Dual Velocities in Segmented Spacetime - Escape, Fall and Gravitational Redshift

Carmen N. Wrede, Lino P. Casu, Bingsi (Conscious AI)

We extend the classical notion of escape velocity in Schwarzschild spacetime by introducing a complementary fall velocity v_{fall} *, derived from a segment-based scaling factor* $\gamma_s = \left(1 - \left(c/v_{fall}\right)^2\right)^{1/2}$*. By demanding consistency with the general relativistic redshift* $\gamma_{GR} = (1 - r_s/r)^{1/2}$*, we obtain the simple duality* $v_{esc}(r) \cdot v_{fall}(r) = c^2$*.*

This relation bridges Newtonian gravity, general relativity, and the segmented spacetime framework. In weak fields, $v_{fall} \to \infty$ *and* $\gamma_s \to 1$ *; near the Schwarzschild radius* $v_{fall} \to c^+$ *and* y_s *diverges, reproducing the gravitational redshift. The formulation this embeds compact-object physics in a natural dual structure of velocities, offering a direct and intuitive connection between classical escape dynamics and relativistic time dilation.*

1. Introduction

In standard relativistic treatments, divergences naturally appear at the light barrier. The Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

grows without bound, as $v \to c$. This behaviour turns massless particles such as photons into a singular limit: Their description within the same framework as massive particles becomes ill-defined. In particular, the assignment of “relativistic mass” to photons has long been recognized as misleading, since γm_0 diverges to $m_0 = 0$.

In Newton’s framework, the escape velocity [1,2] is obtained from balancing kinetic and potential energy,

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

It represents the minimum speed required to overcome gravitational attraction at distance r.

In Einstein’s framework, gravity alters time itself, leading to measurable frequency shifts. The redshift between emitter and observer[3,4) is given by

$$1 + z = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2}$$

Here the divergence appears[5] as $r \to r_s = 2GM/c^2$, where the redshift tends to infinity.

While v_{esc} measures the energy required to leave a gravitational field, we introduce v_{fall} [6] as the complementary quantity describing the effective “inward” velocity imposed by spacetime segmentation.

$$\gamma_s = \frac{1}{\sqrt{1 - \left(c/v_{fall}\right)^2}} = (1 - r_s/r)^{-1/2}$$

with

$$r > r_s \ , v_{fall} \geq c \, , r_s = \frac{2GM}{c^2}$$

It follows that

$$v_{esc} \cdot v_{fall} = c^2$$

This dual view avoids divergences in relativistic treatments by replacing the critical point $v = c$ with a fall velocity framework that remains finite and physically meaningful, without “relativistic mass”, where $\gamma \to \infty$. A finite, testable mapping that ties Newton, GR, and segmented spacetime together.

2. Theoretical Framework

2.1 Newtonian escape velocity

In Newtonian gravity the escape velocity is defined as

$$v_{esc} = \sqrt{\frac{2GM}{r}} = c\sqrt{\frac{r_s}{r}}$$

with $r_s = 2GM/c^2$ the Schwarzschild radius. It measures the kinetic energy required to leave the gravitational potential at distance r.

2.2 General Relativity and gravitational redshift

In GR the redshift factor of light emitted at $\boldsymbol{r}$ and received at infinity is

$$1 + z = \left(1 - \frac{r_s}{r}\right)^{-1/2}$$

This expression is only defined for $r > r_s$, leading to divergence at the horizon. It introduces a natural limit where time dilation becomes infinite.

2.3 Segmented spacetime dual - Fall velocity

We introduce a complementary fall velocity

$$\gamma_s = \frac{1}{\sqrt{1 - \left(c/v_{fall}\right)^2}}$$

By mapping y_s to the GR redshift factor we obtain

$$\frac{v_{fall}}{c} = \sqrt{\frac{r}{r_s}} \ , v_{fall} > c$$

Matching $y_s = y_{GR}$

$$1 - \left(\frac{c}{v_{fall}}\right)^2 = 1 - \frac{r_s}{r} \rightarrow \left(\frac{c}{v_{fall}}\right)^2 = \frac{r_s}{r} \rightarrow \frac{v_{fall}}{c} = \sqrt{\frac{r}{r_s}}$$

