Trapezoidal Rule

The Program:

```
def func(x):
  a=1+x
 b=1/a
  return b
h=0.1
a=int(input("Enter the lower limit:"))
b=int(input("Enter the upper limit:"))
n=(b-a)/h
f=int(n)
suml=0
for i in range(1,f):
  x=a+(i*h)
  suml=suml+(2*func(x))
k=func(a)+suml+func(b)
t = (h/2) *k
print(t)
The Output:
```

Enter the lower limit:0 Enter the upper limit:1 0.6937714031754278

Simpson 1/3rd Method (MP)

The program:

```
def func(x):
  a=1+x
  b=1/a
  return b
h=0.1
a=int(input("Enter the lower limit:"))
b=int(input("Enter the upper limit:"))
n=(b-a)/h
f=int(n)
sume=0
sumo=0
for i in range(1,f):
 x=a+(i*h)
 if(i%2==0):
    sume=sume+(2*func(x))
  else:
    sumo=sumo+(4*func(x))
k=func(a)+sume+sumo+func(b)
t = (h/3) *k
print(t)
The Output :
Enter the lower limit:0
Enter the upper limit:1
0.6931502306889303
```

RK-4 Method (MP)

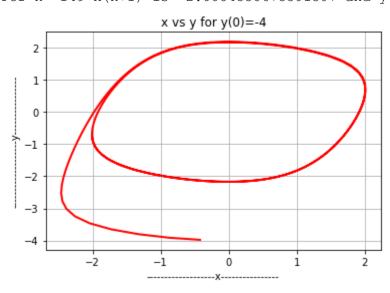
```
The Program:
#Program: Coupled oscillations
(dx/dt=y+x-x^3/3;dy/dt=-x, x(0)=0, y(0)=-1, -2, -3, -4)
def fun(x,y):
  z=y+x-(x**3)/3
  return z
def func(x, y):
  w=-x
  return w
h=0.1
0=0x
y0 = -4
t0 = 0
x=[]
y=[]
t=[]
for i in range(150):
  k1x=h*fun(x0,y0)
  k1y=h*func(x0,y0)
  k2x=h*fun(x0+k1x/2,y0+k1y/2)
  k2y=h*func(x0+k1x/2,y0+k1y/2)
  k3x=h*fun(x0+k2x/2,y0+k2y/2)
  k3y=h*func(x0+k2x/2,y0+k2y/2)
  k4x=h*fun(x0+k3x,y0+k3y)
  k4y=h*func(x0+k3x,y0+k3y)
  y1=y0+((k1y+(2*k2y)+(2*k3y)+k4y)/6)
  x1=x0+((k1x+(2*k2x)+(2*k3x)+k4x)/6)
  y0=y1
  x0=x1
  t0=t0+h
  x.append(x0)
  y.append(y1)
  t.append(t0)
  print("For n=",i,"x(n+1) is", x0, "and y(n+1) is", y1)
import matplotlib.pyplot as plt
plt.plot(x,y,linewidth=2, color='red')
```

```
plt.grid(True)
plt.xlabel('-----')
plt.ylabel('----')
plt.title('x vs y for y(0) = -4')
plt.show()
plt.clf()
The Output:
For n = 0 \times (n+1) is -0.4193715049966315 and y(n+1) is -3.979343135206221
For n = 1 \times (n+1) is -0.8687872455369987 and y(n+1) is -3.9150944846224833
For n = 2 \times (n+1) is -1.3190558415881173 and y(n+1) is -3.805530010349846
For n = 3 \times (n+1) is -1.7241585687039527 and y(n+1) is -3.652789843145498
For n = 4 \times (n+1) is -2.0421461126775586 and y(n+1) is -3.4636309643628485
For n = 5 \times (n+1) is -2.2576728998608497 and y(n+1) is -3.2477939969492846
For n = 6 \times (n+1) is -2.383200854471882 and y(n+1) is -3.0150813879721694
For n = 7 \times (n+1) is -2.443240805125539 and y(n+1) is -2.7733034020385956
For n = 8 \times (n+1) is -2.460678737563513 and y(n+1) is -2.5278212500156934
For n = 9 \times (n+1) is -2.451921925618912 and y(n+1) is -2.2820177278646594
For n = 10 \times (n+1) is -2.4273272911967743 and y(n+1) is -2.0379501361419994
For n = 11 \times (n+1) is -2.3930316213975984 and y(n+1) is -1.7968667313633582
For n = 12 \times (n+1) is -2.352560220762234 and y(n+1) is -1.5595442106166766
For n = 13 \times (n+1) is -2.3079048346023776 and y(n+1) is -1.3264906499352431
For n = 14 \times (n+1) is -2.2601734826104494 and y(n+1) is -1.0980633630304013
For n = 15 \times (n+1) is -2.209964574294552 and y(n+1) is -0.87453673895569
For n = 16 \times (n+1) is -2.1575781595094505 and y(n+1) is -0.6561415872015166
For n = 17 \times (n+1) is -2.103134303356848 and y(n+1) is -0.4430884605973715
For n = 18 \times (n+1) is -2.046639041372445 and y(n+1) is -0.23558202134861395
For n = 19 \times (n+1) is -1.9880206063500285 and y(n+1) is -0.03383044204341115
For n = 20 \times (n+1) is -1.9271484942184478 and y(n+1) is 0.16194787868285906
For n = 21 \times (n+1) is -1.863842280902928 and y(n+1) is 0.3515189513024576
For n = 22 \times (n+1) is -1.7978739474614829 and y(n+1) is 0.5346283644289189
For n = 23 \times (n+1) is -1.728965689146696 and y(n+1) is 0.7109964487892954
For n = 24 \times (n+1) is -1.65678414989181 and y(n+1) is 0.8803130383685358
For n = 25 \times (n+1) is -1.5809313953681778 and y(n+1) is 1.042231491152426
For n = 26 \times (n+1) is -1.5009325322021672 and y(n+1) is 1.196361641512429
For n = 27 \times (n+1) is -1.4162196080806804 and y(n+1) is 1.3422613324579689
For n = 28 \times (n+1) is -1.3261112615717396 and y(n+1) is 1.4794261302071092
For n = 29 \times (n+1) is -1.229787568789064 and y(n+1) is 1.6072767677572297
For n = 30 \times (n+1) is -1.1262597829497016 and y(n+1) is 1.7251438181657741
For n=31 \times (n+1) is -1.0143354619945808 and y(n+1) is 1.8322491014268285
For n = 32 \times (n+1) is -0.892581384030328 and y(n+1) is 1.9276834608980016
For n = 33 \times (n+1) is -0.7592906985414594 and y(n+1) is 2.0103809634665577
For n = 34 \times (n+1) is -0.6124687671914315 and y(n+1) is 2.0790905792932715
For n = 35 \times (n+1) is -0.44986694139609595 and y(n+1) is 2.132348510806971
For n = 36 \times (n+1) is -0.26911851371118434 and y(n+1) is 2.1684584177088686
```

