Chapter 3 Model definition

A popular model studying the interactions between agents with continuous opinions was proposed by Deffuant et al. [4] in 2000. Among the three models they had studied, this dissertation will only focus on the modified versions based on their complete mixing model.

The main task of the complete mixing model in every iteration is to adjust the opinion values of the randomly selected pair. The opinion shifting offset is proportional to the difference between their current opinion values and is controlled by the convergence parameter μ. Although this evolution rule seems very simple, it makes sense to some degree. Considering the possible interaction between agents in real life, Deffuant et al. introduced the threshold by which an adjustment was examined whether to be launched. It is widely accepted that people tend to discuss or interact with those who have similar or close values with them on many subjects. However, there may exist but with fewer possibilities for communications with dissenters.

To be closer to the real-life human interaction pattern, some changes will be applied to that model. The first modified model is called interaction preference model in this dissertation. Firstly, a preference factor relative to opinion difference is introduced to decrease the randomness in pair selection. This is the rationale because two persons with close opinions are more likely to be close to each other. Furthermore, their opinions are with a large probability to get mixed in real life. Therefore, in each time step, if agent A has been chosen, among those adjacent agents, agent B will be selected with a variable possibility, which is a function of the opinion difference between agent A and the one being examined.

For better understanding the preference selection mechanism, a coefficient C is used to control the preference level. The original complete mixing model by Deffuant et al. can be obtained when C=0. The working diagrammatic sketch for the interaction preference model is given in Figure 3.1.

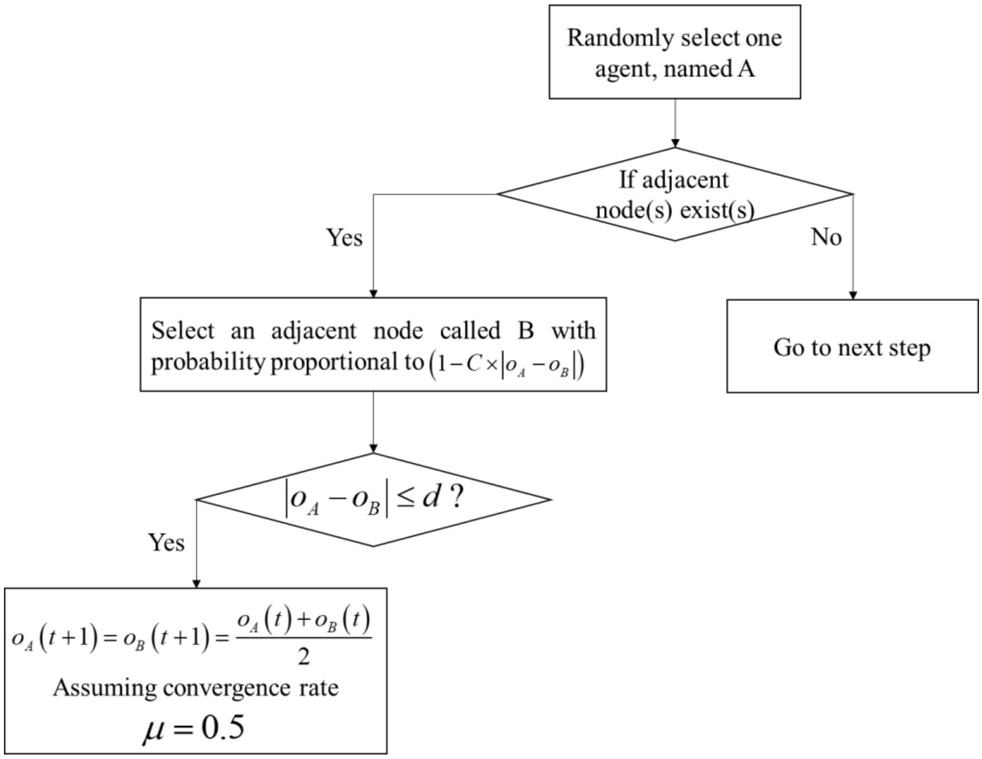


Figure 3.1 Workflow diagram for the interaction preference model

As an extension of the interaction preference model, a rewiring mechanism, idea from [13] is added into the interaction preference model, and the upgraded model is called interaction preference model with rewiring. See Figure 3.2. When the opinion difference of the pair exceeds the threshold, under the condition that agent A's nodal degree is larger than 0 and with a variable probability, rather than doing nothing, the original link between agent A and agent B will be disconnected and a new link will connect agent A to one agent(agent C in Figure 3.2) chosen randomly from the set of all agents with smaller-than-threshold opinion difference with agent A. That probability, a function of the difference of opinion values of the pair chosen, will decide whether the rewiring will happen or not. People may reduce the frequency of the interaction with dissenters[14] and meet new friends who also have similar opinions on some topics. This operation is going to take it into account. A coefficient R is used to adjust the rewiring probability.

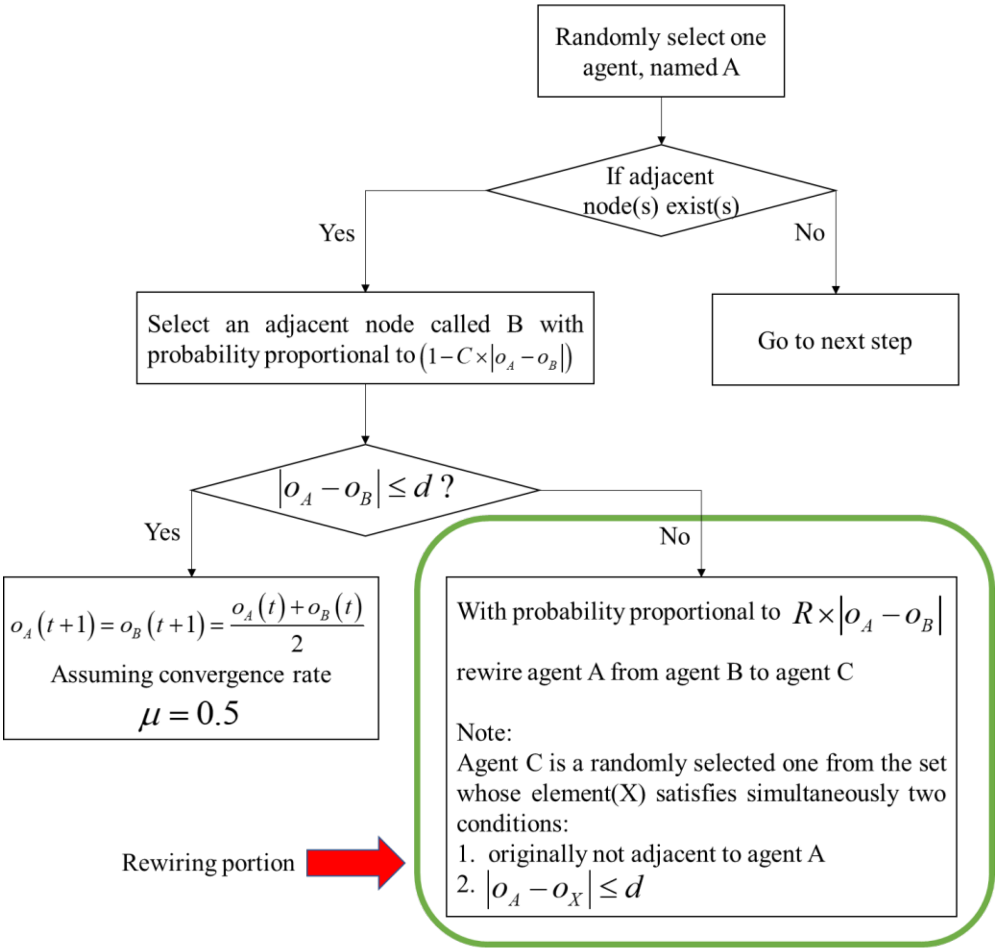


Figure 3.2 Workflow diagram for the interaction preference model with rewiring

Chapter 4 Analysis of Interaction Preference Model

4.1 Introduction

In this chapter, it is going to analyze the interaction preference model by numerical simulation to compare it with the original Deffuant complete mixing model. The main points this chapter cares about are opinion fragmentation, polarization and convergence time. The interaction preference model has been defined in Chapter 3 and the difference has been described. The writer has developed a Python program for this simulation purpose. During the simulation, the program will check each iteration, i.e. one single time pairwise meeting operation no matter the opinions are to be updated or not. Some variables will be utilized to record the opinion change. The routine for them is to find the maximum opinion change in each iteration, and if the value keeps below a pre-set threshold (fixed at 0.00001) for 10,000 iterations, it can be considered to have reached the stead-state. That is the automatic stopping criterion for the interaction preference model.

In the following, a simulation will be executed. To avoid the heavy burden of computing and long time consuming, the population of the network is of size 800. Firstly, a uniform initial distribution of opinions matrix is generated by a Python library according to the system clock as a random seed. Using the function *deepcopy()* provided by Python library *copy*, there will be several identical opinion matrices generated. And then an ER random network[15] of consistent dimension is generated. Similarly, many copies of it will be generated using the same method, although in this chapter it is useless because of no rewiring mechanism involved. Secondly, different threshold d values and coefficient C values will be assigned to sub-simulations.

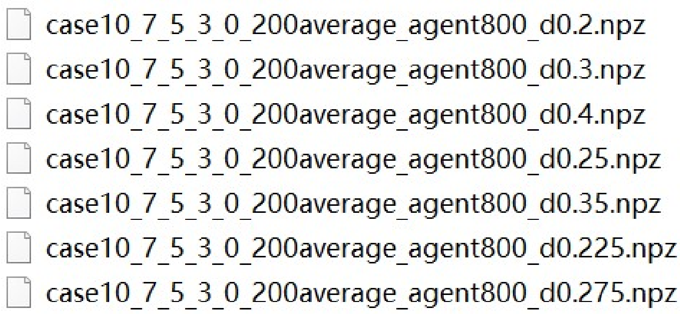


Figure 4.1 snapshot of saved files for interaction preference model simulation

For each simulation task, the initial uniform opinion matrices and adjacent networks used are the same. Each file contains the simulation result under certain conditions. For example, the filename of the first item, “case10_7_5_3_0_200average_agent800_d0.2.npz”, indicates that there are 5 models to be simulated starting from the same opinion matrix and adjacent matrix with C=0.0, 0.3, 0.5, 0.7, 1.0, correspondingly and the tolerance threshold d=0.2. The statistics data are computed over 200 separate simulations for each model. It should be noted that, for instance, in the 50th simulation, all 5 models will start from the same opinion matrix and adjacent matrix.

4.2 Consensus or opinion fragmentation

As we know, the bounded confidence model[4] by Deffuant et al has two significant parameters. The convergence parameter is often fixed at 0.5 in the later works. The tolerance threshold will largely decide the behavior. They have shown the qualitative evaluation of the number of peaks at equilibrium depends largely on the threshold d. They pointed out the maximum number of opinion peaks is inversely proportional to the tolerance value d. According to their explanation, the maxima occur when the minimal distance between peaks is 2d and minimum distance from boundary opinions to the 0 or 1 wall, which leads to $p_{\max} = \dfrac{1}{2d}$.

The Cluster Participation Ratio[16], called CPR for convenience below, is used to quantify the number of clusters.

