Dynamic Nested Sampling with *dynesty*

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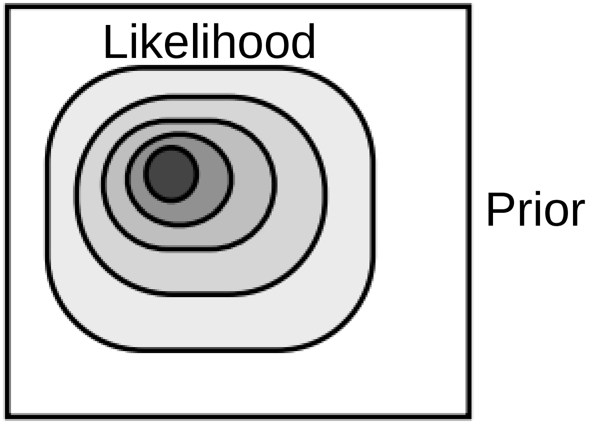
Background

$$\underbrace{\Pr(\boldsymbol{\Theta}|\mathbf{D},\mathrm{M})}^{\text{Posterior}} = \frac{\overbrace{\Pr(\mathbf{D}|\boldsymbol{\Theta},\mathrm{M})}^{\text{Likelihood}}\ \overbrace{\Pr(\boldsymbol{\Theta}|\mathrm{M})}^{\text{Prior}}}{\underbrace{\Pr(\mathbf{D}|\mathrm{M})}_{\text{Evidence}}}$$

Bayes' Theorem

Motivation: Sampling the Posterior



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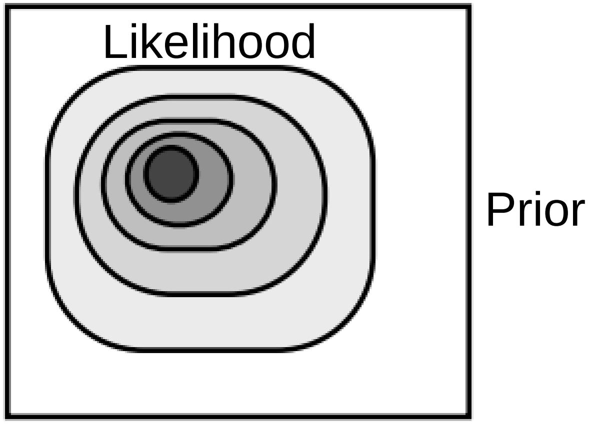

Sampling directly from the likelihood $\mathcal{L}(\boldsymbol{\Theta})$ is **hard**.

Pictures adapted from this 2010 talk by John Skilling.

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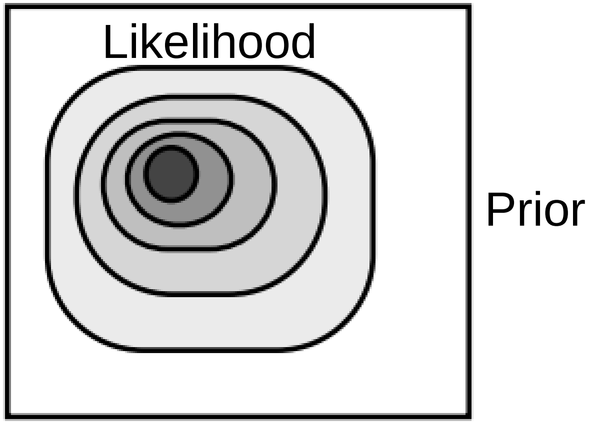

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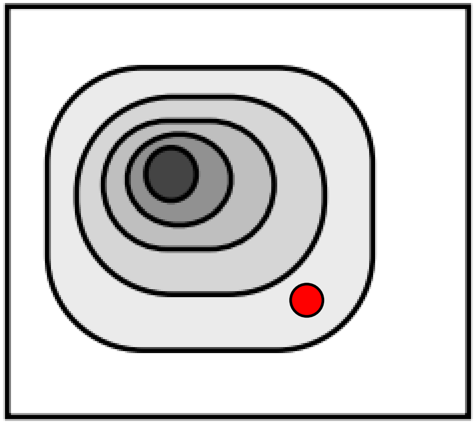

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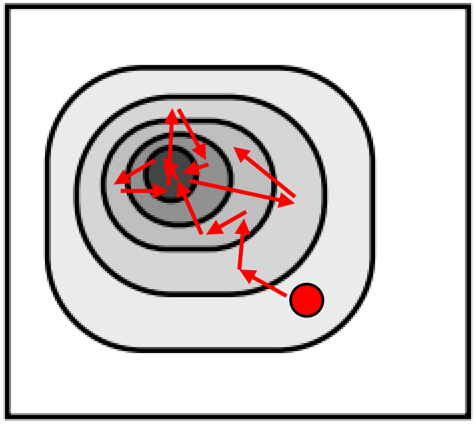

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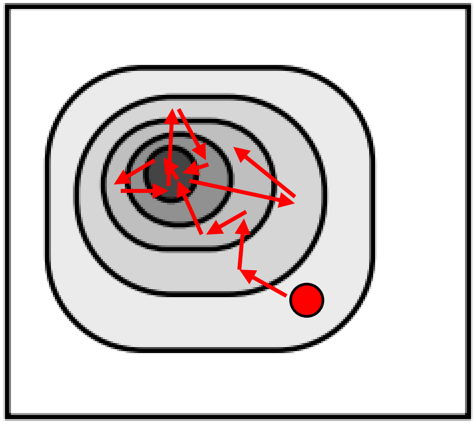

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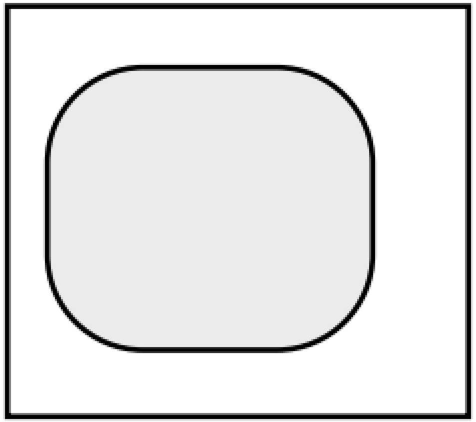
Nested Sampling: Solving an Easier Problem many times.

Sampling uniformly within bound $\mathcal{L}(\boldsymbol{\Theta}) > \lambda$ is **easier**.



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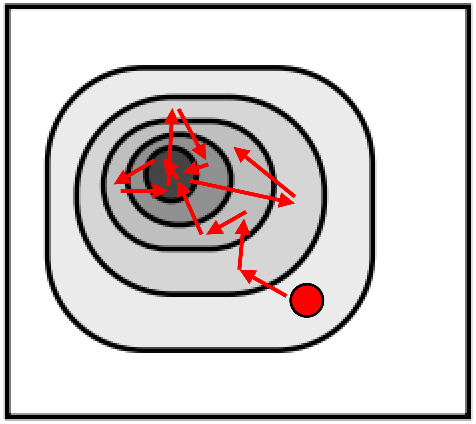

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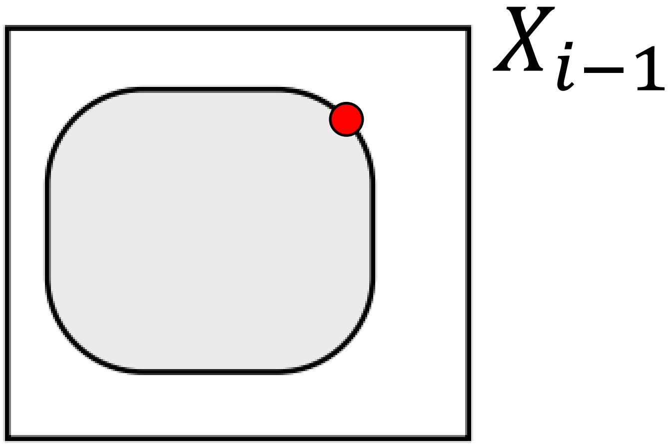

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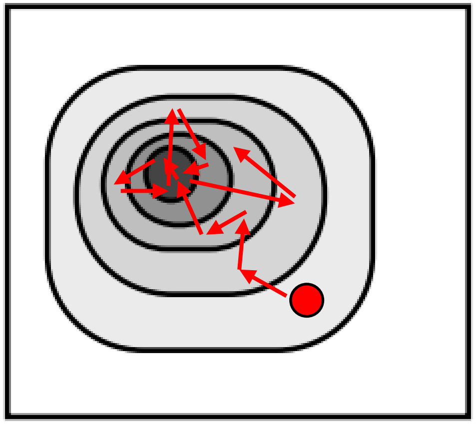

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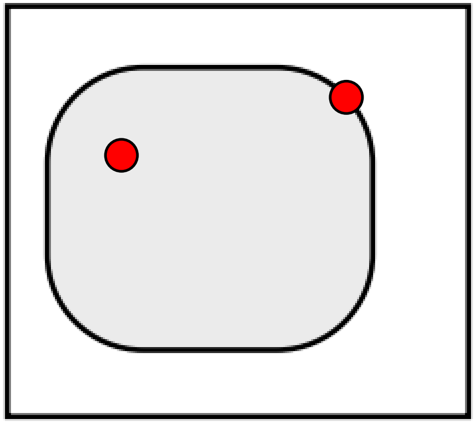

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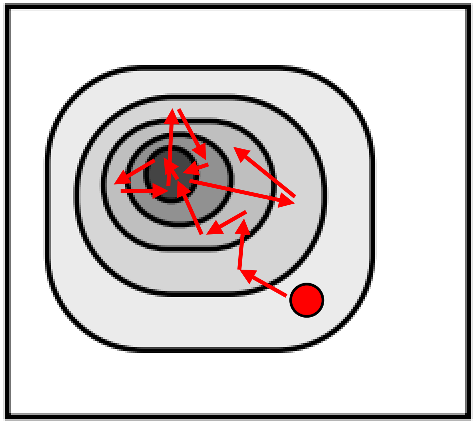

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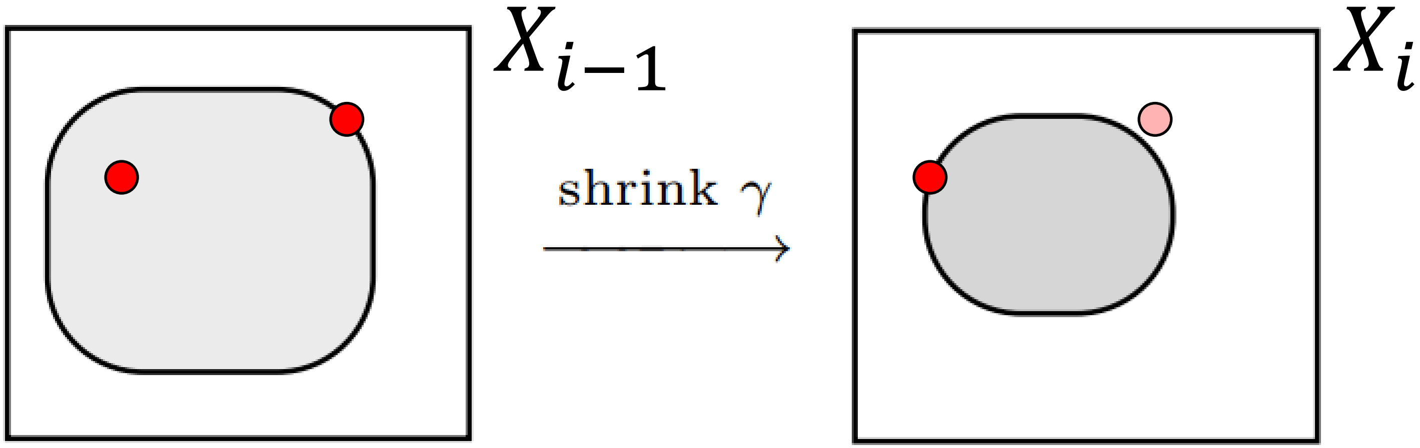

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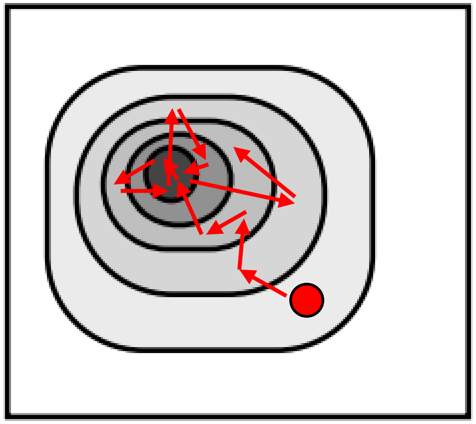

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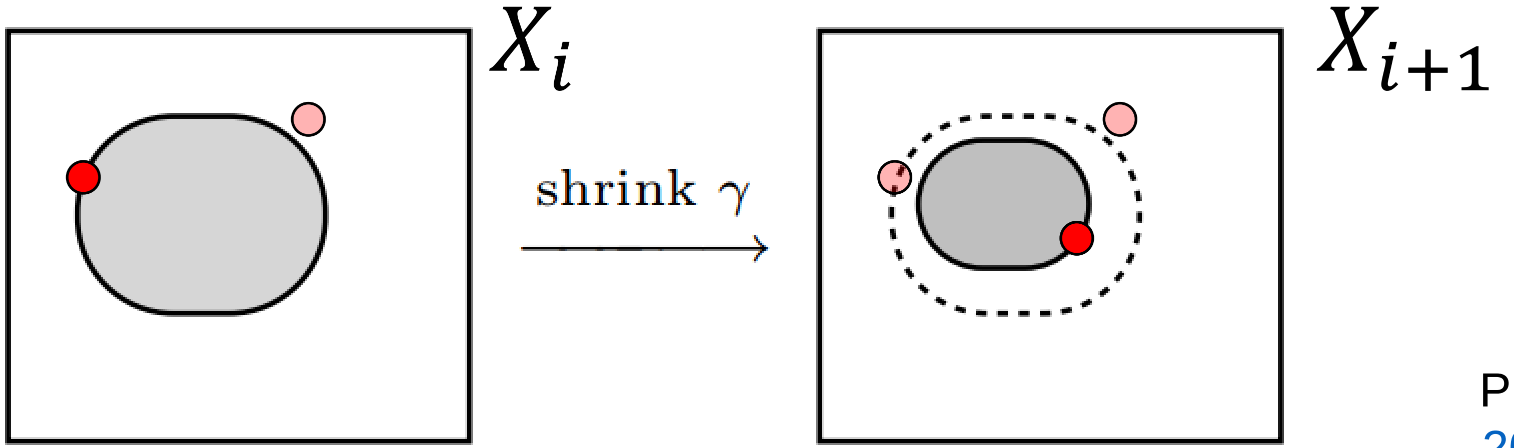

