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ABSTRACT

Stochastic Volatility in Mean Vector Autoregressions (SVMVARs) are popularly used to jointly estimate uncertainty and its economic effects. However, existing studies analyzing macroeconomic and financial uncertainty require the researcher to classify each variable as macroeconomic or financial before estimation, do not consider whether results are sensitive to model size and adopt a specification where results depend on the way the variables are ordered. We overcome these limitations, developing a novel Markov Chain Monte Carlo algorithm for large, order-invariant SVMVARs with unclassified variables. For each unclassified variable, the algorithm determines the appropriate classification at each point in time. Using a simulation study and large U.S. dataset, we uncover the following. Smaller SVMVARs overstate the effects of uncertainty, failing to reveal that only financial uncertainty has an adverse effect on the economy. When using large order-dependent SVMVARs, however, the uncertainty estimates produced depend on the variable ordering, distorting impulse response analysis. Thus, it becomes critical to adopt an order-invariant specification. We also find that many variables change classification with changes often occurring during crisis periods.

1. Introduction

It is widely recognized that economic uncertainty can adversely affect the economy (see Bloom 2009; Bachmann, Elstner, and Sims 2013; Scotti 2016; Baker, Bloom, and Davis 2016 among many others). However, the role played by different types of uncertainty is less well understood. Some theories suggest that macroeconomic uncertainty relating to economic fundamentals dampens economic activity (see Ludvigson, Ma, and Ng 2021 for an overview). Others instead argue that uncertainty primarily arises from or is propagated via financial markets (Gilchrist, Sim, and Zakrajsek 2014; Christiano, Motto, and Rostagno 2014; Bonciani and Van Roye 2016; Alfaro, Bloom, and Lin 2024). Understanding the relative importance of macroeconomic and financial uncertainty is crucial when developing policy prescriptions. If macroeconomic uncertainty dominates, policymakers need to act decisively minimizing uncertainty, for instance, via forward guidance. However, if financial uncertainty dominates, other actions such as increasing liquidity in the financial system are likely to be critical (Caggiano et al. 2021).

In response to these issues, frameworks are being developed to measure and contrast the effects of macroeconomic and financial uncertainty. An important strand of the literature equates uncertainty with Stochastic Volatility (SV) (see Berger, Grabert, and Kempa 2016; Carriero, Clark, and Marcellino 2020; Hou 2020; Mumtaz and Musso 2021 among many others). Within this literature, a popular model is the Stochastic Volatility in

Mean Vector Autoregression (SVMVAR). An attractive property of the SVMVAR is that it jointly estimates uncertainty (through the SV in the errors) and produces an estimate of its impact on the economy (by allowing the SVs to enter the conditional mean of the VAR).

This article contributes to the literature by tackling a number of problems which continue to plague SVMVAR analysis, making it difficult to robustly measure and analyze macroeconomic and financial uncertainty. Closely related to our work is the study by Carriero, Clark, and Marcellino (2018), hereafter CCM, who develop a new SVMVAR model. Using data on 30 variables, they model macroeconomic (financial) uncertainty as the common SV, the common component driving the volatilities of all macroeconomic (financial) variables. However, due to the challenges associated with estimating SVMVARs, several issues require further consideration.

First, existing SVMVAR specifications including CCM rely on using a lower triangular parameterization to model the reduced-form error covariance matrix. This does not relate to structural identification. Rather, it facilitates model estimation and improves computational efficiency (Chan 2023). However, recent studies show that this parameterization results in an order-dependence issue where the results can change depending on the way variables are ordered. This can affect estimation of reduced-form error variances (Chan et al. 2018), forecast performance (Arias, Rubio-Ramírez, and Shin 2023) and

structural analysis (Bognanni 2018; Hartwig 2020), becoming a more important issue in large models (Chan, Koop, and Yu 2024).

Second, Kim, Shephard, and Chib (1998) efficient Markov Chain Monte Carlo (MCMC) algorithm, typically used to estimate VARs with SV, cannot be applied once SV is added to the mean. Alternative approaches involving a Metropolis step (see Jacquier, Polson, and Rossi 2002) or particle Gibbs step (see Andrieu, Doucet, and Holenstein 2010 and Lindsten, Jordan, and Schon 2014) must instead be used. While these algorithms typically work well when estimating small models, the computational burden and poor MCMC mixing properties become more severe as model size increases. Such problems limit the number of variables which can be included in a SVMVAR despite the recent focus on large VARs (see, among many others, Banbura, Giannone, and Reichlin 2010; Koop and Korobilis 2013; Carriero, Clark, and Marcellino 2019; Gefang, Koop, and Poon 2022; and Chan 2022). It is therefore unclear whether the findings obtained from SVMVARs are sensitive to the model's size.

Third, it remains unclear how dozens of series should be classified prior to estimation of the SVMVAR. Should variables relating to money supply, credit, exchange rates, interest rates and stock prices be assigned to the macroeconomic block or assigned to the financial block? If 10 such variables are included in a model, this means there are 10^2 possible classification schemes which the researcher must choose between. In the uncertainty literature, this means that different studies (e.g., CCM, Ludvigson, Ma, and Ng 2021 and Redl 2020) classify key variables differently.

To circumvent these challenges, we develop a novel MCMC algorithm for estimating large SVMVARs which are order-invariant and have unclassified variables. Although we use our model to investigate macroeconomic and financial uncertainty, we stress that our SVMVAR and algorithm can be used in other contexts where large and order-invariant SVMVARs are required. Our starting point is Cross et al. (2023), hereafter CHKP, who propose a fast MCMC algorithm for SVMVARs, exploiting band and sparse matrix algorithms to sample the common SVs and improve efficiency. Our new algorithm uses the computationally efficient approach developed by CHKP to sample the common SVs but is enriched with additional features, leading to several new contributions.

First, we introduce order-invariance. Notably, existing order-invariant algorithms developed by Bertsche and Braun (2022) and Chan, Koop, and Yu (2024) for the VAR with SV cannot be applied to our SVMVAR. We therefore develop a novel algorithm for estimating our proposed point-identified order-invariant SVMVAR (OI-SVMVAR). Second, since we follow CHKP when sampling the common SVs, having contended with order dependence issues, we can robustly estimate very large SVMVARs, assessing how model size influences the results obtained. We can also investigate whether order dependence issues worsen as the size of the SVMVAR increases. Third, we distinguish between macroeconomic, financial and unclassified variables, allowing the algorithm to determine whether each unclassified variable should be included in the macroeconomic or financial block of variables. As the economy's structure evolves or experiences crises, a variable's classification may

change over time. Consequently, we allow for Markov switching time-varying classification.

Our article also contributes to the literature seeking to analyze the effects of macroeconomic and financial uncertainty on the economy. This literature almost exclusively deploys a two step approach. In the first step, uncertainty measures are produced or obtained. In the second step, the effects of uncertainty are analyzed by including the uncertainty proxies in small structural VARs (SVARs). Early studies separately analyze macroeconomic and financial uncertainty (see, e.g., Bloom 2009; Bachmann, Elstner, and Sims 2013 or Scotti 2016). In later work, Casarin et al. (2018) directly contrast macroeconomic and financial uncertainty finding the latter to be more important. However, the two uncertainty types are proxied in different ways via survey data and the VIX, respectively. In their analysis, Ma and Samaniego (2019) use firm-level data, also finding that uncertainty among financial firms has a larger economic impact than non-financial uncertainty. More recently, Ludvigson, Ma, and Ng (2021), hereafter LMN, follow Jurado, Ludvigson, and Ng (2015), producing comparable measures of macro and financial uncertainty by extracting the common variation in the unforecastable component of a large number of macroeconomic and financial time series. Using a SVAR with a novel identification scheme, LMN also find that financial uncertainty rather than macroeconomic uncertainty is a likely source of U.S. output fluctuations.

Importantly, though, Angelini et al. (2019), using a different identification scheme refuted by LMN, and CCM, using a SVMVAR, find that both macro and financial uncertainty have adverse economic effects. Additionally, as outlined in CCM, the two step approach deployed by LMN and others faces limitations: the assumption that variables have time-varying volatility in the first step is inconsistent with the use of a homoscedastic SVAR in the second step; the second step fails to account for measurement error and uncertainty around the uncertainty estimates themselves; and the SVAR itself is likely to suffer from omitted variables bias due to its small size. CCM's deployment of a SVMVAR circumvents these limitations, however, as discussed, issues around order-dependence, model size and classification require investigation.

Our simulation study shows that when smaller SVMVARs are used, uncertainty and its effects are likely to be overstated. Like CCM, we therefore find that when using U.S. data, results from smaller 30 variable SVMVARs indicate that macroeconomic and financial uncertainty both have adverse effects on the economy. In contrast, when a larger 43 variable SVMVAR is deployed, omitted variables bias can be alleviated and it becomes clear that financial not macroeconomic uncertainty has adverse effects. However, to obtain robust estimation results from the 43 variable SVMVAR it is critical to use an order-invariant specification. When using the lower triangular parameterization, many studies order macroeconomic variables before the financial variables. This implicitly assumes that variation in financial variables is not only explained by the volatility of financial variables (i.e., financial uncertainty) but the volatility of macroeconomic variables (i.e., macroeconomic uncertainty). Our simulation study therefore shows that the uncertainty estimates obtained from large SVMVARs depend on the variable ordering, distorting impulse response analysis. In our empirical work, the large

order-dependent SVMVAR thus fails to detect that financial uncertainty adversely affects the economy. Our proposed OI-SVMVAR does not suffer from these issues.

We also show that our algorithm can detect cases in which unclassified variables exhibit constant classification and cases in which time-varying classification is present. In terms of the latter, we find that many variables change classification with changes often occurring during crises. Allowing for unclassified variables therefore ensures that variables are assigned to the appropriate block when our uncertainty measures and their impacts are estimated.

The remainder of this article is organized as follows. Section 2 introduces our OI-SVMVAR model with time-varying classification and MCMC algorithm. Section 3 presents our simulation study. Section 4 outlines the data used and different model specifications before discussing our empirical results. Section 5 concludes. Our Online Appendix contains a Technical Appendix, Data Appendix and supplementary figures.

2. The Order-Invariant Stochastic Volatility in Mean Vector Autoregression

This section describes a new SVMVAR model which is order-invariant and allows for uncertainty in the way variables are classified. In addition, it derives a novel MCMC algorithm which extends the algorithm in CHKP to handle time-varying classification uncertainty and order dependence issues. Full details of the priors and additional details of the algorithm are provided in Online Appendix A (sections A1 and A2).

