M.S. Ramaiah Institute of Technology
(Autonomous Institute, Affiliated to VTU)
Department of Computer Science and Engineering

Course Name: Database Systems

Course Code: CS52

Credits: 3:1:0

UNIT 4

Term: Oct 2021 – Feb 2022

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Inference Rules for FDs

Given a set of FDs F, we can **infer** additional FDs that hold whenever the FDs in F hold

Armstrong's inference rules:

- IR1. (**Reflexive**) If Y *subset-of* X, then X -> Y
- IR2. (**Augmentation**) If X -> Y, then XZ -> YZ
  - (Notation: XZ stands for X U Z)
- IR3. (**Transitive**) If X -> Y and Y -> Z, then X -> Z

IR1, IR2, IR3 form a **sound** and **complete** set of inference rules

- These are rules hold and all other rules that hold can be deduced from these

Inference Rules for FDs

Some additional inference rules that are useful:

- **Decomposition:** If X -> YZ, then X -> Y and X -> Z
- **Union:** If X -> Y and X -> Z, then X -> YZ
- **Psuedotransitivity:** If X -> Y and WY -> Z, then WX -> Z

The last three inference rules, as well as any other inference rules, can be deduced from IR1, IR2, and IR3 (completeness property)

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Proof of Inference Rules

Proof of IR1. Suppose that $X \supseteq Y$ and that two tuples t_1 and t_2 exist in some relation instance r of R such that $t_1 [X] = t_2 [X]$. Then $t_1[Y] = t_2[Y]$ because $X \supseteq Y$; hence, $X \rightarrow Y$ must hold in r.

Proof of IR2 (by contradiction). Assume that $X \rightarrow Y$ holds in a relation instance r of R but that $XZ \rightarrow YZ$ does not hold. Then there must exist two tuples t_1 and t_2 in r such that (1) $t_1 [X] = t_2 [X]$, (2) $t_1 [Y] = t_2 [Y]$, (3) $t_1 [XZ] = t_2 [XZ]$, and (4) $t_1 [YZ] \neq t_2 [YZ]$. This is not possible because from (1) and (3) we deduce (5) $t_1 [Z] = t_2 [Z]$, and from (2) and (5) we deduce (6) $t_1 [YZ] = t_2 [YZ]$, contradicting (4).

Proof of IR3. Assume that (1) $X \rightarrow Y$ and (2) $Y \rightarrow Z$ both hold in a relation r. Then for any two tuples t_1 and t_2 in r such that $t_1 [X] = t_2 [X]$, we must have (3) $t_1 [Y] = t_2 [Y]$, from assumption (1); hence we must also have (4) $t_1 [Z] = t_2 [Z]$ from (3) and assumption (2); thus $X \rightarrow Z$ must hold in r.

Proof of Inference Rules

Proof of IR4 (Using IR1 through IR3).

1. $X \rightarrow YZ$ (given).
2. $YZ \rightarrow Y$ (using IR1 and knowing that $YZ \supseteq Y$).
3. $X \rightarrow Y$ (using IR3 on 1 and 2).

Proof of IR5 (using IR1 through IR3).

1. $X \rightarrow Y$ (given).
2. $X \rightarrow Z$ (given).
3. $X \rightarrow XY$ (using IR2 on 1 by augmenting with X; notice that $XX = X$).
4. $XY \rightarrow YZ$ (using IR2 on 2 by augmenting with Y).
5. $X \rightarrow YZ$ (using IR3 on 3 and 4).

Proof of IR6 (using IR1 through IR3).

1. $X \rightarrow Y$ (given).
2. $WY \rightarrow Z$ (given).
3. $WX \rightarrow WY$ (using IR2 on 1 by augmenting with W).
4. $WX \rightarrow Z$ (using IR3 on 3 and 2).