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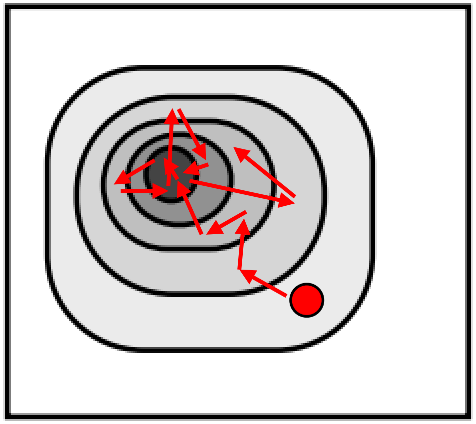

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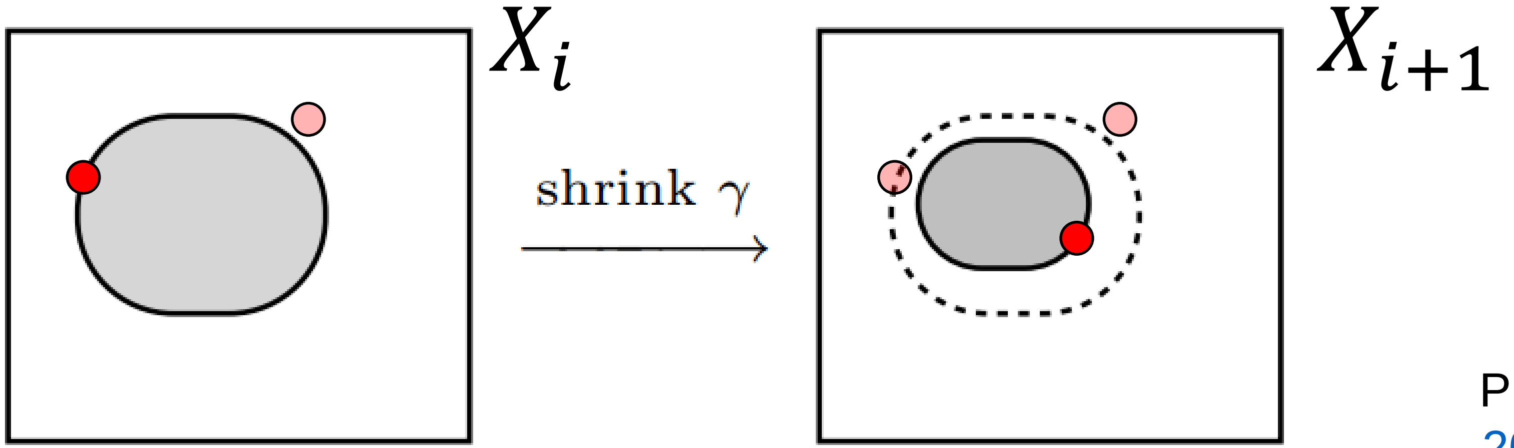

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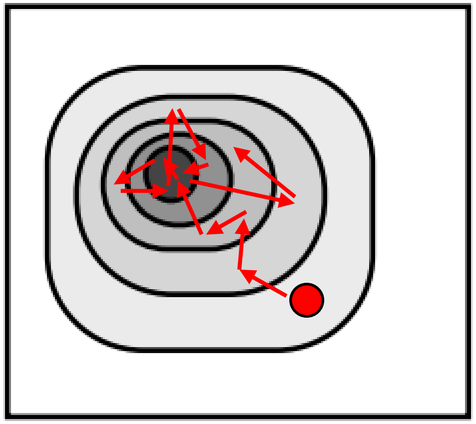

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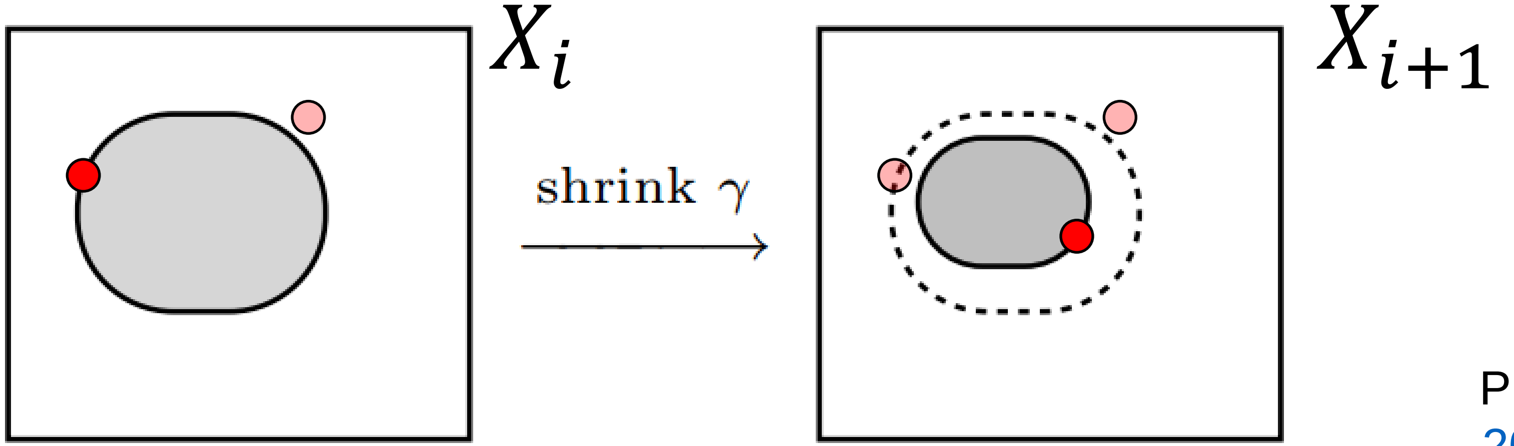
If you have a **prior transform** that converts your priors to look uniform, then this case is equivalent.

Sampling uniformly within bound $\mathcal{L}(\boldsymbol{\Theta}) > \lambda$ is **easier**.



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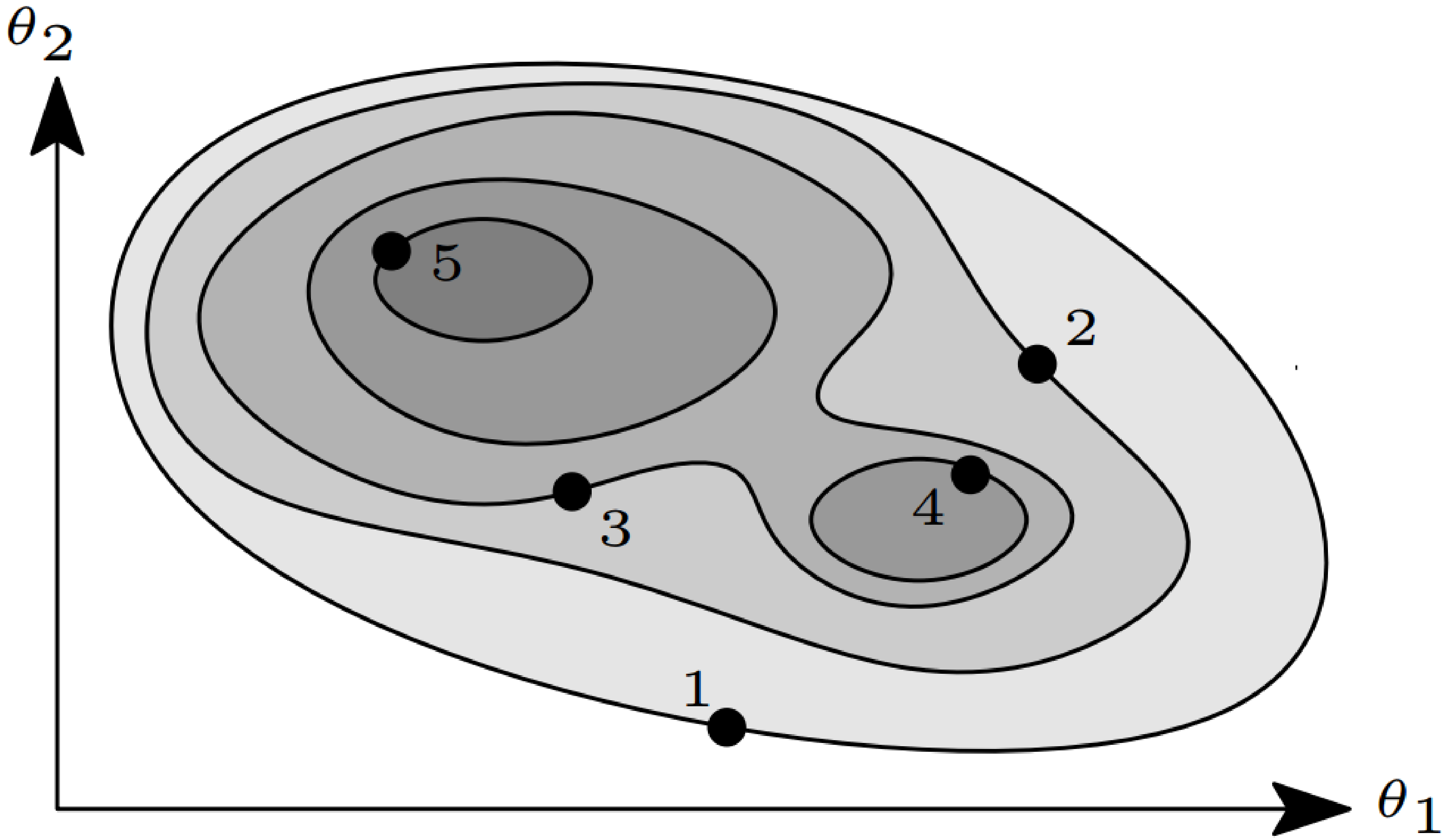
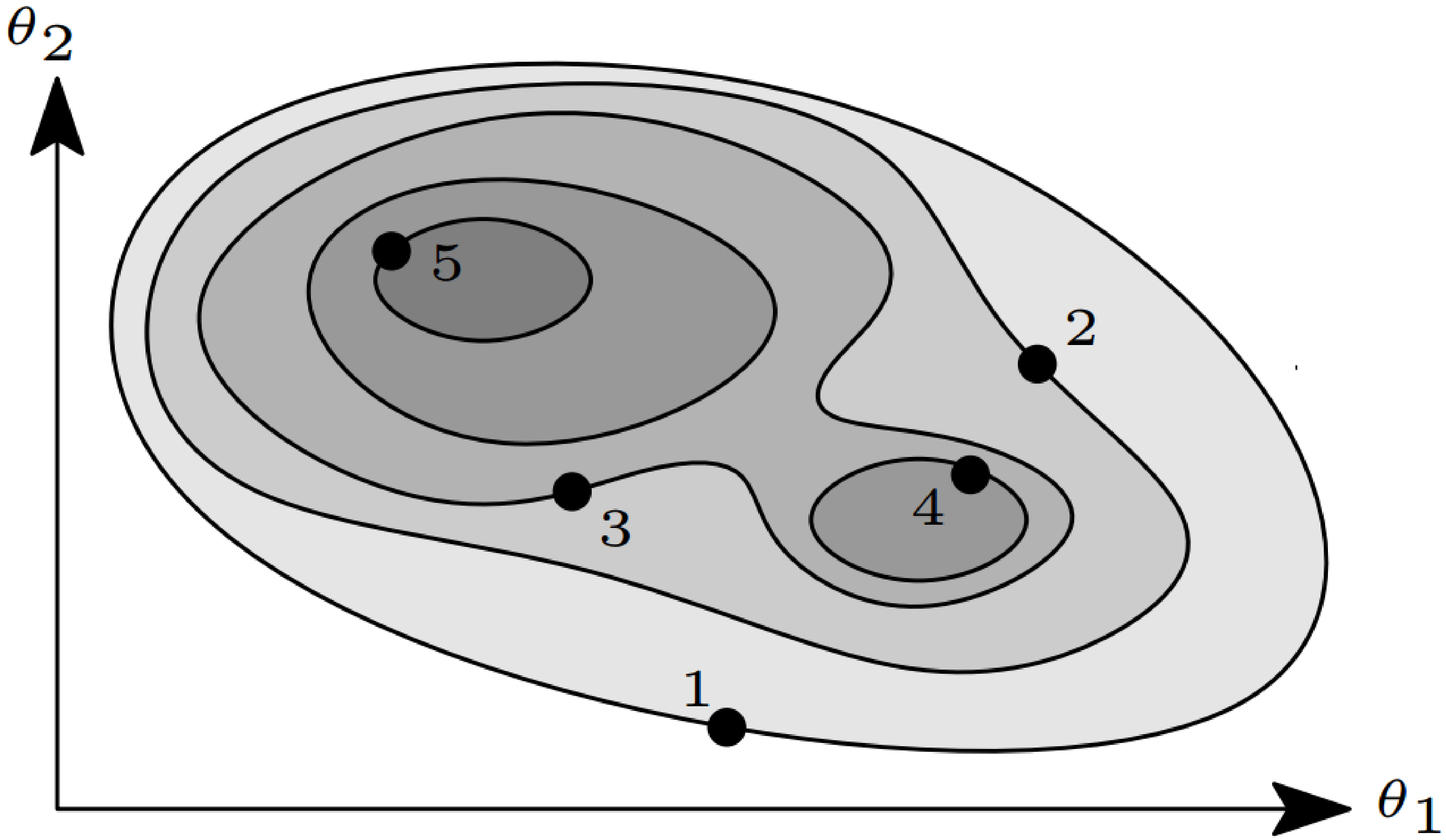
Nested Sampling In Practice