2.1. The Model

While the OI-SVMVAR we develop can be used in different contexts, in this article we will analyze macroeconomic and financial uncertainty. We therefore let $y_t^m = (y_{1,t}^m, \ldots, y_{n_m,t}^m)'$ be an $n_m \times 1$ vector of macro variables, $y_t^f = (y_{1,t}^f, \ldots, y_{n_f,t}^f)'$ be an $n_f \times 1$ vector of financial variables, and $y_t^u = (y_{1,t}^u, \ldots, y_{n_u,t}^u)'$ be an $n_u \times 1$ vector of unclassified variables where each unclassified variable could belong to either the macro or financial block. We consider the following SVMVAR model, denoting $\mathbf{y}_t = (y_t^{m'}, y_t^{u'}, y_t^{f'})'$ and $n = n_m + n_f + n_u$:

$$\mathbf{y}_t = \sum_{i=1}^{p} \mathbf{B}_i \mathbf{y}_{t-i} + \sum_{j=0}^{q} \mathbf{A}_j \mathbf{h}_{t-j} + \mathbf{B}_0^{-1} \boldsymbol{\epsilon}_t^y, \quad \boldsymbol{\epsilon}_t^y \sim \mathcal{N}(\mathbf{0}, \mathbf{U}_t), \quad (1)$$

where $\mathbf{B}_0$ is a nonsingular matrix with ones on the diagonal, $\mathbf{B}_1, \ldots, \mathbf{B}_p$ are $n \times n$ VAR coefficient matrices and $\mathbf{h}_t = (h_{m,t}, h_{f,t})'$ is a 2×1 vector of common log-volatilities which capture the co-movement in the time-varying volatilities of the macro and financial variables respectively. We use $e^{\frac{1}{2}h_{m,t}}$ and $e^{\frac{1}{2}h_{f,t}}$ as our measures of macro and financial uncertainty—these will be described in more detail shortly. The coefficient matrices $\mathbf{A}_0, \ldots, \mathbf{A}_q$ are of dimension $n \times 2$ and capture the effects of the contemporaneous and lagged common log-volatilities on the VAR variables.

A key contribution of our article is to propose an SVMVAR which is order-invariant. We therefore depart from the widely

applied assumption that the impact matrix $\mathbf{B}_0$ has a lower triangular structure. Recent studies show that order dependence issues resulting from this parameterization tend to become serious in large models. Consequently, Bertsche and Braun (2022) and Chan, Koop, and Yu (2024) propose order-invariant approaches, allowing $\mathbf{B}_0$ to be an unrestricted nonsingular matrix which is identified up to permutations and sign changes. We follow Lütkepohl and Woźniak (2020), restricting diagonal elements of $\mathbf{B}_0$ to be ones and leaving other parameters unrestricted. This setup allows us to show that $\mathbf{B}_0$ is point-identified (see Online Appendix A3) and also order-invariant (see Online Appendix A5). We then verify our identification assumption ex-post, demonstrating that this assumption holds in our empirical study (see Online Appendix A.3.2).[1] We also impose a Horseshoe prior on the free parameters of $\mathbf{B}_0$ to achieve sparsity where appropriate.

Returning to our uncertainty measures, the matrix $\mathbf{U}_t$ is assumed to be diagonal:

$$\mathbf{U}_t = \begin{pmatrix} \Omega_{m,t} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Omega_{u,t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Omega_{f,t} \end{pmatrix}, \quad (2)$$

where the volatilities of the macro, financial and unclassified variables are respectively defined as $\Omega_{m,t} = \text{diag}(e^{\omega_{1,t}^m}, \ldots, e^{\omega_{n_m,t}^m})$, $\Omega_{f,t} = \text{diag}(e^{\omega_{1,t}^f}, \ldots, e^{\omega_{n_f,t}^f})$ and $\Omega_{u,t} = \text{diag}(e^{\omega_{1,t}^u}, \ldots, e^{\omega_{n_u,t}^u})$.

For variables in the macro and financial blocks, their log-volatilities are specified as

$$\omega_{i,t}^m = \eta_{i,t}^m + h_{m,t}, \quad i = 1, \ldots, n_m, \quad (3)$$

$$\omega_{i,t}^f = \eta_{i,t}^f + h_{f,t}, \quad i = 1, \ldots, n_f, \quad (4)$$

where $\eta_{i,t}^m$ and $\eta_{i,t}^f$ are the idiosyncratic volatility components associated with the ith macro and financial variables, respectively. Equations (3) and (4) indicate that the volatility of each variable in the macro (financial) block is defined as the sum of its idiosyncratic volatility and the common log-volatility of the macro (financial) variables.[2]

Another contribution of this article lies in the treatment of our unclassified variables. The log-volatilities for the unclassified variables are specified as

$$\omega_{i,t}^u = \eta_{i,t}^u + h_{s_{i,t},t}, \quad i = 1, \ldots, n_u, \quad (5)$$

where $s_{i,t} \in \{m, f\}$ is the indicator variable for the ith unclassified variable, which is assumed to follow a Markov switching process with transition probability $p(s_{i,t} = k | s_{i,t-1} = l) = p_{l,k}^i$, $k, l \in \{m, f\}$. Note that the volatility of the ith unclassified variable is again defined as the sum of two components. The

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[1] Recent studies (see Korobilis 2022 and Chan, Matthes, and Yu 2023) propose efficient algorithms for identifying large SVARs using sign restrictions. However, these methods are not designed for imposing equality restrictions so do not allow for restricting the diagonal elements of $\mathbf{B}_0$ to be one. Thus only set-identification can be achieved.

[2] Equations (3) and (4) can be viewed as a factor model where the factor loadings are set to be one. However, we can modify our algorithm to estimate the factor loadings (see Online Appendix A.2.1). In the unreported estimation results, we find that estimating the factor loadings does not alter the main findings in our empirical study. This is because most of the estimated factor loadings are close to one consistent with CCM.

first component is the idiosyncratic component, $\eta_{i,t}^u$, and the second component, $h_{s_{i,t},t}$, is determined by the indicator variable $s_{i,t}$ and is either the common log-volatility of the macro block or the financial block. For example, if $s_{i,t} = m$, then $h_{s_{i,t},t} = h_{m,t}$, indicating that the ith unclassified variable belongs to the block of macro variables at time t. This specification not only allows each unclassified variable to be assigned to either block, but does so in a time-varying fashion. So it is possible that a variable switches from the macroeconomic to the financial block (or vice versa). This allows us to investigate a range of interesting possibilities. For instance, a variable may be classified in one block during crises and another block during normal times.

Note that the decision to classify a variable in one block as opposed to another is made based on the co-movement of its SV process with the variables which are definitively classified as being in one block or the other. In times when the volatility of an unclassified variable is behaving like the volatilities in the macroeconomic (financial) block it will be classified as a macroeconomic (financial) variable. An important goal of our paper is to robustly measure uncertainty and its effects. Allowing for unclassified variables therefore ensures that, statistically speaking, variables are assigned to the appropriate block when our uncertainty measures and their impacts are estimated.

Recall that $\mathbf{h}_t = (h_{m,t}, h_{f,t})'$ is a 2×1 vector of common log-volatilities used to capture macro and financial uncertainty. We assume they evolve as the following VAR process:

$$\mathbf{h}_t = \sum_{i=1}^{p_h} \boldsymbol{\Phi}_i \mathbf{h}_{t-i} + \sum_{j=1}^{p_y} \boldsymbol{\Psi}_j \mathbf{y}_{t-j} + \boldsymbol{\epsilon}_t^h, \quad \boldsymbol{\epsilon}_t^h \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_h), \quad (6)$$

where $\boldsymbol{\epsilon}_t^h = (\epsilon_{m,t}^h, \epsilon_{f,t}^h)'$ is a 2×1 vector of error terms and the initial state is specified as $\mathbf{h}_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{V}_h)$. Our specification therefore follows CCM and we assume that: financial uncertainty can affect macro uncertainty and vice versa; our uncertainty measures depend not only on their past values but also past values of the VAR variables; and $\epsilon_{m,t}^h$ and $\epsilon_{f,t}^h$ are permitted to be correlated since common shocks may affect both types of uncertainty.

When undertaking impulse response analysis using the SVMVAR, uncertainty shocks are orthogonal to the VAR shocks by construction, providing structural identification. To separately identify the effects of shocks to macro and financial uncertainty, we follow CCM by assuming that macro uncertainty affects financial uncertainty contemporaneously while financial uncertainty affects macro uncertainty with a lag. Specifically, in our main empirical analysis, we assume that the reduced-form errors in (6) can be decomposed as $\boldsymbol{\epsilon}_t^h = \mathbf{L}\mathbf{e}_t^h$ where $\mathbf{L}$ is a lower triangular matrix such that $\mathbf{L}\mathbf{L}' = \boldsymbol{\Sigma}_h$ and $\mathbf{e}_t^h = (e_{m,t}^h, e_{f,t}^h)'$ is a vector of uncorrelated structural shocks with $e_{m,t}^h$ and $e_{f,t}^h$ denoting the macro and financial uncertainty shocks, respectively.

The idiosyncratic log-volatilities associated with each variable do not enter the conditional mean of the VAR and are assumed to follow stationary AR(1) processes:

$$\eta_{i,t}^k = \mu_{k,i} + \rho_{k,i}\eta_{i,t-1}^k + \epsilon_{i,t}^k, \quad \epsilon_{i,t}^k \sim \mathcal{N}(0, \sigma_{k,i}^2), \quad (7)$$

for $i = 1, \ldots, n_k$, $k \in \{m, f, u\}$ and $|\rho_{k,i}| < 1$. The initial state is specified as $\eta_{i,1}^k \sim \mathcal{N}\left(0, \frac{\sigma_{k,i}^2}{1-\rho_{k,i}^2}\right)$. We also note that in (1) and (6) we follow CCM in setting the lag orders of our models to $p = 6$, $q = 2$ and $p_h = 2$ and $p_y = 1$, respectively.

2.2. A New MCMC Algorithm for the OI-SVMVAR

Bayesian inference in VARs with SV is typically undertaken using MCMC methods involving the auxiliary mixture sampler of Kim, Shephard, and Chib (1998). However, once SV is added to the mean the auxiliary mixture sampler can no longer be used. Thus, studies deploying SVMVARs such as CCM use the particle Gibbs algorithm proposed by Andrieu, Doucet, and Holenstein (2010) to draw the log-SVs from the posterior distribution. The particle Gibbs algorithm can be computationally burdensome in large models and suffer from particle degeneracy problems. This is shown by CHKP who develop an MCMC algorithm which uses an independent Acceptance–Rejection Metropolis-Hastings step. While Jacquier, Polson, and Rossi (2002) also use a Metropolis step to draw the log-volatilities this must be done date-by-date. CHKP sample the log-volatilities across all dates in a single step, significantly reducing computation time and improving mixing. This involves a Gaussian candidate generating density with variance depending on the Hessian of the conditional posterior of the log-volatilities. Crucially, this Hessian is block-banded. Band and sparse matrix algorithms can thus be exploited to allow for efficient computation even in large SVMVARs.

