Segmented Spacetime - Geometric Resolution of the Lorentz Indeterminacy at v=0

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In standard relativity, the combination of two sub-light speeds is always well-defined, even when the motions are exactly opposite. The apparent 0/0 problem arises only when the classical velocity fraction is extrapolated to the light limit, which is not physically meaningful. Instead of treating it as a breakdown, we can look at it geometrically. By choosing the reference frame that lies exactly between the two motions - the bisector frame - the composition becomes smooth and symmetric, without singularities. When this idea is applied in curved spacetime, it should be paired with the correct local observer frames (for example, freely falling or static) so that the physical meaning remains consistent.

1. Background

Classical special relativity defines the addition of velocities through a clear and finite relation:

$$v' = (v_1 + v_2) / (1 + \frac{v_1 v_2}{c^2})$$

For all sub-light cases (where the absolute values of both velocities are smaller than the speed of light), this expression remains perfectly regular, even when the two motions are in opposite directions ($v_2 = -v_1$).

The often-mentioned “0/0 problem” at v = 0 is not physical. It appears only when the equation is incorrectly extended to the light limit, where |v| = c. To reveal the underlying structure, it is more natural to describe motion using rapidity rather than velocity. Rapidity provides a linear measure of motion, defined by

$$\chi = atanh\,(v/c)$$

which grows indefinitely as v approaches c. The composition of motions then becomes simple addition:

$$\chi' = \chi_1 + \chi_2$$

In this representation, the so-called “indeterminacy” disappears completely. The midpoint (or bisector) frame between two opposite motions can always be defined and remains smooth, symmetric, and free of algebraic artifacts.

What was previously seen as a breakdown of the Lorentz transformation at zero velocity is now recognized as a mere coordinate effect. When viewed geometrically, the transition through v = 0 is continuous.

In curved spacetime, this reasoning applies locally as well: static and freely falling observers may assign different velocities or frequencies, but the underlying kinematics remain fully consistent. The rapidity formulation thus restores continuity across all sub-luminal motions.

2. The Bisector Frame

When two motions occur in opposite directions, their rapidities can be used to identify a symmetric reference frame located exactly halfway between them.

Let χ_obj represent the rapidity of the outward (or "object") motion, and χ_fall the rapidity of the inward (or "infall") motion. The midpoint frame is defined by:

$$\chi = ½\,(\chi\_obj + \chi\_fall)$$

In this frame, the transformed rapidities appear as equal and opposite values:

$$\chi'_{obj} = +\Delta, \quad \chi'_{fall} = -\Delta$$

where $\Delta = ½\,(\chi_{obj} - \chi_{fall})$.

This construction ensures that both motions are perfectly symmetric in magnitude and opposite in direction.

The special case $v = 0$ occurs only when $\chi_{obj} = -\chi_{fall}$, meaning that the two original motions are exact opposites. In every other case, the frame represents a smooth kinematic midpoint, not a point of rest.

The bisector frame therefore eliminates any sense of "singularity" or "discontinuity." Instead of a breakdown at zero velocity, the transition between opposite motions is continuous and fully defined. This geometric midpoint provides a natural description of symmetry between motion and infall.

3. Static versus True Rest

A true rest state exists only for a freely falling observer following a geodesic. Such an observer experiences no proper acceleration and requires no external energy input to maintain motion.

By contrast, a holding position - such as hovering above a massive body - requires continuous energy expenditure to counteract gravity. Although the observer appears “at rest” in coordinate terms, they are not in dynamical rest: their proper acceleration is non-zero, and energy must constantly be supplied to maintain the position.

This distinction is crucial. The bisector frame we defined belongs to this holding-position category: it describes a symmetric kinematic state between opposing motions, not a dynamically stable or force-free one. Energy can still flow, and forces can still act, yet the geometry of the frame remains regular and smooth.

In this sense, the bisector does not represent true equilibrium, but a balanced kinematic configuration where opposing directions are symmetrically related.

4. Photon-Sphere Context and Curved Spacetime

The bisector construction is purely local and kinematic, independent of global curvature, and can therefore be consistently embedded into curved metrics.

In curved spacetime, particularly near strong gravitational fields such as the photon sphere, the relevant paths are light-like geodesics. Different observers, static or freely falling, assign distinct local velocities and frequencies, but none of these situations create an algebraic singularity at $v = 0$.

The midpoint frame merely removes a coordinate artifact in the velocity expression; it does not imply any physical “null point” or dynamical stability. If stability is to be assessed, it must be done through a proper potential analysis, not by kinematics alone.

In this view, the so-called Lorentz indeterminacy at zero velocity vanishes entirely: the geometry of spacetime remains continuous, and the apparent discontinuity is a product of the algebraic form, not of physical reality.

5. Implications

• Eliminates the apparent singularity of the classical velocity-fraction form.

• Introduces a continuous and symmetric midpoint frame for counter-directed motions.

• Clarifies the distinction between static (accelerated) and freely falling (geodesic) observers.

• Provides a local, coordinate-independent description of relative motion using rapidity space.

• Demonstrates that no physical breakdown occurs at v = 0; only the coordinate form becomes ill-conditioned.

6. Conclusion

The so-called Lorentz indeterminacy at zero velocity is not a failure of relativity but a coordinate illusion caused by expressing motion as a fractional velocity sum.

When reformulated geometrically in rapidity space, the relation between opposing motions remains entirely regular.

The midpoint- or bisector-frame offers a smooth, symmetric way to bridge between motion and infall, preserving continuity and removing any artificial “zero-point” behaviour.

In curved spacetime, this framework remains valid locally, unifying both flat and gravitational settings under a single geometric interpretation. Segmented spacetime extends this principle further, emphasizing that physical continuity is a property of geometry itself, not of the algebra used to describe it.

References

Wrede, C., Casu, L., Bingsi (2025). Dual Velocities in Segmented Spacetime - Escape, Fall and Gravitational Redshift [Preprint]. ResearchGate.