Premixed flames
Homework
Combustion

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1 Introduction

In the present work we shall study the spatial evolution of fuel concentration and temperature within a laminar premixed flame. We shall consider a laminar premixed flame governed by the following equation:

with $\mathcal{C} = \frac{C_F}{C_{F_0}}$ being the dimensionless fuel concentration. The boundary conditions are set:

$$\begin{aligned} x \to -\infty: \quad \mathcal{C} = 1 \\ x \to \infty: \quad \mathcal{C} = 0, \quad \frac{d\mathcal{C}}{dx} = 0 \end{aligned} \tag{1}$$

where u_0 is yet to be determined as the eigenvalue of the system. Scaling the variables with the characteristic deflagration thickness, δ, the following dimensionless parametres appear and define the problem:

$$Pe = \frac{u_0 \delta}{D_F} \qquad Da = \frac{B\delta^2}{D_F} e^{-\frac{E}{R^0 T_\infty}} \tag{2}$$

corresponding to the Peclet and Damköhler numbers respectively.

With the use of dimensionless heat release $Q = \frac{T_\infty}{T_0} - 1$ and streamwise coordinate $\xi = \frac{x}{\delta}$, re-derive the goberning equation in the dimensionless form.

Starting from the equation given in (1) and the dimensionless parameters, we have:

$$\begin{aligned} dx = \delta d\xi \\ \frac{T_0}{T_\infty} = \frac{1}{1+Q} \end{aligned} \tag{3}$$

Introducing the relation in the governing equation, we have:

$$\frac{u_0}{\delta}\frac{d\mathcal{C}}{dx} - \frac{D_F}{\delta^2}\frac{d^2\mathcal{C}}{dx^2} = -B\mathcal{C}e^{-\frac{E}{R^0 T_\infty}} exp\left[-\beta \frac{\mathcal{C}}{1-(1-\frac{1}{1+Q})\mathcal{C}}\right] \tag{4}$$

Multiplying both sides of the equation by δ^2, dividing by D_F and rearranging the terms inside the exponential, we have:

$$\frac{u_0\delta}{D_F}\frac{d\mathcal{C}}{dx} - \frac{d^2\mathcal{C}}{dx^2} = -\frac{B\delta^2}{D_F}e^{-\frac{E}{R^0T_\infty}} exp\left[-\beta\frac{\mathcal{C}(1+Q)}{1+Q(1-\mathcal{C})}\right] \tag{5}$$

which finally yields:

$$\frac{d^2\mathcal{C}}{dx^2} - Pe\frac{d\mathcal{C}}{dx} = Da\ \mathcal{C}exp\left[-\beta\frac{\mathcal{C}(1+Q)}{1+Q(1-\mathcal{C})}\right] \tag{6}$$

Integrate equation 6 knowing that $Q = 1$ and $Da = 1$ with the given boundary conditions using appropriate numerical methods. Compute the value of Pe for $\beta = 5, 20, 25$.

Having a close look to the dimensionless governing equation, it quickly stands out that it is a second order ODE. Thus, two boundary conditions should be enough for the problem. However, three conditions are given by the problem formulation. The third boundary condition will be used to iteratively solve the value of the Peclet number, which is also unknown,

The problem was integrated via Matlab, using Runge-Kutta methods with an arbitrary starting Peclet. A previous stimation of the Peclet number can be given using the expressions given in the slide 8.28 provided in class.

$$Pe = \frac{u_0\delta}{D_F} \approx \frac{\sqrt{\frac{D_F}{B}e^{\frac{E}{2R^0T_\infty}}}\sqrt{D_F B}e^{-\frac{E}{2R^0T_\infty}}}{D_F} = 1 \tag{7}$$

Which gives an estimation of the order of magnitude of the Peclet number. Since the Peclet number is of unity order of magnitud, we will start from $Pe = 0$ and increase its value by 0.01 for each iteration. Once an approximate value is obtained, a finer parametric sweep is going to be carried.