In this article, we use CHKP to efficiently sample the log-volatilities but extend this algorithm to account for possible order-dependence issues. We restrict the diagonal elements of the impact matrix $\mathbf{B}_0$ to be ones, allowing us to demonstrate that $\mathbf{B}_0$ is point-identified. However, this means the order-invariant algorithm for the VAR with SV proposed by Chan, Koop, and Yu (2024) cannot be used to draw $\mathbf{B}_0$. In this subsection, we thus develop a novel parameter transformation sampling approach for drawing $\mathbf{B}_0$. Sampling $\mathbf{B}_0$ in this way does not increase the computational burden thus we retain the speed of CHKP. This opens the door to robust estimation of very large SVMVARs such as those considered in this article.

We also build on CHKP by allowing for variables whose classification is uncertain. Conditional on knowing the way the variables are classified (i.e., conditional on $(s_{i,1}, \ldots, s_{i,T})$ for $i = 1, \ldots, n_u$), we can transform our model, reordering the equations of the unclassified variables so that they can be grouped appropriately with the predetermined macro and financial variables. After this transformation, the transformed model reduces to a standard SVMVAR and the methods of CHKP can be directly applied to sample the log-volatilities $(\mathbf{h}_1, \ldots, \mathbf{h}_T)$. It can then be shown that, given the log-volatilities $(\mathbf{h}_1, \ldots, \mathbf{h}_T)$, draws of the indicator variable $(s_{i,1}, \ldots, s_{i,T}), i = 1, \ldots, n_u$ and the Markov transition probabilities $(p_{m,m}^i, p_{m,f}^i, p_{f,m}^i, p_{f,f}^i)$ can be directly obtained using the algorithm of Chib (1996). Further details about the posterior sampler are provided in Online Appendix A2.

We now present our sampling approach for drawing $\mathbf{B}_0$. To set the stage, we first note that our model specified in (1) can be

rewritten as

$$\widetilde{\mathbf{y}}_t = \mathbf{B}_0^{-1}\boldsymbol{\epsilon}_t^y, \quad \boldsymbol{\epsilon}_t^y \sim \mathcal{N}(\mathbf{0}, \mathbf{U}_t),$$

where $\widetilde{\mathbf{y}}_t = \mathbf{y}_t - \sum_{i=1}^p \mathbf{B}_i \mathbf{y}_{t-i} - \sum_{j=0}^q \mathbf{A}_j \mathbf{h}_{t-j}$. Letting $\widetilde{\mathbf{Y}} = (\widetilde{\mathbf{y}}_1, \ldots, \widetilde{\mathbf{y}}_T)'$ and $\mathbf{E} = (\boldsymbol{\epsilon}_t^{y'}, \ldots, \boldsymbol{\epsilon}_T^{y'})'$, we can express our model more compactly as

$$\widetilde{\mathbf{Y}} \mathbf{B}_0' = \mathbf{E}.$$

Thus, for $i = 1, \ldots, n$, we have

$$\widetilde{\mathbf{Y}} \mathbf{b}_{0,i} = \mathbf{E}_i, \quad \mathbf{E}_i \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}_i), \quad (8)$$

where $\mathbf{E}_i$ is the ith column of $\mathbf{E}$, $\mathbf{b}_{0,i}'$ is the ith row of $\mathbf{B}_0$ and

$$\boldsymbol{\Omega}_i = \begin{cases} \text{diag}(\omega_{i,1}^m, \ldots, \omega_{i,T}^m), \text{ if } 1 \leq i \leq n_m, \\ \text{diag}(\omega_{i-n_m,1}^u, \ldots, \omega_{i-n_m,T}^u), \text{ if } n_m + 1 \leq i \leq n_m + n_u, \\ \text{diag}(\omega_{i-n_m-n_u,1}^f, \ldots, \omega_{i-n_m-n_u,T}^f), \\ \quad\quad \text{if } n_m + n_u + 1 \leq i \leq n_m + n_u + n_f. \end{cases}$$

Since the diagonal elements of $\mathbf{B}_0$ are restricted to be ones (i.e., the ith element of $\mathbf{b}_{0,i}$ is set to be 1), there are $(n-1)$ free parameters in $\mathbf{b}_{0,i}$ that need to be estimated. We let $\widetilde{\mathbf{b}}_{0,i}$ be the $(n-1) \times 1$ vector that collects these free parameters in $\mathbf{b}_{0,i}$, and we then assume an independent Gaussian prior for $\widetilde{\mathbf{b}}_{0,i}$:

$$\widetilde{\mathbf{b}}_{0,i} \sim \mathcal{N}(\boldsymbol{\beta}_i, \mathbf{V}_i), \quad i = 1, \ldots, n. \quad (9)$$

We develop an algorithm for sequentially drawing the free parameters in each row of $\mathbf{B}_0$. Specifically, we propose a novel sampling method that iteratively draws from $p(\widetilde{\mathbf{b}}_{0,i}|\mathbf{B}_{0,-i}, \boldsymbol{\Omega}_i, \widetilde{\mathbf{Y}})$ for $i = 1, \ldots, n$, where $\mathbf{B}_{0,-i} = (\mathbf{b}_{0,1}, \ldots, \mathbf{b}_{0,i-1}, \mathbf{b}_{0,i+1}, \ldots, \mathbf{b}_{0,n})$. Since we can always permute any row of $\mathbf{B}_0$ to be updated to the first row, without loss of generality we will illustrate our method for drawing the conditional posterior of the first row of $\mathbf{B}_0$, $\widetilde{\mathbf{b}}_{0,1}$.

Since the diagonal elements of $\mathbf{B}_0$ are restricted to be ones, we have $\mathbf{b}_{0,1} = (1, \widetilde{\mathbf{b}}_{0,1}')'$. Suppose we write $\widetilde{\mathbf{Y}} = (\mathbf{f}_1, \ldots, \mathbf{f}_n)$, then (8) with $i = 1$ can be expressed as

$$\mathbf{f}_1 + \widetilde{\mathbf{Y}}_{-1} \widetilde{\mathbf{b}}_{0,1} = \mathbf{E}_1, \quad \mathbf{E}_1 \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}_1), \quad (10)$$

where $\widetilde{\mathbf{Y}}_{-1} = (\mathbf{f}_2, \ldots, \mathbf{f}_n)$. Hence, the conditional posterior of $\widetilde{\mathbf{b}}_{0,1}$ is given by

$$p(\widetilde{\mathbf{b}}_{0,1}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}}) \propto |\det\mathbf{B}_0|^T e^{-\frac{1}{2}\left(\mathbf{f}_1 + \widetilde{\mathbf{Y}}_{-1}\widetilde{\mathbf{b}}_{0,1}\right)' \boldsymbol{\Omega}_1^{-1} \left(\mathbf{f}_1 + \widetilde{\mathbf{Y}}_{-1}\widetilde{\mathbf{b}}_{0,1}\right)}$$
$$\times e^{-\frac{1}{2}\left(\widetilde{\mathbf{b}}_{0,1} - \boldsymbol{\beta}_1\right)' \mathbf{V}_1^{-1} \left(\widetilde{\mathbf{b}}_{0,1} - \boldsymbol{\beta}_1\right)}$$
$$\propto |\det\mathbf{B}_0|^T e^{-\frac{1}{2}\left(\widetilde{\mathbf{b}}_{0,1} - \widehat{\widetilde{\mathbf{b}}}_{0,1}\right)' \mathbf{K} \left(\widetilde{\mathbf{b}}_{0,1} - \widehat{\widetilde{\mathbf{b}}}_{0,1}\right)}, \quad (11)$$

where

$$\mathbf{K} = \widetilde{\mathbf{Y}}_{-1}' \boldsymbol{\Omega}_1^{-1} \widetilde{\mathbf{Y}}_{-1} + \mathbf{V}_1^{-1}, \quad \widehat{\widetilde{\mathbf{b}}}_{0,1} = \mathbf{K}^{-1}\left(\mathbf{V}_1^{-1}\boldsymbol{\beta}_1 - \widetilde{\mathbf{Y}}_{-1}'\boldsymbol{\Omega}_1^{-1}\mathbf{f}_1\right).$$

To further simplify the expression of the conditional posterior $p(\widetilde{\mathbf{b}}_{0,1}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$, we partition $\mathbf{B}_0$:

$$\mathbf{B}_0 = \begin{pmatrix} 1 & \widetilde{\mathbf{b}}_{0,1}' \\ \mathbf{b}_{0,21} & \mathbf{B}_{0,22} \end{pmatrix},$$

where $\mathbf{b}_{0,21}$ is $(n-1) \times 1$ and $\mathbf{B}_{0,22}$ is $(n-1) \times (n-1)$. Using a standard property of partitioned matrices, the determinant of $\mathbf{B}_0$ can be expressed as

$$\det\mathbf{B}_0 = \det\mathbf{B}_{0,22} \times (1 - \mathbf{b}_{0,21}' \mathbf{B}_{0,22}'^{-1} \widetilde{\mathbf{b}}_{0,1}). \quad (12)$$

By substituting (12) into (11), we have

$$p(\widetilde{\mathbf{b}}_{0,1}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}}) \propto |1 - \mathbf{b}_{0,21}' \mathbf{B}_{0,22}'^{-1} \widetilde{\mathbf{b}}_{0,1}|^T$$
$$e^{-\frac{1}{2}\left(\widetilde{\mathbf{b}}_{0,1} - \widehat{\widetilde{\mathbf{b}}}_{0,1}\right)' \mathbf{K} \left(\widetilde{\mathbf{b}}_{0,1} - \widehat{\widetilde{\mathbf{b}}}_{0,1}\right)}. \quad (13)$$

Since the conditional posterior $p(\widetilde{\mathbf{b}}_{0,1}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$ specified in (13) is not a standard distribution, direct sampling is not feasible. To facilitate sampling from $p(\widetilde{\mathbf{b}}_{0,1}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$, we introduce a parameter transformation scheme that transforms $\widetilde{\mathbf{b}}_{0,1}$ to $\mathbf{w} = (w_1, \ldots, w_{n-1})'$ as follows

$$\mathbf{w} = \mathbf{d} + \mathbf{Q}\widetilde{\mathbf{b}}_{0,1}, \quad (14)$$

where

$$\mathbf{Q} = \begin{pmatrix} -\mathbf{b}_{0,21}' \mathbf{B}_{0,22}'^{-1} \\ \mathbf{0}_{(n-2)\times 1} \quad \mathbf{I}_{n-2} \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} 1 \\ \mathbf{0}_{(n-2)\times 1} \end{pmatrix}.$$

The parameter transformation given in (14) defines an one-to-one affine transformation from $\widetilde{\mathbf{b}}_{0,1}$ to $\mathbf{w}$. Thus, we can define an inverse mapping from $\mathbf{w}$ to $\widetilde{\mathbf{b}}_{0,1}$:

$$\widetilde{\mathbf{b}}_{0,1} = \mathbf{Q}^{-1}\mathbf{w} - \mathbf{Q}^{-1}\mathbf{d}. \quad (15)$$