Higson et al. (2017)
[arxiv:1704.03459](https://arxiv.org/abs/1704.03459)

Naïve Approach: Sampling from the Prior



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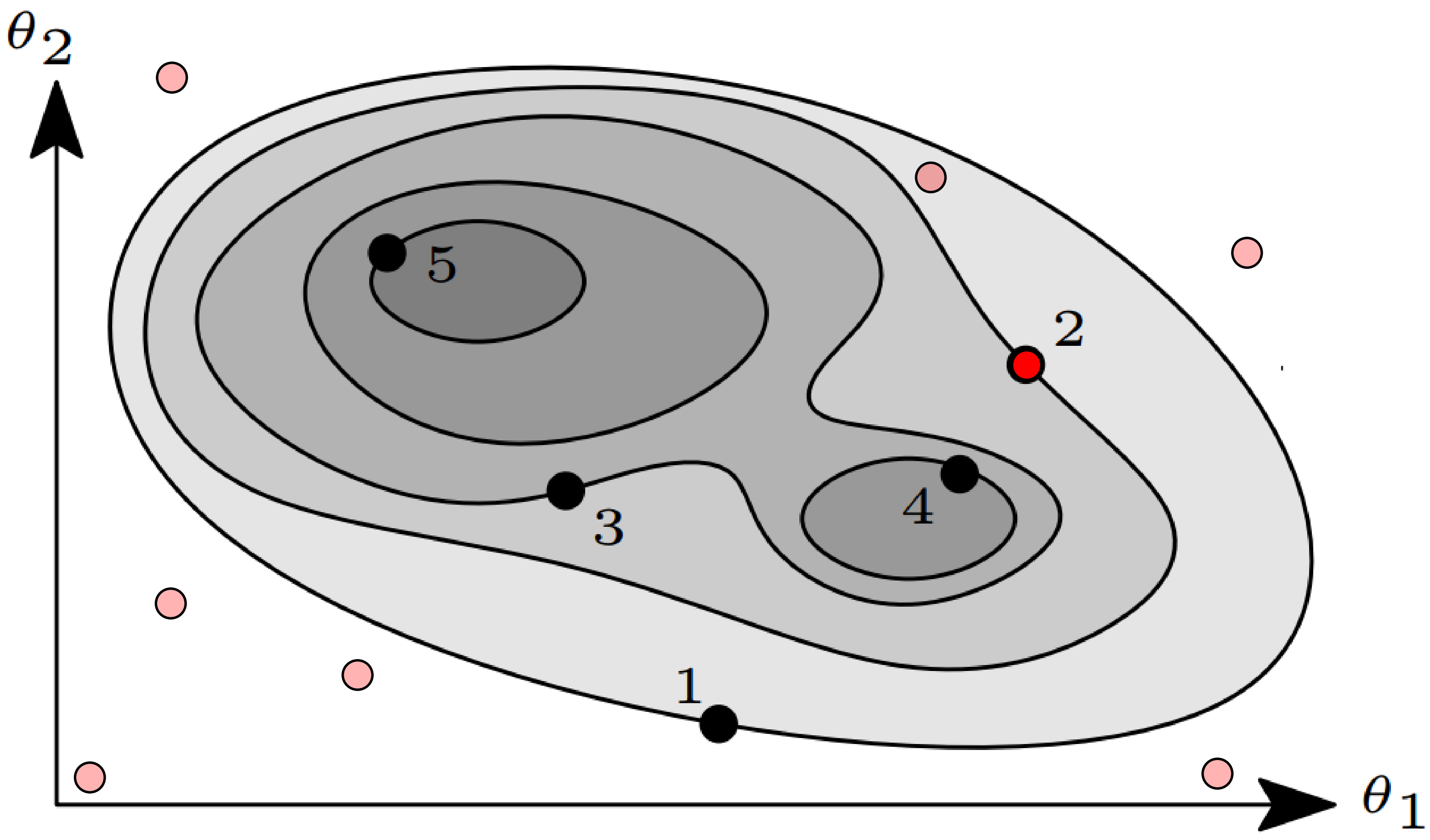

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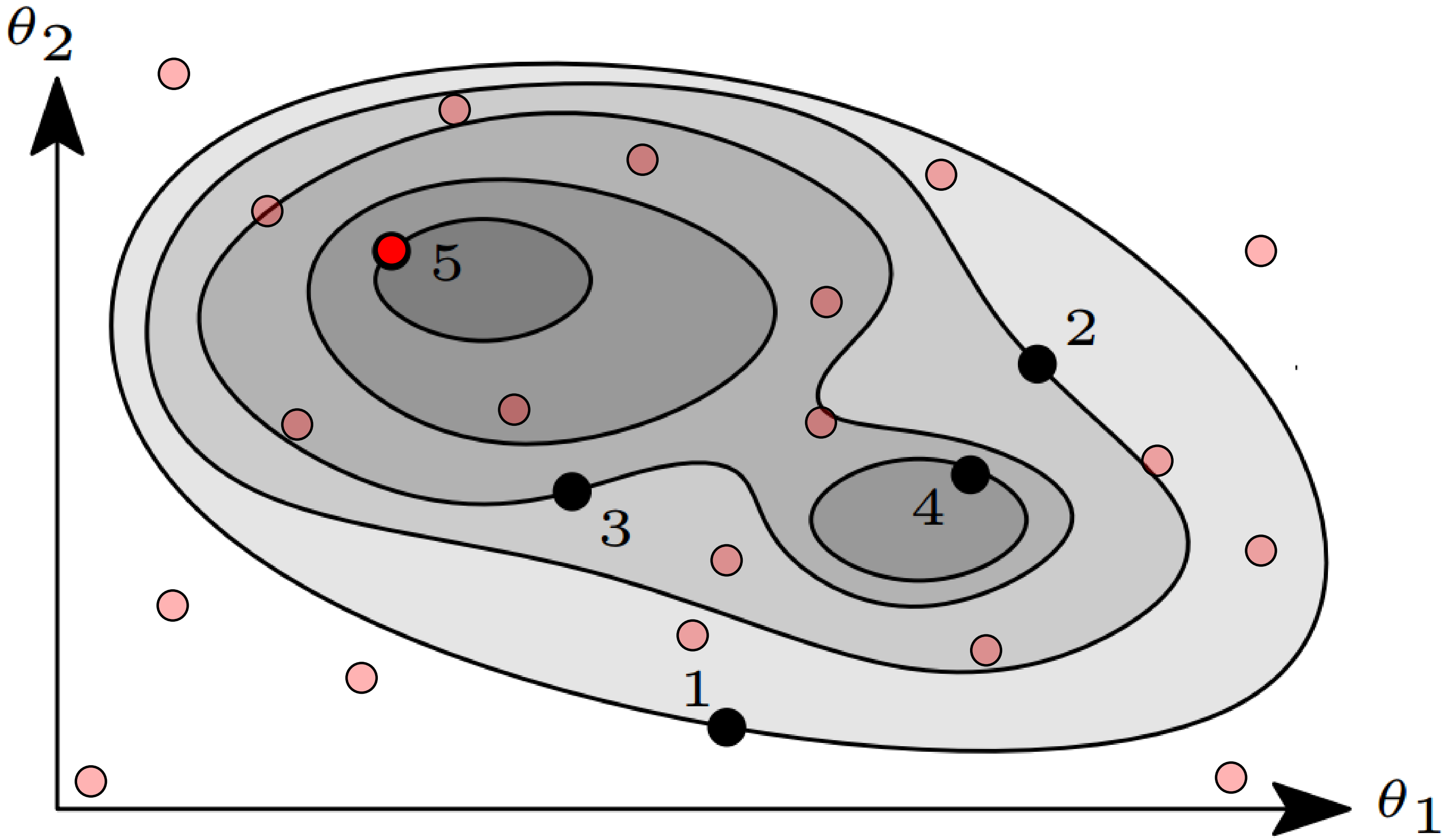

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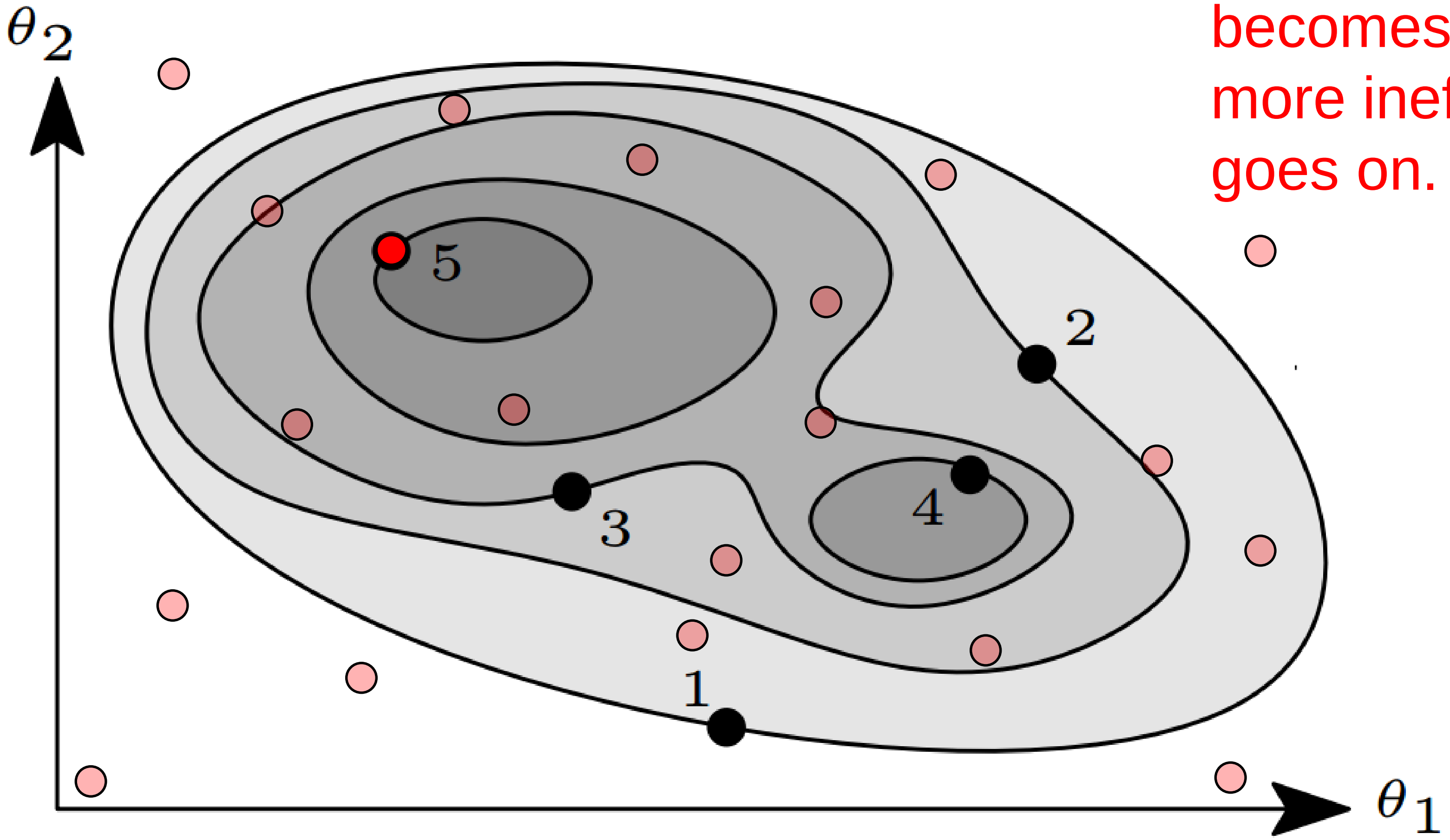

Sampling from the prior becomes **exponentially** more inefficient as time goes on.

