Embedded Control Systems Homework 1 group 20

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March 2015

1 Continuous to Discrete Conversion

The given Transfer functions are illustrated in the figure below;

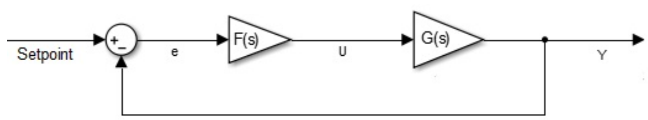

Figure 1: Transfer functions of the system.

1.1 PID

The PID given is on standard form

$$F(s) = K_p + \frac{K_i}{s} + K_d \frac{s}{1 + T_f s}. \tag{1}$$

discretizing the transfer function in 1 using Tustin method, where $s = \frac{2(z-1)}{T_s(z+1)}$, it can be divided into three parts; U_1 = P-part, U_2 = I-Part and U_3 = D-part. By calculating the response for each part individually and summing the three parts up in the end, it gives U. $e(s) = setpoint - y$.

U_1:

$$U_1(s) = K_p * e(s) \quad -> \quad U_1[k] = K_p * e[k] \tag{2}$$

U_2:

$$U_2(s) = \frac{K_i}{s} e(s) \quad -> \quad U_2[k] = U_2[k-1] + \frac{K_i h}{2} e[k] + \frac{K_i h}{2} e[k-1] \tag{3}$$

U_3:

$$U_3(s) = K_d \frac{s}{1 + T_f s} e(s) \tag{4}$$

which after discretization looks like this

$$U_3[k] = \frac{-(-T_f + h)U_3[k-2] - (2h)U_3[k-1] - 2K_d e[k-2] + 2K_d e[k]}{2T_f + h} \quad (5)$$

After calculation, the three parts are summed together as follows

$$U[k] = U_1[k] + U_2[k] + U_3[k] \quad (6)$$

1.2 Plant

The plant given has the following form

$$G(s) = \frac{K}{1 + sT} \quad (7)$$

and it should be discretized using Zero-Order Hold.

The equation for the output y in Laplace-domain looks like this

$$Y(s) = G(s)U(s) \quad (8)$$

and solving for the next value of y and inverse transform to time domain gives the following equation

$$\dot{y}(t) = \underbrace{-\frac{1}{T}}_{A_c} y(t) + \underbrace{\frac{K}{T}}_{B_c} u(t) \quad (9)$$

which can be interpreted as a state-space representation of the plant in continuous time. With $A_c = -\frac{1}{T}$, $B_c = \frac{K}{T}$, $C_c = 1$ and $D_c = 0$.

Discretizing the state-space model using ZOH, gives us the following equation

$$X[k+h] = e^{A_c h} X[k] + (\int_0^h e^{A_c h} B_c \, \partial h) U[k] \quad (10)$$

And after integration, the following equation is received

$$X[k+h] = e^{-\frac{1}{T}h} X[k] + (K - Ke^{-\frac{1}{T}h}) U[k] \quad (11)$$

2 C Program

The equations 6 and 11 are implemented in C program. The program prompts for the file name which has the setpoint values. Once the file name is typed, the program reads the values from each line of the *.txt* file. Output for each input is calculated and written in a file named *output.txt*. The values for k_p, k_i, k_d, T_f, T, h can be changed inside the C program, if required.

3 Conclusion

When testing the program with $K_p = 4$, $K_i = 0.5$, $K_d = 2$, $T_f = 4$, $K = 2$, $T = 3$ and $h = 0.1$, the output looks like in Table 1.

setpoint	output
0	0
0	0
0	0
0	0
0	0
1	0.30
1	0.50
1	0.62
1	0.71
1	0.76
0	0.51
0	0.34
0	0.24
0	0.17
0	0.12

Table 1: Setpoints and outputvalues

When the input is low, the output is also low. When the input increases to a higher value, the output follows but it won't reach the setpoint value in time and it doesn't overshoot 1. Output decreases as the input decreases and won't reach 0 during the given timeframe. Thus the system works as intended.