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For n=37 \times (n+1) is -0.06806723621672325 and y(n+1) is 2.185493941170839
For n = 38 \times (n+1) is 0.15458524187608483 and y(n+1) is 2.1813488912291925
For n = 39 \times (n+1) is 0.39819662692036484 and y(n+1) is 2.153873332057444
For n = 40 \times (n+1) is 0.6587108085333782 and y(n+1) is 2.101139232550323
For n = 41 \times (n+1) is 0.9272782376194695 and y(n+1) is 2.021855415551002
For n = 42 \times (n+1) is 1.190074037433997 and y(n+1) is 1.9158716855078528
For n = 43 \times (n+1) is 1.4305212681636381 and y(n+1) is 1.7845875927258255
For n = 44 \times (n+1) is 1.6337773882396798 and y(n+1) is 1.631015024177863
For n = 45 \times (n+1) is 1.7911443764977273 and y(n+1) is 1.4593706315440842
For n = 46 \times (n+1) is 1.9016308415302954 and y(n+1) is 1.2743536306442464
For n = 47 \times (n+1) is 1.9702071105275123 and y(n+1) is 1.0804407799263833
For n = 48 \times (n+1) is 2.0046363398188465 and y(n+1) is 0.8814460647286849
For n=49 \times (n+1) is 2.0128499092583456 and y(n+1) is 0.680381761934722
For n = 50 \times (n+1) is 2.0015685081538663 and y(n+1) is 0.4795206824560464
For n = 51 \times (n+1) is 1.975918366266696 and y(n+1) is 0.2805427702736145
For n = 52 \times (n+1) is 1.9395636756437857 and y(n+1) is 0.08469059438617085
For n = 53 \times (n+1) is 1.8950094990179769 and y(n+1) is -0.10709905752255855
For n = 54 \times (n+1) is 1.8439043441506044 and y(n+1) is -0.29409474516310674
For n = 55 \times (n+1) is 1.7872831135813836 and y(n+1) is -0.4756974412942552
For n = 56 \times (n+1) is 1.7257430722925335 and y(n+1) is -0.6513885002526402
For n = 57 \times (n+1) is 1.6595635302883915 and y(n+1) is -0.8206922940411189
For n = 58 \times (n+1) is 1.5887830252227777 and y(n+1) is -0.9831485807455129
For n = 59 \times (n+1) is 1.5132456813336597 and y(n+1) is -1.1382909646421262
For n = 60 \times (n+1) is 1.432625246274572 and y(n+1) is -1.2856288169731895
For n = 61 \times (n+1) is 1.346432508221606 and y(n+1) is -1.4246307310343322
For n = 62 \times (n+1) is 1.2540097413542262 and y(n+1) is -1.5547080460887035
For n = 63 \times (n+1) is 1.1545145707207873 and y(n+1) is -1.6751972697286002
For n = 64 \times (n+1) is 1.04689526547009 and y(n+1) is -1.7853404401476394
For n = 65 \times (n+1) is 0.9298602645977251 and y(n+1) is -1.884262698338354
For n = 66 \times (n+1) is 0.8018474349954997 and y(n+1) is -1.970946734858251
For n = 67 \times (n+1) is 0.6610045619425967 and y(n+1) is -2.044204589808898
For n = 68 \times (n+1) is 0.5052042365845566 and y(n+1) is -2.1026489577929777
For n=69 \times (n+1) is 0.33213685796328873 and y(n+1) is -2.1446694056012974
For n = 70 \times (n+1) is 0.1395572487584261 and y(n+1) is -2.168424780016865
For n = 71 \times (n+1) is -0.07420182164601499 and y(n+1) is -2.171872544500389
For n = 72 \times (n+1) is -0.3093218156216561 and y(n+1) is -2.1528682647098334
For n = 73 \times (n+1) is -0.5631593132541607 and y(n+1) is -2.109378070341939
For n = 74 \times (n+1) is -0.8287627725582583 and y(n+1) is -2.039836829366512