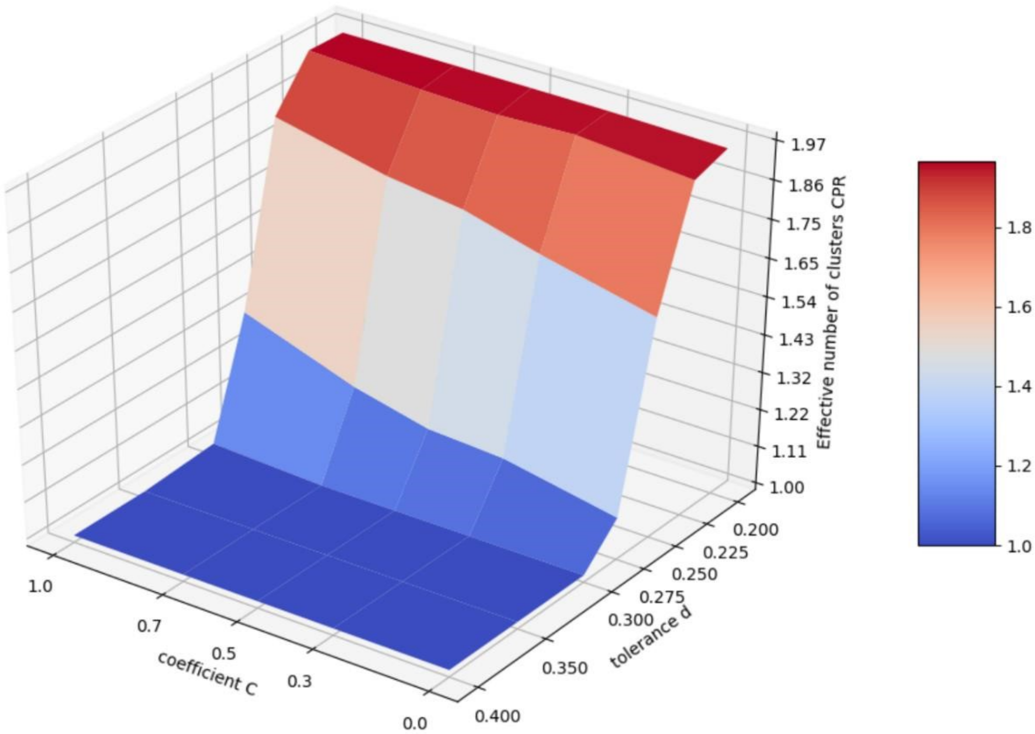


Figure 4.2 Chart of the effective number of clusters obtained for various d and C

Figure 4.2 shows the effective number of clusters for various d and C values with each simulation over 200 runs. The tolerance threshold range is confined within 0.2 to 0.4 because the bigger threshold will always get the number of clusters being 1.

As Figure 4.2 shown, the result of the interaction preference model with C=0.0 is consistent with the original Deffuant model, [16]where the transition between 2 and 1 cluster usually happens for $d \in [0.25, 0.3]$. The numbers of effective clusters indicated in the plot are slightly lower than 2. That is because two clusters with a small amount difference in size will result in a slight drop in cluster participation ratio(CPR) computed. Only two exactly same-size clusters can have CPR=2. Table 4.1 is the raw data used for Figure 4.2 plotting, generated from the simulation.

Table 4.1 Effective number of clusters obtained for various d and C

	C=1.0	C=0.7	C=0.5	C=0.3	C=0.0
d=0.2	1.96779301	1.96637369	1.96491339	1.96318764	1.95604585
d=0.225	1.9699138	1.95247618	1.94514597	1.95284171	1.92512438
d=0.25	1.83998408	1.76939248	1.74131062	1.68337092	1.60931902
d=0.275	1.3420523	1.22862259	1.17708762	1.16384803	1.10195266
d=0.3	1.01446794	1.0	1.0	1.0	1.0
d=0.35	1.0	1.0	1.0	1.0	1.0
d=0.4	1.0	1.0	1.0	1.0	1.0

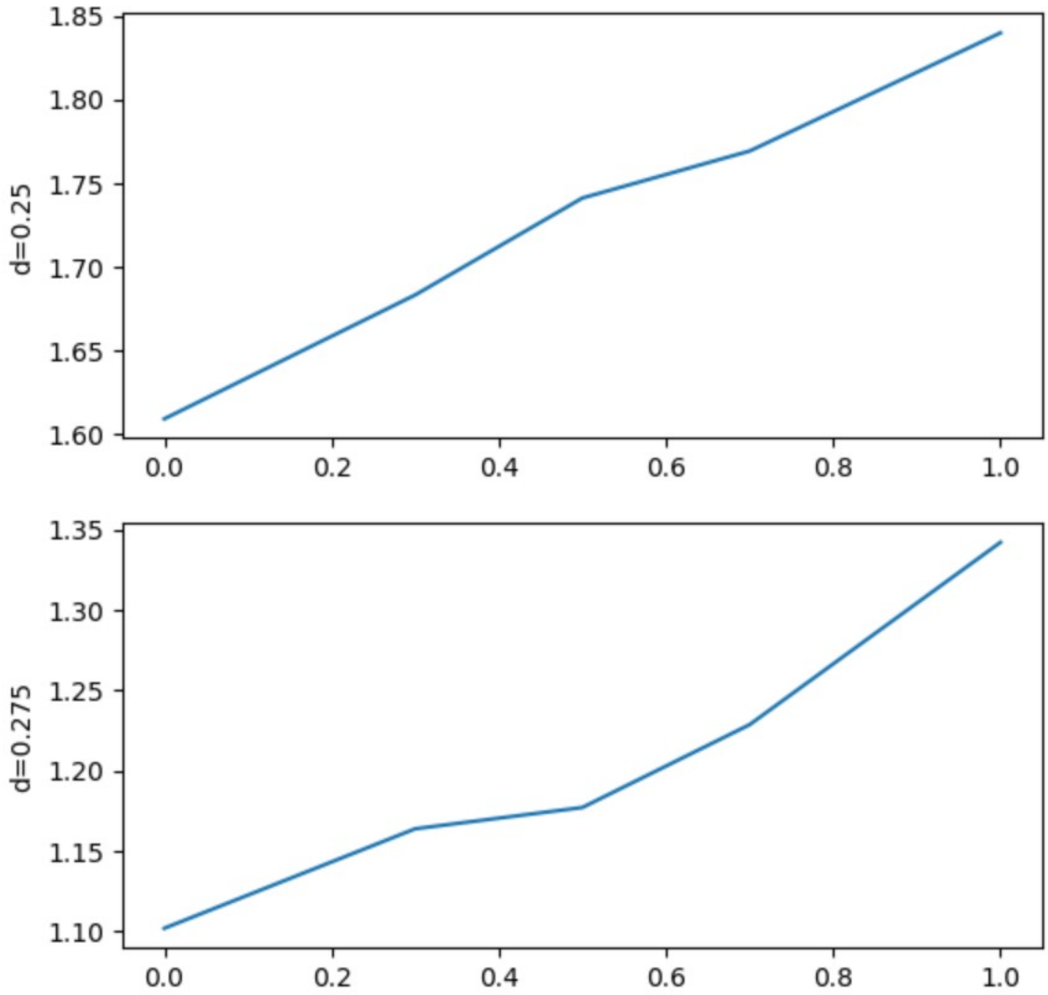


Figure 4.3 cluster participation ratio changes as C varying for d=0.25 and d=0.275

Looking at Figure 4.3, there are two subplots for threshold d=0.25 and d=0.275, respectively, and X-axis is coefficient C while Y-axis is the effective number of clusters. It is obvious that compared to the original Deffuant model where C=0.0 in this case, the effective number of clusters gets increased as the parameter C becomes larger.

The result shown above is consistent with paper[16], the interaction preference mechanism may cause opinion fragmentation through increasing the preference level, coefficient C in this case. In the transition phase, i.e. $d \in [0.25, 0.3]$, the influence is much more significant than other range of threshold value d. The physics behind it may be that people with different hobbies are likely to make up or join in clubs. In the clubs, people rapidly mix their opinions, further this leads to their opinion clusters. Of course, compared to the paper[16], the preference mechanism involved in the interaction preference model seems to have a very weak effect on the increase of the effective number of clusters. The writer thinks it may be due to the different degrees and the difference between model probability mathematic computing equations. It will be discussed later.

In the sixth row the first field (d=0.3, C=1.0) in Table 4.1 there points out a value 1.01446794, which is protruding from the values below or to the right of it. This situation almost not appears in the original Deffuant model given in Figure 4.[4]. However, in the interaction preference model with C=1.0, the preference mechanism has small strength to make it. Also, compared with the algorithmic bias[16] introduced by Sîrbu et al., the preference mechanism shows weak capacity on this. The possible reasons will be discussed later.

Now, some examples of d=0.3 and C=1.0 are shown in Figure 4.4.

