

# Segmented Spacetime (SSZ)

A phi-geometric extension of GR with segment density  $\Xi_i$ , SSZ time dilation, and a no-free-parameters cosmology test scaffold

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**Status:** consolidated paper (merges 'Finales Paper' + 'Drei Ausblicke' and includes reproducibility tests)

**Abstract.** Segmented Spacetime (SSZ) is a falsifiable, phi-motivated extension of General Relativity (GR) formulated via a dimensionless segment-density field  $\Xi_i$  and an SSZ time-dilation factor  $D_{\text{SSZ}} = 1/(1+\Xi_i)$ . SSZ is engineered to reproduce GR in the weak-field regime (g1) and to introduce controlled, saturating deviations in the strong-field regime (g2) without divergences. This paper consolidates the current SSZ axioms, regime definitions, and anti-circularity rules, and it adds a no-free-parameters cosmology scaffold that turns any proposed effective  $\Xi_i(z)$  into immediate contradiction tests against (i) CMB acoustic-scale proxies, (ii) BBN expansion-rate proxies, and (iii) linear growth of structure. We also provide an explicit reproducibility and regression-test report for the included reference implementation (`ssz_cosmo.py` / `run_cosmo.py`) and note one edge-case bug ( $z_{\text{max}}=0$ ) plus a minimal fix.

**Keywords:** segmented spacetime; strong-field gravity; time dilation; phi geometry; falsifiability; cosmology scaffold; CMB; BBN; growth of structure

## 1. Motivation and scope

SSZ is a phenomenological gravity extension designed to stay extremely close to GR in weak fields while remaining falsifiable in strong fields. The central design constraints are: (a) explicit axioms and derived consequences, (b) no hidden free parameters per dataset, (c) regime clarity (g1 vs g2), and (d) reproducible computational predictions with residual-based validation rather than least-squares curve fitting.

This consolidated paper merges the core SSZ definitions (time dilation, segment density, saturation) with the practical next-step roadmap (fixing the regime transition  $r^*$ , cosmological inhomogeneity tests, and an operational g1/g2 classifier), and it appends the current numeric test report for the minimal cosmology scaffold shipped in the accompanying code files.

## 2. SSZ axioms, definitions, and regimes

### 2.1 Segment-density field $\Xi_i$

SSZ postulates a dimensionless scalar field  $\Xi_i$  that represents an effective 'segmentation density' of spacetime.  $\Xi_i$  is intended to be derived from regime formulas and fixed constants rather than tuned per object.

### 2.2 SSZ time dilation

SSZ defines a proper-time factor

$$D_{\text{SSZ}}(r) = 1 / (1 + \Xi_i(r))$$

so that local proper time is  $d\tau = D_{\text{SSZ}} dt$ . This is the primary SSZ input to redshift and clock-comparison observables.

### 2.3 Saturation (no divergence)

SSZ assumes  $\Xi_i$  saturates in the strong field:  $\Xi_i$  is bounded above by a theory-fixed maximum  $\Xi_{i,\text{max}}$  (often tied to the phi construction). This avoids singular divergences in  $D_{\text{SSZ}}$  and provides a controlled strong-field limit.

## 3. Regime formulas and the g1/g2 bridge

### 3.1 Weak-field (g1): GR limit

In g1, SSZ uses a GR-matching segment density proxy such as  $\Xi_{i,\text{weak}}(r) = r_s/(2r)$ . For  $\Xi_i \ll 1$ ,  $D_{\text{SSZ}} \approx 1 - \Xi_i$ , which matches GR behavior in the weak field to leading order.

### 3.2 Strong-field (g2): saturating segmentation

In g2, SSZ uses a saturating functional form (phi-motivated), e.g.  $\Xi_{i,\text{strong}}(r) = \min(1 - \exp(-\phi/r_s), \Xi_{i,\text{max}})$ . This yields a finite horizon value (e.g.  $\Xi_i(r_s) = 1 - \exp(-\phi) \approx 0.8017$ ) and a finite minimum dilation  $D_{\text{min}} = 1/(1 + \Xi_{i,\text{max}})$ .