For n = 75 \times (n+1) is -1.0941164640117274 and y(n+1) is -1.9436280765360834
For n = 76 \times (n+1) is -1.343382314147541 and y(n+1) is -1.8215493179512205
For n = 77 \times (n+1) is -1.5606607917253486 and y(n+1) is -1.6760238993831282
For n = 78 \times (n+1) is -1.7347175354655633 and y(n+1) is -1.5108669869918792
For n = 79 \times (n+1) is -1.8617702652567645 and y(n+1) is -1.3306538519964088
For n = 80 \times (n+1) is -1.944853728728046 and y(n+1) is -1.1399798839953146
For n = 81 \times (n+1) is -1.9909579784376006 and y(n+1) is -0.9429125469407147
For n = 82 \times (n+1) is -2.0080899849945277 and y(n+1) is -0.7427487494619441
For n = 83 \times (n+1) is -2.003444878718768 and y(n+1) is -0.5420150556542285
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```
For n = 84 \times (n+1) is -1.9827072438998337 and y(n+1) is -0.34259161272588734
For n = 85 \times (n+1) is -1.950034500825831 and y(n+1) is -0.145867887161083
For n = 86 \times (n+1) is -1.9083171672487467 and y(n+1) is 0.04711646723037807
For n = 87 \times (n+1) is -1.8594876245547167 and y(n+1) is 0.2355604649023597
For n = 88 \times (n+1) is -1.8047847078251125 and y(n+1) is 0.4188197271338209
For n = 89 \times (n+1) is -1.7449520793330064 and y(n+1) is 0.5963475986995106
For n = 90 \times (n+1) is -1.6803763099163507 and y(n+1) is 0.7676529898124578
For n = 91 \times (n+1) is -1.6111778770219574 and y(n+1) is 0.9322695577896426
For n = 92 \times (n+1) is -1.5372675682840837 and y(n+1) is 1.0897321646579694
For n = 93 \times (n+1) is -1.4583778411514445 and y(n+1) is 1.2395576553195151
For n = 94 \times (n+1) is -1.3740757347687687 and y(n+1) is 1.3812278043605282
For n = 95 \times (n+1) is -1.2837616236718987 and y(n+1) is 1.5141728175194338
For n = 96 \times (n+1) is -1.1866565749313116 and y(n+1) is 1.6377541196784682
For n = 97 \times (n+1) is -1.081780376655478 and y(n+1) is 1.7512453951743625
For n = 98 \times (n+1) is -0.9679226607209348 and y(n+1) is 1.8538110602726428
For n = 99 \times (n+1) is -0.8436115112053844 and y(n+1) is 1.9444816710962451
For n = 100 \text{ x}(n+1) is -0.707088687092349 and y(n+1) is 2.022126416104126
For n = 101 \times (n+1) is -0.5563101260572609 and y(n+1) is 2.0854241804400107
For n = 102 \times (n+1) is -0.38900775098875184 and y(n+1) is 2.1328373241723977
For n = 103 \times (n+1) is -0.20287678408794582 and y(n+1) is 2.1625972311985056
For n = 104 \times (n+1) is 0.004009799187165042 and y(n+1) is 2.1727189411544776
For n = 105 \times (n+1) is 0.2324306599411563 and y(n+1) is 2.1610739714189173
For n = 106 \times (n+1) is 0.48080484608649343 and y(n+1) is 2.125562144718428
For n = 107 \times (n+1) is 0.7436917385296504 and y(n+1) is 2.0644220407075005
For n = 108 \times (n+1) is 1.0106726213188433 and y(n+1) is 1.9766806509543744
For n = 109 \times (n+1) is 1.266832232545864 and y(n+1) is 1.8626457383186859
For n = 110 \times (n+1) is 1.4957736051756565 and y(n+1) is 1.724225598836942
For n=111 \times (n+1) is 1.684333264319482 and y(n+1) is 1.5648458023748995
For n = 112 \times (n+1) is 1.8262678504877212 and y(n+1) is 1.3889210575149127
