Leung code as an example of quantum polar code

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Contents

Consider Leung 4-qubit code

$$|0_{\texttt{leung}}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle),$$
$$|1_{\texttt{leung}}\rangle = \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle). \tag{0.1}$$

Noth that the following circuit (after Hadamard gate) may polarize noisy channel $\mathcal{N}$.

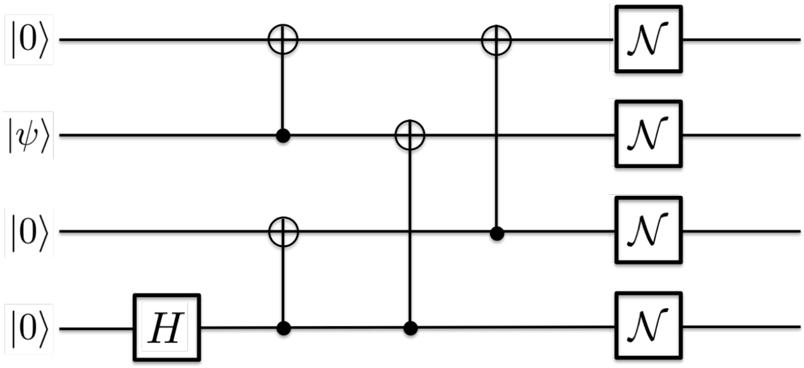

Figure 1: Channel synthesis: Compare this with figure 2 of [1] and figure 2 of [2].

Starting from $|\Phi\rangle = |0\rangle|\psi\rangle|0\rangle|0\rangle = \alpha|0000\rangle + \beta|0100\rangle$, we have

$$\begin{aligned}|\Phi\rangle &\to \alpha\frac{1}{\sqrt{2}}\Big[|0000\rangle + |0001\rangle\Big] + \beta\frac{1}{\sqrt{2}}\Big[|0100\rangle + |0101\rangle\Big]\\ &\to \alpha\frac{1}{\sqrt{2}}\Big[|0000\rangle + |0011\rangle\Big] + \beta\frac{1}{\sqrt{2}}\Big[|1100\rangle + |1111\rangle\Big]\\ &\to \alpha\frac{1}{\sqrt{2}}\Big[|0000\rangle + |1111\rangle\Big] + \beta\frac{1}{\sqrt{2}}\Big[|1100\rangle + |0011\rangle\Big] = \alpha|0_{\texttt{leung}}\rangle + \beta|1_{\texttt{leung}}\rangle.\end{aligned} \tag{0.2}$$

interpretation may be the second qubit gives a good channel (after polarization) and we froze the first and third qubit for amplitude and the fourth for phase.

Bibliography

[1] E. Arkan, *"Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels"*, IEEE Trans. Inf. Theory **55**, 3051 (2009).

[2] Joseph M. Renes, Frdric Dupuis, and Renato Renner, *"Efficient Polar Coding of Quantum Information"*, Phys. Rev. Lett. **109**, 050504 (2012).