A Transformation-Based Definition of Local Lorentz Invariance and Its Connection to Frame Dragging

Carmen N. Wrede, Lino P. Casu, Bingsi

Abstract

We develop a transformation-based formulation of local Lorentz invariance in which the decisive criterion is not geometric flatness but the equality between the local Lorentz transformation in General Relativity and its counterpart in Special Relativity. When this equality holds, a local frame behaves inertially; when it fails, rotational effects arise. In this picture, frame dragging appears as a direct consequence of a mismatch between the GR and SR transformation structures, independent of whether matter itself undergoes physical rotation. This distinction naturally explains why light exhibits apparent high-speed optical motion near rotating spacetimes while matter can remain stationary. The framework accommodates the Sagnac effect as the flat-spacetime analogue of this transformation mismatch, demonstrating that rotation-induced optical asymmetries do not require curvature. Finally, the formulation integrates naturally with segmented-spacetime models (SSZ), where differences in local transformation structure encode effective gravitational rotation. This provides a unified operational criterion for identifying inertial, rotating, and segment-modified local frames.

1. Introduction

Local Lorentz invariance in the literature is usually formulated geometrically:

Any sufficiently small region of a curved spacetime can be approximated by a Minkowski tangent space.

We propose an alternative formulation:

> **Local Lorentz invariance is present exactly when the local Lorentz transformation derived from GR is identical to the Lorentz transformation of SR.**

This definition is operational, mathematically transparent and offers a direct way to characterize the onset of rotational or twisting effects in strong gravitational fields.

2. Formal Background

2.1 Lorentz transformations in SR

In flat spacetime, Lorentz transformations [1,2] satisfy:

$$x'^{\mu} = \Lambda^{\mu}_{\ \nu}\, x^{\nu}$$

with

$$\Lambda^{T}\, \eta\, \Lambda = \eta.$$

2.2 Local inertial frames in GR

In curved spacetime, a tetrad field $e_{\mu}{}^{a}$ defines a local Minkowski metric:

$$g_{\mu\nu}(x) = e^{a}_{\mu}\, e^{b}_{\nu}\, \eta_{ab}.$$

This yields a local Lorentz transformation:

$$\mathrm{LT}_{\mathrm{GR(l)}}: \quad x^{a} = e^{a}_{\mu}\, x^{\mu}.$$

3. Transformation-Based Definition of Local Lorentz Invariance

We define:

$$\mathrm{LT}_{\mathrm{GR(l)}} = \mathrm{LT}_{\mathrm{SR}}.$$

as the necessary and sufficient condition for local Lorentz invariance.

This implies:

- no rotational or twisting components,
- no off-diagonal coupling terms between time and space,
- no inertial drift of local frames.

The definition is independent of the geometric statement “spacetime is locally flat” and instead focuses on the equality of the transformations themselves.

4. Violation of Equality: Origin of Frame Dragging

In rotating GR solutions, such as the Kerr spacetime [3], off-diagonal terms appear:

$$g_{t\phi} \neq 0$$

signifying a coupling between temporal and angular directions.

This directly implies:

$$\mathrm{LT}_{\mathrm{GR(l)}} \neq \mathrm{LT}_{\mathrm{SR}}.$$

We formulate:

> **Theorem (informal).**
>
> **Frame dragging occurs precisely when the local Lorentz transformation of GR deviates from that of SR.**

Consequences:

- Local inertial frames acquire rotation relative to static observers,
- Geodesics exhibit angular drift,
- Freely falling frames are “dragged” by spacetime,
- Structures such as Zero-Angular-Momentum Observers (ZAMOs) emerge naturally.

This yields a transformation-level characterization of frame dragging, without invoking additional geometric axioms.

4.1 Optical Motion vs. Physical Motion in Rotating Spacetimes

An important consequence of the transformation-based framework is the distinction between **optical motion** (the observed motion of light) and **physical motion** (the motion of massive matter). In rotating spacetimes such as Kerr, these two do not coincide.

Although massive matter can remain perfectly stationary with respect to a static observer, the light emitted from that matter does not. Because null geodesics are entirely determined by the local geometry, they respond immediately to the rotation of spacetime. In Kerr spacetime, the off-diagonal term

$$g_{t\phi} \neq 0$$

tilts the local light cones. As a result, photons propagate asymmetrically even when their source is not moving. This effect is known as **optical frame dragging** or the **dragging of light cones**.

Thus:

$$\text{Matter stationary} \neq \text{Light stationary.}$$

Light follows the twisted null directions, while matter due to inertia can remain at rest if external forces or stable orbital conditions permit it. The observer perceives the distorted light-paths as apparent motion, even though no physical motion of the matter occurs.

This creates an observational illusion analogous to gravitational lensing, but with an additional rotational component: Spacetime curvature bends light, and spacetime rotation twists it. The effect is purely geometric and arises from the deviation between $\mathrm{LT}_{\mathrm{GR(l)}}$ and $\mathrm{LT}_{\mathrm{SR}}$.

In this sense, light reveals the Lorentz-structure distortion immediately, while massive matter reacts only through forces and inertial constraints. Optical motion therefore becomes a sensitive indicator of local deviations from Lorentz invariance.