For n = 113 \times (n+1) is 1.9226918431448565 and y(n+1) is 1.201112322470421
For n = 114 \times (n+1) is 1.979686462456489 and y(n+1) is 1.005694996282549
For n = 115 \times (n+1) is 2.005192376948702 and y(n+1) is 0.8062197791230636
For n=116 \times (n+1) is 2.0067828554301705 and y(n+1) is 0.6054481958122367
For n = 117 \times (n+1) is 1.9906463051315582 and y(n+1) is 0.4054492237343623
For n = 118 \times (n+1) is 1.9614039840564528 and y(n+1) is 0.20775191833203335
For n = 119 \times (n+1) is 1.922315075274361 and y(n+1) is 0.013493720496351724
For n = 120 \times (n+1) is 1.875584939965181 and y(n+1) is -0.1764585720418898
For n = 121 \times (n+1) is 1.8226483028275966 and y(n+1) is -0.3614180316750272
For n = 122 \times (n+1) is 1.764388736477414 and y(n+1) is -0.5408120766143258
For n = 123 \times (n+1) is 1.701294560544441 and y(n+1) is -0.7141356416643038
For n = 124 \times (n+1) is 1.633563332646192 and y(n+1) is -0.8809172533095091
For n = 125 \times (n+1) is 1.561168059530094 and y(n+1) is -1.0406935378408
For n = 126 \times (n+1) is 1.4838956711910782 and y(n+1) is -1.1929888878936903
For n = 127 \times (n+1) is 1.4013652359250768 and y(n+1) is -1.3372979119658512
For n = 128 \times (n+1) is 1.3130308558322 and y(n+1) is -1.4730689063788167
For n = 129 \times (n+1) is 1.2181724037126542 and y(n+1) is -1.5996869875369826
For n = 130 \times (n+1) is 1.1158762921537315 and y(n+1) is -1.7164557827981415
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For n = 131 \times (n+1) is 1.005008447895113 and y(n+1) is -1.8225767872283742
For n = 132 \times (n+1) is 0.8841830307944489 and y(n+1) is -1.9171257645524609
For n = 133 \times (n+1) is 0.7517341247369037 and y(n+1) is -1.999026083661003
For n = 134 \times (n+1) is 0.6057053632845932 and y(n+1) is -2.0670199477273137
For n = 135 \times (n+1) is 0.44388696455853904 and y(n+1) is -2.1196406242512356
For n = 136 \times (n+1) is 0.2639542006472738 and y(n+1) is -2.1551928643247544
For n = 137 \times (n+1) is 0.0637966805304103 and y(n+1) is -2.1717557985046616
For n = 138 \times (n+1) is -0.15783753353038454 and y(n+1) is -2.167233394184696
For n = 139 \times (n+1) is -0.40027032332440693 and y(n+1) is -2.139490121552595
For n = 140 \times (n+1) is -0.6594423218746093 and y(n+1) is -2.086614535593255
For n=141 \times (n+1) is -0.9265615936162908 and y(n+1) is -2.0073295457096867
For n = 142 \times (n+1) is -1.1879357271723587 and y(n+1) is -1.9014895079326286
For n = 143 \times (n+1) is -1.4271648937677632 and y(n+1) is -1.7704825675334692
For n = 144 \times (n+1) is -1.6295520558725973 and y(n+1) is -1.617292368699626
For n = 145 \times (n+1) is -1.7864434357988141 and y(n+1) is -1.4460974717106114
For n = 146 \times (n+1) is -1.8967856189298267 and y(n+1) is -1.2615601397491407
For n = 147 \times (n+1) is -1.965433538926796 and y(n+1) is -1.0681295792916927
For n = 148 \times (n+1) is -2.0000391359827714 and y(n+1) is -0.8696039341048776
For n = 149 \times (n+1) is -2.0084550675591597 and y(n+1) is -0.6689892483777681
```



Forward Interpolation (MP)