To sample from $p(\widetilde{\mathbf{b}}_{0,1}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$, we can draw from the conditional posterior of $\mathbf{w}$, $p(\mathbf{w}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$, and then transform the obtained $\mathbf{w}$ back to $\widetilde{\mathbf{b}}_{0,1}$ via the inverse mapping outlined in (15). It is important to note that the Jacobian of the inverse mapping (15) is independent of $\widetilde{\mathbf{b}}_{0,1}$ and $\mathbf{w}$. Hence, we can get the conditional posterior $p(\mathbf{w}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$ by simply substituting (15) into (13), which gives

$$p(\mathbf{w}|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$$
$$\propto |w_1|^T e^{-\frac{1}{2}(\mathbf{Q}^{-1}\mathbf{w} - \mathbf{Q}^{-1}\mathbf{d} - \widehat{\widetilde{\mathbf{b}}}_{0,1})' \mathbf{K}(\mathbf{Q}^{-1}\mathbf{w} - \mathbf{Q}^{-1}\mathbf{d} - \widehat{\widetilde{\mathbf{b}}}_{0,1})}$$
$$\propto |w_1|^T e^{-\frac{1}{2}(\mathbf{w} - \widehat{\mathbf{w}})' \widehat{\mathbf{K}}(\mathbf{w} - \widehat{\mathbf{w}})}, \quad (16)$$

where

$$\widehat{\mathbf{K}}^{-1} = \mathbf{Q}\mathbf{K}^{-1}\mathbf{Q}', \quad \widehat{\mathbf{w}} = \mathbf{d} + \mathbf{Q}\widehat{\widetilde{\mathbf{b}}}_{0,1}.$$

To sample $\mathbf{w} = (w_1, \ldots, w_{n-1})'$, we propose a direct sampling approach for first drawing $w_1 \sim p(w_1|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$, and then drawing $\mathbf{w}_{-1} = (w_2, \ldots, w_{n-1})' \sim p(\mathbf{w}_{-1}|w_1, \mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$. To derive $p(w_1|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$ and $p(\mathbf{w}_{-1}|w_1, \mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}})$, we first partition $\widehat{\mathbf{K}}^{-1}$ and $\widehat{\mathbf{w}}$ as

$$\widehat{\mathbf{K}}^{-1} = \begin{pmatrix} \lambda_{11} & \boldsymbol{\lambda}_{21}' \\ \boldsymbol{\lambda}_{21} & \boldsymbol{\Lambda}_{22} \end{pmatrix}, \quad \widehat{\mathbf{w}} = (\widehat{w}_1, \widehat{\mathbf{w}}_{-1}')',$$

where λ_{11} is a scalar, $\boldsymbol{\lambda}_{21}$ is $(n-2) \times 1$, $\boldsymbol{\Lambda}_{22}$ is $(n-2) \times (n-2)$ and $\widehat{\mathbf{w}}_{-1} = (\widehat{w}_2, \ldots, \widehat{w}_{n-1})'$. Then it can be shown that

$$p(w_1|\mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}}) \propto |w_1|^T e^{-\frac{1}{2\lambda_{11}}(w_1 - \widehat{w}_1)^2}, \quad (17)$$

$$p(\mathbf{w}_{-1}|w_1, \mathbf{B}_{0,-1}, \boldsymbol{\Omega}_1, \widetilde{\mathbf{Y}}) \propto e^{-\frac{1}{2}\left(\mathbf{w}_{-1} - \widehat{\boldsymbol{\mu}}_{\mathbf{w}_{-1}}\right)' \boldsymbol{\Sigma}_{\mathbf{w}_{-1}}^{-1} \left(\mathbf{w}_{-1} - \widehat{\boldsymbol{\mu}}_{\mathbf{w}_{-1}}\right)}, \quad (18)$$

Table 1. Summary of SVMVAR models in simulation study.

Models	6 variables	23 variables	Time-varying classification (TVC)	Order-invariant (OI)	Specification of $\mathbf{B}_0$
1. CCM-6	•				lower triangular
2. OI-6	•			•	unrestricted
3. CCM-TVC-23		•	•		lower triangular
4. OI-TVC-23		•	•	•	unrestricted

where $\widehat{\boldsymbol{\mu}}_{\mathbf{w}_{-1}} = \widehat{\mathbf{w}}_{-1} + \frac{w_1 - \widehat{w}_1}{\lambda_{11}} \lambda_{21}$ and $\boldsymbol{\Sigma}_{\mathbf{w}_{-1}} = \boldsymbol{\Lambda}_{22} - \frac{\lambda_{21}\lambda'_{21}}{\lambda_{11}}$. Note that (18) implies that the conditional posterior of $\mathbf{w}_{-1}$ is a Gaussian distribution with mean $\widehat{\boldsymbol{\mu}}_{\mathbf{w}_{-1}}$ and covariance matrix $\boldsymbol{\Sigma}_{\mathbf{w}_{-1}}$. However, the conditional posterior of w_1 given by (17) is nonstandard. Therefore in Proposition 1 we propose a parameter transformation approach for sampling the conditional posterior of ω_1. The proof of Proposition 1 is in Online Appendix A4. To apply Proposition 1, we sample from the absolute normal distribution, $\widetilde{w}_1 \sim \mathcal{AN}(\frac{\widehat{w}_1}{\sqrt{T\lambda_{11}}}, \frac{1}{T})$, using the normal mixture approximation proposed in Appendix C of Villani (2009).[3]

Proposition 1. Suppose $\widetilde{w}_1 \sim \mathcal{AN}(\frac{\widehat{w}_1}{\sqrt{T\lambda_{11}}}, \frac{1}{T})$, where $\mathcal{AN}(a, b)$ denotes the absolute normal distribution that has the density function

$$f_{AN}(x; a, b) \propto |x|^{\frac{1}{b}} e^{-\frac{1}{2b}(x-a)^2}, \quad x \in \mathbb{R}, b \in \mathbb{R}^+, a \in \mathbb{R}.$$

Let $w_1 = \widetilde{w}_1 \times \sqrt{T\lambda_{11}}$, then w_1 follows a distribution with density function given by (17).

3. Simulation Study

This section undertakes a simulation study to demonstrate the performance of our proposed algorithm. We also show that omitting relevant variables and using an order-dependent specification can distort results.

3.1. Data Generating Process and Models

In our simulation study, we use the SVMVAR specified in (1)–(7) with $p = 2$, $q = 1$, $p_h = 1$, $p_y = 0$ and $T = 600$. We also set the idiosyncratic log-volatilities to be zeros for simplicity, that is, $\eta_{i,t} = 0$ for $i \in \{m, f, u\}$. We work with a 23-variable system with one block of $n_m = 10$ variables, a second block of $n_f = 10$ variables and $n_u = 3$ unclassified variables. We will refer to the first block as the macro variables and the second as the financial variables. For the unclassified variables, we set $s_{1,t} = m$ and $s_{3,t} = f$ for $t = 1, \dots, 600$. This means the first variable is a macro variable and the third variable is a financial variable. We then have $s_{2,t} = f$ when $t = 1, \dots, 200$ or $t = 401, \dots, 600$, and $s_{2,t} = m$ when $t = 201, \dots, 400$. Thus, the second unclassified variable is typically classified as financial but is temporarily assigned to the macro block in the middle of the sample.

The true impact matrix $\mathbf{B}_0$ is assumed to be a full nonzero matrix with ones on its diagonal and off-diagonal elements generated from $\mathcal{N}(0, 0.2)$. For the VAR coefficients, parameters on the diagonal and off-diagonal of $\mathbf{B}_1$ are from $\mathcal{U}(0, 0.6)$ and $\mathcal{U}(-0.2, 0.2)$, respectively. The elements in $\mathbf{B}_i$, $i > 1$, are generated from $\mathcal{N}(0, 0.1^2/i^2)$. The coefficient matrices associated with the common log-volatilities in the mean equation (i.e., $\mathbf{A}_i, i = 0, \dots, q$) are generated from $\mathcal{N}(0, 0.5^2/(i+1)^2)$. For the common log-volatilities, we set $\boldsymbol{\Phi}_1 = 0.95\mathbf{I}$ and $\boldsymbol{\Sigma}_h = 0.05\mathbf{I}$.
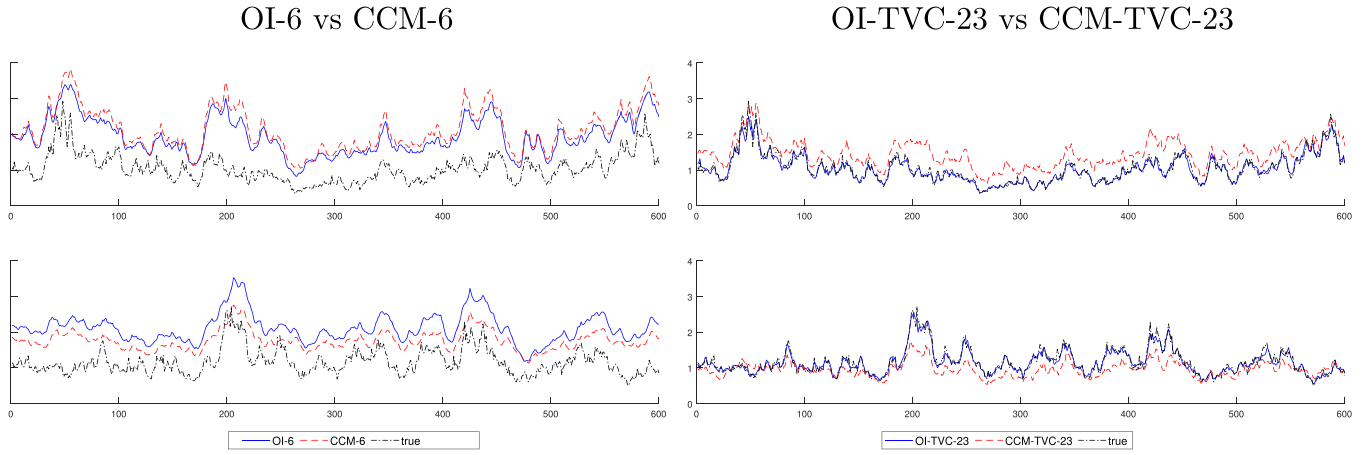
We consider four SVMVAR models in total which are described in Table 1. This includes our true model OI-TVC-23, a 23 variable OI-SVMVAR with unclassified variables, and three misspecified models: CCM-TVC-23, OI-6, and CCM-6. CCM-TVC-23 is the same as OI-TVC-23 but its impact matrix $\mathbf{B}_0$ is restricted to be lower triangular so the variable ordering matters. The two other misspecified models, OI-6 and CCM-6, are small versions of OI-TVC-23 and CCM-TVC-23 respectively, only containing the first three macro and first three financial variables and no unclassified variables. Following common practice, in our order-dependent specifications, we order our macro variables before our financial variables. All results are based on producing 15,000 draws, discarding the first 5000 as burn in.
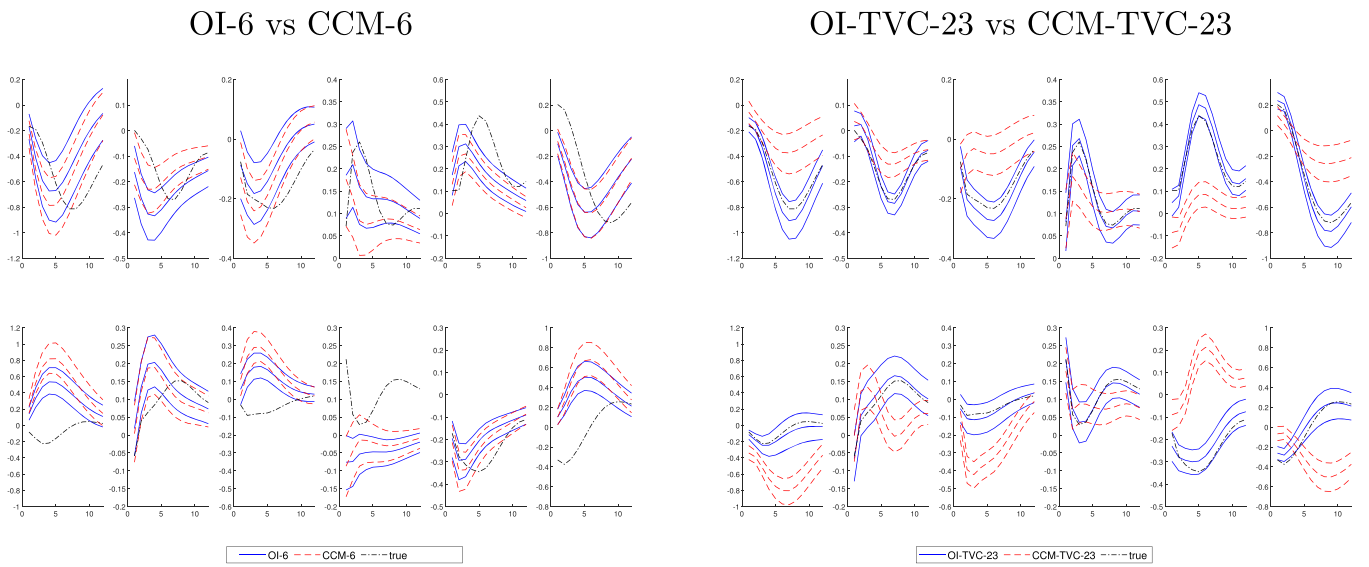
3.2. Algorithm Performance