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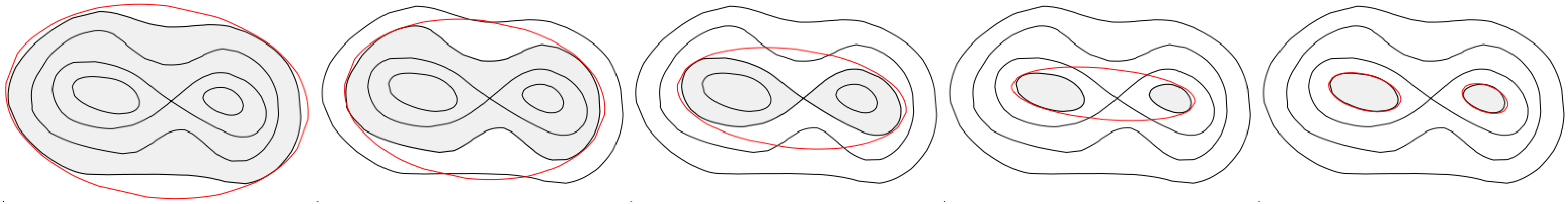
Sampling from the Constrained Prior

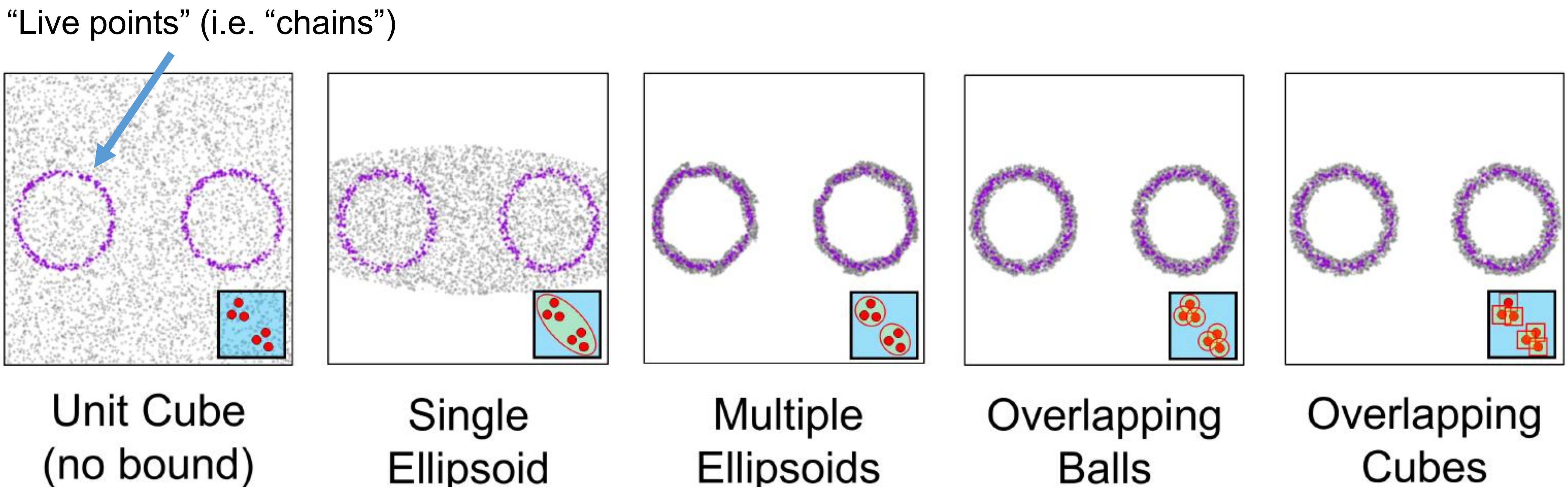
Proposal:
Try to bound the iso-likelihood contours in real time.



(a) (b) (c) (d) (e)

Feroz et al. (2009)

Examples of Bounding Strategies



Bounding Method	**Relevant Papers**
Single ellipsoid (§4.1.2)	Mukherjee et al. (2006)
Multiple ellipsoids (§4.1.3)	Shaw et al. (2007), Feroz & Hobson (2008), Feroz et al. (2009, 2013), Handley et al. (2015)
Overlapping balls/cubes (§4.1.4, §4.1.5)	Buchner (2016, 2017)

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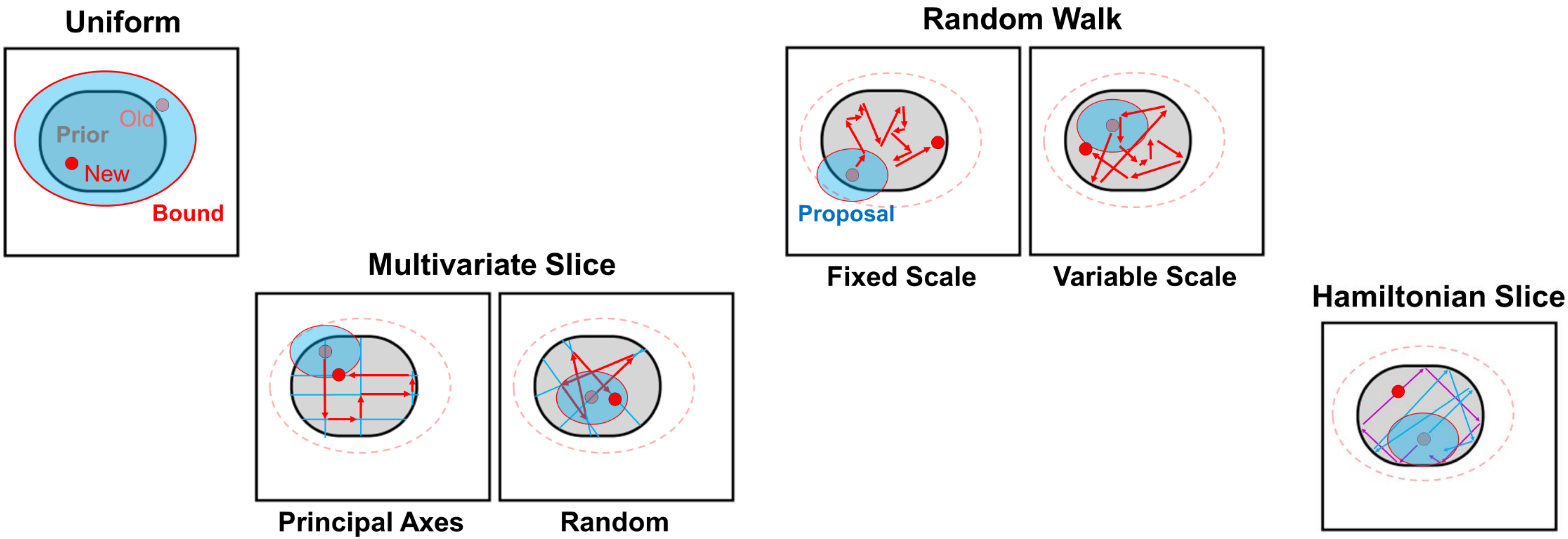

Sampling Method	
Uniform Sampling (§4.2.1)	Mukherjee et al. (2006), Feroz & Hobson (2008), Feroz et al. (2009), Buchner (2016)
Random walks (§4.2.2)	Skilling (2004), Skilling (2006)
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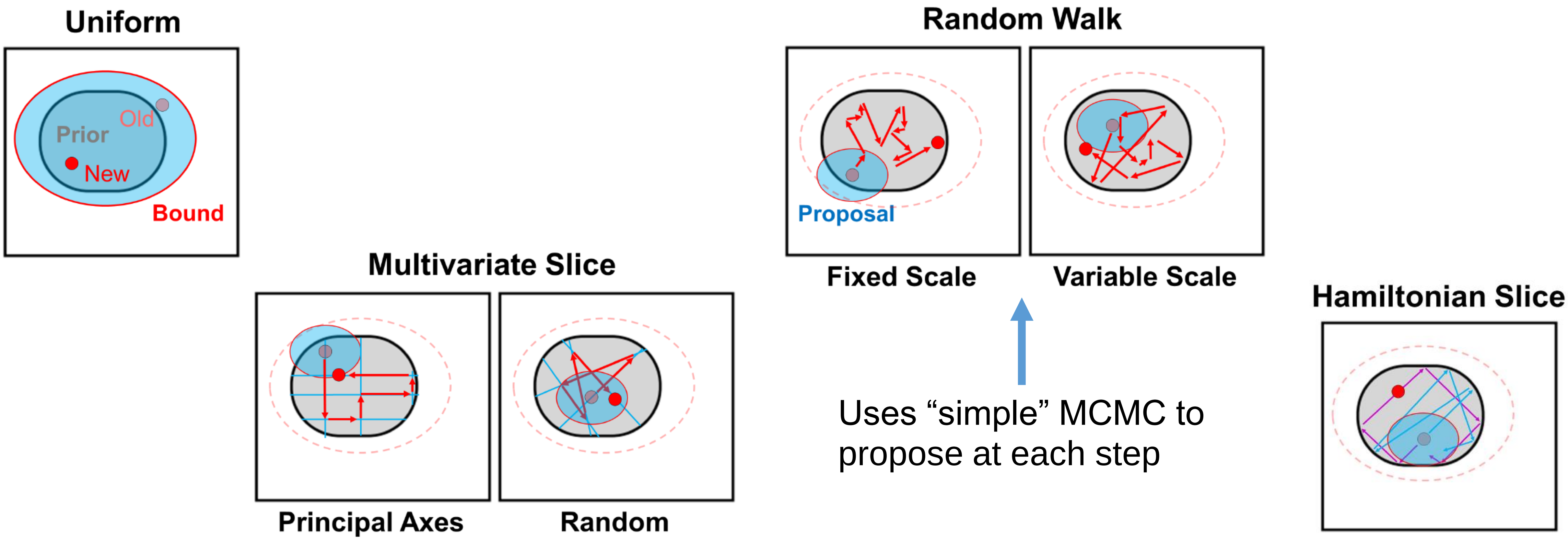

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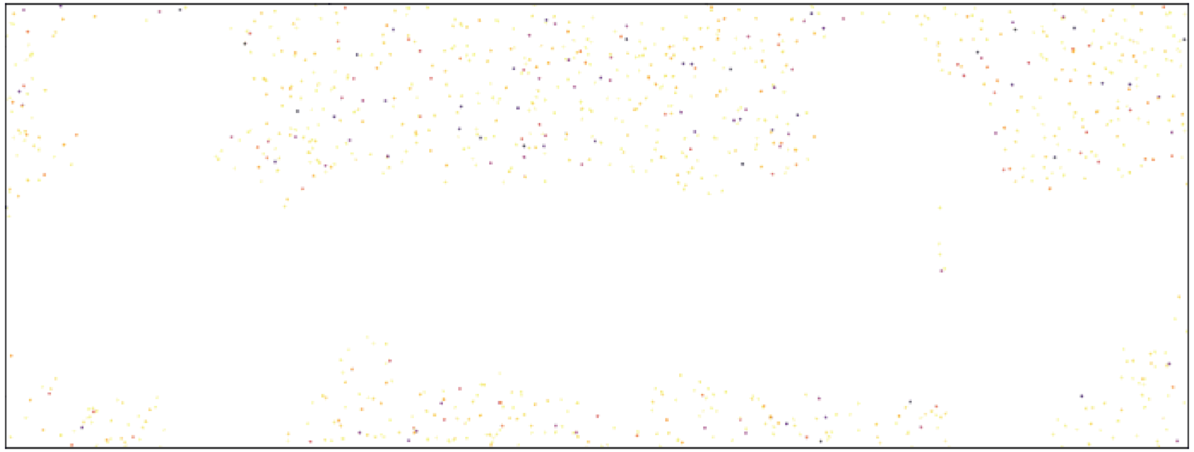

Speagle (2020)

https://dynesty.readthedocs.io

dynesty

```
import dynesty

# "Static" nested sampling.
sampler = dynesty.NestedSampler(loglike, ptform, ndim)
sampler.run_nested()
sresults = sampler.results

# "Dynamic" nested sampling.
dsampler = dynesty.DynamicNestedSampler(loglike, ptform, ndim)
dsampler.run_nested()
dresults = dsampler.results
```

