Radial Scaling Gauge for Maxwell Fields - A geometric reparametrization with invariant local light speed

Carmen N. Wrede, Lino P. Casu, Bingsi

We introduce a minimal geometric reparameterization in which gravitational effects are encoded primarily as a radial scaling of physical distance rather than as a modification of local wave frequencies. The local speed of light remains invariant. Electromagnetic wave propagation is formulated in terms of a scaled radial coordinate, yielding modified spatial derivatives while preserving standard local dynamics. The approach provides a simple and transparent interpretation of gravitational phase accumulation and delay (and a clear pathway to redshift once a proper-time mapping is specified), and related effects as consequences of geometric scaling, and is naturally compatible with phase-based formulations used in quantum theory.

1. Motivation

In standard relativistic descriptions, gravitational effects are commonly expressed via time dilation, leading to frequency shifts when comparing distant clocks. While operationally correct, this representation can obscure the geometric origin of such effects and easily invites misleading interpretations (e.g., "light slows down").

Here we adopt an alternative but equivalent viewpoint: Gravitation is represented as a scaling of physical radial distance, while local temporal dynamics and the local speed of light remain unchanged. Observed frequency and phase effects then arise from geometry alone.

2. Scope and interpretation

Throughout this note we use a time parameter t that appears in the local wave equation and keep the locally measured light speed invariant.

For a static, time-independent scaling $s(r)$, the temporal oscillation frequency ω is taken as constant along propagation, while the spatial phase accumulation becomes position dependent through $k_{eff}(r) = ks(r)$ equivalently $\phi(r,t) = k\rho(r) - \omega t$ with $d\rho = s(r)dr$. The results derived here therefore target geometric phase accumulation and coordinate-level travel-time (delay) integrals.

The travel-time expression $t(r_1 \to r_2) = \frac{1}{c}\int_{r_1}^{r_2} s(r)dr$ should be understood as a convenient parameterization of path length via the scaled radial distance. Comparison to delays measured by stationary observers can be obtained by relating t to observer proper time $\tau(r)$ within a chosen metric embedding; that additional mapping is not required for the phase/delay relations established here.

For clarity the derivation is shown for 1D propagation along a radial line. The corresponding 3D radial operator can be transformed analogously under $d\rho = s(r)dr$; this introduces only the familiar geometric divergence terms and does not change the basic identification $k_{eff}(r) = ks(r)$ for spatial phase accumulation.

Finally, we may define an effective optical factor $n_{eff}(r) \approx s(r)$ so that $d\ell_{opt} = n_{eff}(r)dr$. This is a geometric reparameterization of path length, not a physical medium and not a modification of the locally measured c.

2. Radial scaling definition

Let r denote a coordinate radius. We define a physical radial distance $\rho(r)$ by

$$d\rho = s(r)\, dr$$

where $s(r) > 0$ is a dimensionless radial scaling function.

In Segmented Spacetime a natural choice is

$$s(r) \;=\; 1 + \;\Xi(\mathrm{r})$$

with $\Xi(\mathrm{r})$ describing the local segment density. No specific form is required in what follows.

Spatial derivatives transform accordingly

$$\frac{\partial}{\partial \mathrm{r}} = \frac{\mathrm{d}\rho}{dr}\frac{\partial}{\partial \rho} = \mathrm{s(r)}\frac{\partial}{\partial \rho}$$

3. Electromagnetic phase and wave propagation

Consider a monochromatic electromagnetic wave propagating radially. In physical distance ρ, the field takes the standard form

$$E(\rho, t) = E_0\, cos(k\rho - \omega t)$$

With the vacuum relation

$$\omega = ck$$

Here:

- ω is the temporal oscillation frequency with respect to the chosen time parameter t
- k is the spatial wave number per unit physical distance
- c is the locally measured speed of light, invariant by construction.

Expressed in the coordinate variable r,

$$E(r, t) = E_0 cos(k\rho(r) - \omega t)$$

The effective wave number with respect to r becomes

$$k_{eff}(r) = \frac{d}{dr}\big(k\rho(r)\big) = ks(r)$$

Thus, the spacing of wavefronts is geometrically scaled, while local oscillation dynamics remain unchanged.