### 3.3 Fixing the regime transition $r^*$

A central 'no-free-parameters' requirement is that the regime transition  $r^*$  is not a hand-chosen blending point. The recommended definition is an invariant equality condition (e.g.  $\Xi_{i,\text{weak}}(r^*) = \Xi_{i,\text{strong}}(r^*)$ ) or an equivalent observable-level equality, solved once and then frozen globally.

## 4. Anti-circularity and no-free-parameters rules

SSZ commits to an explicit anti-circularity protocol: **No free parameters per object**:  $\phi$ ,  $X_{i,\max}$ , regime formulas, and transition logic are global. **Matching points like  $r^*$** : determined once by an invariant criterion, not per dataset. **No least-squares fitting as core evidence**: use exact algebra where possible; validate via residuals and consistency checks. **Calibration vs validation separation**: any calibration step must be declared and excluded from hold-out validation.

## 5. Cosmology extension as a minimal falsification scaffold

### 5.1 Principle

The cosmology module is intentionally minimal: it accepts any effective  $X_i(z)$  and evaluates immediate contradiction tests against three standard pillars: CMB acoustic-scale proxies, BBN expansion-rate proxies, and linear growth of structure. The module explicitly separates two time conventions for how  $D_{SSZ}(z)$  maps into  $H(z)$ .

### 5.2 Two time conventions (A/B)

With  $d\tau = D_{SSZ}(z) dt$ , there are two consistent conventions depending on which time variable is taken as the cosmological 'clock':

**Mode 'divide'**:  $H_{SSZ} = H_{GR} / D_{SSZ} = H_{GR} (1+X_i)$

**Mode 'multiply'**:  $H_{SSZ} = H_{GR} \cdot D_{SSZ} = H_{GR} / (1+X_i)$

### 5.3 Minimal observables

The reference implementation computes: (i) a CMB acoustic-scale proxy via the sound horizon  $r_s(z^*)$  and angular diameter distance  $D_A(z^*)$ , (ii) a BBN proxy via the ratio  $H_{SSZ}/H_{GR}$  at  $z \approx 4 \times 10^9$ , and (iii) linear growth  $D(z)$  and  $f(z)$  via the standard growth ODE with  $E(z)$ .

## 6. Three next steps (roadmap) — from the 'Drei Ausblicke' working paper

This roadmap is included to reduce attack surface and increase falsifiability: **(1)  $r^*$  fixation as a physics constraint**: define  $r^*$  by an invariant condition (weak = strong) and derive constraints from it. **(2) Cosmology as an inhomogeneity problem**: treat SSZ signatures via a potential/lensing and runtime mapping rather than a homogeneous  $X_i(z)$  background. **(3) g1/g2 operationalization**: provide a deterministic regime classifier (indicators, thresholds, observable routing) with explicit test hooks.

## 7. Reproducibility and regression-test report (this release)

### 7.1 Files in this bundle

The following files were tested together: ssz\_cosmo.py, run\_cosmo.py, ssz\_cosmo\_results\_divide.csv, ssz\_cosmo\_results\_multiply.csv, ssz\_cosmo\_summary\_all.json, and the two reference plots growth\_both.png and Hratio\_both.png.

### 7.2 Deterministic reproduction

Re-running the reference pipeline reproduces the provided CSV outputs to machine precision.

Maximum absolute deviation (divide): 2.220e-16. Maximum absolute deviation (multiply): 1.110e-16. Identity checks (algebraic invariants): max |(Hratio\_div·Hratio\_mul)-1| = 2.220e-16; max |Hratio\_div - 1/D| = 2.220e-16; max |Hratio\_mul - D| = 2.220e-16.

### 7.3 Summary metrics (CMB/BBN)

Metric	divide	multiply
CMB theta_GR (rad)	11.3467	11.3467
CMB theta_SSZ (rad)	11.3468	11.3467
CMB theta ratio SSZ/GR	1	0.999999
CMB r_s ratio SSZ/GR	0.999988	1.00001
CMB D_A ratio SSZ/GR	0.999988	1.00001
BBN H ratio at z=4e9	1.00001	0.99999