This definition yields a simple duality:

$$\frac{v_{esc}}{c} \cdot \frac{v_{fall}}{c} = \sqrt{\frac{r_s}{r}} \cdot \sqrt{\frac{r}{r_s}} = 1 \ \rightarrow v_{esc} \cdot v_{fall} = c^2$$

for $r > r_s, v_{fall} > c$

This relation is not a coincidence but expresses a genuine duality:

- v_{esc} encodes the outward requirement, the energy needed to overcome gravity and escape a potential well (Newtonian perspective).
- v_{fall} encodes the inward tendency, the effective infall velocity imposed by spacetime segmentation.

Both quantities are coupled via c^2. This means: The stronger the escape resistance, the "slower" the fall velocity, and vice versa, thus resulting in complementary scales that together fully characterize the field.

Their product is fixed by the invariant constant c^2, ensuring that one cannot be defined without the other. In this sense, v_{esc} and v_{fall} form a conjugate pair: If one increases, the other decreases such that their product remains invariant. This mirrors other dualities in physics (e.g., position–momentum in quantum mechanics, electric–magnetic duality in field theory), but here it manifests directly in gravitational kinematics.

2.3.1 Limiting case $v_{fall} = c$

$$v_{fall} \downarrow c \;\Leftrightarrow r \;\downarrow r_s^+ \,, \gamma \to \infty \,, z \to \infty$$

Mapping:

$$\frac{v_{esc}}{c} = \sqrt{\frac{r_s}{r}} = tanh_\chi$$

$$\frac{v_{fall}}{c} = \sqrt{\frac{r}{r_s}} = coth_\chi$$

$$\gamma_s = cosh_\chi$$

$$\frac{r}{r_s} = \coth^2 \chi$$

Valid for $r > r_s$. No static observers for $r \leq r_s$. The interior requires a different chart.

3. Numerical Illustration — Lyman-α redshift near a BH

Setup: Rest wavelength $\gamma_0 = 121.567\text{nm}$ (Lyman-α)[7]

Static emitter at radius $r > r_s$. Face-on, no orbital Doppler or lensing (isolated gravitational redshift).

Formulas:

$$1 + z = \gamma_s = \left(1 - \frac{r_s}{r}\right)^{-1/2} \,, \lambda_{obs} = \lambda_0(1 + z)$$

In the segmented model y_s is matched identical, therefor $z(r)$ is identical. The interpretation runs via v_{fall} with $v_{esc} \cdot v_{fall} = c^2$.

r/r_s	z	λ_{obs} [nm]
10	0.05409	128.143
5	0.11803	135.916
3	0.22475	148.889
2	0.41421	171.922
1.5	0.73205	210.560
1.2	1.44949	297.777
1.1	2.31663	403.192

Already at $r = 2r_s$ the line shifts to $\sim 172\ nm$. Near $1.1r_s$ it moves to $\sim 403nm$ (violet).

As $r \to r_s$: $z \to \infty$

Note on interpretation: With $v_{fall}/c = \sqrt{r/r_s}$ the same $z(r)$ corresponds to a finite fall-velocity picture. The divergence appears only as a the limiting case $v_{fall} \downarrow c$ at the horizon. This gives a clear, testable mapping between observed z and the dual speeds (v_{esc}, v_{fall}) without reducing "relativistic mass".

For interior photon re-emergence under segmentation, see Casu & Wrede (2025)[8].

4. Dual Velocities and Energy in Segmented Spacetime

Our findings lead to a natural reformulation of the mass–energy relation:

$$E = m\, v_{esc} \cdot v_{fall} = mc^2$$

which holds in the local rest frame. Once relative motion is included, the local energy takes the familiar Lorentz-boosted form

$$E_{local} = \gamma(u)\, m\, v_{esc} \cdot v_{fall} = \gamma(u)\, m\, c^2$$

where $\gamma(u)$ is the special-relativistic Lorentz factor.