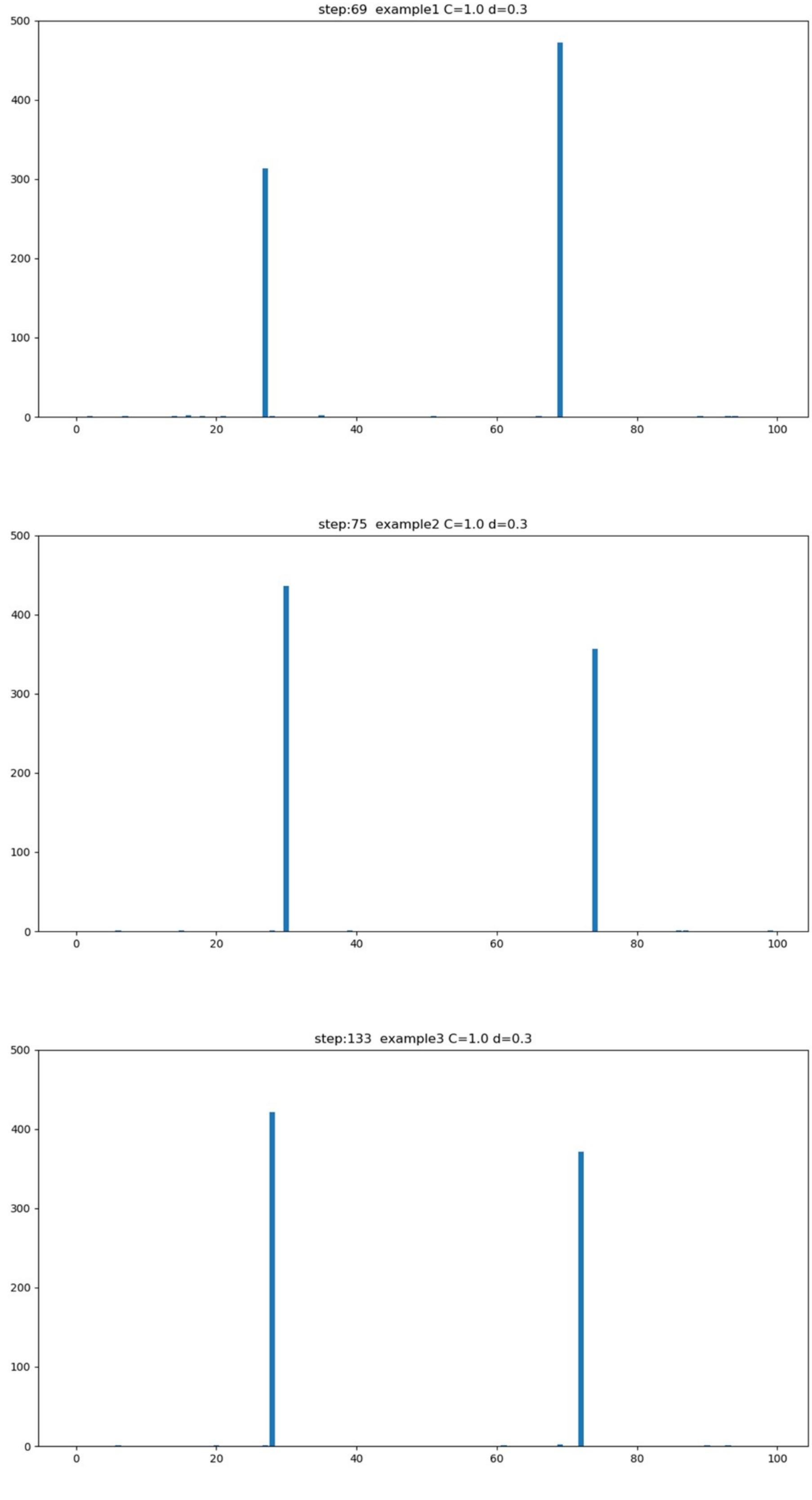
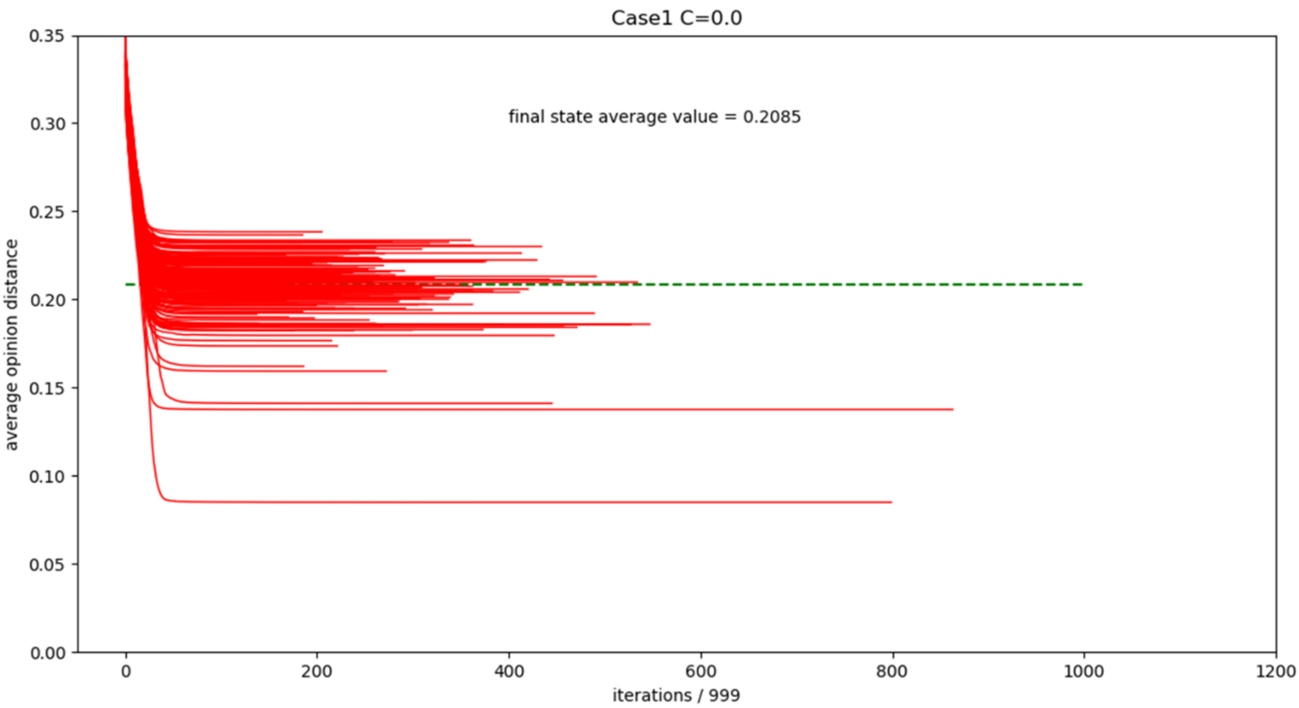


Figure 4.4 three examples of two clusters obtained for d=0.3 and C=1.0

4.3 Polarization

To investigate the polarization among agents regarding the preference mechanism introduced, the Python program mentioned above has been designed to track the average pairwise opinion distance every 999 iterations and at the last iteration at each simulation for all 5 models, i.e. different C values for interaction preference model.

Figure 4.5.1, Figure 4.5.2 and Figure 4.5.3 show average pairwise opinion distance evolution versus iteration diagram of 200 simulations for interaction preference model with different coefficient C and a fixed threshold d=0.2. The pairwise means two agents connected by a link at the current iteration. The X-axis represents the index of iteration divided by 999. For each subplot, a green horizontal line indicates the average opinion distance value in the final state over those 200 simulations. For facilitating compare them from an overall perspective, the plots are rearranged in terms of the range of the X-axis and Y-axis.



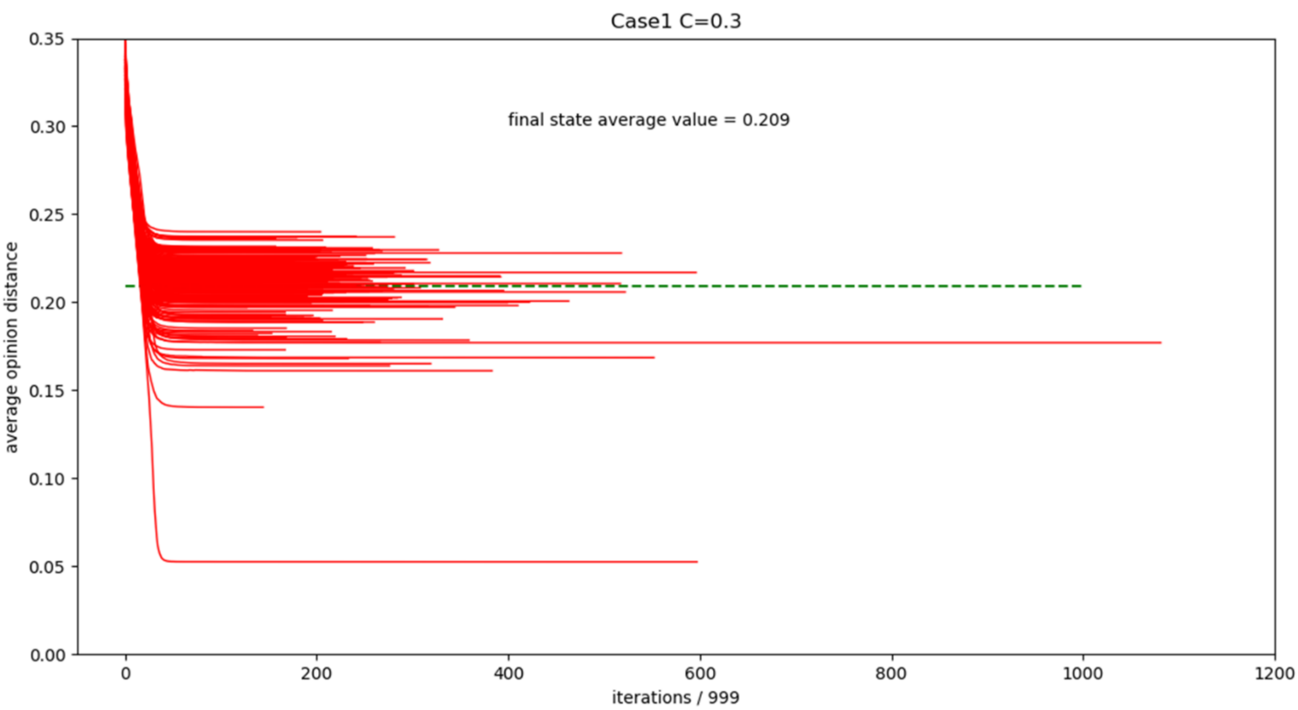
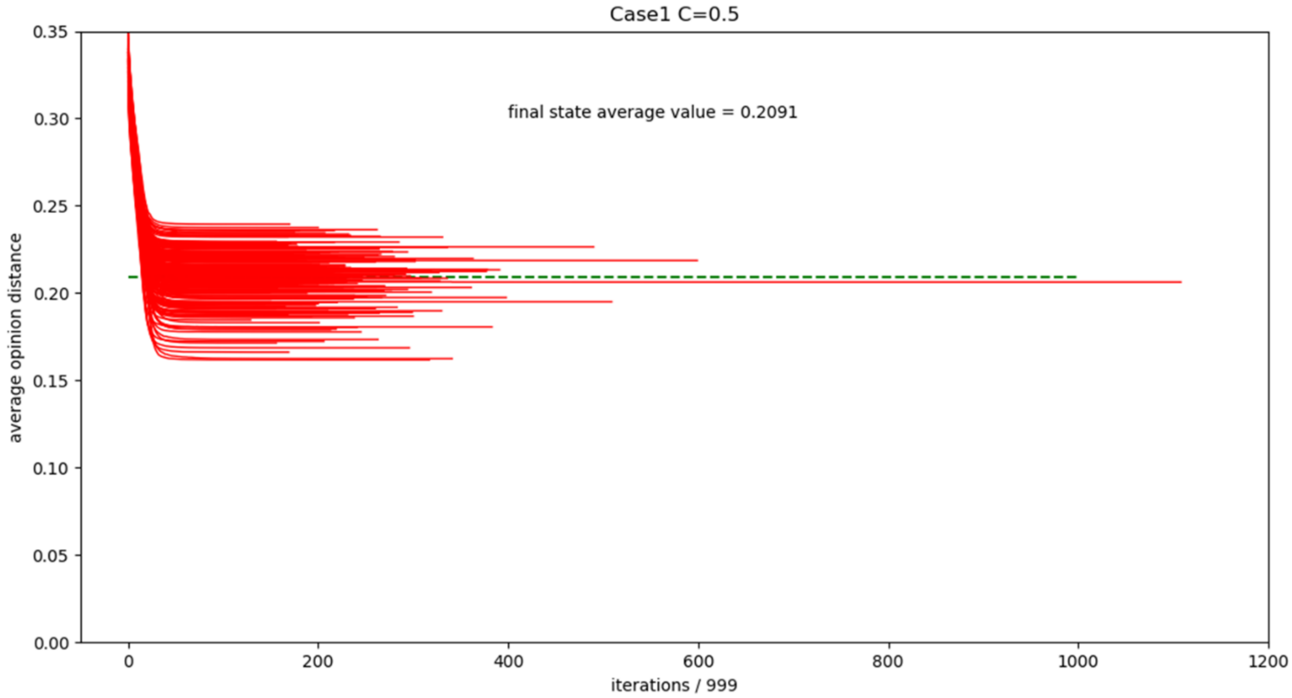


Figure 4.5.1 Average pairwise opinion distance evolution along with the iterations for C=0.0 and C=0.3(d=0.2)

(Note: Case1 is the interaction preference model)