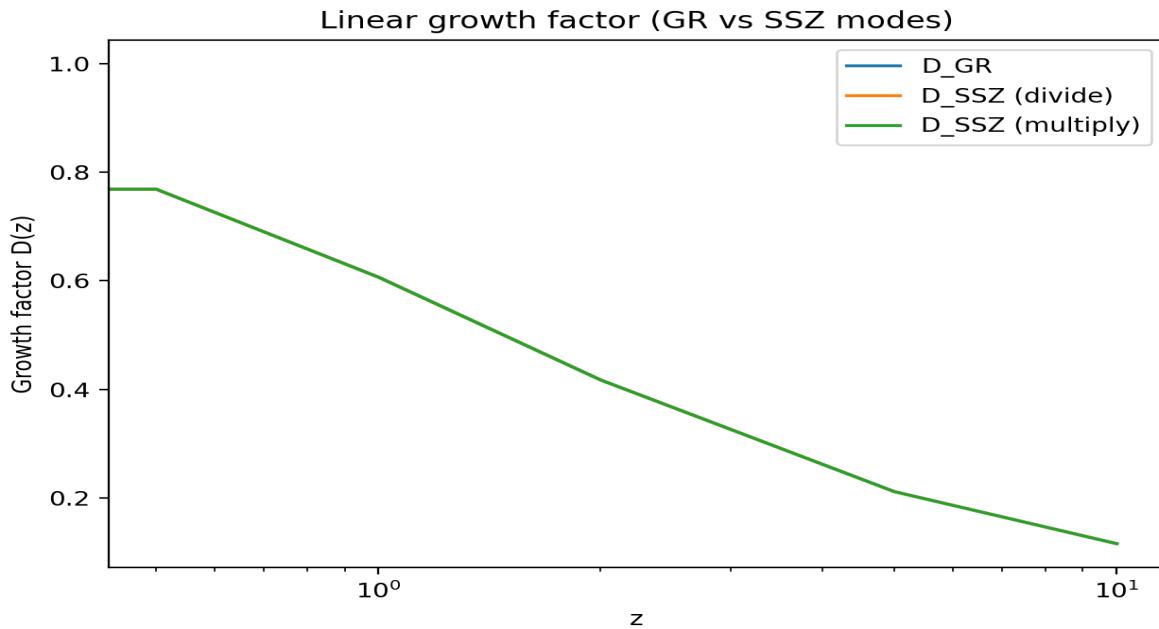
### 7.4 Growth and H(z) ratios at benchmark redshifts

Mode: divide					
z	D_GR	D_SSZ	f_GR	f_SSZ	Hratio
0.0	1.000000	1.000000	0.263819	0.263816	1.000010
0.5	0.768876	0.768878	0.761267	0.761256	1.000012
1.0	0.606721	0.606726	0.876514	0.876501	1.000012
2.0	0.417160	0.417165	0.958186	0.958171	1.000013
5.0	0.211250	0.211255	0.993448	0.993433	1.000013
10.0	0.115406	0.115409	0.996863	0.996848	1.000013

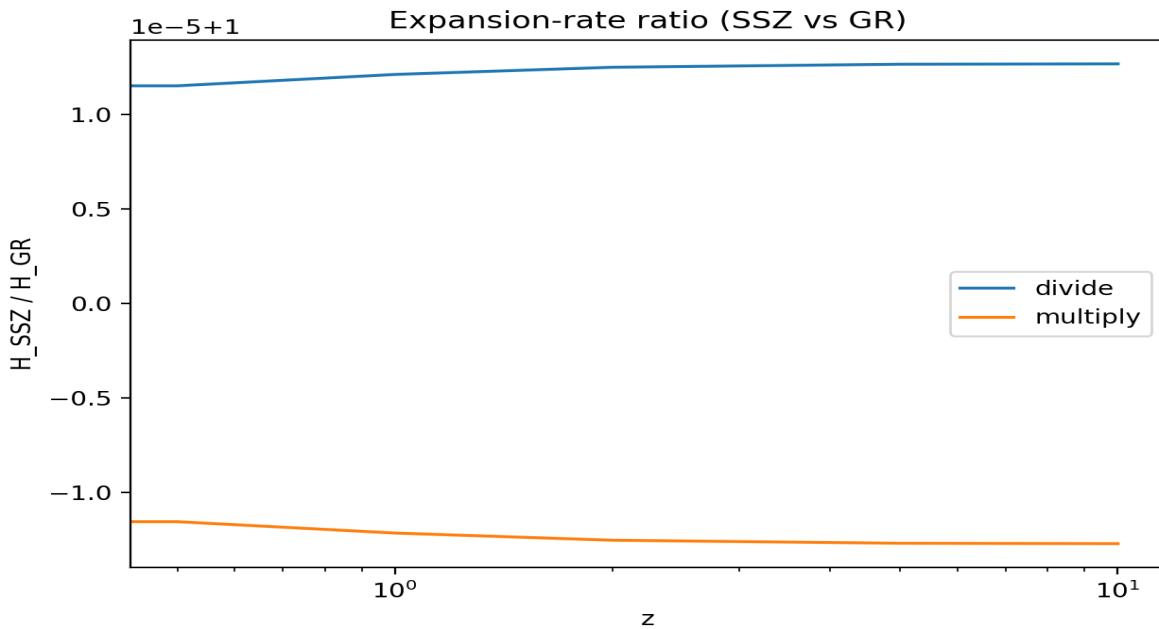
Mode: multiply					
z	D_GR	D_SSZ	f_GR	f_SSZ	Hratio
0.0	1.000000	1.000000	0.263819	0.263822	0.999990
0.5	0.768876	0.768873	0.761267	0.761277	0.999988
1.0	0.606721	0.606717	0.876514	0.876527	0.999988
2.0	0.417160	0.417155	0.958186	0.958200	0.999987
5.0	0.211250	0.211245	0.993448	0.993464	0.999987
10.0	0.115406	0.115403	0.996863	0.996878	0.999987