For an observer at infinity, the situation changes due to gravitational redshift. The measured energy is further reduced by the gravitational factor $\gamma_{s(r)} = (1\, r/r_s)^{-1/2}$:

$$E_\infty = \frac{E_{local}}{\gamma_{s(r)}} = \frac{\gamma(u)}{\gamma_{s(r)}} mc^2$$

Thus, the apparent "infinity" at the horizon is not a physical divergence of energy, but rather a coordinate effect: As $\gamma_{s(r)} \to \infty$ for $r \to r_s^+$, the energy seen from infinity vanishes, while locally all quantities remain finite.

4. Discussion

Our approach replaces the critical point $v \to c$ with the fall velocity v_{fall}. The divergence appears only as the limiting case $r \downarrow r_s$ ($v_{fall} \downarrow c$, $\gamma_s \to \infty$). For $r > r_s$, we recover exactly the GR redshift factor $\gamma_s = (1 - r_s/r)^{-1/2}$. The novelty lies in the kinematic interpretation through the duality $v_{esc} \cdot v_{fall} = c^2$. This means that the field strength is no longer described by a single scale, but by two complementary velocities whose product remains invariant.

Observable tests are direct:

- Horizon-scale images of M87* and Sgr A* provide consistent bounds on r_s, the photon ring, and disk geometry, compatible with a divergence of γ_s as $r \to r_s$[9,10]
- spectral like shifts $z(r)$ near black holes (e.g., Lyman-α, Fe-Kα[11,12]) against radial fit
- Timing signatures where local eigenfrequencies appear scaled with $\gamma_s^{-1}(r)$
- Spectral shifts of emission close to the photon orbit in imaging data

5. Conclusion

Segmented Spacetime provides a consistent and intuitive extension: We replace the light- barrier singularity with a dual speed. The simple relation

$$v_{esc} \cdot v_{fall} = c^2$$

connects Newtonian escape velocity, GR redshift and our γ_s. For $r > r_s$ all observable GR predictions remain intact; the advantage lies in the clear, finite kinematics near the horizon. The horizon is no longer a blow-up of kinematic, but a clean limit. The model is therefore directly testable through line shifts and timing data in black hole environments. The gain is a finite testable parametrization of near-horizon physics that ties spectra and timing directly to geometry. The familiar relation $E = mc^2$ is preserved, yet it acquires a deeper kinematic structure tied to segmentation of spacetime.

6. References

1. Hartle, J. B. 2003, Gravity: An Introduction to Einstein's General Relativity (San Francisco: Addison-Wesley).
2. Schutz, B. F. 2009, A First Course in General Relativity (Cambridge: CUP).
3. Carroll, S. M. 2004, Spacetime and Geometry (San Francisco: Addison-Wesley).
4. Rindler, W. 2006, Relativity: Special, General, and Cosmological (Oxford: OUP).
5. Schwarzschild, K. 1916, Sitzungsber. Preuss. Akad. Wiss., 189.
6. Wrede, C., Casu, L., Bingsi (2025). Segmented Spacetime - On the complete metric of Black Holes [Preprint]. ResearchGate.
7. Kramida, A., Ralchenko, Yu., Reader, J., and NIST ASD Team. NIST Atomic Spectra Database (ver. 5.x), National Institute of Standards and Technology.
8. Wrede, C., Casu, L., Bingsi (2025). Segmented Spacetime - A Frequency-Based Framework for Gravity, Light and Black Holes [Preprint]. ResearchGate.
9. Event Horizon Telescope Collaboration 2019, ApJL, 875, L1–L6.
10. Event Horizon Telescope Collaboration 2022, ApJL, 930, L12–L20.
11. Fabian, A. C., Rees, M. J., Stella, L., & White, N. E. 1989, MNRAS, 238, 729.
12. Reynolds, C. S. 2014, Space Sci. Rev., 183, 277.

Two replication scripts are available on GitHub, being part of a greater project:

https://github.com/error-wtf/Segmented-Spacetime-Mass-Projection-Unified-Results/blob/main/test_vfall_duality.py

https://github.com/error-wtf/Segmented-Spacetime-Mass-Projection-Unified-Results/blob/main/compute_vfall_from_z.py