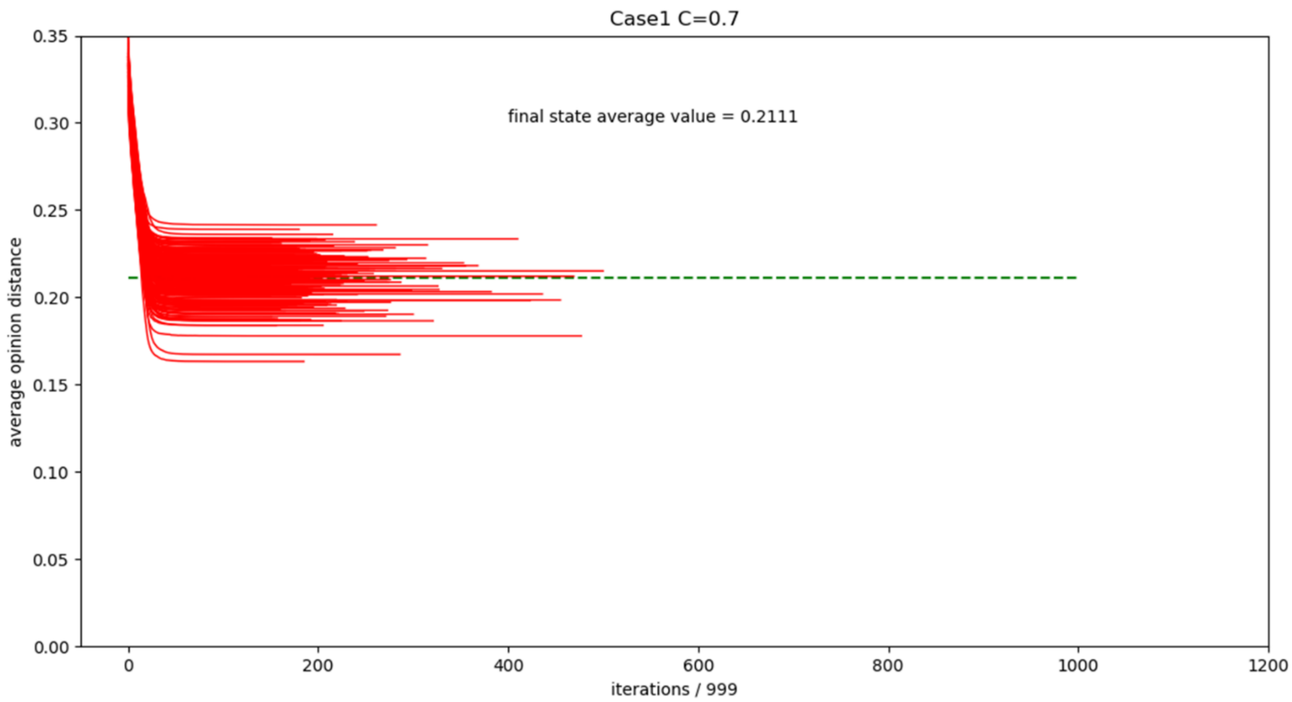


Figure 4.5.2 Average pairwise opinion distance evolution along with the iterations for C=0.5 and C=0.7(d=0.2)

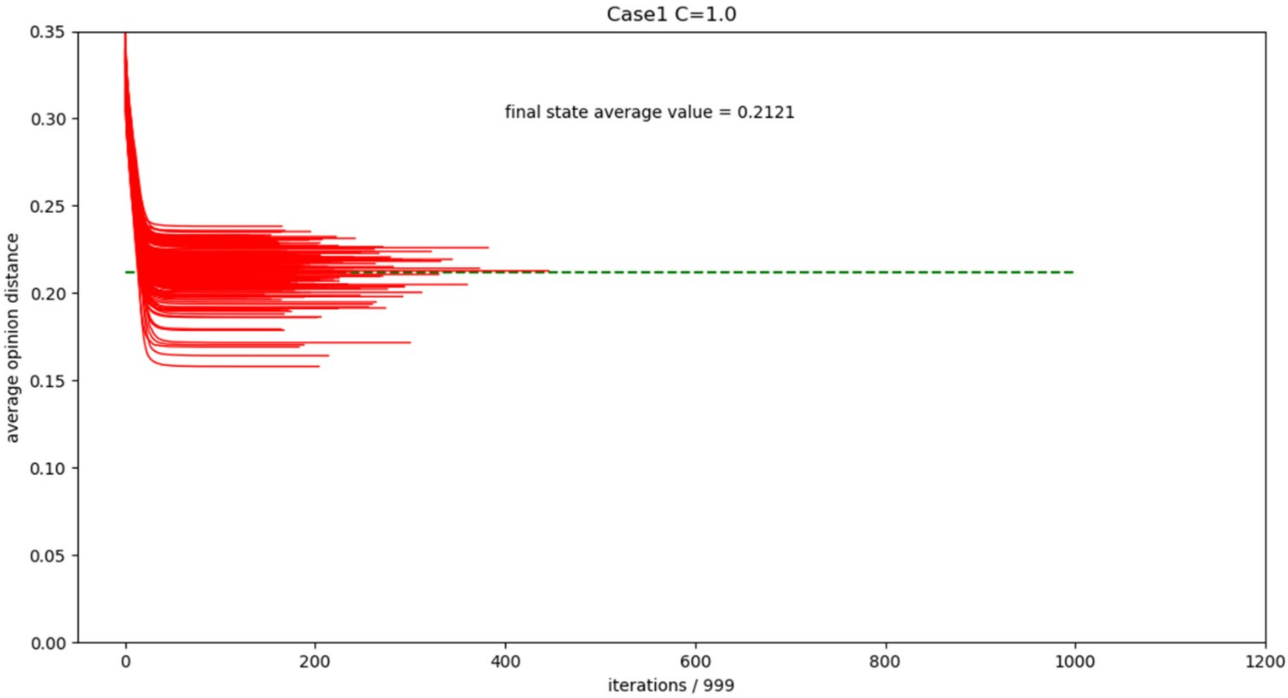


Figure 4.5.3 Average pairwise opinion distance evolution along with the iterations for C=1.0(d=0.2)

(Note: Case1 is the interaction preference model)

Those separate curves have tails with various lengths. This is because of the random time for the model to find out links pending to be processed. In other words, in some simulations, the maximum opinion change value is kept below 0.00001 for at least 10,000 iterations while in other simulations, the value can only be kept below 0.00001 after the very deep iterations. The randomness is the main cause.

It can be seen that from C=0.0 to C=1.0, the average opinion value in the final state gets a very slight increase. However, that is only for d=0.2, what about the situation for other threshold d values?

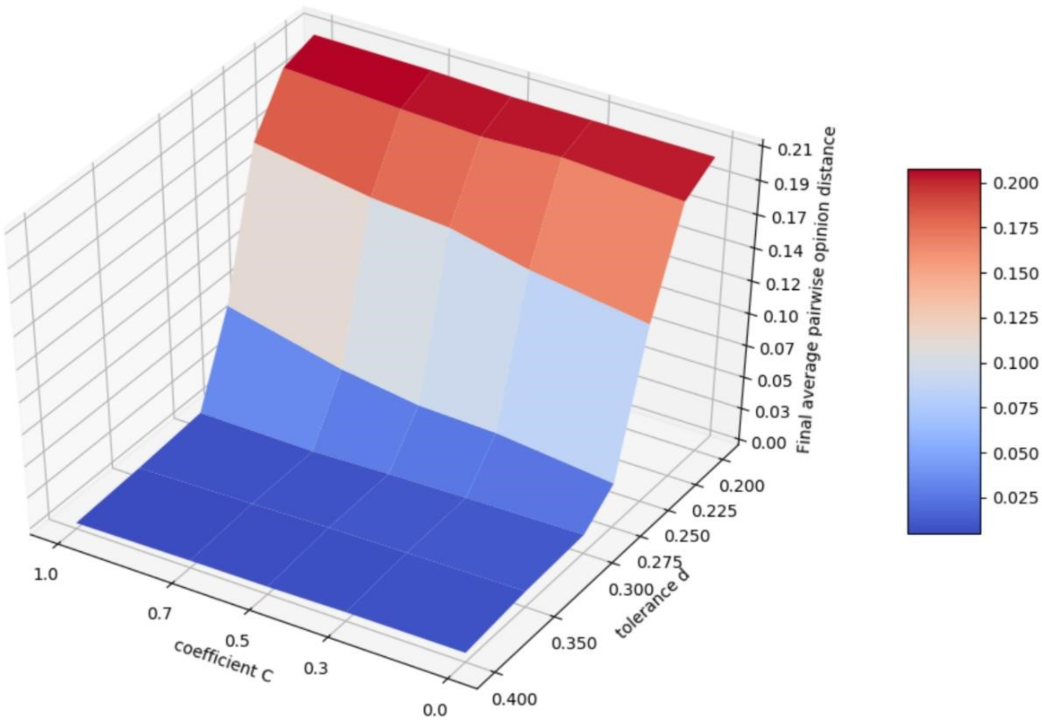


Figure 4.6 Final average pairwise opinion distance for various threshold d and coefficient C

Figure 4.6 displays the final average opinion distance changes for seven different threshold d values. It can be observed that the preference mechanism does have a small effect on the final average opinion distance. For d=0.275 and d=0.25, the increase caused by larger coefficient C is very clear. If carefully looking through this graph and connecting with Figure 4.2, one can easily find that the final average opinion distance increases almost along with the growth of the effective number of clusters in Figure 4.2. For threshold d larger than 0.3, the models will almost no chance to grow the second community, i.e. there is a consensus among the majority in most cases, so the average opinion distance can get the minimum. For the same reason, when tolerance threshold d value set smaller like 0.225 and 0.2, there will be more likely to contain two clusters in the final state, compared with reaching consensus, the final average pairwise opinion distance is definitively to be higher.

4.4 Convergence time

In the physical world, if the opinion consensus requires tremendous depth iterations to get, it may barely occur in real human life, as paper[16] said. It arouses the interest to investigate the time in terms of iterations needed to obtain the consensus or opinion clusters in a steady-state. This section will measure the number of iterations from the initial network to the equilibrium. The simulation result is also concurrently collected and saved in the files shown in Figure 4.1.