Problem on Inference Rules:

1. Prove or disprove the following inference rules for functional dependencies. A proof can be made by using inference rules R1 through R3. A disproof should be performed by demonstrating a relation that satisfies the conditions and functional dependencies in the left-hand side of the inference rule but does not satisfy the dependencies in the right-hand side.

Prove:

a. $\{W \Rightarrow Y, X \Rightarrow Z\} \models WX \Rightarrow Y$

b. $\{X \Rightarrow Y$, Z is a subset of $Y\} \models X \Rightarrow Z$

c. $\{X \Rightarrow Y, X \Rightarrow W, WY \Rightarrow Z\} \models X \Rightarrow Z$

d. $\{X \Rightarrow Y, XY \Rightarrow Z\} \models X \Rightarrow Z$

e. $\{X \Rightarrow Y, Z \Rightarrow W\} \models XZ \Rightarrow YZ$

f. $\{X \Rightarrow Y, Y \Rightarrow Z\} \models X \Rightarrow YZ$

Disprove:

g. $\{XY \Rightarrow Z, Y \Rightarrow W\} \models XW \Rightarrow Z$

Closure of a set F of FDs

- **Closure** of a set F of FDs is the set F^+ of all FDs that can be inferred from F.
- **Closure** of a set of attributes X with respect to F is the set X^+ of all attributes that are functionally determined by X. X^+ can be calculated by repeatedly applying IR1, IR2, IR3 using the FDs in F.

Algorithm 10.1: Determining X^+, the Closure of X under F

$X^+ := X$;
repeat
    old$X^+ := X^+$;
    for each functional dependency $Y \rightarrow Z$ in F do
    if $X^+ \supseteq Y$ then $X^+ := X^+ \cup Z$;
until (X^+ = oldX^+);

The following set F of functional dependencies that should hold on EMP_PROJ;

$$F = \{SSN \longrightarrow ENAME,$$
$$PNUMBER \longrightarrow \{PNAME,\ PLOCATION\},$$
$$\{SSN,\ PNUMBER\} \longrightarrow HOURS\}$$

we calculate the following closure sets with respect to F;

$$\{SSN\}^+ = \{SSN,\ ENAME\}$$
$$\{PNUMBER\}^+ = \{PNUMBER,\ PNAME,\ PLOCATION\}$$
$$\{SSN,\ PNUMBER\}^+ = \{SSN,\ PNUMBER,\ ENAME,\ PNAME,\ PLOCATION,\ HOURS\}$$

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Problem:

Consider the following relation schema and set of functional dependancies:

Emp-Dept (SIN, E_Name, B_Date, Address, D_Num, D_Name, D_Manager)

F={SIN→{E_Name, B_Date, Address, D_Num},

D_Num → {D_Name, D_Manager}

}

Calculate the closure of $\{SIN\}^+$ and $\{D\_Num\}^+$ with respect to F.

R(A,B,C,D,E,F)

FDs S = {AB→C, BC→AD, D→E, CF→B}

What is the closure of:

$\{A,B\}^+$

Equivalence of Sets of Functional Dependencies

Two sets of FDs F and G are **equivalent** if:

- Every FD in F can be inferred from G, and
- Every FD in G can be inferred from F
- Hence, F and G are equivalent if $F^+ = G^+$

Definition (**Covers**):

- F **covers** G if every FD in G can be inferred from F
  - (i.e., if G^+ *subset-of* F^+)

F and G are equivalent if F covers G and G covers F.

Equivalence of Sets of Functional Dependencies

Definition. A set of functional dependencies F is said to **cover** another set of functional dependencies E if every FD in E is also in F^+; that is, if every dependency in E can be inferred from F; alternatively, we can say that E is **covered by** F.

Definition. Two sets of functional dependencies E and F are **equivalent** if $E^+ = F^+$. Hence, equivalence means that every FD in E can be inferred from F, and every FD in F can be inferred from E; that is, E is equivalent to F if both the conditions E covers F and F covers E hold.

