Optimization of a Two-bar Truss Model Using Particle Swarm Optimization

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1 Problem Definition

In engineering and design, the optimization of structural components is a critical task to ensure that the final design meets performance requirements while minimizing material usage and costs. In this context, we consider a two-bar truss model as shown in Figure 1, which is subject to various physical constraints and design variables.

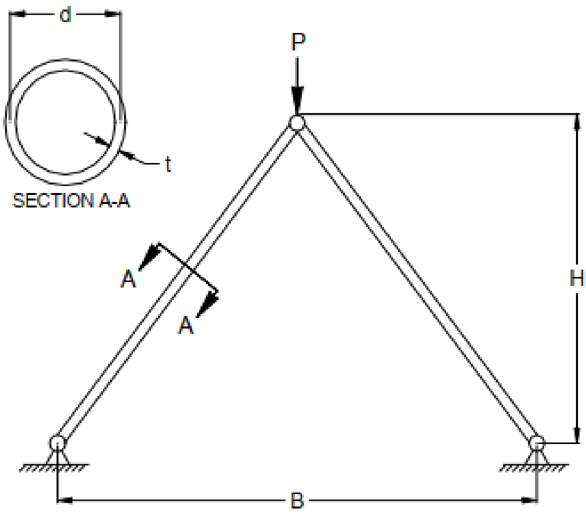

Figure 1: Two-Bar Truss Model

The model is governed by a set of mathematical equations that describe its behavior, including the weight, stress, buckling stress, and deflection. These equations are derived from the fundamental principles of mechanics and material properties. The problem can be defined as follows:

$$\text{Weight} = \rho \cdot 2 \cdot \pi \cdot d \cdot t \cdot \sqrt{(B/2)^2 + H^2} \tag{1}$$

$$\text{Stress} = \frac{P \cdot \sqrt{(B/2)^2 + H^2}}{2 \cdot t \cdot \pi \cdot d \cdot H} \tag{2}$$

$$\text{Buckling Stress} = \frac{\pi^2 \cdot E \cdot (d^2 + t^2)}{8((B/2)^2 + H^2)} \tag{3}$$

$$\text{Deflection} = \frac{P \cdot ((B/2)^2 + H^2)^{3/2}}{2 \cdot t \cdot \pi \cdot d \cdot H^2 \cdot E} \tag{4}$$

The analysis variables that define the geometry and material properties of the truss model are detailed in Table 1. The analysis functions are shown in Table 2.

The objective of the optimization problem is to find the values of thickness (t) and diameter (d) that minimize the stress in the truss structure. This optimization is subject to the following constraints: the weight must not exceed 50 lbs, and the deflection must be limited to 0.25 inches. These constraints ensure that the structural component meets the desired performance while respecting practical limitations.

In addressing this optimization problem, we will employ Particle Swarm Optimization (PSO), a powerful and versatile optimization technique that can handle complex engineering design challenges effectively. PSO is

Table 1: Analysis Variables

Variable	Symbol	Unit	Value
Height	H	in	30
Diameter	d	in	3
Thickness	t	in	0.15
Separation distance	B	inches	60
Modulus of elasticity	E	1000 lbs/in^2	30000
Density	ρ	lbs/in^3	0.3
Load	P	1000 lbs	66

Table 2: Initial Model Characteristics

Property	Value	Unit
Weight	35.99	lbs
Stress	33011.60	psi
Buckling Stress	185517.72	psi
Deflection	0.066	in

inspired by the social behavior of birds or fish and is known for its ability to explore solution spaces efficiently and find optimal solutions in the presence of multiple design variables and constraints.

2 Optimization Methods

Optimization is a fundamental problem-solving technique used to find the best solution from a set of possible alternatives. Various optimization methods have been developed to address different types of optimization problems. Here, we will explore four of the most commonly used optimization methods:

1. **Gradient Descent**: Gradient Descent is a widely employed method for minimizing or maximizing functions. It iteratively updates the solution by following the negative gradient direction of the objective function. This method is especially effective for solving unconstrained continuous optimization problems.

2. **Simulated Annealing**: Simulated Annealing is a probabilistic optimization technique inspired by the annealing process in metallurgy. It explores the solution space by probabilistically accepting less optimal solutions to escape local optima. This method is valuable for solving complex, multi-modal optimization problems.

3. **Particle Swarm Optimization (PSO)**: PSO is a population-based optimization method that simulates the social behavior of birds or fish. Particles in the search space communicate with one another, updating their positions based on personal and global best solutions. PSO is commonly used for optimization problems involving continuous variables and is known for its simplicity and ease of implementation.

4. **Genetic Algorithm (GA)**: Genetic Algorithms are evolutionary optimization techniques that mimic the process of natural selection and genetic evolution. In a GA, a population of potential solutions undergoes selection, crossover, and mutation operations over several generations to improve the quality of solutions. GAs are versatile and effective for solving a wide range of optimization problems, including those with discrete variables, non-linear objectives, and constraints.

In the context of our problem, we will utilize the Particle Swarm Optimization as our chosen optimization method. PSOs have demonstrated success in handling complex engineering and design problems, making them a suitable choice for our specific optimization task.

3 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a population-based optimization method that simulates the collective behavior of particles. Each particle represents a potential solution and moves through the solution space while adjusting its position based on its own experience and the experience of neighboring particles. PSO is guided by the concepts of velocity and position, which help particles explore and exploit the solution space efficiently.

