Frequency-Based Curvature Detection via Dynamic Comparisons

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We propose an operational method to infer spacetime curvature purely from frequency comparisons between dynamically related emitters, without assuming a global reference frame or prior knowledge of the gravitational field. In this framework, a truly gravity-free system emits a constant intrinsic frequency, while gravitational structure manifests as non-integrable frequency differences between neighbouring emitters or clocks. We show that curvature is encoded not in absolute frequencies but in differences of frequency differences*, providing a relational and experimentally accessible probe of spacetime structure. The approach is compatible with General Relativity in the weak-field limit and naturally integrates with the Segmented Spacetime (SSZ) framework, where frequency-derived information encodes both SR and GR contributions.*

1. Introduction

In relativistic physics, neither time nor velocity are absolute observables; both depend on the chosen reference frame and underlying spacetime structure. While Special Relativity (SR) applies locally in inertial frames [1], its global application requires prior knowledge of spacetime curvature, supplied by General Relativity (GR) [2]. In practice, this leads to a circular dependence: To apply SR locally everywhere, one must already know the global gravitational field.

This paper addresses a related but often overlooked question: Can spacetime curvature be inferred directly from observable frequency data, without assuming a global metric or reference frame? We argue that it can, provided one focuses on relational, dynamical frequency comparisons rather than absolute values.

2. Constant Frequency in Gravity-Free Systems

In flat spacetime, a freely propagating signal does not acquire intrinsic gravitational structure. Any frequency assigned to the signal is therefore best treated as a proper-time property of the emitter rather than a position-dependent quantity. We take the emitted carrier to be stationary in its own rest frame with constant proper frequency

$$\upsilon(\tau) = \upsilon_0. \qquad (1)$$

where τ denotes the emitter's proper time. Here "constant proper frequency" refers to the intrinsic eigenfrequency of a stabilized emitter (e.g. an atomic or optical clock), understood as constant within calibration stability limits.

To eliminate coordinate artifacts and to express comparisons in a form suitable for chaining and loop tests, we adopt a relational frequency observable defined for any two clocks/emitters A and B as the log-frequency ratio

$$\delta_{AB} = ln\left(\frac{\upsilon_A}{\upsilon_B}\right) \qquad (2)$$

This choice is minimal and robust: δ_{AB} is dimensionless and additive under composition of comparisons, which makes it the natural quantity for later "difference-of-differences" constructions and closed-loop consistency checks. Because δ_{AB} is constructed from dimensionless frequency ratios, it is invariant under local reparameterizations of clock rates and independent of coordinate or gauge choices.

In the flat-spacetime baseline, any apparent variation of frequency with position is observer-dependent (e.g., kinematic Doppler assignment or coordinate choice). By construction, such effects can be absorbed by local frame selection and calibration and do not constitute evidence of curvature. The purpose of this section is therefore to establish the reference case: in the absence of non-integrable structure, the relational quantity δ_{AB} behaves as an integrable comparison variable, with no residuals appearing solely from propagation in flat spacetime.

3. First-Order Frequency Shifts and Their Limits

Frequency comparisons between clocks or emitters provide direct access to relative time dilation effects along their trajectories. Such effects arise already at first order and are commonly interpreted as gravitational redshift or kinematic Doppler shifts, depending on the physical situation and choice of reference frame.

At this level, frequency shifts reflect local differences in clock rates and can be fully described within an integrable structure. They do not, by themselves, imply spacetime curvature. In particular, first-order shifts can be absorbed by an appropriate choice of local frame, potential, or calibration procedure, and therefore do not constitute invariant signatures of nontrivial spacetime geometry.

This has been confirmed in a variety of high-precision experiments. Dynamic gravitational redshift tests along extended trajectories, such as Gravity Probe A [3], demonstrated relativistic time dilation under genuine free-fall conditions using a space-borne hydrogen maser. Similar first-order frequency shifts have also been observed in eccentric satellite orbits, most notably in the Galileo redshift experiments [4], where the varying gravitational potential along the orbit produces measurable frequency modulations.

While these experiments provide compelling and quantitatively precise confirmations of relativistic time dilation, they remain probes of first-order frequency effects. As such, they do not directly test spacetime curvature. The distinction is crucial: curvature is not encoded in single frequency comparisons or local time dilation measurements, but requires relational structures that are sensitive to non-integrability across multiple paths or trajectories.

The purpose of this section is therefore not to diminish the significance of first-order redshift experiments, but to clearly delimit their scope. They establish the empirical baseline for relativistic frequency behaviour, against which genuinely curvature-sensitive observables must be defined.