dynesty

```python
from dynesty import plotting as dyplot

# Plot a summary of the run.
rfig, raxes = dyplot.runplot(results)

# Plot traces and 1-D marginalized posteriors.
tfig, taxes = dyplot.traceplot(results)

# Plot the 2-D marginalized posteriors.
cfig, caxes = dyplot.cornerplot(results)
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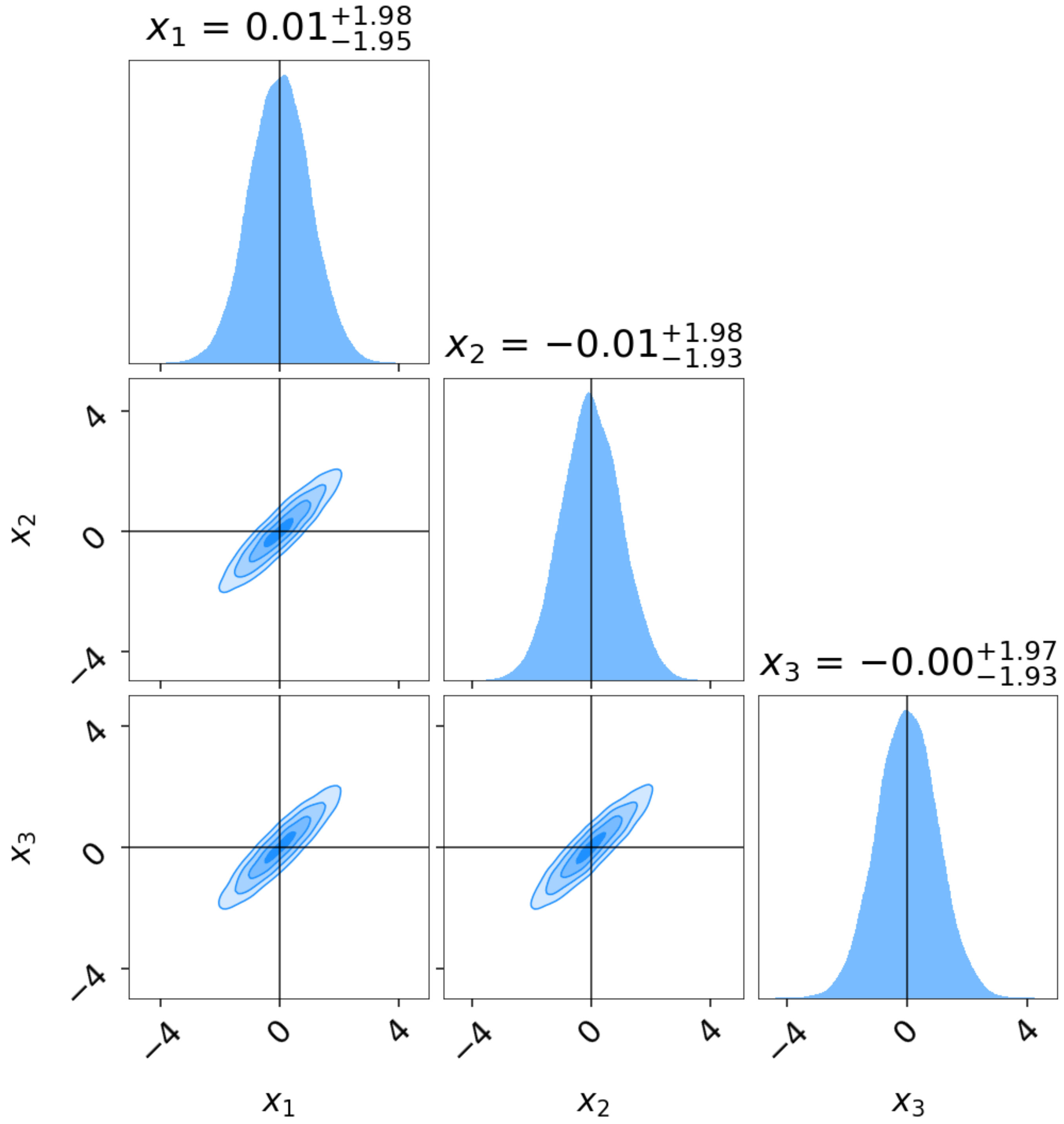

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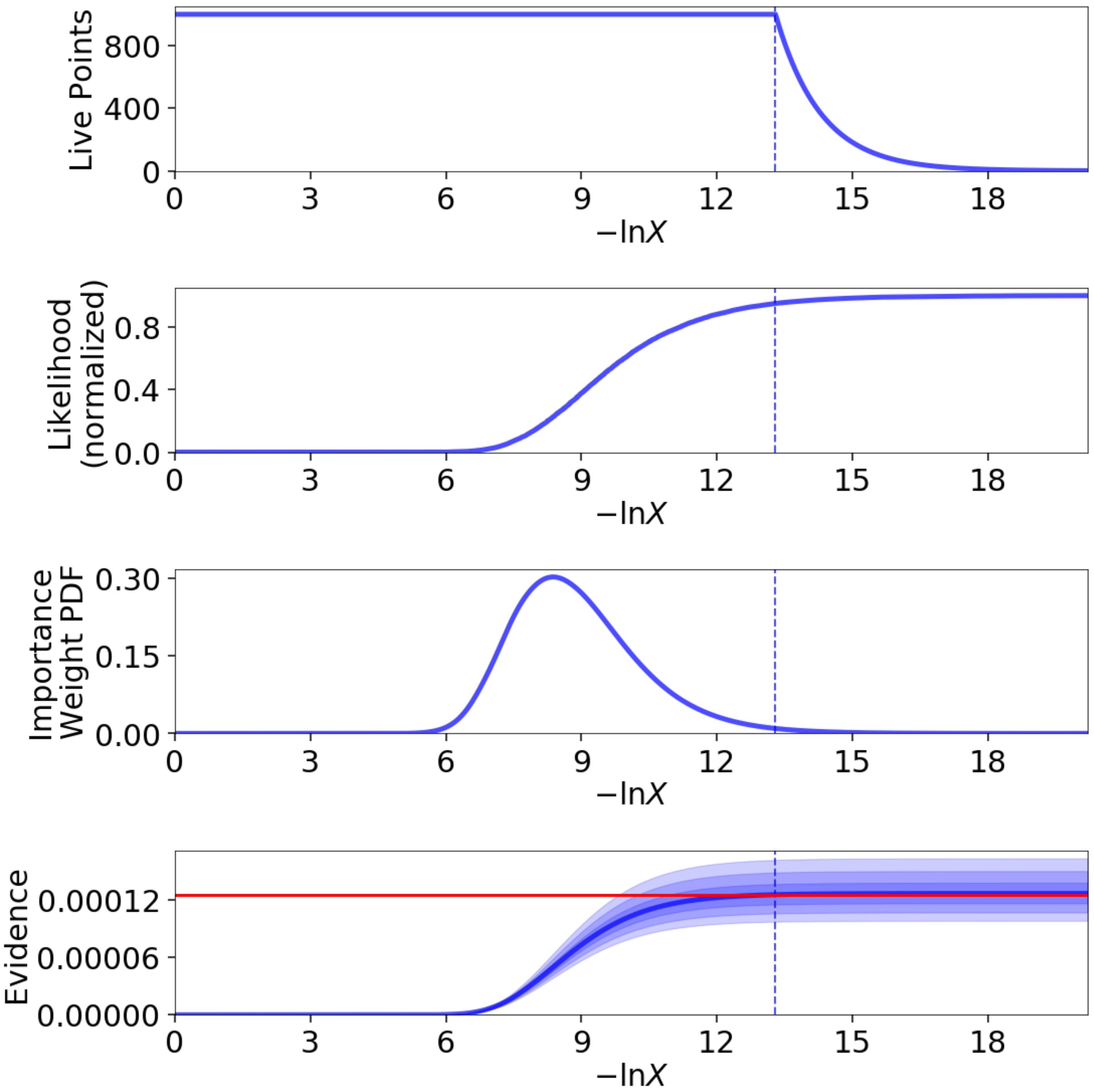

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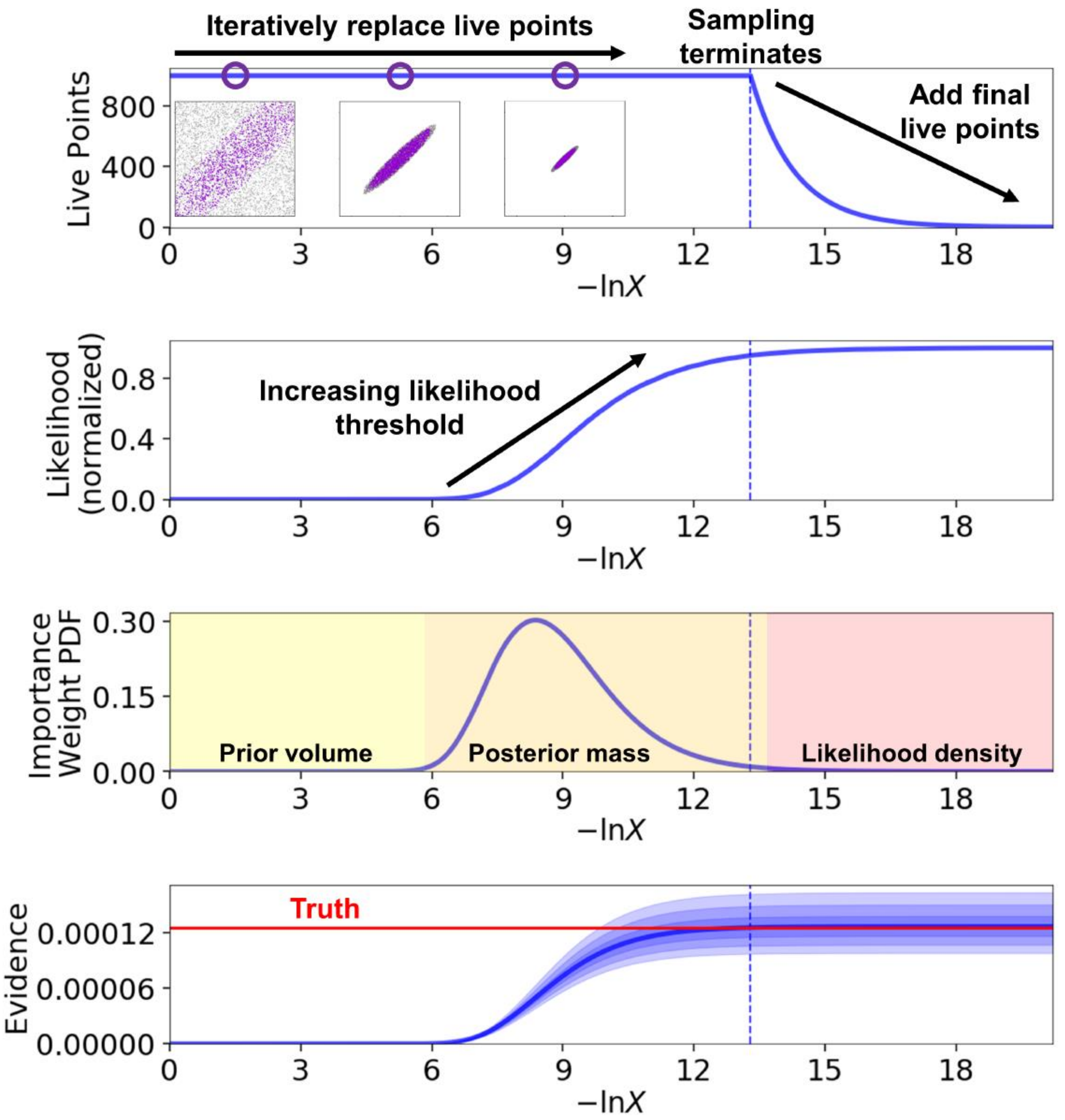

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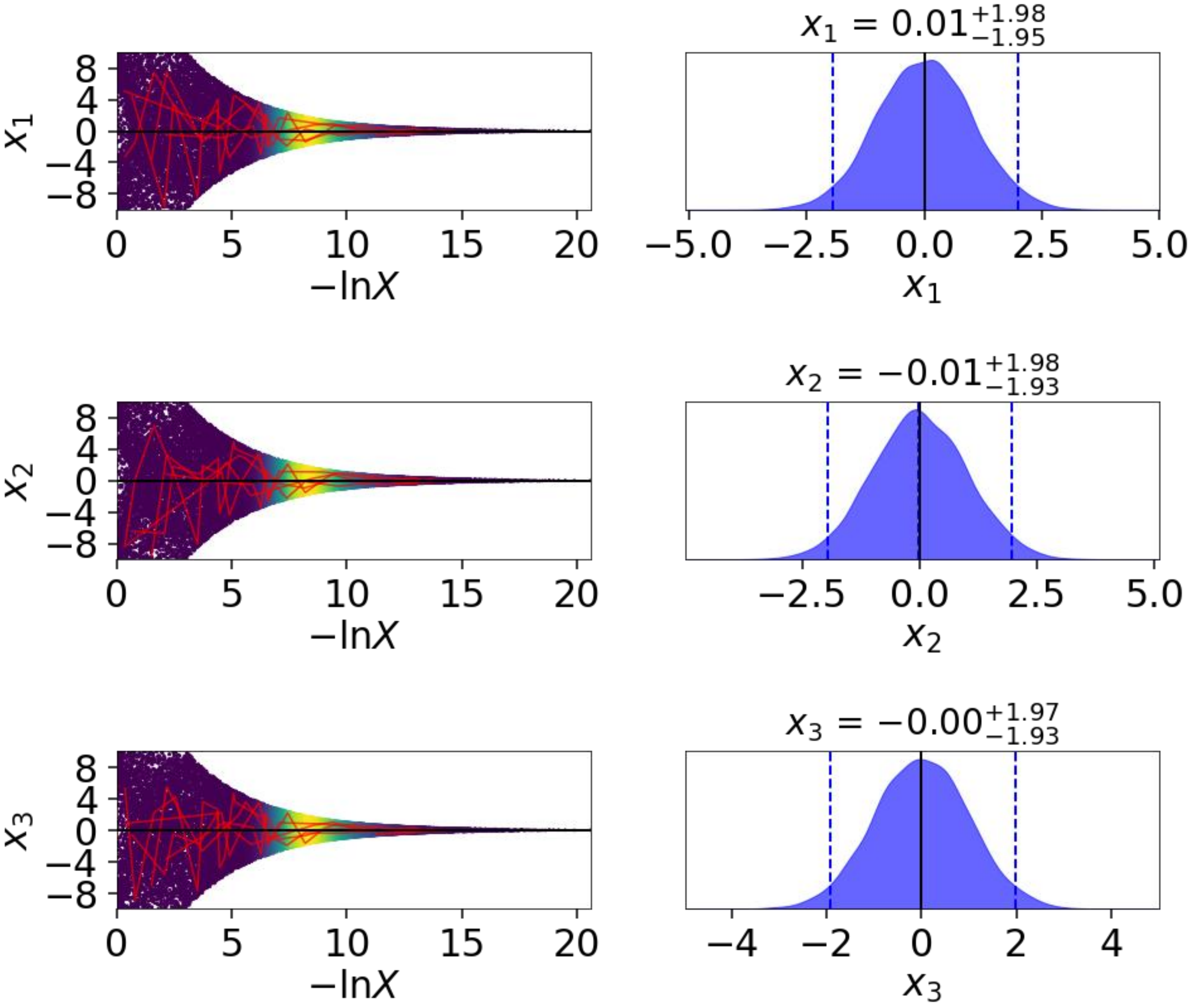