Figure 1 reports our estimates of macro and financial uncertainty for all four models. It is clear that OI-TVC-23 recovers the true values well, demonstrating the sound performance of our algorithm. Two further points are clear from our analysis. First, relative to the true values, CCM-TVC-23 overestimates macro uncertainty and underestimates financial uncertainty. This is because the true $\mathbf{B}_0$ is a full nonzero matrix but CCM-TVC-23 imposes a lower triangular parameterization. Since our macro variables are ordered before our financial variables, the variation in financial variables will be explained not only by financial uncertainty (proxied by the common SV of financial variables) but also macroeconomic uncertainty (proxied by the common SV of macroeconomic variables). Thus, macro uncertainty explains a larger share of variation in the data than financial uncertainty.

Second, OI-6 and CCM-6 suffer from omitted variables bias, generating estimates of macro and financial uncertainty larger than the true values. This result is intuitive since the proportion of variation in the data explained by omitted variables is instead captured by the models' residuals. This increases the residuals' volatility, in turn leading to the overestimation of macro and financial uncertainty. Order dependence issues are also present in our smaller models: relative to OI-6, order-dependent CCM-6 generates slightly higher values of macro uncertainty and lower values of financial uncertainty.

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[3] Our algorithm for drawing $\mathbf{B}_0$ and the one by Braun (2023), which extends Waggoner and Zha (2003), both implement a parameter transformation, facilitating estimation. This involves an approximation step using a mixture of two Gaussians in the sampling procedure. While our algorithm involves approximating an absolute normal distribution, Braun (2023) approximates a generalized absolute normal distribution.

OI-6 vs CCM-6

OI-TVC-23 vs CCM-TVC-23

Figure 1. Simulation study uncertainty estimates: posterior medians of the macro, $e^{\frac{1}{2}h_{m,t}}$, (top panel) and financial uncertainty, $e^{\frac{1}{2}h_{f,t}}$, (bottom panel).

OI-6 vs CCM-6

OI-TVC-23 vs CCM-TVC-23

Figure 2. Simulation study impulse responses for one standard deviation macro (top) and financial (bottom) uncertainty shocks: posterior medians and 70% credible intervals.

Next, Figure 2 presents impulse responses to a one standard deviation macro and financial uncertainty shock in our four models. To facilitate comparisons, we report the posterior medians and 70% credible intervals for the six variables common across models. Again, our results show that OI-TVC-23 recovers the true values well with the 70% credible intervals containing the true values across almost all horizons. In contrast, impulse responses obtained using CCM-TVC-23, OI-6, and CCM-6 clearly deviate from the path of the true values. These deviations, however, arise from different forms of model misspecification. CCM-TVC-23 fails to recover the true responses due to order dependence issues. These issues are so severe that OI-6 and CCM-6 tend to perform better in approximating the true responses.

In contrast, the responses obtained from OI-6 and CCM-6 are very similar, demonstrating that order dependence issues are less severe in small models. These similarities, however, do not mean results are robust. With fewer variables in the model, larger effects are more likely to be detected. Consequently, relative to OI-TVC-23, OI-6, and CCM-6 can overestimate the effects of shocks, particularly when the true response is near zero. For
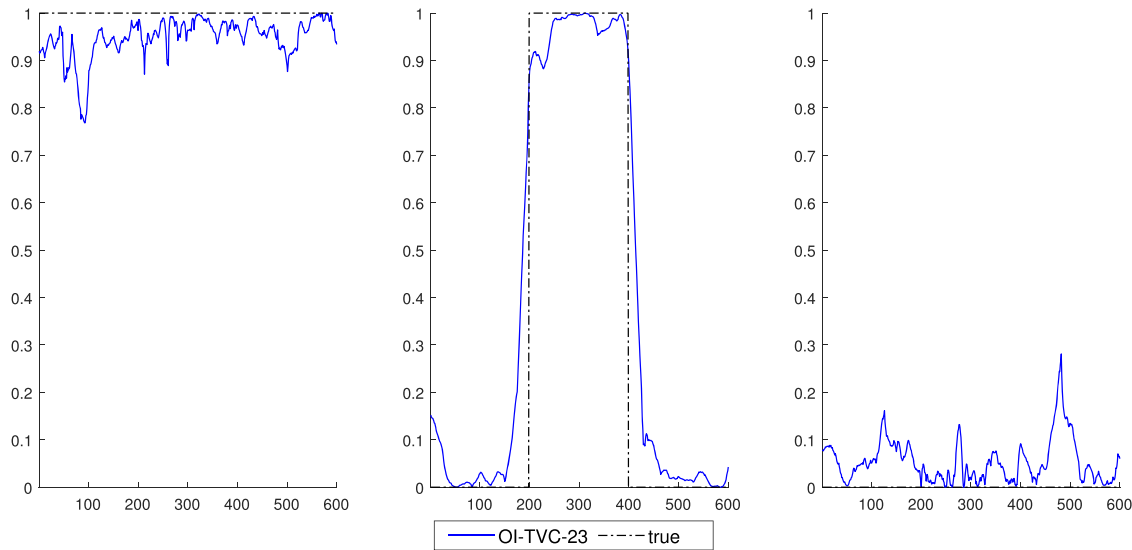
example, results from OI-TVC-23 indicate that when the first and third variables respond to a financial uncertainty shock the credible bands span zero for most horizons. However, CCM-6 and OI-6 detect a positive nonzero response.

Finally, Figure 3 plots the estimated posterior probabilities that each unclassified variable is assigned to the macro block. We show that our algorithm can effectively detect the appropriate classification. For the variable with time-varying classification, the algorithm performs well and for variables with constant classification, our algorithm assigns each variable to the appropriate block at each point in time with high probability.

4. Measuring and Analyzing the Effects of Macroeconomic and Financial Uncertainty

A deeper understanding of the relative importance of macro and financial uncertainty is required so that appropriate policy prescriptions can be administered. While there is growing evidence suggesting that financial uncertainty dominates, existing studies typically deploy a two step approach, first estimating uncertainty

Figure 3. The blue lines are the estimated posterior probability $p(s_{i,t} = m|\mathbf{y})$ for each of the unclassified variables $i = 1, 2, 3$. The black dashed lines indicate the true values.

Table 2. Summary of SVMVAR models in empirical analysis.

	Main empirical analysis				
Models	30 variables	43 variables	Time-varying classification (TVC)	Order-invariant (OI)	Specification of $\mathbf{B}_0$
1. CCM-30	●				lower triangular
2. OI-30	●			●	unrestricted
3. CCM-TVC-43		●	●		lower triangular
4. OI-TVC-43		●	●	●	unrestricted
	Assessing robustness to reversing variable ordering				
5. CCM-TVC-RO-43	same as CCM-TVC-43 but the ordering of the variables is reversed				
6. OI-TVC-RO-43	same as OI-TVC-43 but the ordering of the variables is reversed				