In Python, we modeled the PSO algorithm using the DEAP (Distributed Evolutionary Algorithms in Python) framework, which provides a flexible and extensible platform for evolutionary and swarm-based optimization. Our PSO implementation involved defining the particle's attributes, the evaluation function, and the optimization parameters, such as the number of particles, maximum iterations, and neighborhood topology.

4 Entropy Calculation

Entropy is a measure of uncertainty, randomness, or disorder. In the context of an optimization algorithm, it can be used to quantify the spread or diversity of the solutions in the search space. The entropy $H(X)$ of a random variable X with probability mass function $P(x)$ is calculated using the formula:

$$H(X) = -\sum_{i=1}^{n} P(x_i) \log_e P(x_i) \tag{5}$$

In our case, we estimate $P(x)$ as a Gaussian distribution with mean and standard deviation calculated from the current positions of the particles in the swarm. The entropy is then calculated at each iteration of the Particle Swarm Optimization (PSO) algorithm.

5 Rate of Convergence

The rate of convergence of an optimization algorithm refers to how quickly the algorithm approaches the optimal solution. One way to assess the rate of convergence is to plot a measure of the diversity or spread of the solutions, such as entropy, at each iteration. As the algorithm converges to an optimal solution, we would expect this measure to decrease.

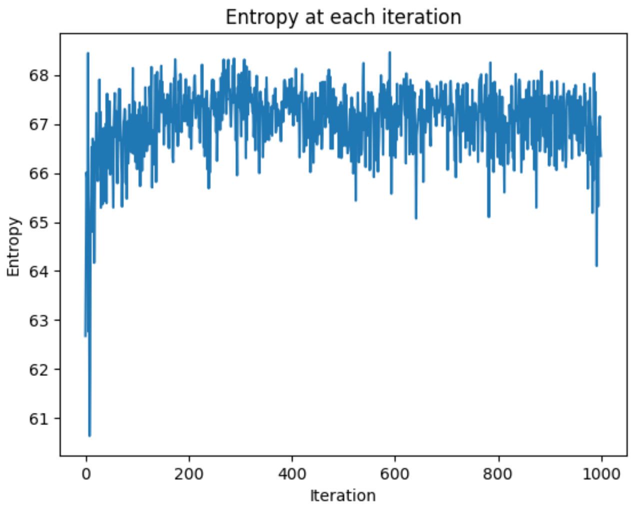

Figure 2: Plot of entropy at each iteration

The plot above shows the entropy at each iteration for our PSO algorithm. As can be seen, the entropy decreases over time, indicating that the algorithm is converging towards an optimal solution.

6 Sensitivity Analysis

In our study, we performed a sensitivity analysis around the optimal thickness and diameter values. Sensitivity analysis is a technique used to determine how different values of an independent variable impact a particular dependent variable. In this case, we are interested in understanding how small changes in the thickness and diameter can affect the stress.

We used a Monte Carlo simulation for the sensitivity analysis. This method involves running many simulations by randomly sampling from the input parameter space. For each simulation, we perturbed all parameters simultaneously, which allows us to capture the combined effect of changes in multiple parameters.

The histogram above shows the distribution of stress values obtained from the Monte Carlo simulation. Each bar in the histogram represents the number of simulations that resulted in a stress value within a certain range. The red dashed line indicates the mean stress value, and the green dashed lines indicate one standard deviation above and below the mean.

From this analysis, we can observe how sensitive the stress is to changes in thickness and diameter around their optimal values. This information can be useful for decision-making and risk assessment.

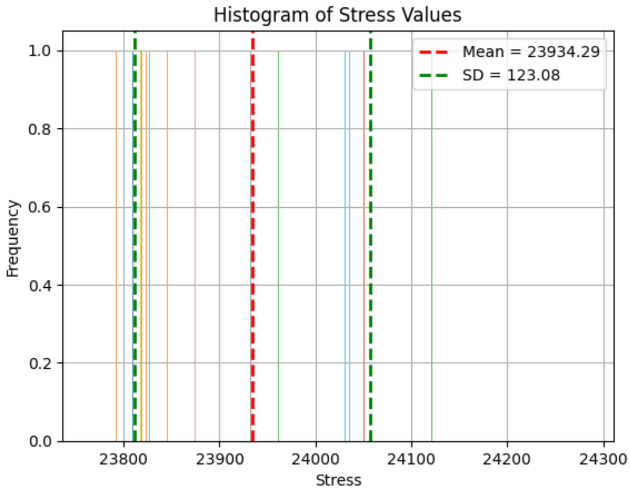

Figure 3: Histogram of stress values from the Monte Carlo simulation

7 Conclusion

After applying Particle Swarm Optimization (PSO) to our two-bar truss model, we successfully optimized its design to minimize stress while satisfying weight and deflection constraints. The optimized results are as follows:

Table 3: Optimized Model Characteristics

Property	**Value**	**Unit**	**Variation (%)**
Optimal Diameter	0.994	in	-66.9%
Optimal Thickness	0.629	in	+419.4%
Optimal Weight	50.00	lbs	+38.4%
Optimal Stress	23760.00	psi	-28.0%
Optimal Buckling Stress	28443.90	psi	-84.7%
Optimal Deflection	0.048	in	-27.3%

hese optimized values represent a significant improvement over the initial model. The diameter has decreased by approximately -66.9%, resulting in a smaller component. On the other hand, the thickness has increased by approximately +419.4%, indicating the use of more structural material. The weight has also increased, as expected due to the increase in thickness. The stress and buckling stress have decreased, demonstrating that the structure is now operating within safer limits. The deflection has been reduced, further enhancing the performance of the truss design. The optimization has effectively balanced the trade-offs between these characteristics.