta \approx \theta}$

$\boxed{\sin\theta \approx \theta = 2\left(\frac{1}{\sqrt{2^n}}\right)}$

put in eq 2.2 in place of θ

$\approx \frac{\pi}{4}\sqrt{2^n}$

$\approx O(\sqrt{n})$

so our expected (w) could be search in $\underline{\text{Big-oh}\ (\sqrt{n})}$

quadriatic times more speed up in searching is the speciality of the Grover's search Algorithm