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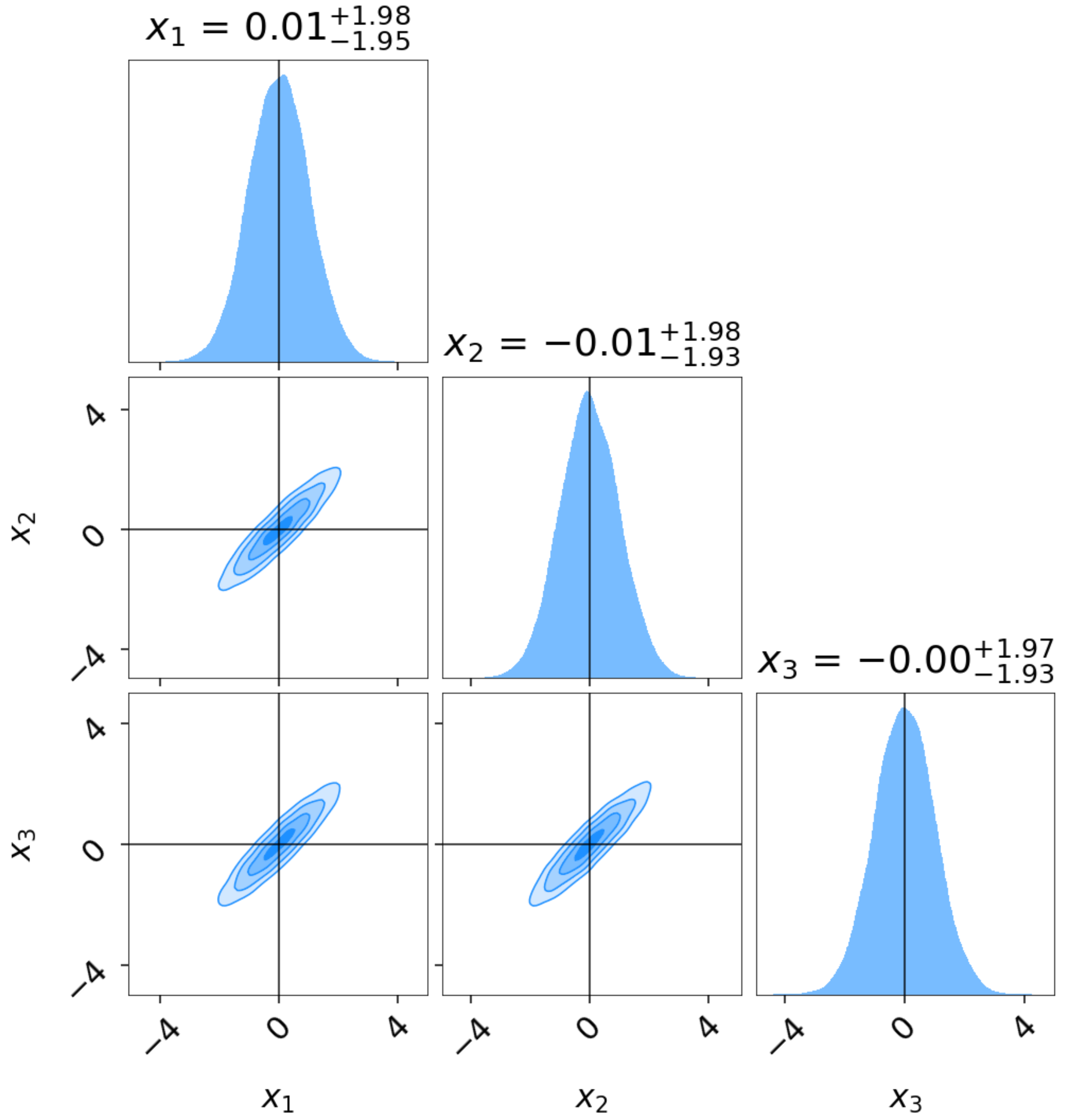
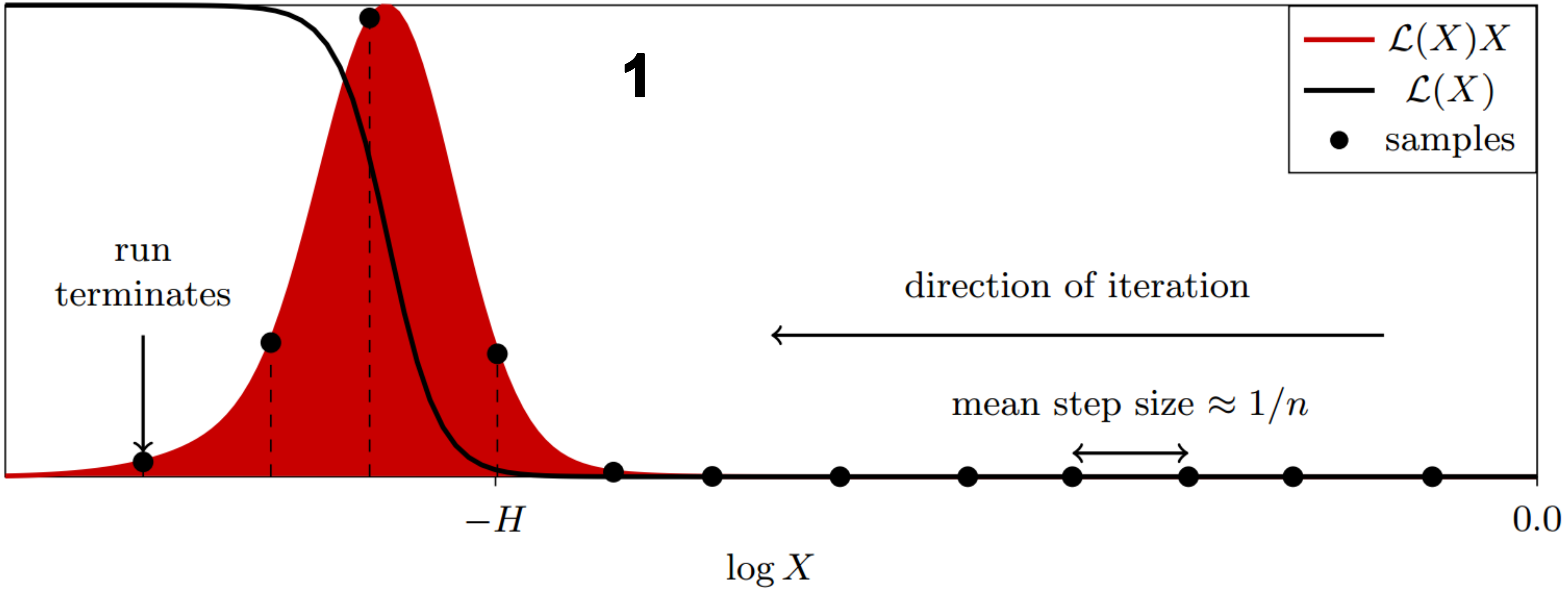


Dynamic Nested Sampling



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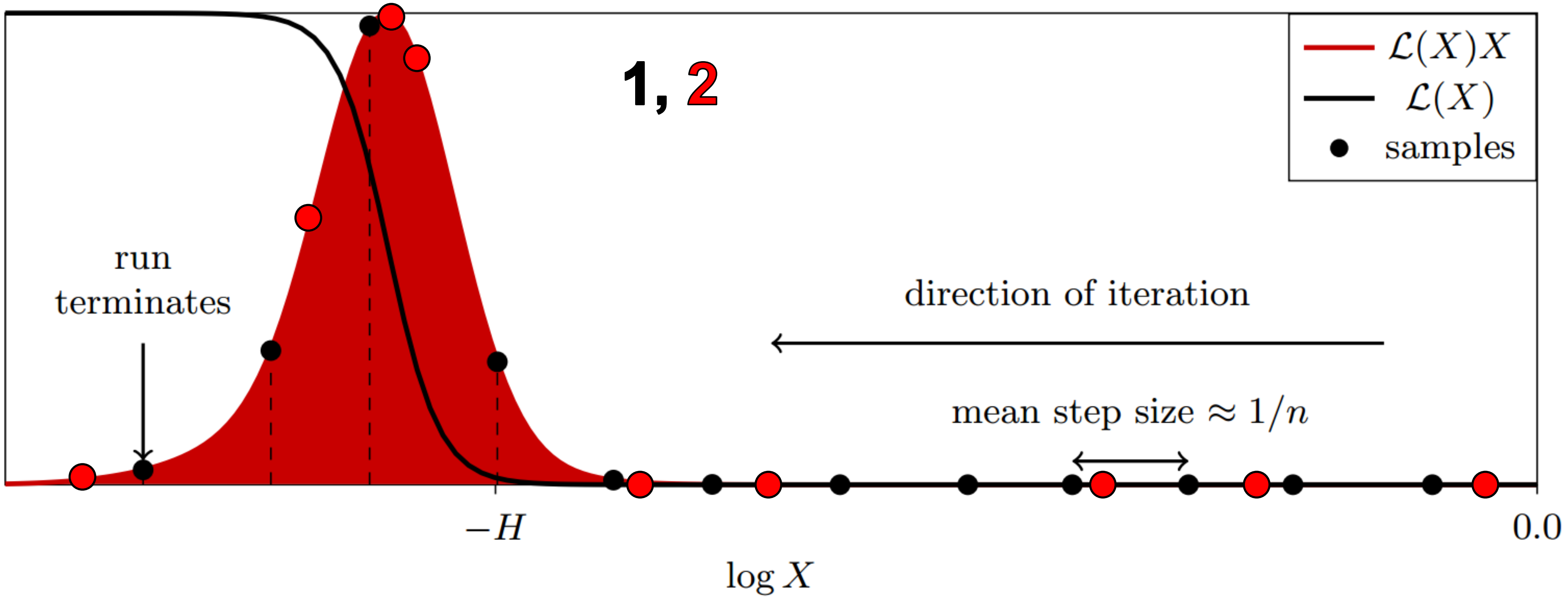

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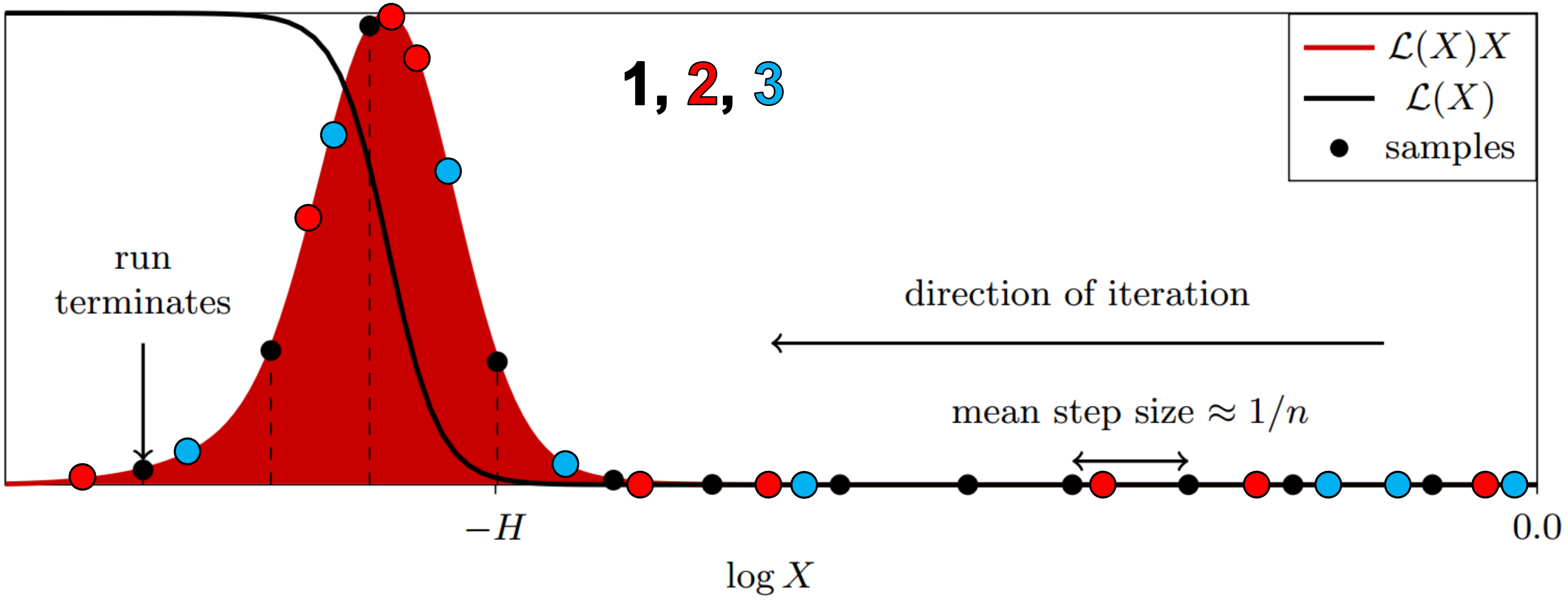

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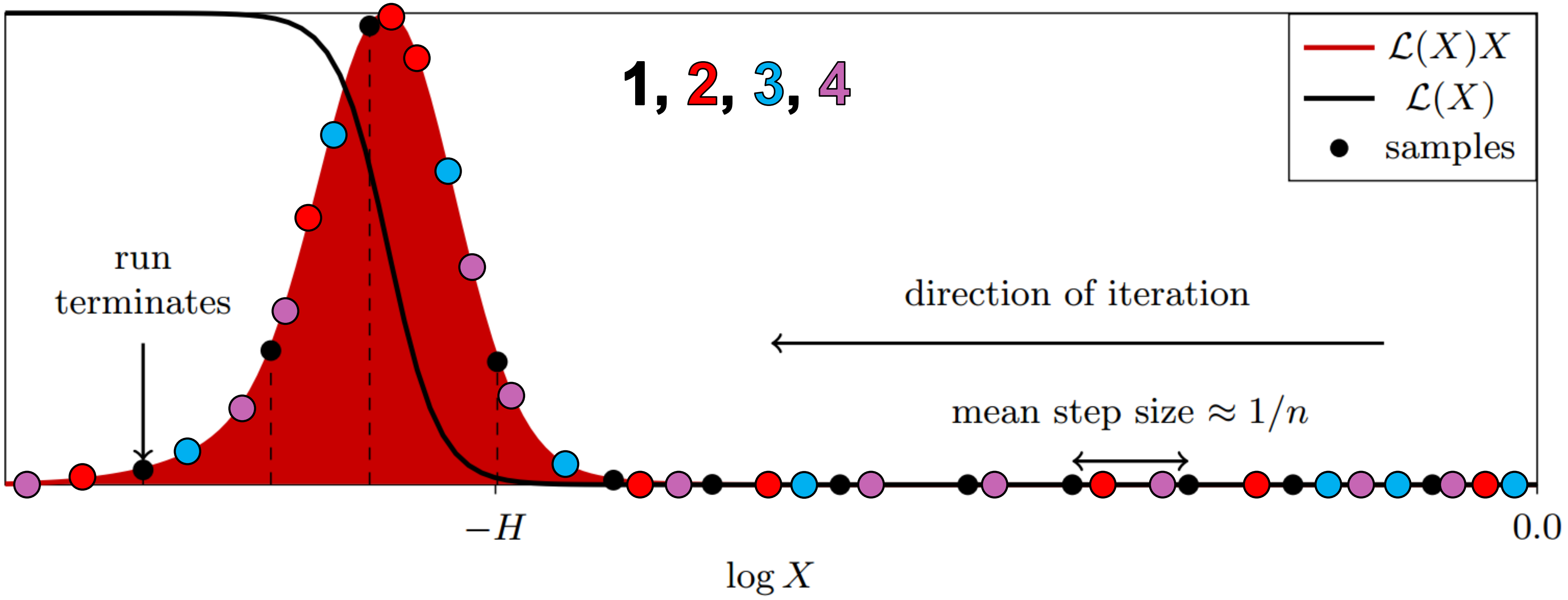

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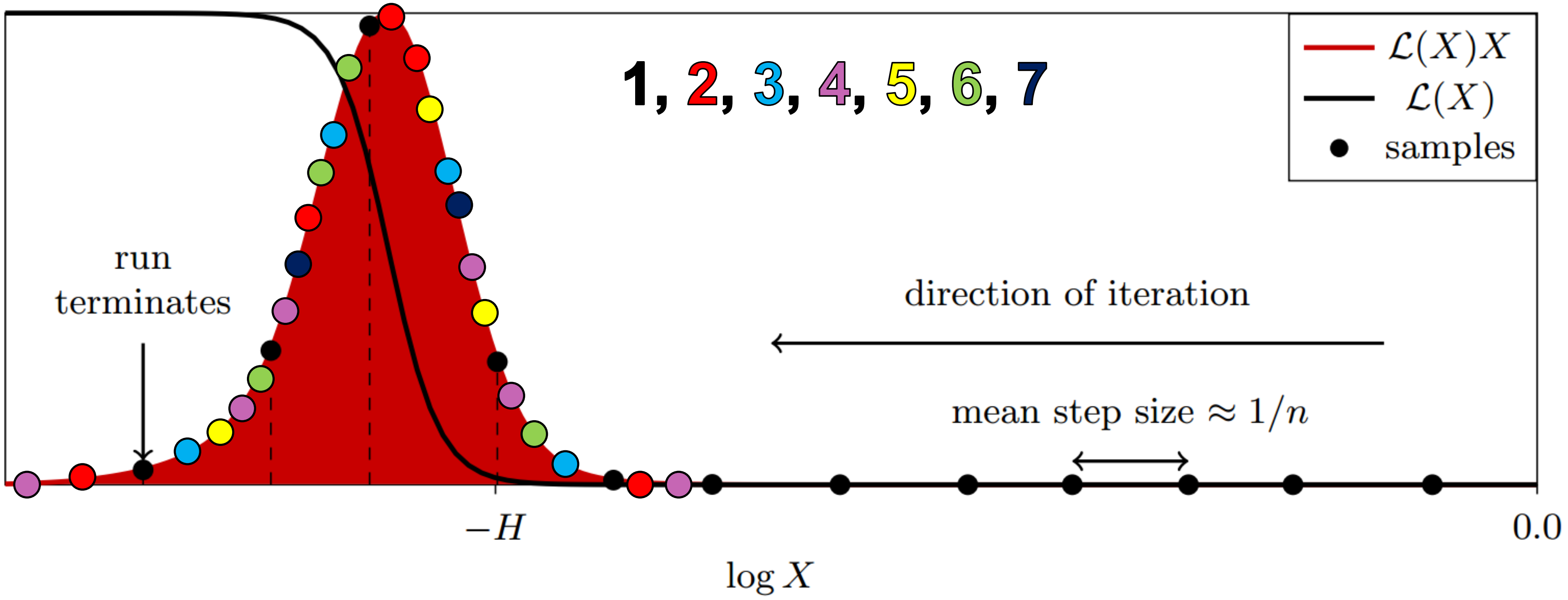

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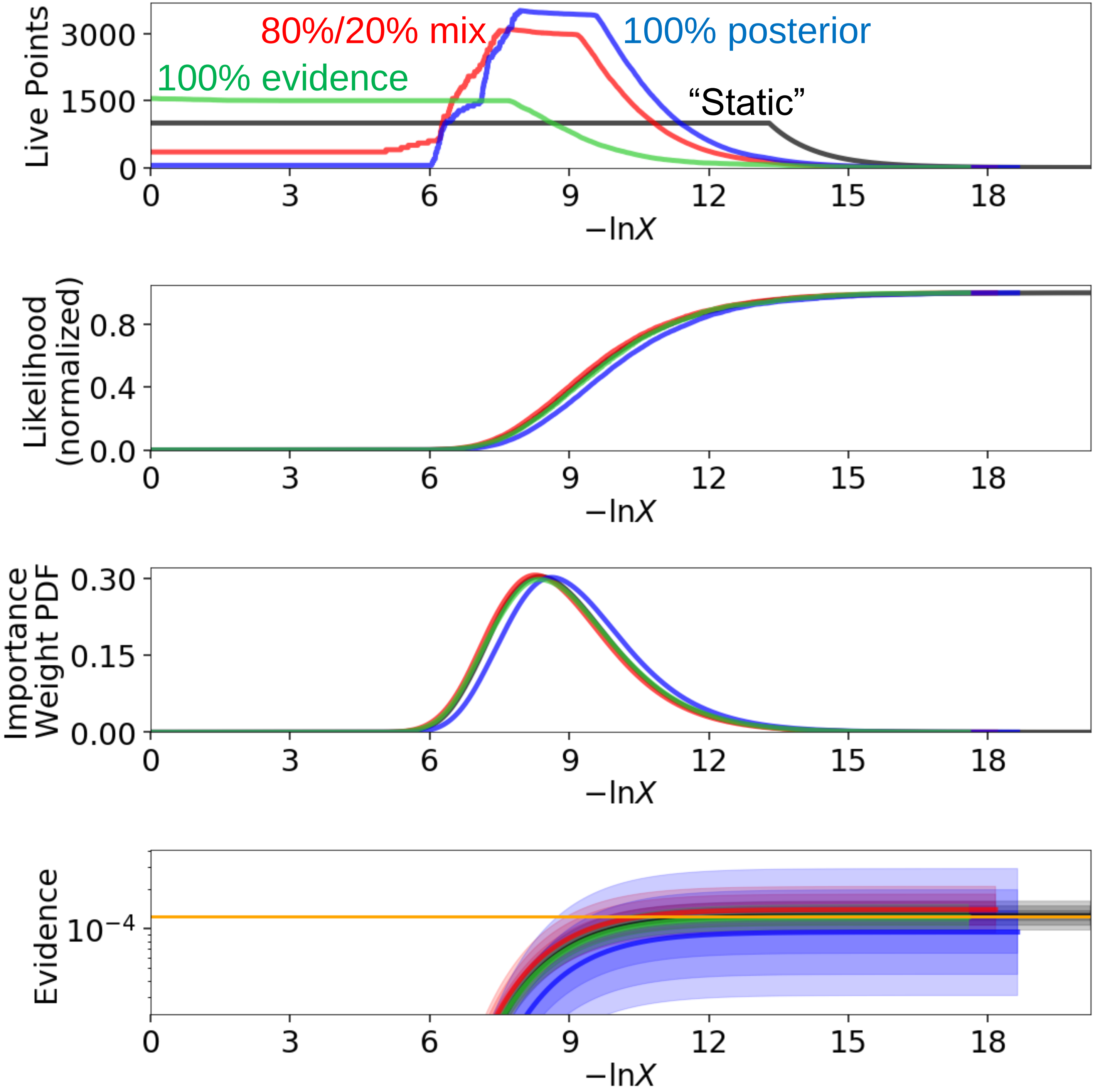
Comparisons

- Same 2-D Gaussian example as before with **fixed number of samples**.