In order to discern wether a value of the Peclet is valid or not, the integration will start in the positive side of the spatial coordinate and will integrate backwards. This will enable to change the value of Pe as:

$$abs(C(-\infty) - 1) < \epsilon \tag{8}$$

with ϵ being an arbitrarily small parameter.

The results obtained in this problem are:

$$\begin{aligned} \beta = 5 &\rightarrow Pe = 0.305 \\ \beta = 20 &\rightarrow Pe = 0.0704 \\ \beta = 25 &\rightarrow Pe = 0.0563 \end{aligned} \tag{9}$$

The behaviour of the Peclet number in respectof β has been studied, showing an asimptotic behaviour both in β tending to zero, where Pe will tend to infinity, and in β tending to infinity, where Pe will tend to zero. Results are shown in figure 1.

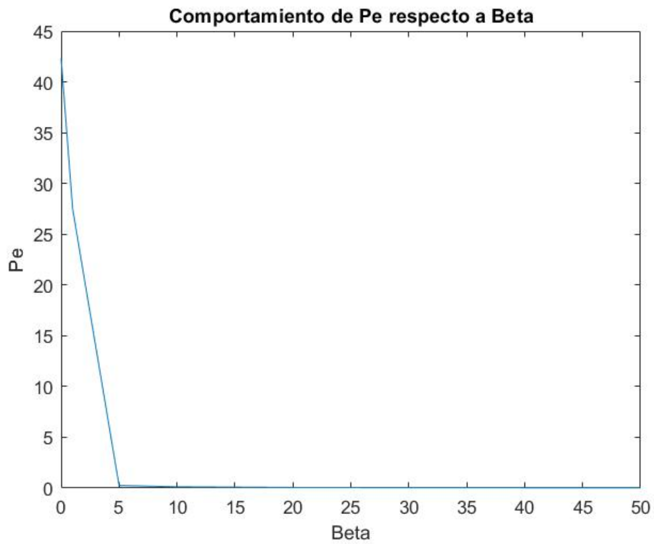


Figure 1: β vs Peclet behaviour

After having solved the ODE, results will be shown afterwards, we can see that for higher values of β and, thus, lower values of Pe, the fall of the fuel concentration is more abrupt and displaced to the right side of the problem. For higher valued of β, the descent of the concentration is smoother, and starts earlier.

Defining the dimensionless reaction rate as

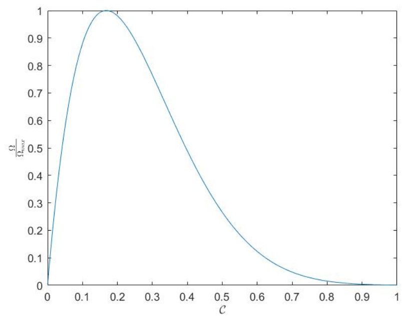
$$\Omega = \mathcal{C}exp\left[-\beta\frac{\mathcal{C}(1+Q)}{1+Q(1-\mathcal{C})}\right] \tag{10}$$

determine the value of C at which Ω peaks and the value of Ωmax for the parameters selected in the previous item.

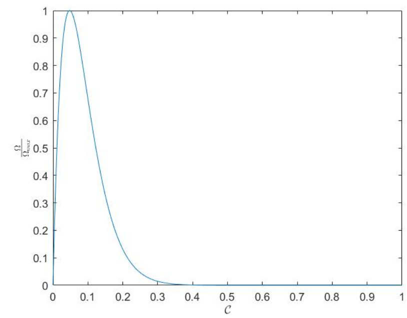
For this section is not necessary to have solved the problem, since it is known beforehand that the solution is delimited within 0 and 1 so, $\mathcal{C}$ could be evenly spaced and introduced in the equation above. However, since the governing equation has already been solved, the data obtained in the previous section will be used.

The results obtained are shown in figure 2, a more peak can be found when increasing β. However, we should keep in mind that the values shown are normalised. Thus, even the more acute peaks may be smaller in absolute terms.