and then considering its effects in small SVARs—this can lead to measurement error and omitted variables bias. CCM overcome this by developing a SVMVAR framework, finding that both macro and financial uncertainty have adverse economic effects. Nonetheless, issues around model size, order-dependence and classification require investigation. In this section, we therefore measure and analyze the effects of macro and financial uncertainty using a large dataset and different SVMVAR specifications. We show that in smaller SVMVARs order dependence issues are less severe. However, when a smaller information set is used, both macro and financial uncertainty are found to have adverse effects. In contrast, when using large SVMVARs order dependence issues worsen. Thus. it only becomes clear that financial uncertainty dominates using our large order-invariant SVMVAR.
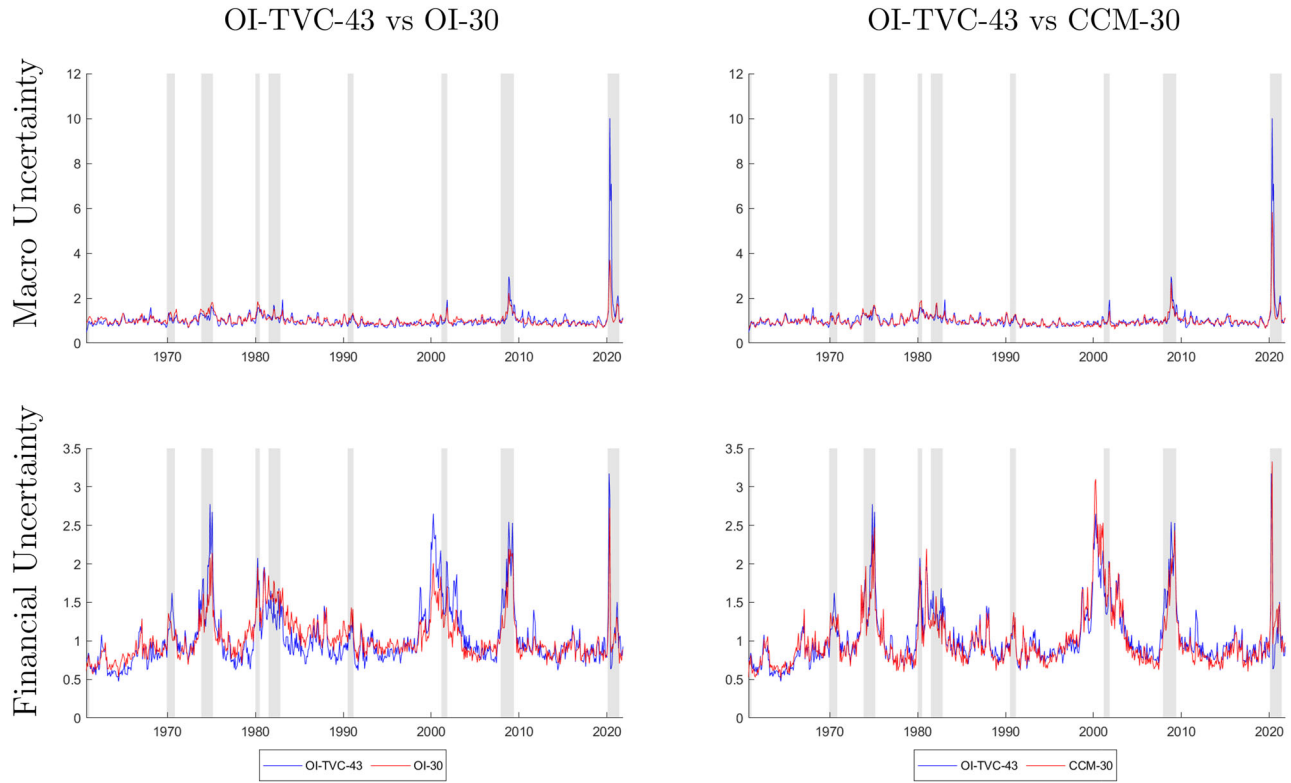
4.1. Large Dataset, Unclassified Variables and Models

In our analysis, we use an updated and extended version of the dataset considered in CCM. The sample spans January 1960 to October 2021, allowing us to capture the coronavirus pandemic. All variables are obtained from the FRED-MD and FRED datasets with historical house price data available from Robert Shiller. Like CCM and LMN, all models are estimated with standardized data. The complete list of variables and their classi-

fication, abbreviations and transformations are given in Online Appendix B.

We estimate six SVMVARs as outlined in Table 2.[4] Model 1 is order-dependent, includes the same 30 variables as CCM and classifies them in the same way with 18 variables treated as macroeconomic and the remaining 12 as financial. Model 2 is almost identical but uses our order invariant specification. Models 3 and 4 are very large including 43 variables and introducing time-varying classification. The only difference between them is that Model 4 is order invariant. As previously emphasized, when $\mathbf{B}_0$ is order dependent, variable ordering matters. For Models 1 and 3 which are order dependent, we order macro variables before financial variables as is common practice in the literature. This implies that the variation in financial variables is explained by the volatility of macroeconomic variables with the remaining variation explained by the volatility of financial variables. Models 5 and 6 therefore mirror Models 3 and 4 but the ordering of the variables is reversed to examine whether

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[4]Recall that for all six SVMVARs, we follow CCM, assuming that macro uncertainty has a contemporaneous impact on financial uncertainty while financial uncertainty reacts to macro uncertainty with a lag. However, in Online Appendix C4, we consider an alternative identification scheme where financial uncertainty affects macro uncertainty contemporaneously while macro uncertainty affects financial uncertainty with a lag. Using this alternative identification scheme, our overarching findings remain unchanged.

Figure 4. Uncertainty estimates for different model sizes: posterior medians of the macro ($e^{\frac{1}{2}h_{m,t}}$) and financial uncertainty ($e^{\frac{1}{2}h_{f,t}}$). Gray shading indicates NBER recession periods.

the results obtained are robust to this change.[5] Models 4 and 6 should produce the same results; we include both just to confirm this empirically.

Determining the classification of the federal funds rate, the S&P 500 and credit spread is challenging. CCM suggest that as the instrument of monetary policy the federal funds rate should be treated as a macro variable although studies as recent as Redl (2020) include it in the financial block. For the S&P 500 and credit spread "the distinction between macro and finance is admittedly less clear" (CCM, p. 805) but ultimately CCM place them in the financial block. Consequently, in Models 3–6 we treat the federal funds rate, credit spread and S&P 500 as unclassified variables. We also include data on an additional 13 unclassified variables spanning: money supply, credit, house prices, interest rates and exchange rates.

Over the next three subsections, we consider the importance of model size, order-invariance and time-varying classification. For brevity, we will generally focus on Models 1–4, however, results from Models 5 and 6 are presented in Section 4.3 as well as in Online Appendix C2. All results presented are based on producing 15,000 draws discarding the first 5000 as burn-in. It takes approximately 10 (30) hr to estimate each 30 (43) variable

model.[6] Throughout, we report cumulated impulse response functions.[7]
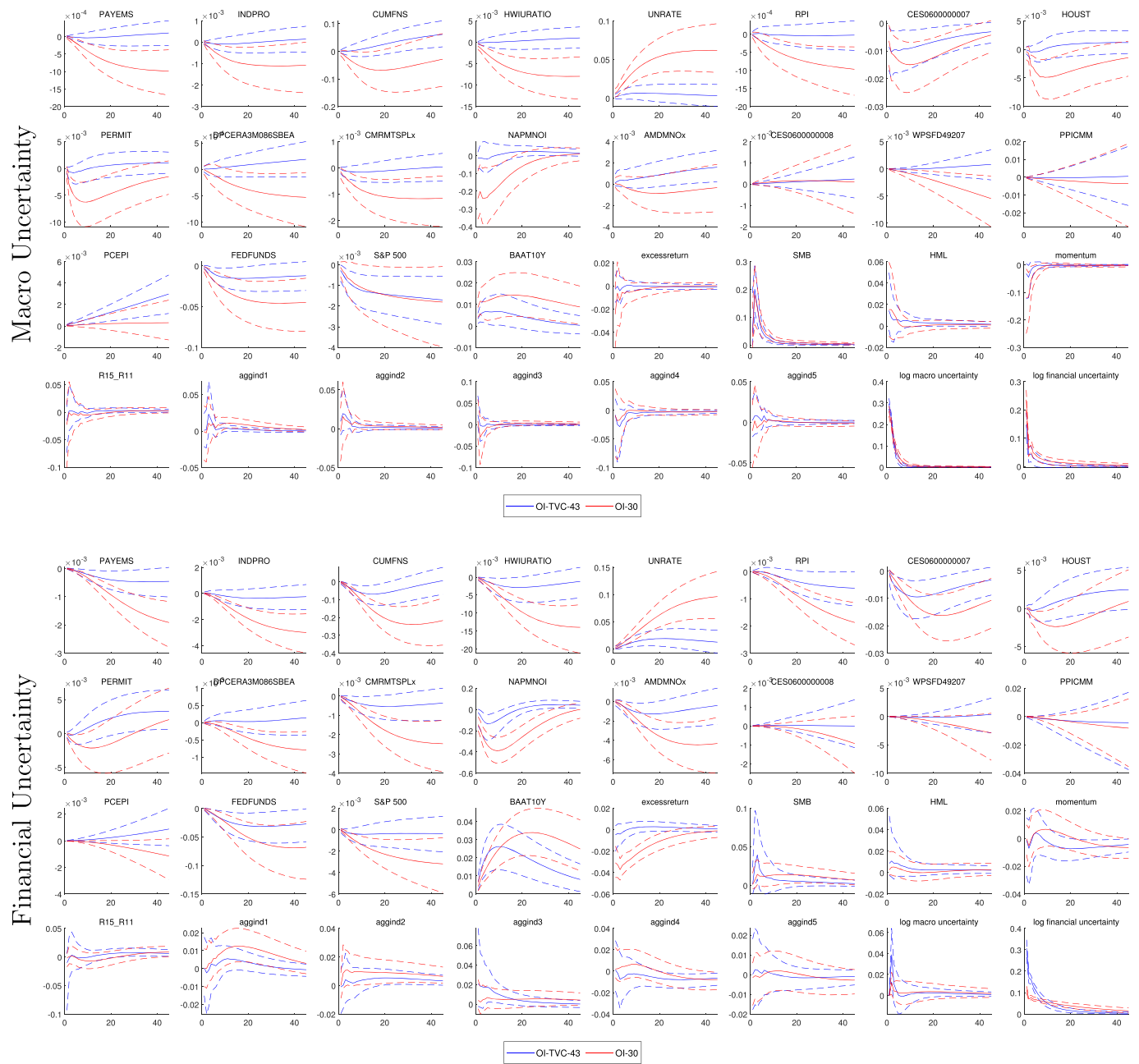
4.2. The Importance of Model Size

To investigate the role of model size, we begin by contrasting results from our smaller models, CCM-30 and OI-30, and our large order invariant model, OI-TVC-43. Our uncertainty estimates are shown in Figure 4. We can see that the broad trends observed are similar across our two smaller models and OI-TVC-43. This suggests that when estimating macro and financial uncertainty omitted variables bias is not as severe as the case considered in our simulation study. There is, however, evidence to suggest that a larger information set is required to capture the key features of certain crisis periods. For example, the coronavirus pandemic was a crisis that was not financial in nature. Instead, there was considerable disruption to supply, demand and productivity (Lin, Londono, and Ma 2022). We would expect this to manifest as macro rather than financial uncertainty. Smaller 30 variable models suggests that macro and financial uncertainty spiked to a similar degree during the pandemic. However, OI-TVC-43 provides a more intuitive result: macro uncertainty surged during the pandemic dwarfing the increase in financial uncertainty.

We now contrast impulse responses in Figure 5, focusing on our small and large order-invariant specifications, OI-30 and OI-

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[5]For Models 5 and 6, the reduced-form errors of the common log-volatilities are decomposed as $\epsilon_t^h = \mathbf{L}\mathbf{e}_t^h$ where the structural uncertainty shocks are given by $\mathbf{e}_t^h = (e_{f,t}^h, e_{m,t}^h)'$. Thus, the financial uncertainty shock is ordered before the macro uncertainty shock, and $\mathbf{L}$ is an upper triangular matrix with $\mathbf{L}\mathbf{L}' = \Sigma_h$. This is consistent with our identification assumption that macro uncertainty has a contemporaneous impact on financial uncertainty while financial uncertainty reacts to macro uncertainty with a lag.

[6]We use MATLAB on a desktop with AMD Core Ryzen 9 @340GHz processor.

[7]Specifically, using the standard deviations obtained from standardizing our data before estimation, we scale the response of each variable. We then cumulate responses to obtain the response in levels or log levels.