- Only change is in overall Dynamic Nested Sampling **strategy**.



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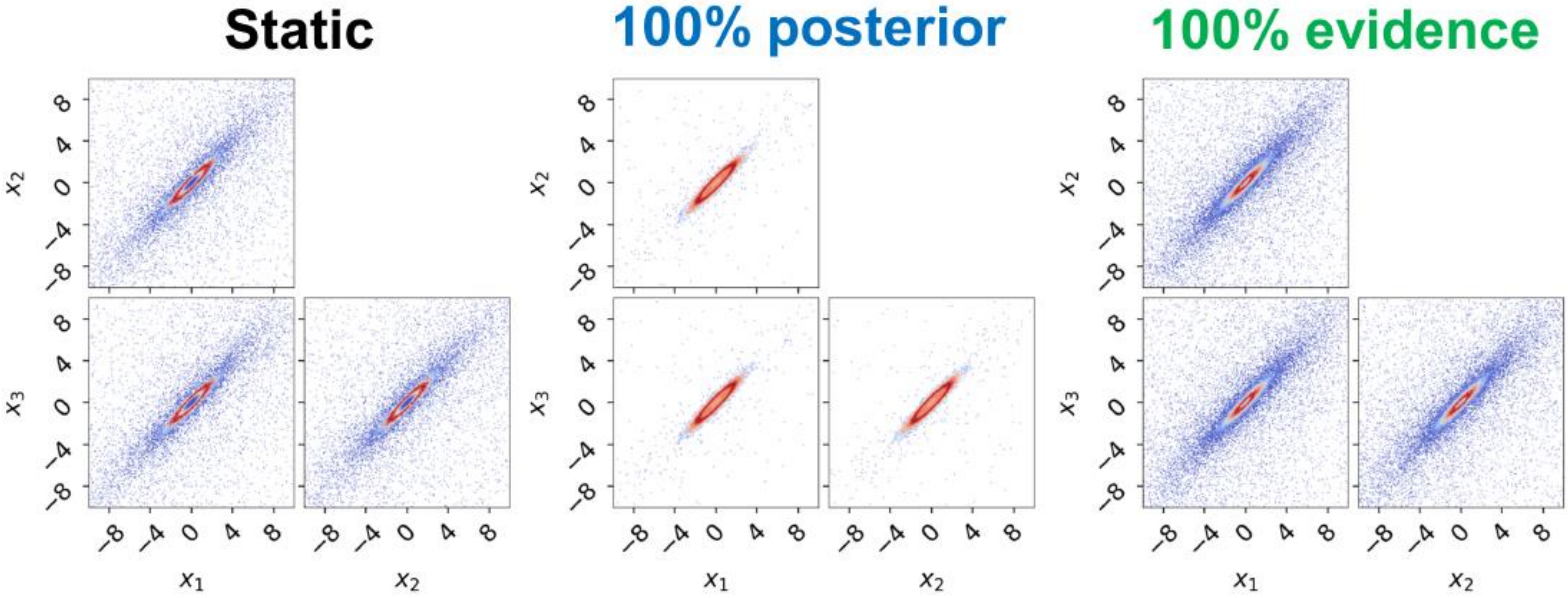
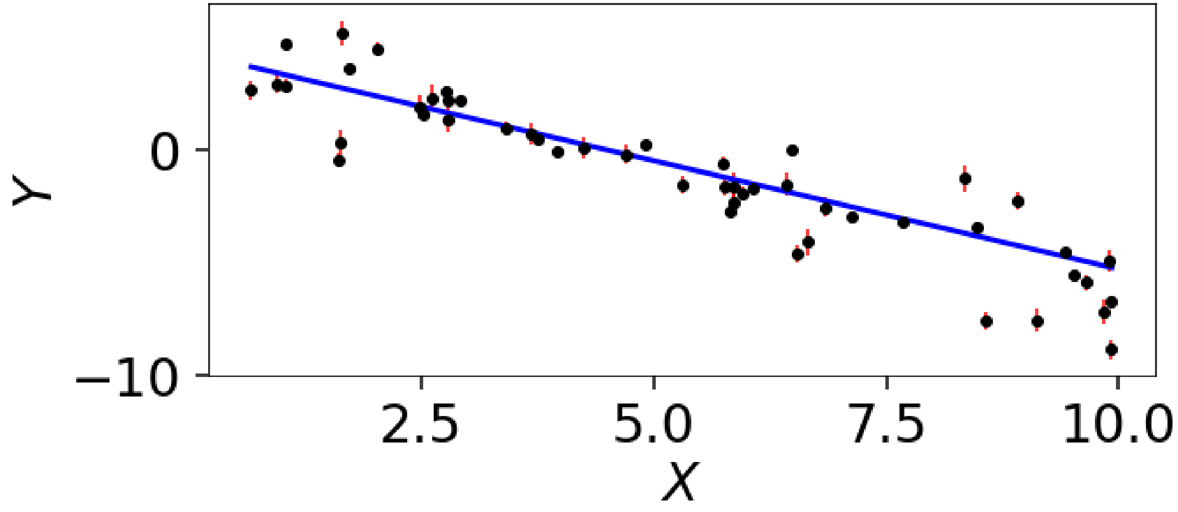
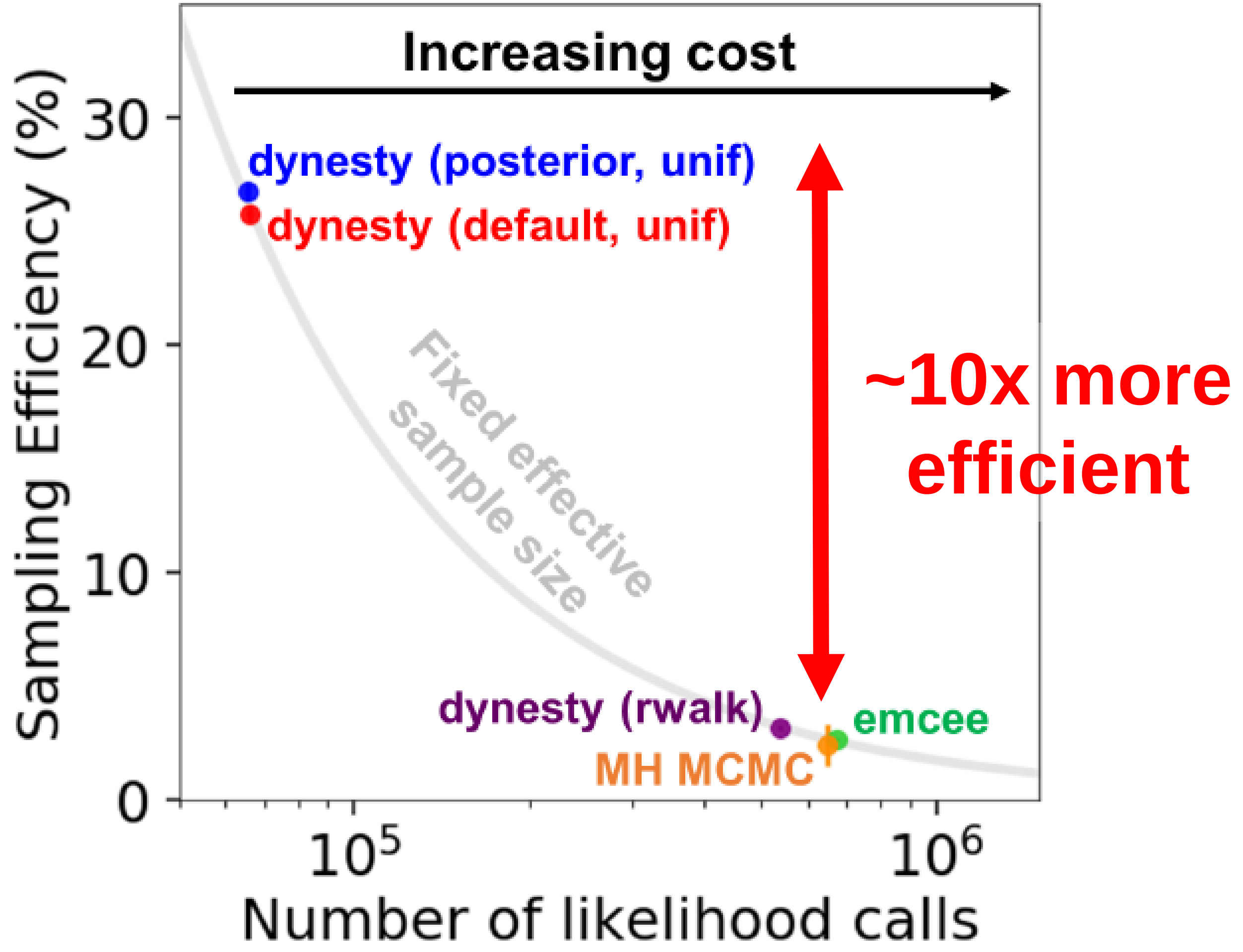


Speagle (2020)

Comparisons with emcee

- Simple 3-parameter linear regression problem with reasonable prior constraints.

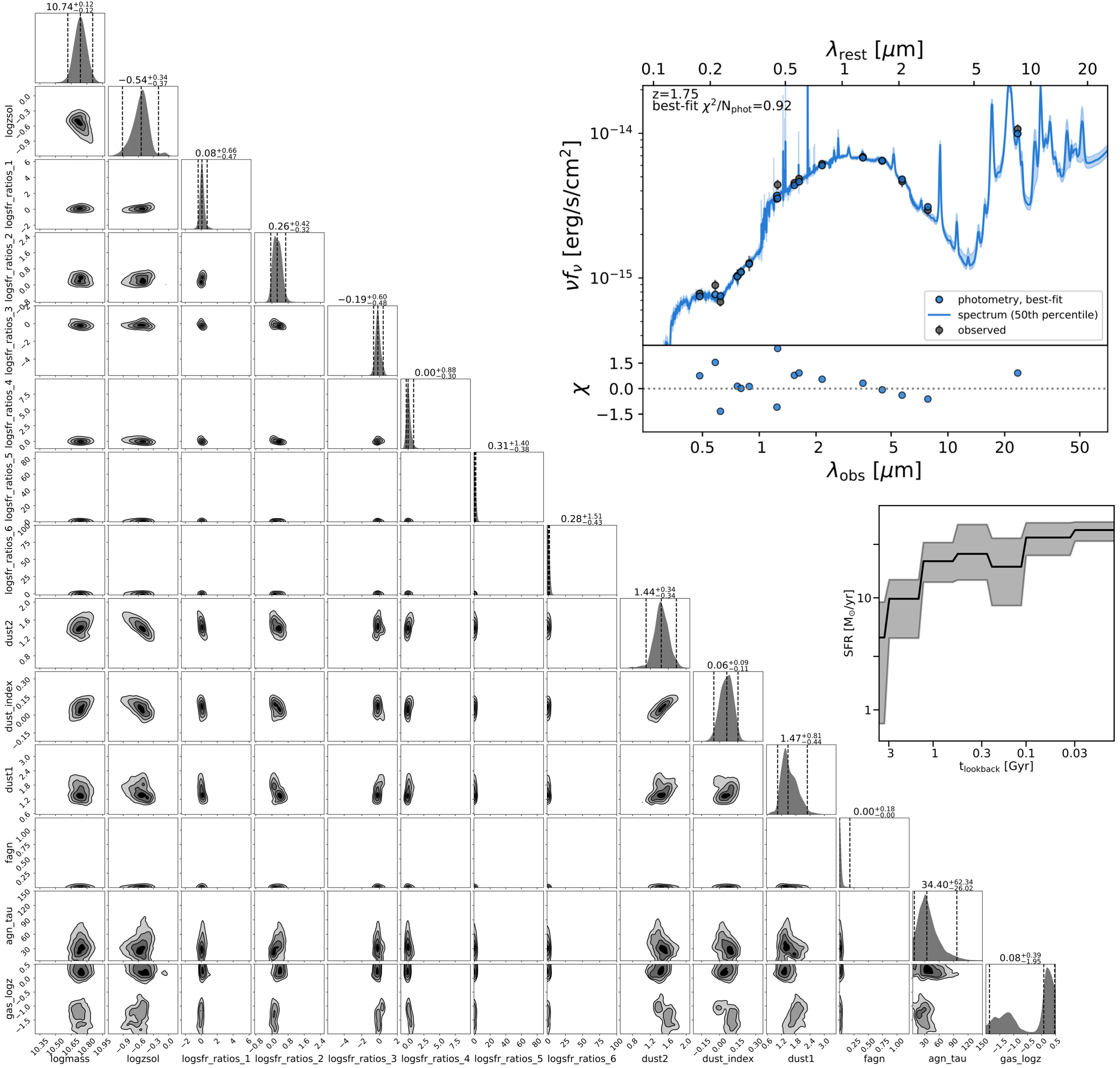




Speagle (2020)

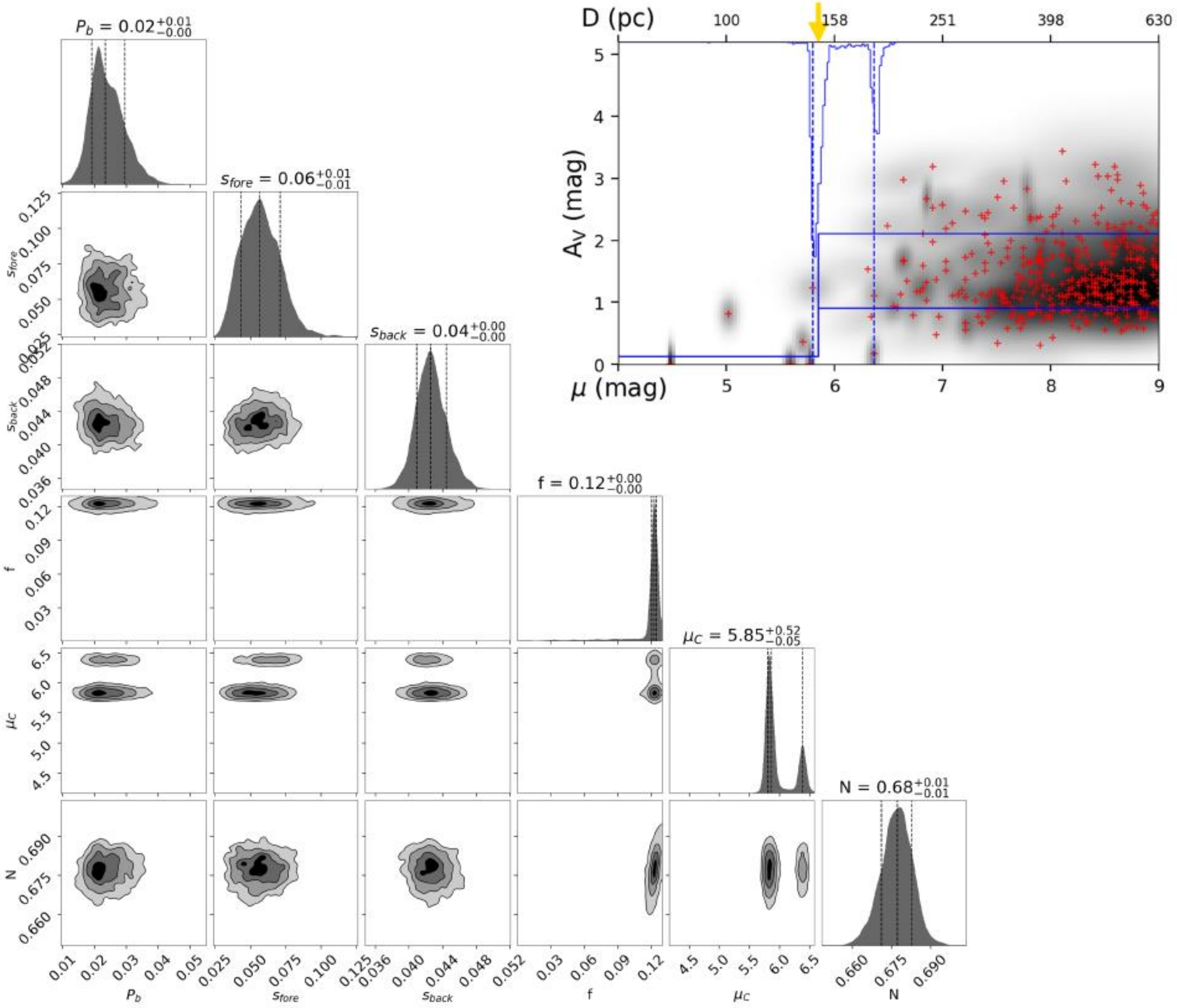
Applications

- Galaxy SED modeling with *Prospector* (Leja et al. 2019, Johnson et al. in prep.).



Applications

- 3-D dust mapping (Zucker & Speagle et al. 2019)



