

# DEGREES OF FREEDOM BOOKKEEPING

*Why 4 images give an exact solution for  $m=2$  model*

## EQUATIONS (Constraints):

- Each image gives 2 equations (x and y components)
- 4 images → 8 equations total

## UNKNOWN (Parameters):

- $\theta_E$  (Einstein radius): 1 parameter
- (a, b) quadrupole amplitudes: 2 parameters
- $\phi_\gamma$  quadrupole phase: 1 parameter (NONLINEAR)
- ( $\beta_x$ ,  $\beta_y$ ) source position: 2 parameters
- Total: 6 parameters (5 linear + 1 nonlinear)

## SOLUTION STRATEGY:

1. Fix  $\phi_\gamma$  → 5 linear unknowns, 8 equations
2. Use 5 equations to solve for 5 unknowns exactly
3. Remaining 3 equations give consistency condition
4. Find  $\phi_\gamma$  where consistency = 0 (rootfinding)

8 equations - 6 unknowns = 2 redundant equations  
→ System is OVERDETERMINED → Consistency check possible!