4. Radial wave equation

In terms of the physical distance ρ, the vacuum wave equation is

$$\frac{\partial^2 \mathrm{E}}{\partial \rho^2} - \frac{1}{c^2}\frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2} = 0$$

Transforming to the coordinate radius r yields

$$\frac{1}{s(r)}\frac{\partial}{\partial \mathrm{r}}\left(\frac{1}{s(r)}\frac{\partial \mathrm{E}}{\partial \mathrm{r}}\right) - \frac{1}{c^2}\frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2} = 0$$

All deviations from the flat-space form arise solely from the spatial scaling factor $s(r)$. The local propagation speed remains c.

5. Physical interpretation

In this representation:

- Gravitation does not modify local frequencies or the local speed of light.
- Observable effects such as delay and phase accumulation (and the redshift interpretation via clock comparison in an embedding) result from integrating spatial scaling along a path:

$$\Delta\phi \sim \mathrm{k}\int \mathrm{s(r)dr}$$

- Energy flux variations are naturally associated with geometric effects (e.g. beam focusing or spreading), not with changes in intrinsic wave speed.

This removes the need for ad hoc “phase shifts” or frequency reinterpretations while preserving local Maxwell dynamics and the path-integral observables within the present scope.

6. Relation to quantum theory

Quantum mechanics is fundamentally phase-based. Replacing coordinate distance by a scaled physical distance,

$$x \to \rho(x)$$

modulates phase accumulation without altering local temporal dynamics. This structure is compatible with:

- WKB and semiclassical approximations,
- path-integral formulations via geometric action weighting,
- phase-based interpretations of gravitational redshift.

No modification of quantum postulates is required.

7. Limiting case

For $s(r) \to 1$, one recovers standard Maxwell theory in flat space immediately, demonstrating consistency with known limits.

8. Conclusion

Encoding gravitation as a radial scaling of physical distance provides a minimal, transparent reformulation of electromagnetic propagation. The approach preserves local invariants while shifting gravitational effects entirely into geometry, offering a clean bridge between classical field theory and phase-based quantum descriptions.

A) Addendum: Shapiro delay and weak-field lensing in the radial gauge

A.1 Travel time and Shapiro-like delay as a path integral

With the physical radial distance defined by

$$d\rho = s(r)\, dr$$

A null signal propagating locally with invariant speed c has the travel-time element

$$dt = \frac{d\ell_{phys}}{c}$$

For a purely radial segment (minimal case),

$$dt = \frac{d\rho}{c} = \frac{s(r)}{c}\, dr$$

Hence the coordinate travel time between r_1 and r_2 becomes

$$t(r_1 \to r_2) = \frac{1}{c}\int_{r_1}^{r_2} s(r) dr$$

Defining a flat reference time $t_0 = \frac{1}{c}\int_{r_1}^{r_2} dr$, the excess delay is

$$\Delta t = t - t_0 = \frac{1}{c}\int_{r_1}^{r_2} (s(r) - 1)\, dr$$

If $s(r) = 1 + \Xi(\mathrm{r})$, this simplifies to

$$\Delta t = \frac{1}{c}\int_{r_1}^{r_2} \Xi(r)\, dr$$

Interpretation: The delay arises from an increased physical path length (ρ) per coordinate radius, while local propagation remains luminal. Travel-time delays for null propagation in curved geometry are discussed in standard GR treatments (e.g. MTW; Carroll).

A.2 Weak deflection (lensing) as a transverse gradient of the optical path

In the eikonal (geometric optics) limit, the accumulated phase is proportional to physical path length:

$$\phi \sim k \int n_{eff}(x)\, d\ell$$

with an effective "optical factor" (not a change of local c) given here by the geometric scaling. Here n_{eff} is an effective 'optical factor' encoding the geometric scaling; in the radially symmetric case we write $n_{eff}(r)$. In the simplest weak, slowly varying case one may treat

$$n_{eff}(r) \approx s(r)$$

so that the stationary-path condition yields the usual small-angle deflection estimate:

$$\alpha \approx \int \nabla\perp \ln n_{eff}\, d\ell \;\approx\; \int \nabla\perp \ln s(r) d\ell$$

For weak scaling $s(r) = 1 + \Xi(\mathrm{r})$ with $|\ \Xi\ | \ll 1$

$$\alpha \approx \int \nabla\perp\ \Xi(\mathrm{r})\ \ d\ell$$

Interpretation: lensing appears as a refraction-like bending due to spatial variation of the physical-length scaling, again without invoking any change of locally measured c. A full 3D derivation follows by using the corresponding spatial line element and applying the eikonal approximation. This uses the standard eikonal/geometric-optics stationary-path approximation (see e.g. Born & Wolf; Schneider–Ehlers–Falco).

