Additive Decomposition of the Observed Light-Travel Time

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In gravitational lensing, the observed light-travel time t_{obs} is traditionally expressed as the sum of a geometric contribution and a gravitational (Shapiro) delay,

$$t_{\text{obs}} = t_{\text{geom}} + t_{\text{grav}}$$

The observed time consists of a geometric part plus a gravitational part.

This representation, used in the Fermat potential formalism, is effective but it mixes several distinct physical mechanisms into only two broad terms.

Here we propose a more explicit additive decomposition in which the geometric, gravitational, and kinematic contributions are separated through distinct Lorentz-type factors.

$$c\, t_{\text{obs}} = \underbrace{\gamma_{\text{GR}}\, L}_{\text{gravitational (Shapiro + geometry)}} + \underbrace{\gamma_{\text{SR}}\, v\, \tau}_{\text{special relativistic (kinematic)}}$$

where:

- **L** is the baseline geometric distance in the instantaneous rest frame of the gravitational environment,
- $\boldsymbol{\gamma_{G_R}}$ is the gravitational scaling factor (encoding gravitational time dilation, Shapiro delay, and local curvature effects),
- $\boldsymbol{\gamma_{S_R}}$ is the special-relativistic Lorentz factor associated with the peculiar velocity of the source, lens, or observer,
- $\boldsymbol{v}$ is the relative velocity along the line of sight,
- $\boldsymbol{\tau}$ is the proper time interval associated with the emitting system.

This provides a physically transparent decomposition:

- $\boldsymbol{\gamma_{G_R} \cdot L}$ contains all gravitational distortions of spacetime affecting light propagation,
- $\boldsymbol{\gamma_{S_R} \cdot v \cdot \tau}$ captures the special-relativistic kinematic effects (including Doppler shifts and time dilation due to motion).

This formulation is structurally analogous to the additive split of relativistic energies,

$$E_{\text{tot}} = E_{\text{GR}} + E_{\text{SR}},$$

highlighting the conceptual similarity between the decomposition of energy contributions and the decomposition of observed light-travel times in lensing scenarios.

In future work, we will apply this decomposition to observational data from strong gravitational lenses to test:

- whether gravitational and kinematic components can be empirically isolated,
- whether this formulation offers practical advantages over the standard Fermat potential approach,
- whether it provides insight into anomalies such as the H_0 tension.

Visibility Threshold and Segment-Based Interpretation

The additive decomposition

$$ct_{obs} = \gamma_{GR} L + \gamma_{SR} v\tau$$

makes an additional phenomenon explicit that is usually hidden in the standard lensing equations:

Light becomes observable only when the special-relativistic (SR) coherence of the wave exceeds the gravitational (GR) segmentation imposed by the local spacetime curvature.

In classical relativity, gravitational contributions are often written with a global constant c^2.
However, physically the relevant scale in the GR term is the local free-fall velocity

$$\mathrm{v_{fall}} = \sqrt{\frac{2\mathrm{GM}}{\mathrm{r}}}$$

which quantifies the local gradient of spacetime segmentation. Thus the gravitational contribution to the effective energy is not mc^2 but:

$$\mathrm{E_{GR}} = \mathrm{m v_{fall}}^2$$

Electromagnetic waves (especially radio waves) become measurable only when:

$$\mathrm{E_{SR}} > \mathrm{E_{GR}}$$

or, equivalently,

$$\boldsymbol{\gamma}_{\mathrm{SR}}\, \mathrm{v}\tau = \boldsymbol{\gamma}_{\mathrm{GR}} \mathrm{L}$$

Near strong gravitational fields (black holes, compact objects), the GR segmentation term $\boldsymbol{\gamma}_{\mathrm{GR}}\mathrm{L}$ becomes so large that the SR coherence of the photon wave packet is suppressed.

The wave is physically present, but not observable, because its internal structure is over-segmented by the curvature of spacetime.

As the wave propagates outward and $\mathrm{v_{fall}}$ decreases, the segmentation relaxes.
Once the SR coherence term overtakes the GR segmentation term, the electromagnetic wave becomes observable again.

We can define a visibility radius $\mathrm{r_{vis}}$ as the radius at which the special–relativistic coherence of an electromagnetic wave becomes equal to the gravitational segmentation imposed by the local spacetime curvature. Formally, this corresponds to the threshold condition

$$\mathrm{E_{SR}} = \mathrm{E_{GR}}$$

Using the free–fall velocity $\mathrm{v_{fall}(r)} = \sqrt{2\mathrm{GM/r}}$ and $\mathrm{E_{GR}} \sim \mathrm{m\, v_{fall}}^2 = 2\mathrm{GMm/r}$, we obtain:

$$\mathrm{r_{vis}} = \frac{2\mathrm{GM}}{\mathrm{E_{SR}/m}}$$

If we parameterise the SR contribution by an effective coherence scale v_* via $E_{SR} \sim m v_*^2$, this becomes

$$r_{vis} = \frac{2GM}{v_*^2}$$

Low–energy (radio) waves with small E_{SR}(small v_*) thus have a large visibility radius and only re-emerge far outside the strong–gravity region, while high–energy photons remain observable much closer to the compact object.

This provides a natural explanation for:

- why radio emission near compact objects appears "missing,"
- why certain frequencies reappear only beyond a specific radius,
- why low-energy photons emerge from regions that classically appear causally suppressed.

Interpretation in the segmented-spacetime model

In a segmented spacetime, gravity corresponds to a spatially varying segmentation density.
Light becomes observable when the wave packet has enough internal coherence to form a stable segment pattern.

Thus:

- **GR determines how much a wave is segmented**,
- **SR determines how much coherence the wave can maintain**,
- **visibility arises when SR-coherence exceeds GR-segmentation**.

This is fully consistent with the decomposition

$$E_{\text{tot}} = E_{\text{GR}} + E_{\text{SR}}$$

and clarifies why the observed light-travel time naturally contains two independent contributions.

Implications for gravitational lensing analysis

This refined structure suggests several testable predictions:

- Radio-band visibility thresholds depend on local curvature, not solely on redshift.
- Light-curve delays may contain hidden GR/SR crossover signatures.
- Multi-frequency lensing observations can separate GR segmentation from SR coherence.
- The "missing radio flux" near compact lenses could be explained without exotic mechanisms.

These predictions will be explored in the forthcoming analysis section using strong-lens systems.