Advantages and Disadvantages

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1. Can characterize complex uncertainties in real-time.
2. Can allocate samples much more efficiently in some cases.
3. Possesses well-motivated stopping criteria (Skilling 2006; Speagle 2020).

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Disadvantages to Nested Sampling:

1. Implementations require a prior transform.
2. Runtime sensitive to size of prior.
3. Overall approach can sometimes miss certain types of solutions.
4. Sampling is more involved.

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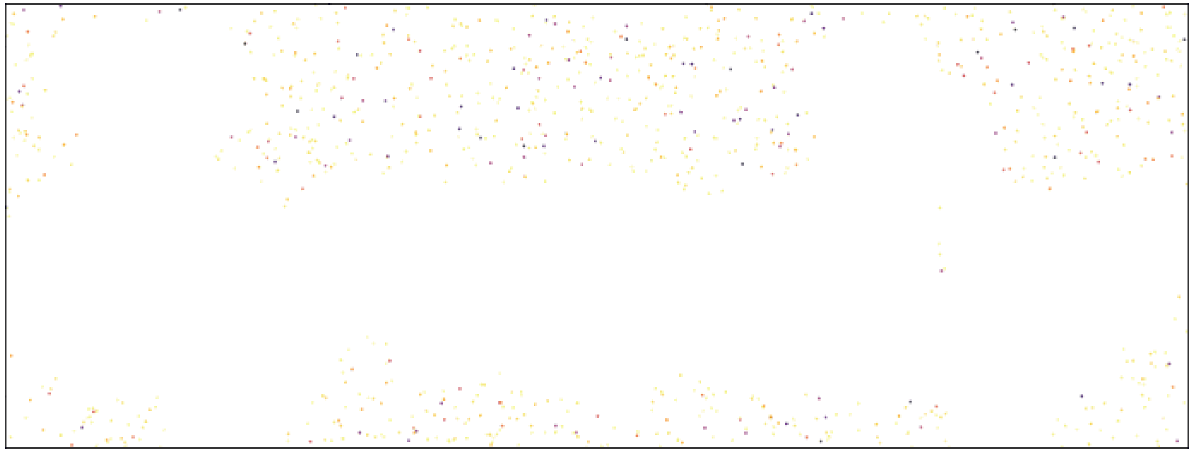

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