Sensitivity Analysis and Correlation Matrix with Uncertainties

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1 Sensitivity Analysis with Uncertainties

In this document, we extend the sensitivity analysis to include uncertainties for a Heating, Ventilation, and Air Conditioning (HVAC) system. We analyze a set of functions f_0, f_1, f_2, and f_3 with respect to the parameters x_0, x_1, x_2 and uncertainties u_0, u_1, u_2.

1.1 Inputs and Uncertainties

The inputs and uncertainties to our analysis are as follows:

- x_0: Temperature control setting with a value of 2.0.
- x_1: Insulation level of the building with a value of 2.0.
- x_2: HVAC system efficiency with a value of 2.0.
- u_0: Uncertainty in x_0 with a value of 0.1.
- u_1: Uncertainty in x_1 with a value of 0.2.
- u_2: Uncertainty in x_2 with a value of 0.1.

1.2 Functions with Uncertainties

We have four functions that depend on the parameters and uncertainties:

- $f_0(x, u) = (x_0 + u_0)^1 \cdot (x_1 + u_1)^2 \cdot (x_2 + u_2)^3$
- $f_1(x, u) = (x_0 + u_0) + (x_1 + u_1) + (x_2 + u_2)^3$
- $f_2(x, u) = (x_1 + u_1) \cdot (x_2 + u_2)$
- $f_3(x, u) = \frac{(x_0+u_0)\cdot(x_1+u_1)}{(x_2+u_2)^2}$

1.3 Jacobian Matrix and Normalized Sensitivity

The Jacobian matrix represents the partial derivatives of the functions with respect to the parameters and uncertainties at a given point x and u. The normalized sensitivity matrix k_{0ij} is calculated using the formula:

$$k_{0ij} = \frac{\partial y_i}{\partial x_j} \cdot \frac{x_j}{y_i}$$

1.4 Correlation Matrix of Normalized Sensitivity

The correlation matrix Cf of the normalized sensitivity matrix is shown below:

	f_0	f_1	f_2	f_3
f_0	1.000	0.891	0.860	−0.897
f_1	0.891	1.000	0.534	−1.000
f_2	0.860	0.534	1.000	−0.546
f_3	−0.897	−1.000	−0.546	1.000

2 Conclusion

In this extended sensitivity analysis with uncertainties, we considered the effects of uncertainties in the functions f_0, f_1, f_2, f_3 and the parameters x_0, x_1, x_2. The correlation matrix Cf provides insights into the correlations between the normalized sensitivities of these functions with respect to the parameters and uncertainties.