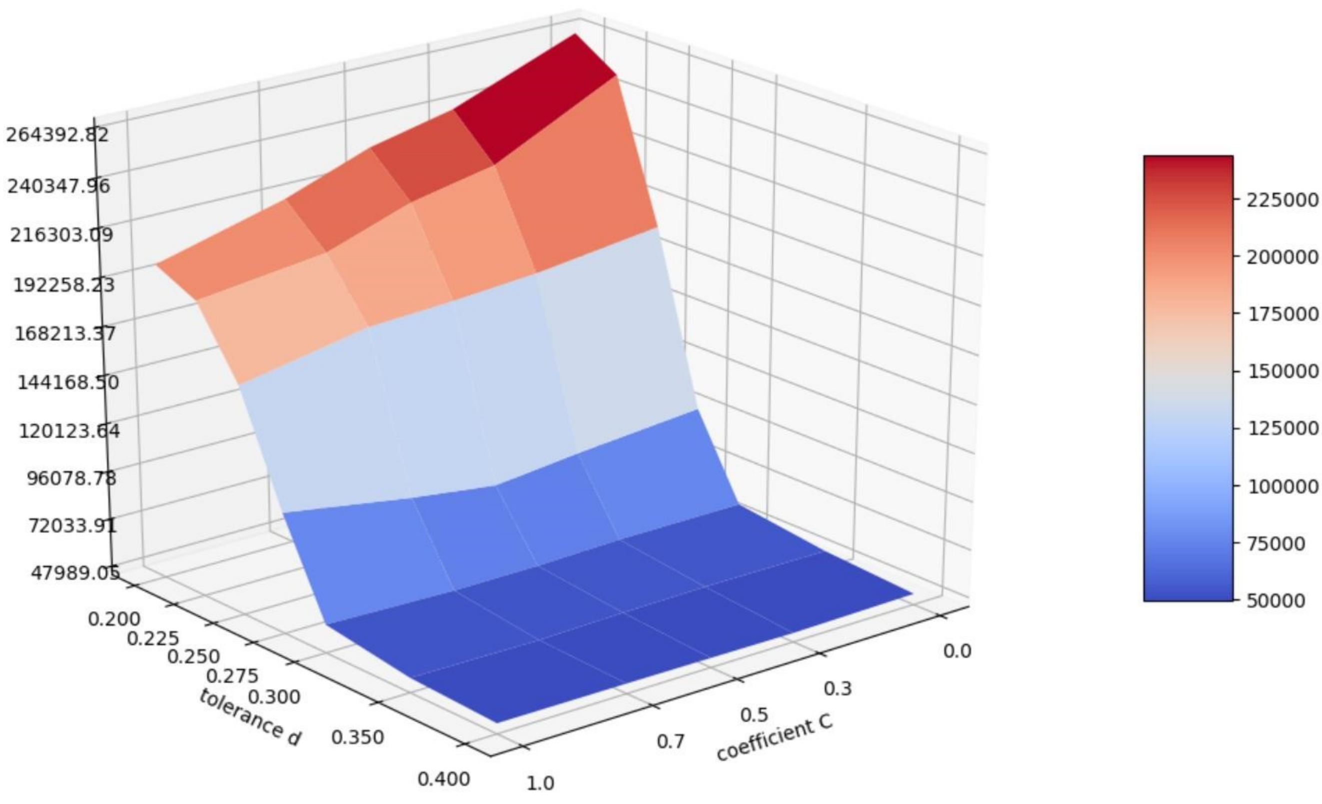


Figure 4.7 Total iterations to convergence for various threshold d and coefficient C

Table 4.2 Raw data for plotting Figure 4.7

	C=1.0	C=0.7	C=0.5	C=0.3	C=0.0
d=0.2	199496.285	216083.66	231371.99	240663.9	264392.82
d=0.225	189904.36	197535.68	212007.73	220409.325	250736.4
d=0.25	157225.535	169296.485	171487.825	174661.475	182470.375
d=0.275	103686.455	93359.295	87801.985	92460.815	98155.105
d=0.3	57621.025	55950.595	56169.485	57386.85	57624.95
d=0.35	50848.62	50320.82	51451.21	52075.755	51949.875
d=0.4	48693.02	48519.03	48283.1	47989.05	48978.055

Figure 4.7 is obtained as an average of 200 simulations for each different tolerance threshold d and coefficient C value. The Z-axis represents the average number of iterations required to get the equilibrium.

There are two points to be noted in Figure 4.7. At the first, when d value is quite small, i.e. 0.2 and 0.225, the total convergence iterations are expected to be deeper than the situations under the conditions of larger d values, such as d=0.3, 0.35 and 0.4. This may be due to two clusters in most cases rather than a consensus that will be made. The more opinion groups, the longer time for agents to make a confirmed decision. That makes sense in our daily life. If we have many choices on a matter, people may hesitate to set their hearts. If only one possibility, people will know that it is useless to spend a long time on it, so quick confirm can be made immediately or in no time compared to the situation when multiple choices available.

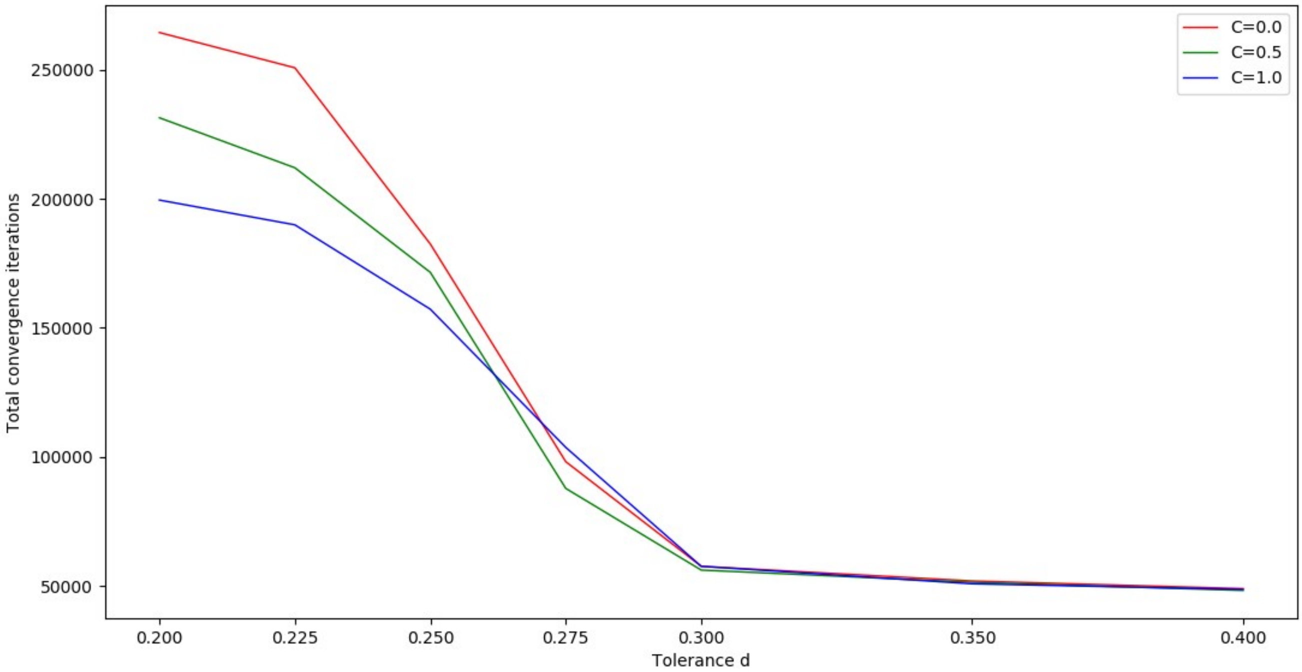


Figure 4.8 The impact of different level of preference mechanism on total convergence iterations (C=0.0, C=0.5, C=1.0)

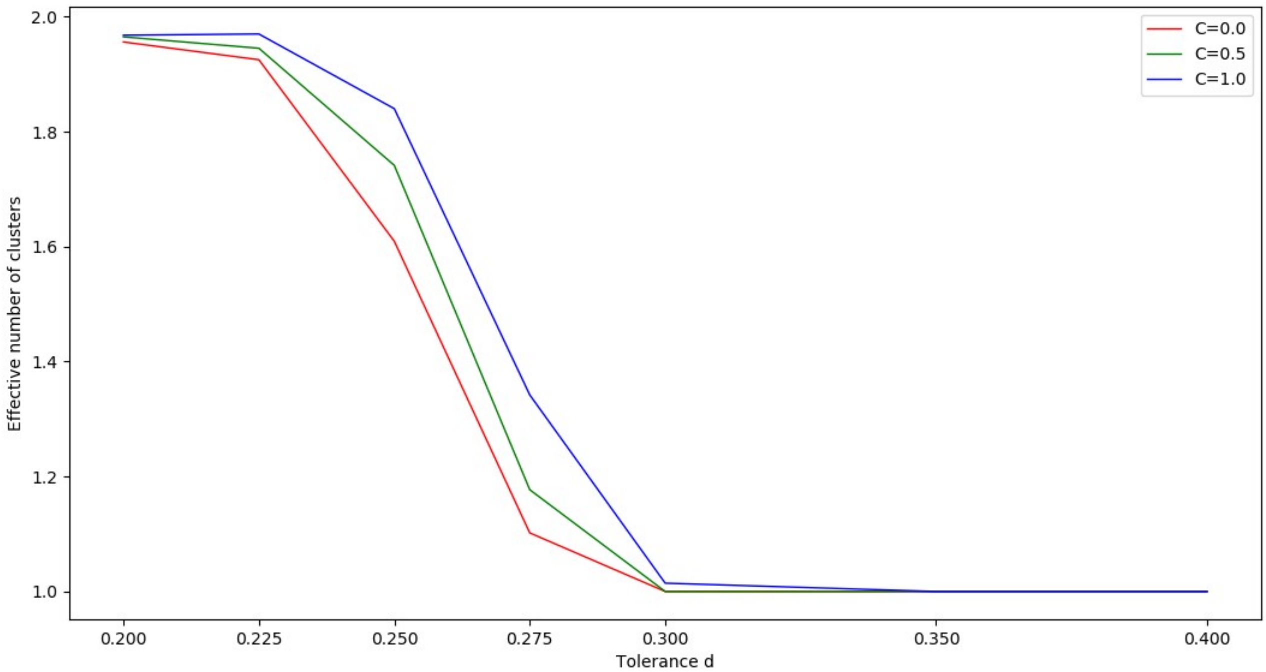


Figure 4.9 The impact of different level of preference mechanism on the effective

number of clusters (C=0.0, C=0.5, C=1.0)

Secondly, it can be observed that larger C values tend to decrease the convergence time compared to smaller or zero C for lower d area(from d=0.2 to d=0.225). That means with smaller d, the model can converge at a faster speed given a larger C value. This can be easily seen in Figure 4.8.

Besides, one can distinguish from Figure 4.7 and Figure 4.8, that for d=0.275 the convergence time value of model with C=0.5 is slightly lower than that of both C=1.0 and C=0.0. It can refer to Figure 4.9, associating with which one can understand the philosophy that the cluster participation ratio, effective number of clusters, in the final state can largely influence the convergence time. However, the coefficient C value, representing the level of preference mechanism, is also trying its best to increase the convergence speed. When d=0.275 and C=1.0, the model facing the overwhelming pressure from the value in Figure 4.9 compared with C=0.5 and C=0.0, thus cannot perfectly show its strength to accelerate the convergence. And when the d value becomes small, the cluster participation ratio gets small. The factor of cluster participation ratio contributes to all three models(C=0.0, 0.5 and 1.0) are almost the same at d=0.2. At the time, the preference mechanism makes out of its advantages to cut down the convergence time. So, the growth of C values significantly decreases the convergence time used.

Some previous works[16] have proved that the sum of pairwise opinion distance will decrease or at least not increase as the simulation going on. The convergence process can be, hence, understood by observing the change of average pairwise opinion distance over the iterations, for example, the population go to the equilibrium as the average pairwise opinion distance reaches its bottom plane. Next, some observations from the perspective of the evolution of average pairwise opinion distance will be shown.