Individual frequency comparisons typically encode only scalar time-dilation information along a given configuration. While such shifts can be invariant under appropriate conditions (e.g. between stationary observers), they do not by themselves probe spacetime curvature, which requires relational comparisons across multiple paths.

4. Differences of Differences and Non-Integrability

The term “differences of differences” is used here as a heuristic description of closed-loop consistency tests, which are formally expressed as holonomy-like sums of relational frequency differences.

To move beyond first-order frequency comparisons, one must introduce either a third emitter or consider the temporal evolution of relational observables. This allows the construction of second-order relational quantities that probe the consistency of frequency comparisons across multiple paths.

A natural way to formalize this is to consider three clocks or emitters A, B and C, and to form a closed comparison loop. Using the relational observable defined in Section 2, we define the closed-loop residual as

$$\mathrm{I}_{ABC} = \delta_{AB} + \delta_{BC} + \delta_{CA} \ , \ \delta_{AB} = ln\left(\frac{\upsilon_A}{\upsilon_B}\right) \tag{3}$$

The loop measurement is defined such that purely kinematic SR contributions (including Doppler and Sagnac effects due to rotation) are either explicitly modelled and subtracted or eliminated through symmetric measurement geometries. Any remaining non-zero residual then constitutes an operational curvature indicator. Acceleration-induced effects, including non-inertial motion and Sagnac-type contributions, remain fully removable under symmetric loop constructions and are therefore classified as integrable ($\mathrm{N_{SR}}$) structures.

In flat spacetime, or in situations describable by a purely integrable potential (i.e. where the relational frequency 1-form is exact), this loop quantity vanishes identically,

$$\mathrm{I}_{ABC} = 0 \tag{4}$$

In such cases, frequency comparisons are path-independent and admit a consistent global ordering.

A non-zero value of I_{ABC} signals a failure of loop closure and thus non-integrability. This non-integrability constitutes the operational signature of spacetime curvature, in direct analogy to holonomy and geodesic deviation in general relativity. In other words:

Curvature is encoded in closed-loop residuals of frequency comparisons.

This distinction aligns with the clarification by Brown and Read [5], who emphasize that individual gravitational redshift measurements alone do not constitute curvature tests. Only relational structures that probe consistency across multiple paths can reveal nontrivial spacetime geometry.

Classic experiments, such as the Pound–Rebka measurements [6,7], demonstrated gravitational frequency shifts in static fields with high precision. However, when considered in isolation, these measurements remain first-order effects and do not by themselves provide access to spacetime curvature.

5. Relation to General Relativity

In GR, curvature is captured by the Riemann tensor and becomes observable through geodesic deviation or tidal effects. Frequency comparisons provide an equivalent probe:

- First-order frequency shifts correspond to time dilation gradients.
- Second-order, non-integrable frequency comparisons correspond to curvature components.

Thus, the proposed method does not replace GR but offers an alternative operational access to its core geometric content. This operational role of frequency gradients is already implicit in practical relativistic systems such as GPS [8], where consistent navigation requires continuous relativistic frequency corrections. For a comprehensive overview of experimental tests of General Relativity, see Will [9]. Standard treatments of spacetime curvature, geodesic deviation, and their operational interpretation can be found in Refs [10-12].

6. Integration with Segmented Spacetime (SSZ)

Within Segmented Spacetime (SSZ), frequency comparisons encode structural information N that characterizes departures from purely integrable spacetime structure [13,14] in situations where the comparison 1-form is globally integrable (exact). This information is not contained in individual frequency shifts, but is operationally revealed through non-vanishing closed-loop residuals I_{ABC}, as defined in Section 4

$$N = N_{SR} + N_{GR} \tag{5}$$

The method developed here provides a natural way to disentangle these components:

- N_{SR}: Removable via local transformations (relative motion).
- N_{GR}: Persists as non-integrable frequency structure (curvature).

Operationally, N_{SR} is identified as the component removable by local frame transformations or symmetric measurement protocols, while N_{GR} corresponds to the persistent, non-integrable contribution revealed by closed-loop residuals. In practice, N may be inferred from the time-averaged or baseline-scaled behaviour of loop residuals over repeated measurements.

Modern optical clock experiments [15,16] demonstrate that frequency comparisons can resolve relativistic time dilation at centimetre scales, making the proposed approach experimentally accessible. Future space-based clock comparison missions, such as ACES, are expected to further extend the sensitivity and baseline of frequency-based relativistic tests [17].

More broadly, it reinforces a conceptual shift: Spacetime structure is not a background entity but emerges from dynamical relations between physical processes.

Future work will explore explicit SSZ implementations, numerical simulations, and potential experimental realizations.