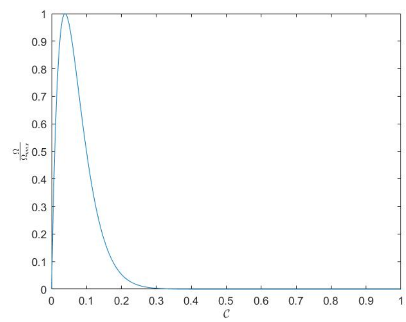
$$\begin{aligned} \beta = 5 &\rightarrow \Omega_{max} = 0.0672 \\ \beta = 20 &\rightarrow \Omega_{max} = 0.0529 \\ \beta = 25 &\rightarrow \Omega_{max} = 0.0144 \end{aligned} \tag{11}$$



(a) $\beta = 5$



(b) $\beta = 20$



(c) $\beta = 25$

Figure 2: Dimensionless reaction rate for different values of β.

Ω peaks for each β at:

$$\begin{aligned} \beta = 5 &\rightarrow \mathcal{C}(\Omega_{max}) = 0.1578 \\ \beta = 20 &\rightarrow \mathcal{C}(\Omega_{max}) = 0.018 \\ \beta = 25 &\rightarrow \mathcal{C}(\Omega_{max}) = 0.04 \end{aligned} \tag{12}$$

Plot the dimensionless variables for the fuel concentration $\mathcal{C}$, temperature $\mathcal{T} = \frac{T}{T_0}$ and the scaled reaction rate $\frac{\Omega}{\Omega_{max}}$.

Last but not least, the variables of interest will be overlaid so spatial variations of each can be better understood. But first, an expression for the dimensionless temperature should be proposed.

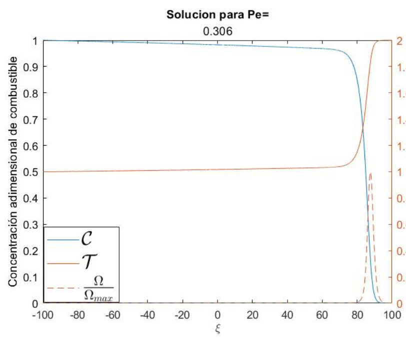
Assuming Lewis number has a value of unity, which has been done throughout the entire exercise, the following expression can be derived:

$$\frac{T_\infty - T}{T_\infty - T_0} = \mathcal{C} \tag{13}$$

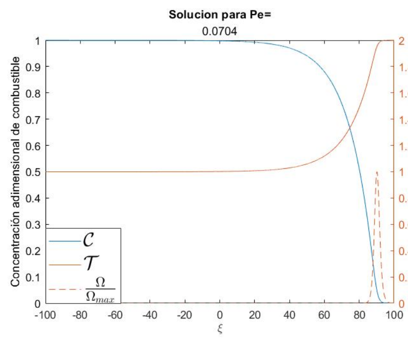
Which, properly rearranging the terms and using the expression for the dimensionless heat given above, the expression can be transformed into:

$$\mathcal{T} = \frac{T}{T_0} = (1 - \mathcal{C})Q + 1 \tag{14}$$

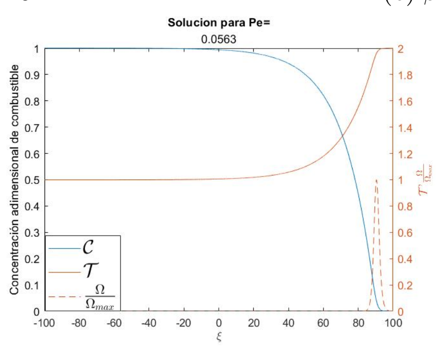
In figure 3 an overlay of the dimensionless temperature, the dimensionless fuel concentration and the scaled reaction rate are represented. The different aspects of the influence of β in the different variables that have already been assessed can be graphically seen there. A similar parametric sweep could be carried for other parameters such as the dimensionless heat release or the Damköhler number.



(a) $\beta = 5$



(b) $\beta = 20$



(c) $\beta = 25$

Figure 3: Solution for different β.