Figure 5. Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals of selected variables for OI-30 and OI-TVC-43.

TVC-43. If we first consider OI-30, we find that both macro and financial uncertainty adversely affect the economy which is consistent with CCM's findings. In response to a macro uncertainty shock, there is a decline in economic activity, little movement in prices and a subsequent monetary policy easing. The response of financial indicators is muted apart from the credit spread. A financial uncertainty shock has similar consequences but affects financial indicators such as the credit spread and excess returns more strongly.

Our larger model, OI-TVC-43, however, yields different results. Macro uncertainty has little effect on the economy with most credible intervals spanning zero. In contrast, a financial uncertainty shock still has nonzero, adverse effects on indicators of economic activity such as earnings, employment, unemployment and new orders. The credit spread also reacts

and monetary policy eases. In Figure 11 (Online Appendix C), we also plot the differences in the responses obtained from OI-30 and OI-TVC-43 and the corresponding 70% credible interval. Our results show that for many variables the 70% credible intervals exclude zero for most horizons. Thus, it is evident that the size of the model has a substantial influence on estimation results and responses to both macro and financial uncertainty shocks.

Turning to historical decompositions (HDs) and forecast error variance decompositions (FEVDs) from OI-TVC-43 (see Online Appendix C2, Figures 14 and 15), we have two sets of findings.[8] First, consistent with CCM, our FEVDs and HDs both indicate that uncertainty shocks play a small role in explaining

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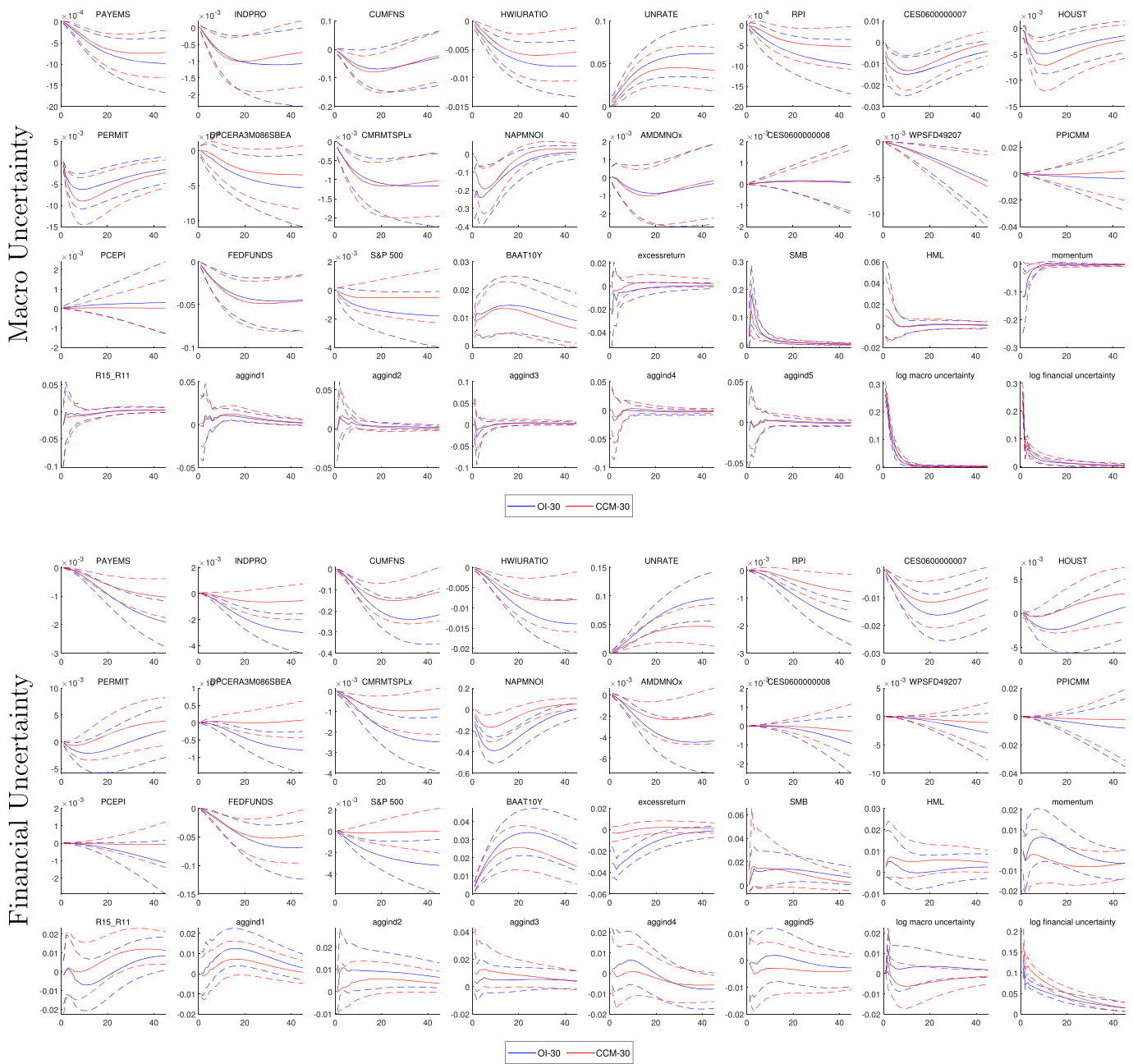[8] See Online Appendix A6 for details of how to compute the HDs and FEVDs.

Figure 6. Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals for OI-30 and CCM-30.
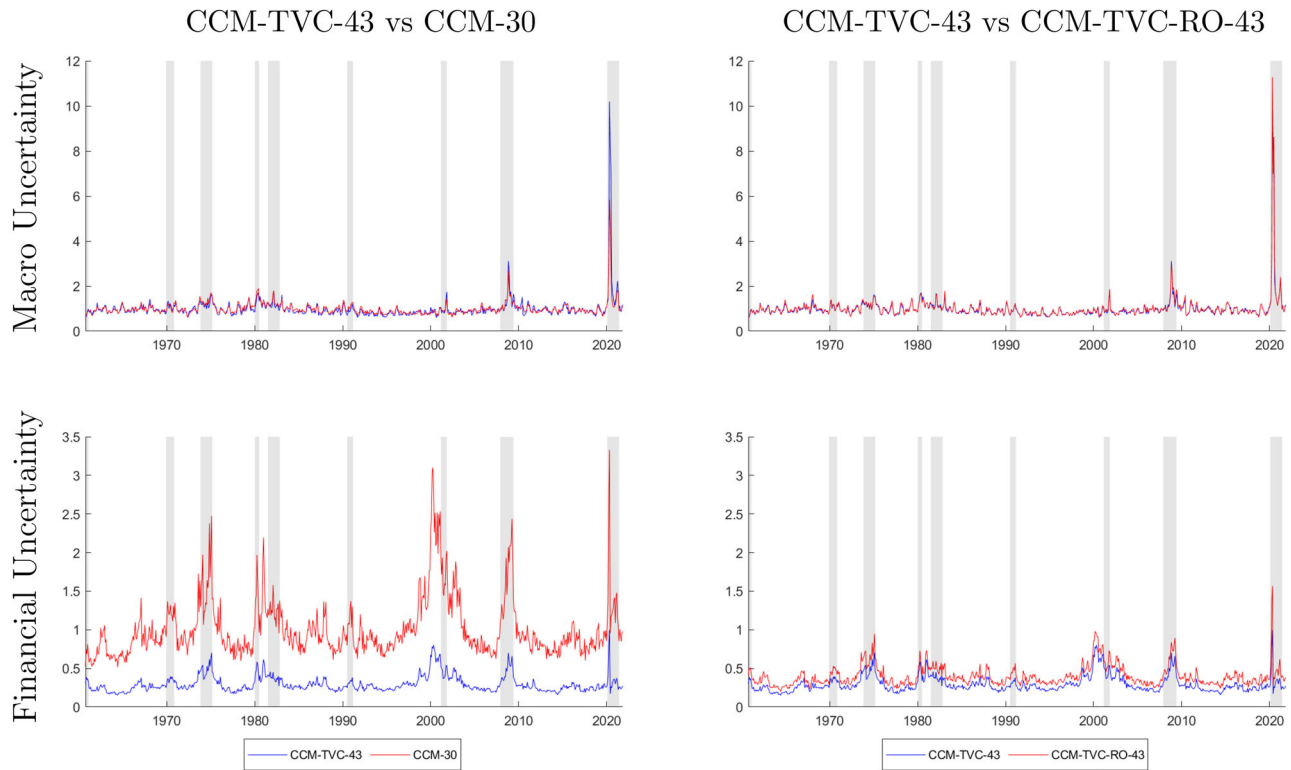
fluctuations in economic activity. As expected, studies deploying small-scale VARs including Jurado, Ludvigson, and Ng (2015) and LMN tend to detect larger contributions since there are fewer total variables in the system.

Second, if we consider the relative importance of macro and financial uncertainty, our HDs do not uncover a systematic pattern. However, our FEVDs indicate that financial uncertainty plays a larger role in explaining variation in headline indicators such as industrial production, unemployment, new orders, the federal funds rate and credit spread. This aligns with our impulse responses which suggest financial uncertainty has a more pronounced effect on economic activity.

Our results are supported by our simulation study which demonstrated that in smaller models with omitted variables bias we are more likely to find that impulse responses are large

when they are in fact near zero. In practice, when using OI-30, this results in both macro and financial uncertainty having adverse effects. However, OI-TVC-43 is less likely to suffer from omitted variables. Using this larger model we show that financial not macro uncertainty has an adverse impact on the economy. This aligns with a growing body of literature including LMN. However, LMN find that macro uncertainty has a positive effect on output in the short-run in line with "growth options" theories. This could be induced by omitted variables bias. Inclusion of other key variables in their SVAR such as the federal funds rate could eliminate this puzzle. Similarly, relative to OI-TVC-43, OI-30 as well as other studies using small SVARs may overstate the effects of financial uncertainty due to omitted variables bias. Together, our findings underscore the advantage of using our OI-SVMVAR which can accommodate a large dataset.

Figure 7. Uncertainty estimates for order-dependent models: posterior medians of macro ($e^{\frac{1}{2}h_{m,t}}$) and financial uncertainty ($e^{\frac{1}{2}h_{f,t}}$). Gray shading indicates NBER recession periods.

4.3. The Importance of Order Invariance

Having revealed the dominant role played by financial uncertainty in affecting the economy, we now assess the severity of order dependence issues in models of different sizes. We first contrast results obtained from CCM-30 and its order invariant counterpart OI-30. We saw in Figure 4 that they produce similar measures of uncertainty. Figure 6 also shows that the overarching results are similar but the effects of financial uncertainty are more pronounced if using OI-30. Plotting the difference in the responses and corresponding 70% credible interval (Online Appendix C, Figure 12) reveals that some differences are nonzero—see, for instance, industrial production, new orders, the stock price and excess returns. Thus, order dependence issues are present but less severe when we use relatively smaller models.