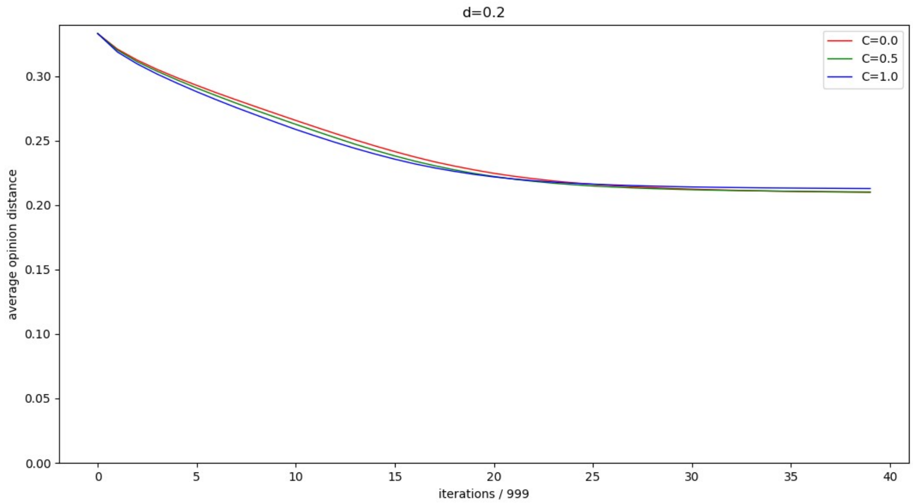


Figure 4.10.1 Evolution of average opinion distance over iterations for d=0.2

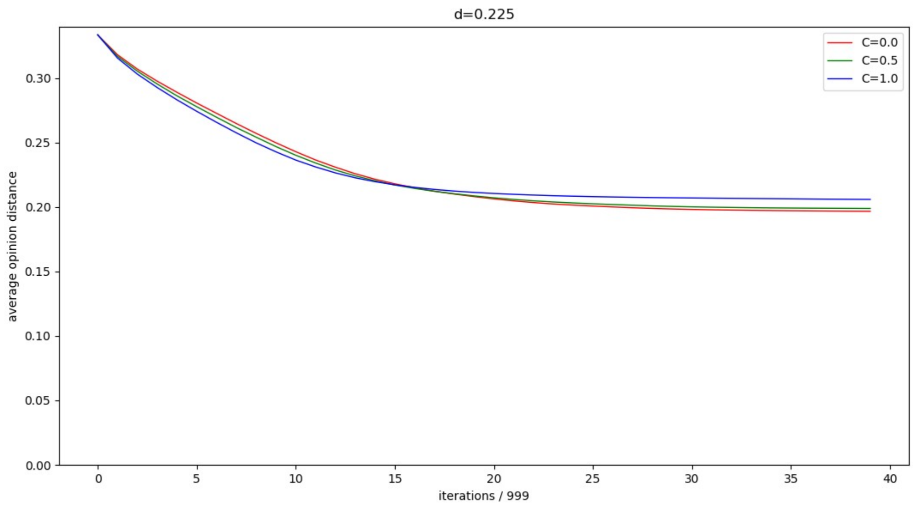


Figure 4.10.2 Evolution of average opinion distance over iterations for d=0.225

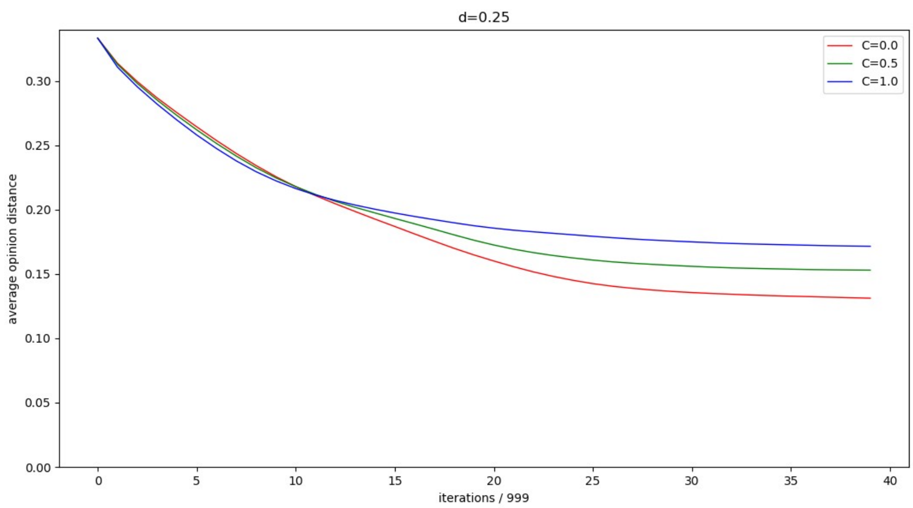


Figure 4.10.3 Evolution of average opinion distance over iterations for d=0.25

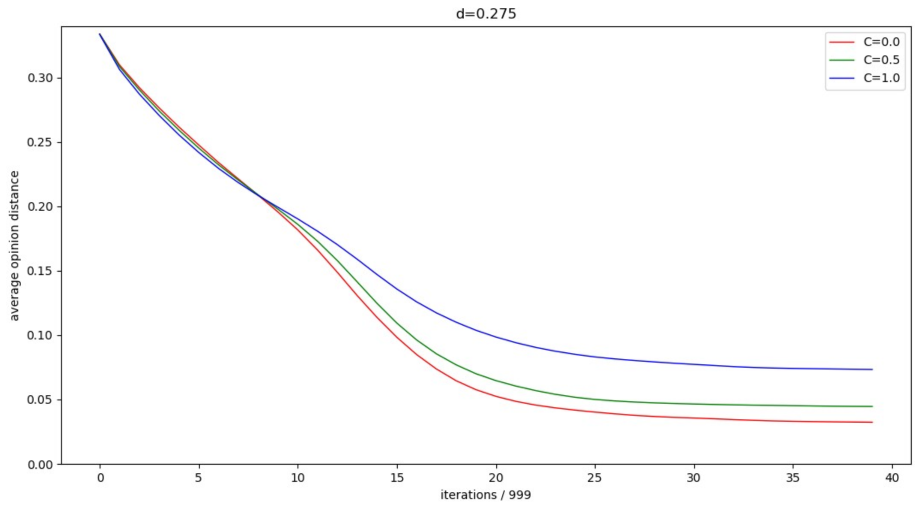


Figure 4.10.4 Evolution of average opinion distance over iterations for d=0.275

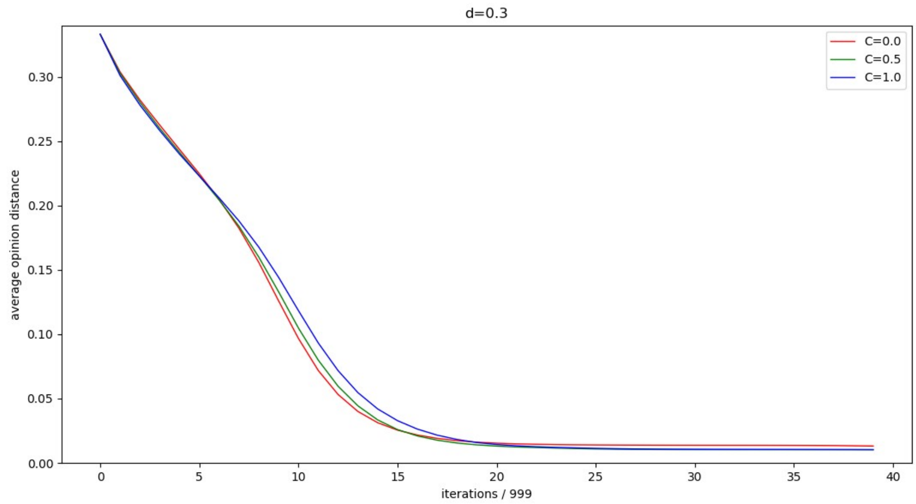


Figure 4.10.5 Evolution of average opinion distance over iterations for d=0.3

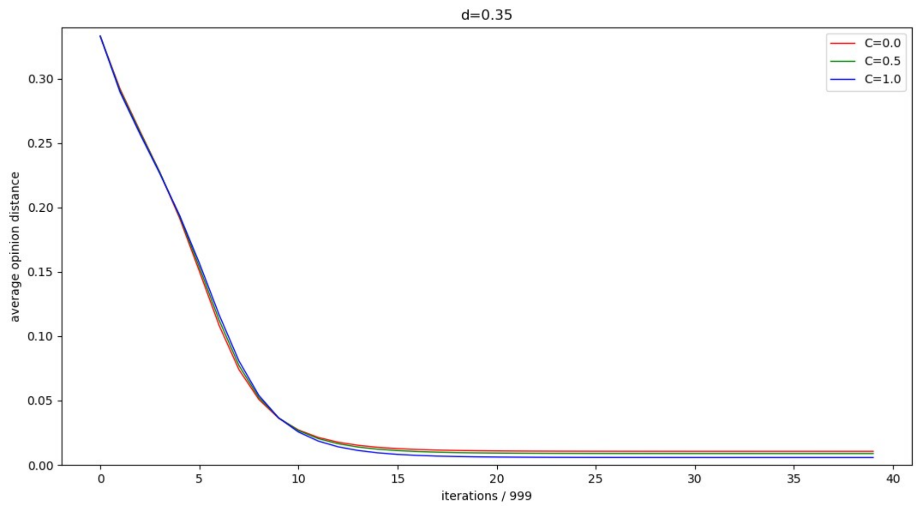


Figure 4.10.6 Evolution of average opinion distance over iterations for d=0.35

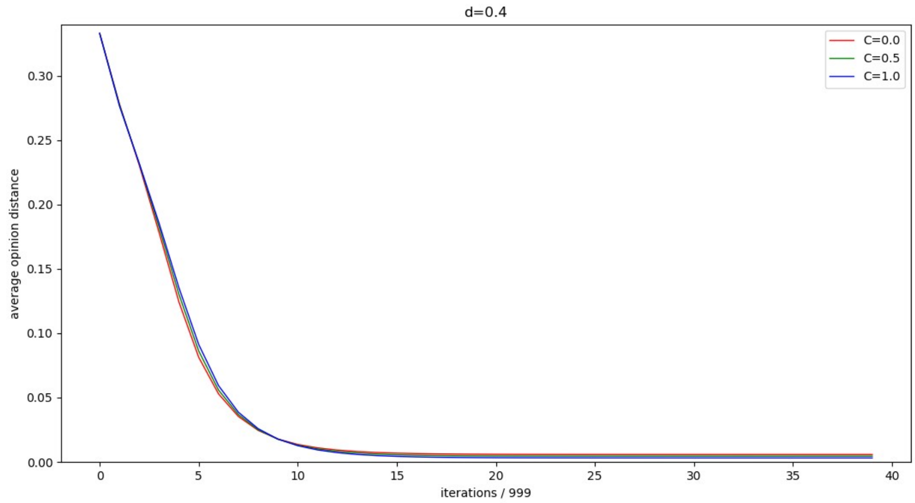


Figure 4.10.7 Evolution of average opinion distance over iterations for d=0.4

Figure 4.10.1-7 illustrate the evolution of the average opinion distance for C=0.0, 0.5 and 1.0 within the starting 40*999 iterations. Each curve in pictures is an average of 200 simulations.

As seen, when d value is small, the power of preference mechanism can be explained by the different decline rates on the average opinion distance, especially in Figure 4.10.1-3. The decline rate on the average opinion distance can be considered, in some sense, as the convergence speed. It is clear from Figure 4.10.1-3 that the decline rate of the blue line is slightly larger than the green one, and then the red one in the early stage.

But from Figure 4.10.4-7, it looks like someone is grabbing the wires and squeezing them to the left(the initial stage). The straightforward result of it is that the overall convergence time reduces. So given the large tolerance value d, the overall effect of the preference mechanism gets weak, even vanished.