4.2 The Sagnac Effect as a Flat-Spacetime Analogue

The transformation-based view of local Lorentz invariance is not restricted to strongly curved spacetimes or black holes. A closely related phenomenon appears already in flat spacetime when described in a rotating frame: The Sagnac effect [4].

In its standard form, the Sagnac effect is observed in a ring interferometer placed on a rotating platform. Two coherent light beams are sent in opposite directions around a closed loop and recombined after one round trip. Even though the spacetime is globally flat, the rotating frame induces a measurable time delay

$$\Delta t = \frac{4A\Omega}{c^2}$$

where A is the area enclosed by the loop, Ω the angular velocity of the platform and c the invariant causal propagation rate. One beam returns earlier than the other, as if the light had been "dragged" by the rotation [5].

In the language of our framework, this can be interpreted as follows:

1. In an inertial frame, the metric is strictly Minkowski and the Lorentz transformation is the standard $\mathrm{LT}_{\mathrm{SR}}$.

2. In the rotating frame, the effective line element acquires off-diagonal terms mixing time and angle, leading to a nontrivial local transformation $\mathrm{LT}_{\mathrm{rot}}$.

3. Although the underlying spacetime remains flat, the local relation between time and space coordinates is altered by the rotation of the frame itself.

$$\mathrm{LT}_{\mathrm{rot}} \neq \mathrm{LT}_{\mathrm{SR}}.$$

The Sagnac effect can therefore be understood as a **flat-spacetime analogue of frame dragging**. No curvature is required, only a departure of the local transformation from the inertial SR Lorentz transformation. Light immediately reflects this deviation, producing an observable phase shift, while matter may remain at rest with respect to the rotating apparatus.

In both the Kerr case and the Sagnac setup, the key structure is the same:

- a nontrivial mixing of temporal and angular directions in the local description,
- a deviation of the local transformation from the inertial SR Lorentz transformation,
- and a resulting asymmetry in the propagation of light.

This supports our central claim that **optical phenomena such as frame dragging and Sagnac-type phase shifts are direct signatures of a mismatch between the local GR (or rotating-frame) transformation and the SR Lorentz transformation, rather than requiring curvature alone**.

5. Connection to Segmented Spacetime (SSZ)

In SSZ, spacetime is structured by a segmentation density. We identify:

$$\text{Segmentation} = 0 \ \Rightarrow \ \text{LT}_{\text{GR(l)}} = \text{LT}_{\text{SR}}.$$

$$\text{Segmentation}_{\text{GR}} = \ \text{Segmentation}_{\text{SR}} \ \Leftrightarrow \text{effective gravitational field } = 0 \ \Leftrightarrow \text{LT}_{\text{GR(l)}} = \text{LT}_{\text{SR}}.$$

In this framework, nulling gravity in SR corresponds exactly to equating the segmentation structure of the involved frames.

Thus, frame dragging corresponds to increased segmentation density in this model.

6. Conclusion

By defining local Lorentz invariance through the equality of local Lorentz transformations in GR and SR, we obtain a new, operational perspective on how spacetime behaves in rotating environments. Instead of relying solely on geometric flatness, this framework characterizes local physics through the transformation properties themselves. Whenever $LT_{GR(l)} = LT_{SR}$ local Lorentz invariance holds, and no rotational or twisting effects arise.

In contrast, the violation of this equality provides a direct, transformation-based explanation of frame dragging: the local mixing of temporal and angular directions (e.g., $g_{t\phi} \neq 0$) leads to tilted light cones and asymmetric propagation of light, even when massive matter remains stationary. This distinction between optical motion and physical motion clarifies why light can exhibit apparent high-speed behaviour around rotating bodies while matter does not.

Importantly, this perspective extends beyond curved spacetimes. The Sagnac effect demonstrates that similar transformation mismatches arise even in flat spacetime when described in a rotating frame. Here, too, the deviation from the inertial SR Lorentz transformation produces measurable optical asymmetries without requiring curvature.

Thus, both Kerr frame dragging and the Sagnac effect emerge as manifestations of the same underlying principle: Optical phenomena act as sensitive indicators of local deviations from the SR Lorentz transformation, whether caused by spacetime curvature, frame rotation, or segmentation in SSZ models. This unifying viewpoint suggests that transformation-based methods may be a powerful tool for analysing rotational phenomena across classical GR, alternative spacetime models, and experimental setups.

References

1. Einstein, A. (1905), *Zur Elektrodynamik bewegter Körper*, Annalen der Physik 17, 891–921.

2. Carroll, S. M. (2004), *Spacetime and Geometry: An Introduction to General Relativity*, Addison Wesley, San Francisco.

3. Kerr, R.P. (1963), *Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics*, Physical Review Letters 11, 237–238.

4. Post, J. (1967), *Sagnac Effect*, Reviews of Modern Physics 39, 475–493.

5. Rizzi, G. & Ruggiero, M. L. (2004), *The Sagnac effect: Correctly interpreting the relativistic time delay*, in Relativity in Rotating Frames, edited by G. Rizzi and M. L. Ruggiero, Kluwer Academic Publishers, Dordrecht.