B) Quantum-connection page: phase, action, and WKB in the scaled-distance picture

B.1 Phase accumulation as the primary observable

Quantum evolution is phase-based. For a stationary wave,

$$\psi \propto\ e^{i\theta}, \qquad \theta(x,t) = k\,x - \omega t$$

Replacing coordinate distance by physical distance in the radial direction,

$r \mapsto \rho(r), \qquad d\rho = s(r)dr$

the phase becomes

$$\theta(r,t)\ =\ k\rho(r)\ - \omega t$$

Thus, spatial phase accumulation is geometrically modulated:

$$\frac{d\theta}{dr} = k\frac{d\rho}{dr} = ks(r)$$

while local temporal oscillation remains governed by ω.

B.2 WKB (Wentzel–Kramers–Brillouin) form: Effective local wave number without modifying ω

In WKB language,

$$\psi(r) \sim A(r)\, exp\left(i \int^{r} k_{eff}(r')\, dr'\right)$$

with

$$k_{eff}(r) = ks(r)$$

This is precisely the "radial scaling gauge" statement: geometry changes the spatial phase rate, not the intrinsic local propagation law. For the semiclassical/WKB phase–action connection, see e.g. Landau & Lifshitz.

B.3 Action-based view (path integral / semiclassical limit)

In semiclassical mechanics the phase is proportional to action:

$$\theta = \frac{1}{\hbar} S$$

If the physical path element is stretched by $s(r)$, then the geometric contribution to S (and hence to phase) acquires the same scaling. In the simplest optical analogy for massless propagation,

$$\Delta\theta \sim \frac{\omega}{c} \int d\ell_{phys} = \frac{\omega}{c} \int s(r)\, d\ell$$

so gravitationally induced phase differences can be treated as geometric path-weighting in the action, rather than as any local change in c.

B.4 Practical consequence: Interferometry-friendly formulation

Any experiment that compares phase along two paths (interferometers, timing arrays, cavity modes, etc.) can be expressed as

$$\Delta\theta \propto \int_{\gamma_1} s(r)\, d\ell \; - \int_{\gamma_2} s(r)\, d\ell$$

making the model directly testable in phase-sensitive settings without invoking ambiguous "frequency changes of light in flight".

References

1. Born, M., Wolf, E., *Principles of Optics*, 7th ed., Cambridge University Press (1999).
2. Schneider, P., Ehlers, J., Falco, E. E., *Gravitational Lenses*, Springer (1992).
3. Misner, C. W., Thorne, K. S., Wheeler, J. A., *Gravitation*, W. H. Freeman (1973).
4. Carroll, S.M., *Spacetime and Geometry: An Introduction to General Relativity*, Addison-Wesley (2004).
5. Landau, L. D., Lifshitz, E. M., *Quantum Mechanics: Non-Relativistic Theory*, 3rd ed., Pergamon Press (1977).

Appendix: Observer proper time and clock comparison

This note has treated gravitational effects via a geometric radial scaling of physical distance. To connect the formalism to observer clock measurements, we introduce an explicit mapping between the wave/coordinate time t used above and observer proper time τ.

A. Proper time mapping (general)

We define a position-dependent clock rate

$$d\tau = D(r)dt$$

where $D(r)$ encodes the gravitational time dilation for a stationary observer at radius r. No assumption is made at this stage about the relation between $D(r)$ and the spatial scaling $s(r)$.

B. Relation to metric embedding (GR-style)

If the framework is embedded into a static, spherically symmetric metric, the clock factor is identified as

$$D(r) = \sqrt{-g_{tt}(r)}$$

For a stationary observer ($dr = 0$), proper time follows directly from the metric time component. The spatial scaling $s(r)$ corresponds to the radial part of the spatial line element.

C. Segmented Spacetime implementation

Within the segmented-spacetime framework, one may instead identify

$$D(r) = DSSZ(r)$$

using the independently defined SSZ time-dilation function. This keeps spatial scaling and clock rate conceptually distinct while allowing both to be tested against observations.

D. Observables

With the proper-time mapping in place, standard observables follow immediately:

- **Gravitational redshift (clock comparison):**
  frequency ratios between stationary observers at different radii are given by ratios of $D(r)$.

- **Measured signal delays:**
  coordinate-level travel times derived from spatial scaling can be converted into clock readings via $d\tau = D(r)\,dt$.

- **Phase–clock consistency:**
  spatial phase accumulation (via $s(r)$) and clock rates (via $D(r)$) can be tested independently or jointly.