The large tolerance value favors consensus in the final state rather than multiple communities. From the above figures, the larger tolerance value d, the smaller final average pairwise opinion distance, because the equilibrium is reached with from multiple clusters to a consensus. For smaller d, for example, d=0.2 and 0.225, the change of average opinion distance from the initial state to the final state is lesser compared with larger d, such as d=0.3, 0.35 and 0.4, the preference pair selection mechanism can quickly shorten the distance between agents with small opinion difference to form clusters in the vicinity. However, when d becomes large, the evolution of average opinion distance has a long way to go. The preference mechanism may not effective. Even if the mechanism can have an impact on the decline rate in the early stage, the time will not last long. Roughly, when the average opinion distance decrease to 0.20 approximately, the preference mechanism starts to act as a little resistance because the preference mechanism will have a comparative larger probability to select the link with opinion difference equal to zero or small. It does not know there are jobs undone. At this time, the plain model(C=0.0) using a random method with uniform probability is more effective in finding out those links with opinion difference satisfying the tolerance constraint. That's why the red line(C=0.0) can be seen to lead the decrease since the average opinion distance at around 0.2.

Do remember the preference mechanism tends to select the link with opinion difference as small as possible all the time, while the plain model(C=0.0) always keeps fair to deal with this job. At the initial distribution, the model with preference mechanism, i.e. with C>0, is like a hungry tiger and the preference mechanism helps it to easily find out the targets, the links with small opinion distance in this case. When the simulation goes deep, the mechanism guides the tiger to select most links it cannot have a bite because the link opinion distance is already zero. Fortunately, the power of the preference mechanism is moderate, not so extreme that it will not fall behind the plain model too much at the end.

4.5 Discussion

This chapter mainly focuses on the original Deffuant complete mixing model[4] and its modified version, i.e. the introduction of preference mechanism and variable coefficient parameter C. The purpose is to investigate how the parameter C changes the behavior of evolution and the convergence speed of the original model. The prime assumption for those models is only connected agent pairs can be chosen to get opinion updating. The unconnected agents are considered without the possibility to make interaction. It exactly makes sense in real life.

In section 4.2, the coefficient C is found to increase the Cluster Participation Ratio, namely, CPR, i.e. the effective number of clusters as it grows, especially when d=0.25 and d=0.275. More importantly, the cluster participation ratio got 1.01446794, which although is just slightly larger than 1.0, however, it means some simulation samples captured at least 2 clusters in its equilibrium state, as Figure 4.4 shown. This also supports that the preference mechanism contributes a little to the increase of the effective number of clusters, compared with the original model with C=0. At the same time, it can be seen that the final average pairwise opinion distance shares the same growing pattern at d=0.275 and d=0.25 in Figure 4.6. The parameter C can also have a positive proportional relationship with the final average pairwise opinion distance. In paper[16], the multiple clusters mean fragmentation and the opinion difference of connected agent pairs illustrates polarization. So, this dissertation applies this terminology to the section titles. Further, in section 4.4, the convergence time of each model is measured. The obvious result is when d is small, the preference mechanism can be dominant. As d gets large, the power of preference mechanism will be affected by the mainstream trend, i.e. the final state is going to contain only a consensus rather than two opinion communities. Therefore, the final average distance will be reduced to very low, the preference mechanism functions as an obstacle in the later stage. But fortunately, only one opinion peak at the end, so the agents will not waste too much time on changing among multiple communities.

In the writing of this chapter, paper[16] becomes a very important and useful reference. The ideas of the extended models are very similar. Most of the time, the simulation result can reflect almost the same phenomena. Apart from the different degrees of the phenomena, the apparent disagreement may happen in section 4.4 about the convergence speed. Overall, the main difference of the models between the interaction preference model (used in this dissertation) and paper[16] is the agent pair selection probability formula.

To make the interaction of individuals with similar opinions values more likely, they introduced the so-called algorithm bias. The detailed implementation is given by the following equation[16], see formula(4.1):

$$p_i(j) = \frac{d_{ij}^{-\gamma}}{\sum_{k \neq i} d_{ik}^{-\gamma}} \tag{4.1}$$

In their model, the agent i is firstly selected, and the selection of agent j should follow this probability equation. The γ can be seen as the strength of the algorithm bias[16]. This is reasonable because the probability formula did help to facilitate the encounters of similar agents. And when γ is set to zero, the model can be easily reduced to the original Deffuant model, since the second agent to be chosen can be any of other agents among the network with the same possibility $p_i(j) = \frac{1}{N\text{-}1}$. However, this is not the end of the story. Attention please to the followed specification. For the risk of undefined issue, they[16] prepared a lower bound for d_{ik}, that is d_ε, and $d_\varepsilon = 0.0001$ designed by them. That means, if $d_{ik} < d_\varepsilon$, the opinion distance of these agent pairs will be replaced by d_ε (0.0001) and contributes to the calculation of encounter probability.

Recalling Chapter 3, to introduce the preference selection mechanism, the second agent B is to be chosen according to the probability $\left(1 - C \times \left|o_A - o_B\right|\right)$, where C is the coefficient aforementioned, indicating the level of the preference mechanism and o_A, o_B represent the current opinion value of agent A and agent B, respectively. To sum up, the overall formula can be given by formula(4.2):

$$p_i(j) = \frac{\left(1 - C \times d_{ij}\right)}{\sum_{k \neq i} \left(1 - C \times d_{ik}\right)} \tag{4.2}$$

Where assuming d_{ij} is the absolute opinion distance of agent i and agent j at the current iteration. It is easy to see the interaction preference model can be transformed into the original Deffuant complete mixing model if C=0. As seen, in the interaction preference model, there will be no undefined problem.

To illustrate the dramatic difference brought by the possibility formulas, there firstly are some assumptions. 1. Only the connected agent pairs can be chosen to perform opinion operation. 2. For purely demonstration purposes, let's say the first agent i, currently, with opinion value 0.3, we pick has ten links at the current iteration, which are involved in the following calculation work, although in a real situation the number can be smaller or larger. 3. Given the first agent i is selected, no other factor can affect the selection of agent j except from those ten links. Originally, the opinion distance of those links are 0.6, 0.5, 0.5, 0.3, 0.3, 0.2, 0.1, 0.1, 0.07, 0.05, separately .

Next, a trial will simulate the first two pairs selection for agent i. By calculating the selection probability difference between the two models, one can easily understand the huge difference in their effect strength. Assuming the two selections will take agent i as the first node. The configuration of opinion value and link distance with selection possibility for two separate times is given below. And we assume at the first selection, agent 10 was selected because of the biggest probability since the opinion distance is shortest.

Table 4.3 Original configuration of agent i and its neighbors

o_i	0.3									
o_k	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.4	0.23	0.25
d_{ik}	0.6	0.5	0.5	0.4	0.3	0.2	0.1	0.1	0.07	0.05

Table 4.4 Selection probability for different models for the first time.

	1	2	3	4	5	6	7	8	9	10
IPM C =0.3	0.08958	0.09286	0.09286	0.09613	0.09941	0.10269	0.10596	0.10596	0.10695	0.1076
Paper[16] $\gamma = 0.3$	0.06993	0.07386	0.07386	0.07898	0.0861	0.09723	0.11971	0.11971	0.13323	0.14738
IPM C = 0.5	0.08149	0.08731	0.08731	0.09313	0.09895	0.10477	0.11059	0.11059	0.11234	0.1135
Paper[16] $\gamma = 0.5$	0.05304	0.05811	0.05811	0.06496	0.07501	0.09187	0.12993	0.12993	0.15529	0.18375
IPM C = 0.7	0.07227	0.08099	0.08099	0.08971	0.09843	0.10715	0.11587	0.11587	0.11849	0.12023
Paper[16] $\gamma = 0.7$	0.03908	0.0444	0.0444	0.05191	0.06349	0.08433	0.13699	0.13699	0.17585	0.22255
IPM C = 1.0	0.05571	0.06964	0.06964	0.08357	0.09749	0.11142	0.12535	0.12535	0.12953	0.13231
Paper[16]	0.02355	0.02825	0.02825	0.03532	0.04709	0.07064	0.14127	0.14127	0.20182	0.28254

$\gamma = 1.0$										

(Note: IPM is the interaction preference model)

Table 4.5 Configuration of agent i and its neighbors after the first operation

o_i	0.275									
o_k	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.4	0.23	0.275
d_{ik}	0.625	0.525	0.525	0.425	0.325	0.225	0.125	0.125	0.045	0.00

Table 4.6 Selection probability for different models for the second time.

	1	2	3	4	5	6	7	8	9	10
IPM C =0.3	0.08912	0.09241	0.09241	0.09571	0.099	0.10229	0.10558	0.10558	0.10821	0.10969
Paper[16] $\gamma = 0.3$	0.03844	0.04051	0.04051	0.04316	0.04677	0.05223	0.0623	0.0623	0.08465	0.52914
IPM C = 0.5	0.08062	0.08648	0.08648	0.09235	0.09821	0.10408	0.10994	0.10994	0.11463	0.11727
Paper[16] $\gamma = 0.5$	0.01056	0.01152	0.01152	0.0128	0.01464	0.0176	0.02361	0.02361	0.03935	0.83478
IPM C = 0.7	0.07086	0.07968	0.07968	0.08849	0.09731	0.10613	0.11495	0.11495	0.122	0.12597
Paper[16] $\gamma = 0.7$	0.00211	0.00238	0.00238	0.00276	0.00333	0.00431	0.0065	0.0065	0.01329	0.95645
IPM	0.05315	0.06733	0.06733	0.0815	0.09568	0.10985	0.12403	0.12403	0.13536	0.14174

C = 1.0										
Paper[16] $\gamma = 1.0$	0.00016	0.00019	0.00019	0.00023	0.00031	0.00044	0.0008	0.0008	0.00221	0.99468

(Note: IPM is the interaction preference model)

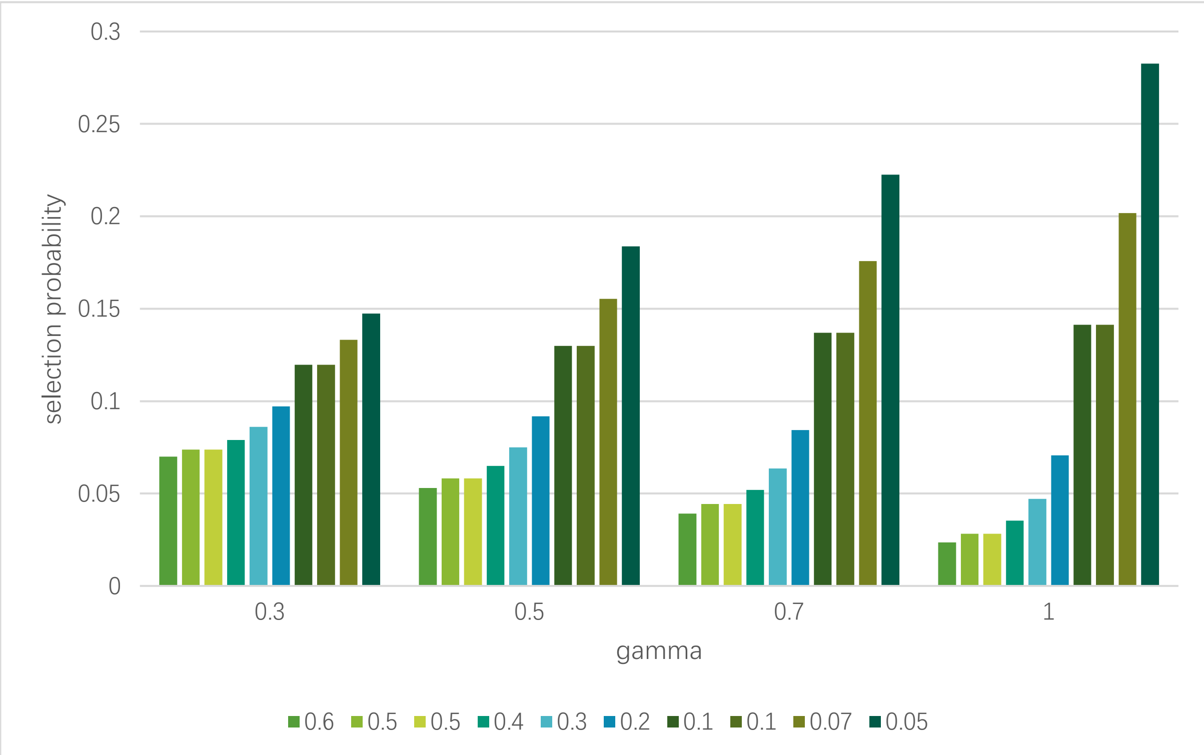


Figure 4.11 Selection probability for candidate links with varying gamma values at the first selection