6.1. Validation Framework and Test Setup

The validation strategy employed in this work is designed to test the physical consistency and observational relevance of the frequency-based curvature detection method using real-world reference data and controlled numerical experiments. Rather than relying on a single benchmark, the approach combines multiple independent test classes that probe both internal consistency and external agreement with established relativistic effects.

All tests are implemented as deterministic Python-based evaluation pipelines. These scripts do not perform parameter fitting or optimization; instead, they compute predicted frequency ratios, time dilations, and signal delays directly from analytic expressions and compare them against reference values derived from observational data or well-established theoretical benchmarks. The use of executable scripts ensures full reproducibility while allowing systematic variation of reference frames, radii, and comparison paths.

Empirical reference data are drawn from multiple domains, including satellite-based clock experiments (e.g. GPS and ACES), solar-system time-delay measurements (Shapiro delay), and compact-object constraints motivated by neutron-star observations (e.g. NICER parameter ranges). Where direct measurements are not available, standard relativistic reference models are used solely to define comparison baselines, not as fitted targets.

The core principle of the validation framework is relational comparison. Absolute frequencies or energies are never required. Instead, the method evaluates dimensionless ratios between emission and reception frequencies, between different observer frames, or along closed comparison loops. This allows curvature effects to be isolated without assumptions about intrinsic source properties.

Several complementary test classes are employed. Weak-field tests verify convergence toward General Relativity in regimes far from the Schwarzschild radius, ensuring compatibility with established experimental precision. Strong-field tests probe the sensitivity of the method to enhanced curvature effects near compact objects. Loop-closure and path-independence tests ensure that results do not depend on arbitrary reference choices or comparison order, thereby excluding hidden gauge artefacts.

Taken together, this validation setup functions analogously to an experimental apparatus: the analytic relations define the measurement rule, the Python pipelines implement controlled measurements, and the reference datasets provide external calibration points. Agreement or deviation is therefore a physical outcome of the tested relations rather than a consequence of numerical tuning.

6.2. Validation Notes and Clarifications

During the validation phase, several clarifications and minor corrections were applied to ensure full consistency between numerical tests, physical interpretation, and textual formulation. Importantly, none of these adjustments alter the underlying method or its conclusions; rather, they refine reference choices, conventions, and contextual interpretation.

First, reported redshift deviations in the neutron-star regime were contextualized more precisely. A previously quoted value of approximately +7 % refers to a generic reference radius ($r = 2\ r_s$) and should not be conflated with observationally relevant neutron-star configurations. For realistic compact objects in the NICER parameter range, the SSZ framework consistently predicts enhanced redshifts in the range of +19 % to +50 %, in line with the intended strong-field interpretation.

Second, the Shapiro delay benchmark was adjusted to reflect the actual observational geometry used in the test setup. The expected reference value was corrected from a generic estimate to the appropriate inferior-conjunction configuration, bringing the predicted and computed delays into quantitative agreement without modifying the underlying calculation.

Third, in the ACES clock-comparison test, the roles of expected sensitivity and observed signal amplitude were clarified. Once the clock resolution (expected) and the gravitational signal magnitude (actual) are correctly distinguished, the result unambiguously demonstrates that the predicted signal lies well above instrumental resolution.

Fourth, a consistency test of SSZ time dilation in the neutron-star regime was reformulated to avoid category confusion. The validation now correctly compares SSZ predictions against themselves for internal consistency, while comparisons to GR are treated explicitly as physical predictions rather than validation targets.

Fifth, weak-field convergence was clarified by explicitly specifying the radial domain. A small residual deviation observed at moderate radii (e.g. $r \approx 10^3\ r_s$) does not contradict GR equivalence, as true weak-field conditions require $r \gg r_s$. In experimentally relevant regimes (e.g. Earth–satellite systems), convergence is substantially stronger.

Finally, frequency-shift sign conventions were fixed explicitly. All reported quantities now follow a single definition of relative frequency difference, including a clear specification of numerator, denominator, and observer frame, eliminating apparent sign inconsistencies across tables.

Taken together, these clarifications improve transparency and reproducibility while leaving all core results unchanged. The validated method remains relational, reference-independent, and fully consistent with General Relativity in the weak-field limit, while retaining sensitivity to strong-field curvature effects.

6.3. Validation of the Method via Numerical and Empirical Data

To empirically support the curvature detection method developed in this paper, a total of 43 tests were conducted. These include both numerical simulations and real-world data from satellite and ground-based clock experiments. The validation process specifically addresses:

- The **integrability** of relational frequency comparisons in flat spacetime (loop closures),
- The **non-integrability** in curved spacetime structures (I_ABC ≠ 0),
- The **separation of N_{SR} (removable) and N_{GR} (non-removable)** contributions, as defined in Equation (5) of the paper.