We next compare uncertainty estimates from our two order-dependent models, CCM-TVC-43 and CCM-30 in Figure 7 (left panel). In doing so, we indirectly compare OI-TVC-43 and CCM-TVC-43 since uncertainty estimates produced by CCM-30, OI-30 and OI-TVC-43 have similar trends. In our simulation study, the true impact matrix $\mathbf{B}_0$ was a full nonzero matrix. In our empirical work, $\mathbf{B}_0$ may be more sparse leading to slightly different findings. Nonetheless, when using a large order-dependent specification, our financial uncertainty measure is smaller in magnitude relative to CCM-30, OI-30, and OI-TVC-43. In CCM-TVC-43, macro variables are ordered before financial variables thus variation in financial variables is explained by the volatility of both financial and macro variables. Our findings illustrate that order dependence issues are more acute in CCM-TVC-43 than CCM-30.

Our simulation study demonstrated that when the true $\mathbf{B}_0$ is not lower triangular, adopting this parameterization can severely distort impulses. If we consider Figure 8, CCM-TVC-43 incorrectly detects that financial uncertainty does not have adverse effects on the economy. Additionally, if we contrast OI-TVC-43 and CCM-TVC-43 (Figure 13, Online Appendix C), there are nonzero differences in the responses of variables such as unemployment, new orders, the federal funds rate and credit spread to financial uncertainty shocks.

To investigate further, we consider whether results from our larger models depend on the way variables are ordered. Figure 7 (right hand panel) shows that the magnitude of our financial uncertainty measure increases if we reverse the ordering of variables in CCM-TVC-43 and consider CCM-TVC-RO-43. This is because variation in financial variables is now explained by the volatility of financial variables but not macro variables. Changing the variable ordering, however, simply introduces a new form of order dependence and CCM-TVC-RO-43 still yields different uncertainty estimates from OI-TVC-43.

Turning to Figure 8, these ordering issues have important repercussions. While CCM-TVC-43 detects that financial uncertainty does not have adverse effects on the economy, if we reverse the variable ordering and consider CCM-TVC-RO-43 the results suggest that financial uncertainty has nonzero effects on several variables (Figure 16, Online Appendix C). In contrast, by adopting our order-invariant specification, we obtain the same results (subject to MCMC approximation error) even when we reverse the variable ordering (Figure 17, Online Appendix C). Thus, an order-invariant approach is required to obtain robust results in larger models.

Figure 8. Impulse responses for one standard deviation uncertainty shocks: posterior medians and 70% credible intervals of selected variables for CCM-TVC-43 and CCM-30.
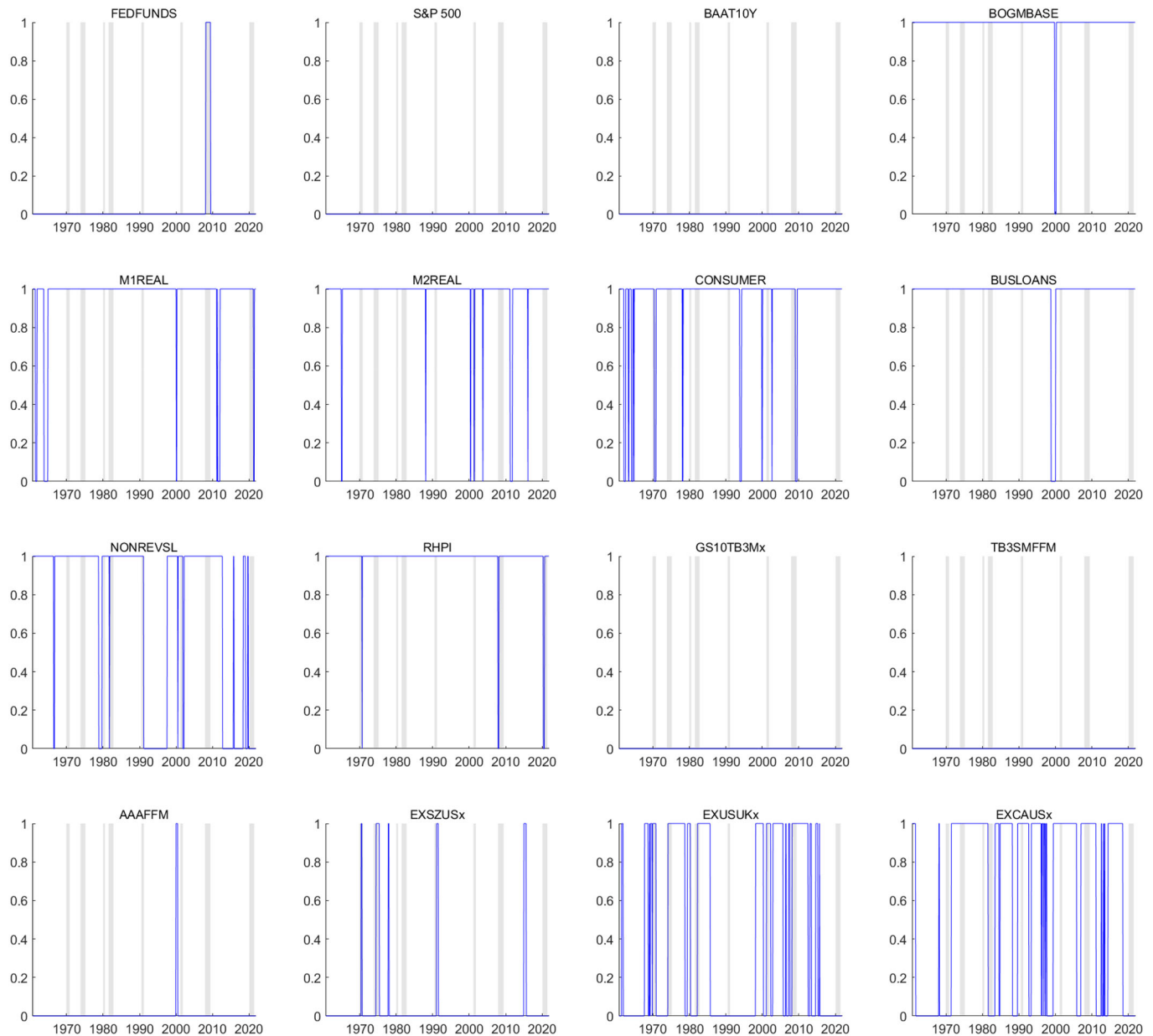
4.4. The Importance of Time-Varying Classification

In addition to developing a novel algorithm to estimate large OI-SVMVARs, another contribution of our article lies in the treatment of unclassified variables. In the OI-TVC-43 we treat 16 variables as unclassified with the algorithm determining whether each of these variables is assigned to the macro block or the financial block at each point in time. Figure 9 reports the estimated classification obtained from OI-TVC-43 (see Figure 18 in Online Appendix C for the posterior probabilities). If the posterior probability of the indicator variable associated with unclassified variable i at time t is greater than 0.5, $p(s_{i,t} = m|\mathbf{y}) > 0.5$, then unclassified variable i is classified as a macro variable at time t and has a value of one in Figure 9. Otherwise, the variable is classified as financial and has a value of zero in Figure 9. We can see that the algorithm can effectively discern

between variables which either have constant classification, limited regime switching or considerable regime switching.

Our algorithm has detected that similar variables are grouped in the same block. For example, all interest rate spreads are typically classified as financial variables together with the stock price and credit spread. This seems intuitive since such variables are commonly included in indices of financial conditions (see Hatzius et al. 2010). The federal funds rate is also typically classified as financial by our algorithm. Interestingly, all these variables either have constant classification or regime switching confined to a single crisis period. With the stock price, credit spread and federal funds rate all typically classified as financial variables, our findings differ from classification schemes selected by other studies such as CCM and LMN.

Turning to our macro variables, our algorithm again groups similar variables together. All three monetary aggregates, busi-

Figure 9. Estimated time-varying classification results obtained from OI-TVC-43. A value of zero (one) indicates that a variable is classified as a financial (macro) variable. Gray shading shows NBER recession periods.

ness loans, and consumer loans and nonrevolving consumer credit are, on average, classified as macro variables. The house price index is also typically assigned to the macro block of variables. That said, none of these variables have constant classification instead exhibiting limited to moderate regime switching. Exchange rates, whose fluctuations are very difficult to predict (Rossi 2013), are the one class of indicator which tend to alternate between the macro and financial block more rapidly.

If we consider the timing of regime switches, we see that changes often coincide with crisis periods. For instance, the federal funds rate is briefly assigned to the macro block as it sharply declines and hits the zero lower bound during the great recession. Similarly, real house prices are assigned to the financial block during three crises including the global financial crisis which was triggered, in part, by the subprime mortgage crash. To give another example, several variables such as business loans,

nonrevolving consumer credit and the Moody's bond yield relative to the federal funds rate change classification during the dotcom crash and the short recession which ensued.

5. Conclusion

The SVMVAR has emerged as an attractive model, allowing for joint estimation of macroeconomic and financial uncertainty and their impact on the economy. However, SVMVARs are difficult to estimate and computationally demanding. To overcome these challenges, existing approaches make a number of assumptions. First, a lower triangular parameterization is used for the reduced-form error covariance matrix, improving computational efficiency but introducing an order dependence issue. Second, a limit is placed on the number of variables which can be included in the model. Third, the researcher must use

expert judgment to classify each variable as macroeconomic or financial prior to estimation. Each of these assumptions may have an impact on empirical results.

The models and posterior simulation methods developed in this article avoid making such assumptions. We introduce a new OI-SVMVAR model which relaxes the lower triangularity assumption, allowing for order-invariant inference. As existing order-invariant algorithms cannot be applied to our SVMVAR, we derive a novel MCMC algorithm. Having accounted for order dependence issues, we adopt the efficient approach developed in CHKP for sampling the common SVs. Thus, we can robustly estimate very large SVMVARs. In our SVMVAR, we also allow the classification of some variables to be uncertain. The algorithm can then decide whether to classify them as macro or financial variables in a time-varying fashion.

We compare our very large OI-SVMVAR with unclassified variables with alternative SVMVARs, investigating the importance of model size, order-invariance and time-varying classification. We show that smaller SVMVARs can overstate the effects of uncertainty. Using our large SVMVAR, however, it becomes clear that financial not macro uncertainty adversely affects the economy, aligning with LMN. Importantly, though, large order-dependent SVMVARs produce measures of uncertainty which depend on the way variables are ordered. In practice, this leads to results which imply that financial uncertainty does not impact the economy. Thus, it is critical to use an order-invariant specification like the one proposed in this article. We also find that when unclassified variables change classification, changes often occur during crises. Allowing for time-varying classification ensures variables are assigned to the appropriate block when our uncertainty measures and their impacts are estimated.

Supplementary Materials

This supplement consists of the Technical Appendix, Data Appendix, and supplementary figures (Online Appendix.pdf).

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