Figure 4.12 Selection probability for candidate links with varying C values at the first selection

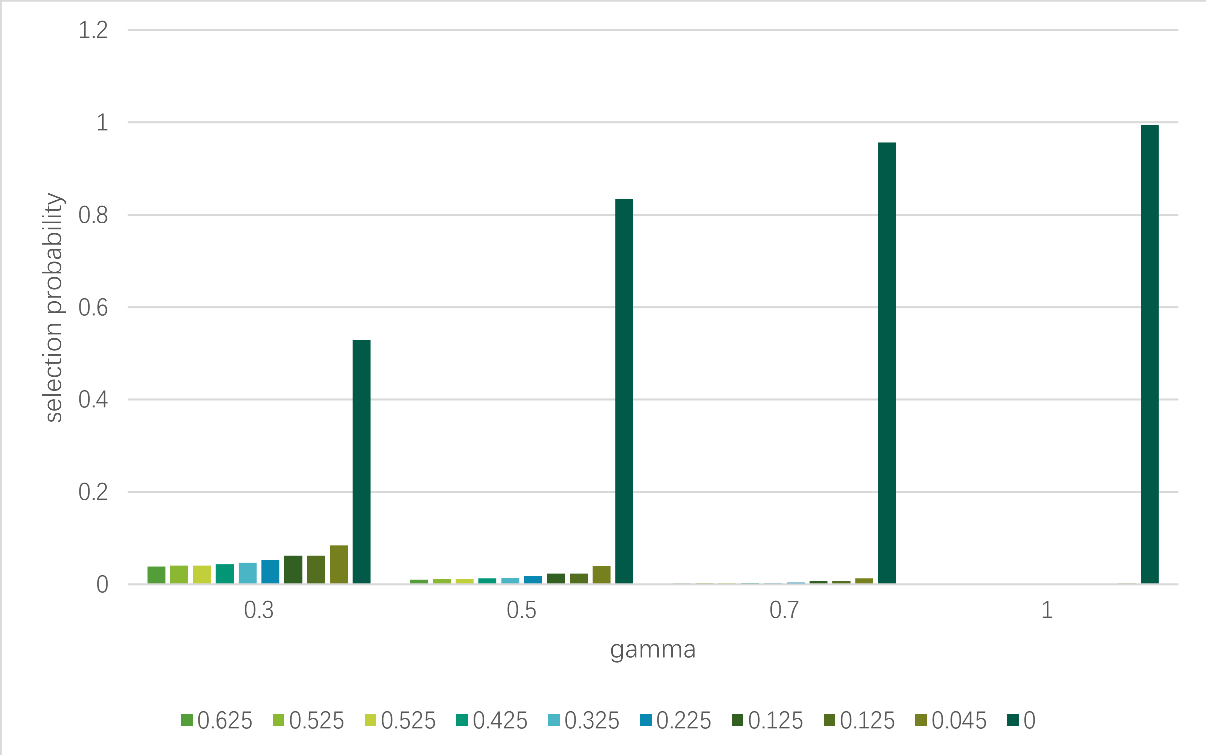


Figure 4.13 Selection probability for candidate links with varying gamma values at the second selection

Figure 4.14 Selection probability for candidate links with varying C values at the second selection

The trial is beginning with the initial settings provided by Table 4.3. From the left to right, index 1-10 are assigned to that ten agents. Table 4.4 shows the selection probability calculation results by two models and for different γ (gamma) and C values. For simplicity and following the maximum probability rule, we assume the agent 10 is selected in this round(iteration). And then, agent i and agent 10 get opinion exchange. Since the default convergence rate is 0.5, agent i and agent 10 compromise their opinion values to 0.275, that is $o_i = o_{10} = 0.275$. At this time, the opinion distance of them is updated to 0.0, as Table 4.5 shown. However, the model by the paper[16] will replace it with a lower bound d_ε, which is 0.0001 when selection probability calculation is performed. Next, according to Table 4.5, the selection probability for the second selection will be what Table 4.6 displays. If there is an opinion distance value extremely close to zero, the selection probability calculator by the paper[16] will produce an overwhelming probability for it. From the comparison of the values in red and those normal black values, it is easy to understand this numerical relationship is disastrous for the other non-close-to-zero values.

To intuitively express the quantitative relationship among the data, Figure 4.11-14 can be helpful. Comparing Figure 4.12 and Figure 4.14, the preference mechanism introduced by the interaction preference model behaves consistently and basically will not be affected by that single opinion exchange. The increase of coefficient C can marginally adjust the probability for links with different opinion distance.

Figure 4.11 and Figure 4.13 demonstrate the selection probability calculated by the paper[16] for the first and second selection. It can be known from Figure 4.11, that the parameter γ does well in widening the probability gap. And then, Figure 4.13 tells its power can support those close-to-zero values to get the preponderant position in selection.

It seems that the impact of algorithm bias[16] is comparatively significant than the preference mechanism introduced by the interaction preference model in this dissertation. This effect causes a higher degree of change in opinion fragmentation and polarization. as for the convergence time, the algorithm bias obviously will slow down the convergence speed due to the extraordinarily unbalanced pair selection scheme. It prefers to meet the after-processing links, which is a waste of time. However, the preference mechanism is milder. Therefore, the results shown are not that dramatic, easy to accept and the convergence speed can be controlled by not only the preference mechanism but only the properties of the original model.

When it comes to the tolerance threshold value setting, in this chapter the range from d=0.2 to d=0.4 is the default used. This is because, for d >= 0.4, the final stage will be always a consensus, which is verified by many previous works[4, 16]. If the tolerance threshold is assigned to a small number, the statistical number of final opinion clusters will increase.

This is easy to understand by an analogy: a string cut off into short sections will give rise to more ends of lines and the ends of these new short lines can be considered as the possible birthplaces for new peaks. The small threshold can be considered as the short length of each cut-off line.

Figure 4.15 should show the blurred relationship between opinion tolerance and the possible maximum number of peaks. If there is a small opinion threshold value d set, the width range of the opinion attraction domain of each community will become small, which allow more possible peak center position in the whole opinion range axis, as Figure 4.15(a) depicts. However, in the second case shown in Figure 4.15(b), less possible positions of suitable birthplace for peaks in this opinion world can be guaranteed due to the large opinion tolerance pre-set.

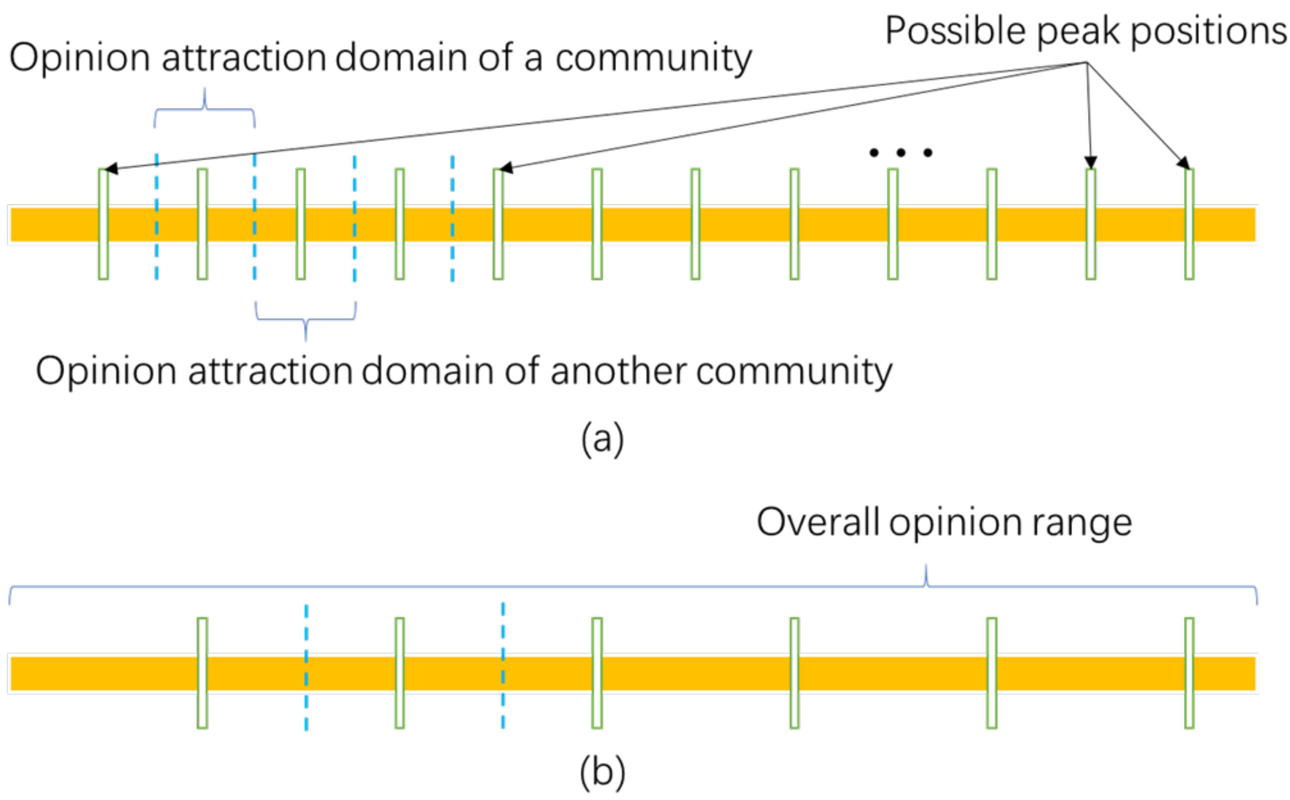


Figure 4.15 Simple illustration for how opinion tolerance affects the possible maximum number of peaks

But if given the tolerance threshold d is very small, the number of opinion clusters in the equilibrium may be easily affected jointly by the randomness. Figure 4 in paper[4] also displays that the frequency for sudden changes in the number of peaks at equilibrium will increase as the threshold value get small.

Of course, there may be a lot of factors affecting the number of peaks in the equilibrium state, such as the randomness of initial network configuration and evolution interaction. Sometimes, the steady-state will show different modularity configurations even if the same tolerance d has been set. This is because the random conditions are not perfectly identical in each simulation. While the positions of peaks in opinion value axis are slightly changed, the living place for new peaks may emerge by chance or be