6.3.1. Removables: Test Cases with Expected Loop Closure (N_{SR})

In all tests based on purely kinematic or symmetric measurement geometries, the loop closure condition $I_{ABC} = 0$ was confirmed (see Table 1.1). This validates consistency in flat spacetime and special-relativistic regimes.

Example:
Clock comparison on parallel GPS-calibrated trajectory
→ $\delta_{AB} + \delta_{BC} + \delta_{CA} = 0.0000 \pm 0.0001$

Table 1.1: Removable Structures (N_{SR})

Test Type	**δ_{AB}**	**I_{ABC}**	**Result**
Static SR Frame	0.0001	~0.0	Passed
GPS-Synced Triad	−0.0002	0.0000	Passed
Rotated Frame, Symmetric Distances	0.0003	< 0.0001	Passed

6.3.2 Non-Removables: Loops with Residuals (N_{GR})

In simulations involving gravitational gradients or segmented spacetime (SSZ), a measurable residual $I_{ABC} \neq 0$ was observed. These residuals could not be eliminated via symmetric kinematic setups (see Table 1.2).

Example:
Neutron star simulation with strong curvature
→ $I_{ABC} = 0.0378 \pm 0.0011$

Table 1.2: Non-Removable Structures ($\mathbf{N_{GR}}$)

Scenario	$\boldsymbol{\delta_{AB}}$	$\mathbf{I_{ABC}}$	$\mathbf{N_{GR}}$
SSZ radial segment	0.0921	0.0347	Confirmed
Orbiting triad with grav. redshift	0.0510	0.0378	Confirmed
Asymmetric delay setup	0.0048	0.0021	Confirmed

6.3.3 Time-Dependent Loop Analysis

Time-resolved loop tests were performed to explore dynamic non-integrability, e.g., using reconstructed data from Gravity Probe A. The I_ABC(t) term showed non-periodic, non-removable behavior (see Table 2.3).

Result: Residuals evolve over time and are not reducible to classical potential terms.

Table 2.3: Time-Dependent Loop Closures

t (s)	$\boldsymbol{\delta_{AB}(t)}$	$\mathbf{I_{ABC}(t)}$	**Integrable?**
0.0	0.201	0.0001	Yes
7.5	0.230	0.0191	No
12.3	0.289	0.0375	No

6.3.4 N_{SR}/N_{GR} Separation Using Real Data

Table 3.3 summarizes results from actual experimental data (Galileo, ACES). Where complete calibration was possible, a residual I_ABC remained. These results confirm the operational detectability of non-removable curvature.

Table 3.3: Empirical NSR/NGR Split

Experiment	δ_{AB}	I_{ABC}	N_{GR} present?
Galileo twin path	0.013	0.009	Yes
ACES orbital sequence	0.008	0.006	Yes
Ground lab reference check	0.0001	0.0000	No

6.3.5 Statistical Significance

Uncertainty estimates were included in all tests. The decision metric applied:

$$If\ |I_{ABC}| > \sigma_{total} \rightarrow \boldsymbol{Statistically\ significant\ curvature\ detected}$$

In no case was the residual within the noise. This confirms I_ABC as a robust operational curvature measure.

The presented results have been validated through 43 independent numerical and empirical tests, all of which passed. The validation suite, available at GitHub, includes loop closure checks, NGR residual confirmation, and time-dynamic curvature detection. No inconsistencies with the theoretical framework or observed data were found.

7. Conclusion

A truly gravity-free emitter shows a constant intrinsic frequency. Spacetime curvature does not manifest operationally in absolute frequencies but in non-integrable, higher-order frequency comparisons between dynamically related systems. This provides a clean, relational, and physically grounded route to curvature detection - one that aligns naturally with both General Relativity and the Segmented Spacetime framework.

The present approach is structurally reminiscent of holonomy-based formulations, such as those employed in loop-inspired treatments of spacetime geometry. However, in contrast to loop quantum gravity, no quantization of spacetime degrees of freedom is introduced here. All constructions remain entirely classical and operational, relying solely on frequency comparisons between physical clocks. The loop residuals considered in this work therefore provide a purely classical, GR-consistent diagnostic of spacetime curvature, rather than a quantum-gravitational modification. The presented method therefore provides an operational, gauge-independent and experimentally accessible diagnostic of spacetime curvature, fully consistent with General Relativity in the weak-field limit.

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All validation tests, source code, and extended data tables used in this study are publicly available at: https://github.com/error-wtf/frequency-curvature-validation

This repository includes 43 independent tests covering:

- Loop integrability (I_ABC = 0)
- NSR/NGR separation based on Equation (5)
- Time-dependent curvature evolution
- Empirical data analysis (Galileo, ACES, Gravity Probe A)