```
The Program:
def uc(u,n):
  for i in range(1,n):
   t=t*(u-i)
  return t
def factorial(n):
  f=1
  for i in range (2, n+1):
   f=f*i
  return f
X=[0.10,0.15,0.20,0.25,0.30]
n=len(X)
y=[[0 for i in range(n)]
   for j in range(n)]
y[0][0]=0.1003
y[1][0]=0.1511
y[2][0]=0.2027
y[3][0]=0.2553
y[4][0]=0.3093
for i in range(1,n):
  for j in range(n-i):
    y[j][i]=y[j+1][i-1]-y[j][i-1]
for i in range(n):
  print(X[i],end="\t")
 for j in range (n-1):
    print(y[i][j],end="\t")
  print(" ")
Value=0.12
suml=y[0][0]
u = (Value - X[0]) / (X[1] - X[0])
for i in range(1,n):
```

```
suml=suml+(uc(u,i)*y[0][i])/factorial(i)
print("\nValue at", Value,"is", suml)
```

The Output:

```
      0.1
      0.1003
      0.05080000000000000
      0.0007999999999999674
      0.0002000000000000089

      0.15
      0.1511
      0.051599999999999
      0.00100000000000564
      0.00039999999999999044

      0.2
      0.2027
      0.052600000000000036
      0.00139999999999568
      0

      0.25
      0.2553
      0.0539999999999999
      0
      0

      0.3
      0.3093
      0
      0
```

Value at 0.12 is 0.12052848000000002

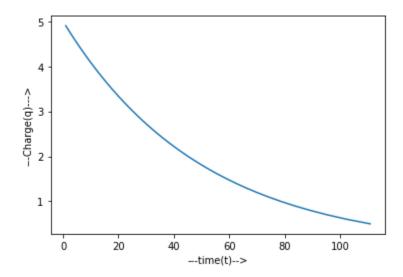
Euler's Method (MP)

The Program:

```
import matplotlib.pyplot as plt
def RCD(q):
  r=10
  c=5
  z=-(r*c)/q
 return z
a=int(input("Charge of the capacitor at time t=0, i.e. Qo:"))
b=float(input("Charge at which you wish to calculate the time t, i.e.
Qn:"))
h=float(input("Enter step:"))
n=(b-a)/h
a=int(n)
t=0
q=5
A=[]
B=[]
for i in range(0,a):
 t=t+(h*RCD(q))
 q=q+h
 A.append(t)
 B.append(q)
print("At time",t,"value of q is:",q)
plt.plot(A,B)
plt.xlabel("---time(t)-->")
plt.ylabel("---Charge(q)--->")
```

The Output:

Charge of the capacitor at time t=0, i.e. Qo:5 Charge at which you wish to calculate the time t, i.e. Qn:0.5 Enter step:-0.1 At time 110.79360024980448 value of q is: 0.500000000000001 Text(0, 0.5, '---Charge(q)--->')



Backward Interpolation (MP)

The Program:

```
def uc(u,n):
  t=u
  for i in range(1,n):
   t=t*(u+i)
  return t
def factorial(n):
  f=1
  for i in range (2, n+1):
    f=f*i
  return f
X=[15,20,25,30,35,40]
n=len(X)
y=[[0 for i in range(n)]
   for j in range(n)]
y[0][0]=0.2588190
y[1][0]=0.3420201
y[2][0]=0.4236183
y[3][0]=0.5
y[4][0]=0.5735764
y[5][0]=0.6427276
for i in range(1,n):
  for j in range (n-i, i-1, -1):
    y[j][i]=y[j][i-1]-y[j-1][i-1]
for i in range(n):
 print(X[i],end="\t")
  for j in range(i+1):
    print(y[i][j],end="\t")
  print(" ")
Value=38
suml=y[n-1][0]
u = (Value - X[n-1]) / (X[1] - X[0])
```

```
for i in range(1,n):
    suml=suml+((uc(u,i)*y[n-1][i])/factorial(i))
print("\nValue at", Value, "is", suml)
```

The Output:

15	0.258819					
20	0.3420201	0.08320109999999997				
25	0.4236183	0.08159820000000001	-0.0016028999	9999999627		
30	0.5	0.0763817	-0.005216500000000013		-0.0036136000000005	
35	0.5735764	0.07357639999999999	-0.0028053000	000000106	0	0
40	0.6427276	0.06915119999999997	0	0	0	0

Value at 38 is 0.61506712

RUNGE-KUTTA(RK-2) Method

Program:-

```
#Given equation is:dy/dx=x+y^2 given that x0=0, y(x0)=1;
import matplotlib.pyplot as plt
def f(x, y):
  z=x+y**2
  return z
x0=0
v_0 = 1
h = 0.1
X = []
Y = []
X.append(x0)
Y.append(y0)
for i in range (0, 10):
  k1=h*f(x0,y0)
  k2=h*f(x0+h,y0+k1)
  y1=y0+(k1+k2)/2
  x0+=h
  y0=y1
  X.append(x0)
  Y.append(y1)
  print("When the value of n is",i,"x0=",x0,"y0=",y1)
plt.plot(X,Y,linewidth=2,color="blue")
plt.grid(True)
plt.xlabel("<---x--->")
plt.ylabel("<---y--->")
plt.show()
```

> Output

```
When the value of n is 0 \times0= 0.1 \times0= 1.1155
When the value of n is 1 \times0= 0.2 \times0= 1.270833765842635
When the value of n is 3 \times 0 = 0.4 \text{ y0} = 1.7768104862553225
When the value of n is 4 \times 0=0.5 \text{ y0}=2.2070444937896987
When the value of n is 5 \times 0 = 0.6 \times 0 = 2.8821144595837636
When the value of n is 6 \times 0 = 0.7 \text{ y0} = 4.074134390866686
```