### 7.5 Reference plots

Figure 1 shows the linear growth factor  $D(z)$  for GR and SSZ under both time conventions. Figure 2 shows the ratio  $H_{\text{SSZ}}/H_{\text{GR}}$  under both conventions. Note: in Figure 2, the y-axis uses a small-offset notation; values slightly below 1 may appear as negative tick labels when plotted as  $(1 + 1e-5 \text{ offset})$ .



**Figure 1.** Growth factor  $D(z)$ : GR vs SSZ (divide and multiply).



**Figure 2.** Hubble ratio  $H_{\text{SSZ}}/H_{\text{GR}}$ : divide gives  $>1$  by  $O(1e-5)$ , multiply gives  $<1$  by  $O(1e-5)$ .

## 8. Known issues and minimal fixes

### 8.1 Edge case: `Xi_cosmo` when `z_max = 0`

When  $\text{Xi}(z)$  is generated from a normalized  $(1+z)/(1+z_{\text{max}})$  form, calling `Xi_cosmo` on a single-point array  $z=[0]$  yields  $z_{\text{max}}=0$  and currently returns  $\text{Xi}=1$  and  $D_{\text{SSZ}}=0.5$ . This does not affect the shipped CSV benchmarks (which evaluate multiple  $z$  values at once), but it can break single-point queries and unit tests. Observed:  $\text{Xi}_{\text{cosmo}}([0])=1$ ,  $D_{\text{SSZ}}([0])=0.5$ . Recommended fix: if  $z_{\text{max}}==0$ , return  $\Phi_0$  for all entries.

### 8.2 Notation caution: acoustic-scale definition

The current implementation computes a CMB 'theta' proxy using  $r_s(z^*)$  and  $D_A(z^*)$ . In cosmology literature,  $\text{theta}_*$  is often defined as  $r_s/D_M$  (or equivalently  $r_s/[(1+z^*)D_A]$ ). For SSZ/GR ratio tests this distinction mostly cancels, but for absolute comparison to Planck-style numbers the definition should be stated explicitly.

## 9. Reproducibility instructions

To reproduce the cosmology scaffold outputs:

- 1) Ensure Python 3 with numpy/pandas installed.
- 2) Run:

```
python run_cosmo.py
```

This produces two CSV files and a combined JSON summary for both coupling modes. The included regression checks in Section 7 confirm that the current bundle is deterministic to machine precision.

## 10. Conclusion

This consolidated document provides a single coherent SSZ paper that covers: axioms and regimes ( $g_1/g_2$ ), anti-circularity rules, the three-step roadmap to reduce scientific attack surface, and a tested no-free-parameters cosmology scaffold with reproducibility evidence. The next work should prioritize (i) a physics-fixed  $r^*$  criterion, (ii) explicit inhomogeneity-based cosmology signatures (CMB lensing / ISW), and (iii) a deterministic regime classifier that routes each observable through the correct SSZ regime without tuning.