Singapore Polytechnic **MS0240**

AY20xx/20xx SEMESTER ONE CLASS TEST (MOCK)

School of Computing (SOC)

Diploma in Applied AI and Analytics (DAAA)

2nd Year Full Time

MATHEMATICS FOR AI

Time allowed: 1 hour

Instructions:

1. The Singapore Polytechnic Examination rules are to be complied with.
2. This paper consists of **9** printed pages, inclusive of cover page.
3. All solutions are to be written in the space provided in this booklet.
4. Unless otherwise stated, all answers given should be correct to at least **three** significant figures.
5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply will result in loss of marks.

Student Name	:			
Student ID	:		Module Class	: DAAA/FT/2A/

Question Answered	Marks
A1–A5 (10 marks)	
B1 (10 marks)	
B2 (8 marks)	
B3 (14 marks)	
B4 (8 marks)	
TOTAL (50 marks)	

SECTION A (10 marks)

Answer ALL 5 questions. Each question carries 2 marks. No marks will be deducted for incorrect answers. Write your selected option for each question in the following table:

Question A1	Question A2	Question A3	Question A4	Question A5

1. Given the function $f(x) = \sin 2x$, for $n = 0,1,\cdots$, the $(4n+2)$-th order derivative of f, $f^{(4n+2)}(x) =$ _____.

   (a) $2^{4n}\sin 2x$ (b) $2^{4n+1}\cos 2x$

   (c) $-2^{4n+2}\sin 2x$ (d) $-2^{4n+3}\cos 2x$

2. What is the output of the given code from Jupyter Notebook?

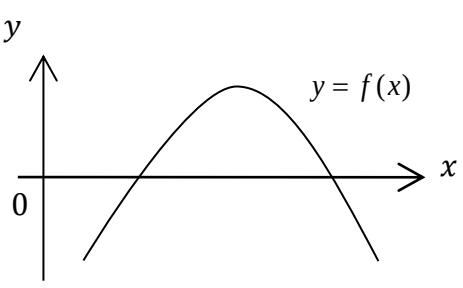
```
import sympy as sp
x = sp.Symbol('x')
sp.diff(10*sp.log((2*x+2),2), x)
```

   (a) $\dfrac{10}{2x+2}$ (b) $\dfrac{20}{2x+2}$

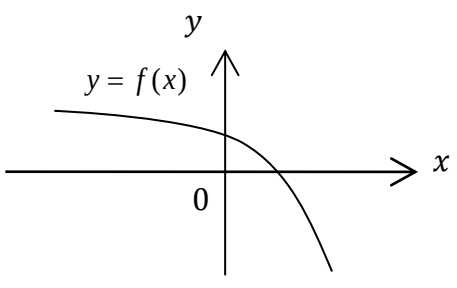
   (c) $\dfrac{20}{(2x+2)\ln(2)}$ (d) $\dfrac{20}{(2x+2)\log(2)}$

3. Which of the following graphs best represents the function $f(x)$, where $f'(x) > 0$ and $f''(x) < 0$ for all values of x?

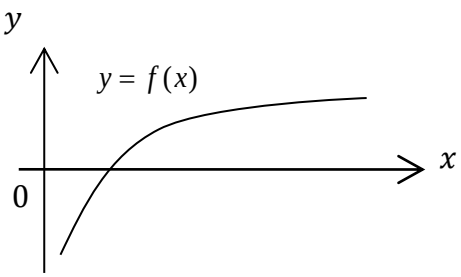
   (a)



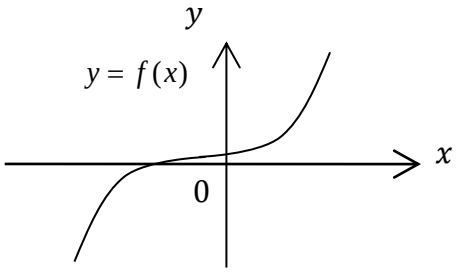
   (b)



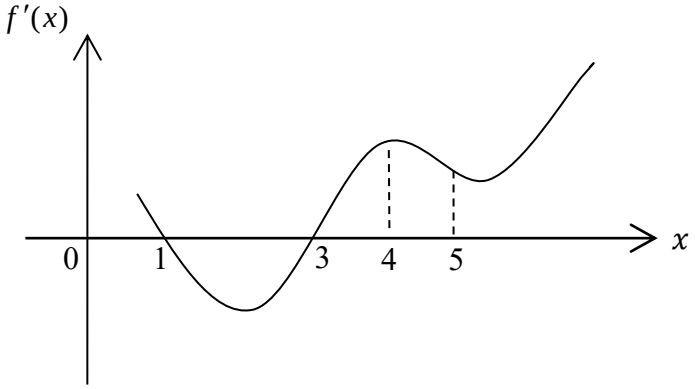
(c)



(d)



4. The graph of the **derivative** of the function $f(x)$ is shown below. Which of the following statements is true?



(a) f is decreasing at $x=1$.

(b) $f(3)$ is a local minimum.

(c) $f(4)$ is a local maximum.

(d) f is decreasing at $x=5$.