Examples:

R=(A,B,C,D,E,F)

F1={A->BC, B->CDE, AE->F}

F2={A->BCF, B->DE, E->AB}

Check whether F1 and F2 are equivalent or not.

Solution

To check F1 covers F2 –

A^{+}={A,B,C,D,E,F} contains B,C,F

B^{+}={B,C,D,E} contains D,E

E^{+}={E} contains A,B

So F1 does not cover F2.

Hence F1 and F2 are not equivalent.

Consider another example where two functional dependencies are equivalent.

R=(A,C,D,E,H)

F1={A->C, AC->D, E->AD, E->H},

F2={A->CD, E->AH}

Check whether F1 and F2 are equivalent or not?

Solution

To check F1 covers F2 –

A^{+}={A,C,D} contains C,D

E^{+}={A,D,E,H} contains A,H

So F1 covers F2

To check F2 covers F1:

A^{+}={A,C,D} contains C

{ A,C}+={A,C,D} contains D

E^{+}={A,C,D,E,H} contains A,D,H

So F2 covers F1.

Hence F1 and F2 are equivalent.

Minimal Sets of FDs

A set of FDs is **minimal** if it satisfies the following conditions:

1. Every dependency in F has a single attribute for its RHS.
2. We cannot replace any dependency X -> A in F with a dependency Y -> A, where Y proper-subset-of X (Y subset-of X) and still have a set of dependencies that is equivalent to F.
3. We cannot remove any dependency from F and have a set of dependencies that is equivalent to F.

Minimal Sets of Functional Dependencies

Algorithm 10.2: Finding a Minimal Cover F for a Set of Functional Dependencies E

1. Set $F := E$.
2. Replace each functional dependency $X \rightarrow \{A_1, A_2, \ldots, A_n\}$ in F by the n functional dependencies $X \rightarrow A_1, X \rightarrow A_2, \ldots, X \rightarrow A_n$.
3. For each functional dependency $X \rightarrow A$ in F

   for each attribute B that is an element of X

   if $\{ \{ F - \{X \rightarrow A\} \} \cup \{(X - \{B\}) \rightarrow A\} \}$ is equivalent to F,

   then replace $X \rightarrow A$ with $(X - \{B\}) \rightarrow A$ in F.
4. For each remaining functional dependency $X \rightarrow A$ in F

   if $\{ F - \{X \rightarrow A\} \}$ is equivalent to F,

   then remove $X \rightarrow A$ from F.

In Chapter 11 we will see how relations can be synthesized from a given set of dependencies E by first finding the minimal cover F for E.

Minimal Sets of FDs

- Every set of FDs has an equivalent minimal set.
- There can be several equivalent minimal sets.
- There is no simple algorithm for computing a minimal set of FDs that is equivalent to a set F of FDs.
- To synthesize a set of relations, we assume that we start with a set of dependencies that is a minimal set.

Find Minimal Cover/ Canonical Cover of E

Let the given set of FDs be E : {B → A, D → A, AB → D}.We have to find the minimum cover of E.

- All above dependencies are in canonical form; so we have completed step 1 of Algorithm 10.2 and can proceed to step 2.

In step 2 we need to determine if AB → D has any redundant attribute on the left-hand side; that is, can it be replaced by B → D or A → D?

- Since B → A, by augmenting with B on both sides (IR2), we have BB → AB, or B → AB

(i). However, AB → D as given (ii).

- Hence by the transitive rule (IR3), we get from (i) and (ii), B → D. Hence AB → D may be replaced by B → D.
- We now have a set equivalent to original E , say E′ : {B → A, D → A, B → D}. No further reduction is possible in step 2 since all FDs have a single attribute on the left-hand side.
- In step 3 we look for a redundant FD in E′. By using the transitive rule on B → D and D → A, we derive B → A. Hence B → A is redundant in E′ and can be eliminated.
- **Hence the minimum cover of E is {B → D, D → A}**

Thank you