5. Which of the following is a good approximation of $f''(c)$, where c is a constant, given $f(x)=\sin(2x+2)$?

(a) $\dfrac{\sin(2c+2.001)-\sin(2c+1.999)}{0.002}$

(b) $\dfrac{\sin(2c+2.02)-\sin(2c+1.98)}{0.02}$

(c) $\dfrac{\cos(2c+2.002)-\cos(2c+1.998)}{0.002}$

(d) $\dfrac{\cos(2c+2.02)-\cos(2c+1.98)}{0.01}$

SECTION B (40 marks)

Answer ALL questions.

1. Find $\frac{dy}{dx}$ for the following expressions, simplifying your answers wherever possible:

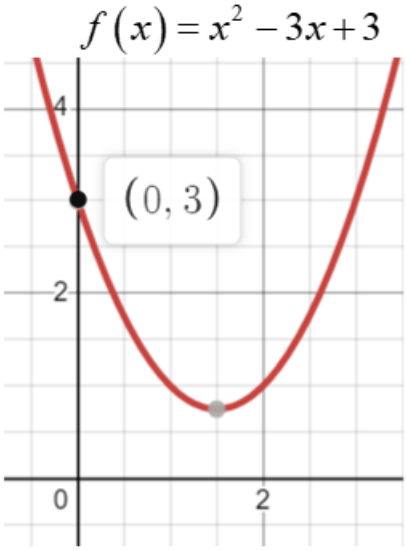
   (a) $y = 3x^5 \cos\left(x^2 + 1\right)$ (b) $y = \frac{\ln x}{2x^3}$ (10 marks)

2. Consider the graph of the function $y = e^{ax}$, where $a < 0$ is a constant.

   (a) Find the equation of the tangent line to this curve at any given point $x = b$.
   (b) Consider the gradient of the tangent line. Is the gradient decreasing, increasing or stationary with respect to b?

(8 marks)

3. (a) Use the gradient descent algorithm to estimate the minimum point of the curve $f(x) = x^2 - 3x + 3$. Given that the initial x, $x_0 = 0$, the learning rate is 0.4 and stopping criterion is when the absolute difference between two *x*-values is less than 0.05. (6 marks)



3. (b) The volume of a cylindrical tank with base radius r is 1000 m^3. The cost of making the cylindrical body is \$25 per square metre and the cost is thrice for the top and bottom of the tank. Show that the cost of making the tank is $C = \frac{50000}{r} + 150\pi r^2$. Find the radius *r* such that cost of making the tank is at a minimum.

(8 marks)

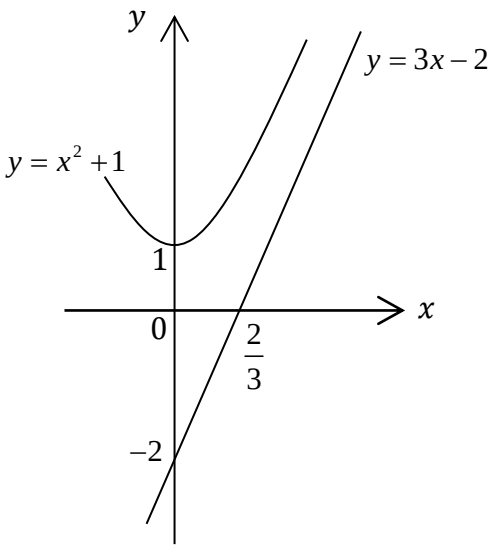
Given formula:

Surface area of a closed cylindrical tank of radius r and height h is $2\pi r^2 + 2\pi rh$

Volume of cylindrical tank of radius r and height h is $\pi r^2 h$

4. A car starts moving from the point $(0,1)$ along the graph $y = x^2 + 1$. At the same time, another car starts moving from the point $(0,-2)$ along the graph $y = 3x - 2$. The distances are measured in kilometres. Both cars are moving in a way that the speed to the right is 10 km/h. At what rate is the distance between the two cars changing at $x = 5$ km ? (Hint: $x^2 + 1 > 3x - 2$ for all x)

(8 marks)



Standard Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x \qquad \frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x \qquad \frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x \qquad \frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

Rules of Differentiation

Let $u \equiv u(x)$, $v \equiv v(x)$ and $y \equiv y(u)$

- Constant Multiple Rule $\frac{d}{dx}\left(ku\right) = k\frac{du}{dx}$
- Sum Rule $\frac{d}{dx}\left(u+v\right) = \frac{du}{dx} + \frac{dv}{dx}$
- Product Rule $\frac{d}{dx}\left(u+v\right) = \frac{du}{dx} + \frac{dv}{dx}$
- Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
- Chain Rule $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Numerical Differentiation

For a differentiable function *f(x), f'(a)* can be approximated by

- Symmetric difference quotient: $f'(a) \approx \frac{f(a+\epsilon) - f(a-\epsilon)}{2\epsilon}$
- Forward difference quotient: $f'(a) \approx \frac{f(a+\epsilon) - f(a)}{\epsilon}$
- Backward difference quotient: $f'(a) \approx \frac{f(a) - f(a-\epsilon)}{\epsilon}$

where $\epsilon > 0$ is a very small number.

END OF PAPER