CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)

(b) Find the solution with $a_0 = 1$.
(Solution)

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)

Problem 1:

$a_n = 3a_{n-1} + 2^n$

a) $a_n = -2^{n+1}$ implies $a_{n-1} = -2^{(n-1)+1} = \underline{-2^n}$

$3a_{n-1} + 2n = 3.(-2^n) + 2n$

$= -3.(2^n) + 2^n = 2^n \underbrace{(-3+1)}_{-2}$

$= -2^{n+1}$

$= a_n$ is solution of $\boxed{3a_{n-1} + 2^n}$

b) $a_0 = 1$

$a_0 = 1 = \alpha - 2$

$3 = \alpha$

$a_n = \alpha . 3^n - 2^{n+1}$

$a_n = 3 . 3^n - 2^{n+1}$

$\boxed{a_n = 3^{n+1} - 2^{n+1}}$ solution of nonhomogeneous recurrence relation.

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Problem 2

$f(n) = 4 \cdot f(n-1) - 4f(n-2) + n^2$ for $f(0) = 2$, $f(1) = 5$

$g(n) = an^2 + bn + c.$

$g(n) = 4 \cdot g(n-1) - 4 \cdot g(n-2) + n^2.$

$an^2 + bn + c = 4[a(n-1)^2 + b(n-1) + c] - 4[a(n-2)^2 + b(n-2) + c] + n^2$

$an^2 + bn + c = 4(an^2 + (b-2a)n + (a-b+c))$

$-4(an^2 + (b-4a)n + (4a-2b+c) + n^2$

$an^2 + bn + c = (4a - 4a + 1)n^2 + (4b - 8a - 4b + 16a)n + (4a - 4b + 4c - 16a +$

$8b - 4c)$

$an^2 + bn + c = n^2 + 8an + (-12a + 4b)$

$a = \underline{1}$

$b = 8$

$c = 20$

$g(n) = n^2 + 8n + 20$

$f(1) = 5$

$f(0) = 2$

characteristic equation $\Rightarrow t^2 - 4t + 4 = 0$

$t = 2$

$f(n) = 2^n(dn + e) + n^2 + 8n + 20.$

$f(0) = 2^0 \cdot (e) + 20 = e + 20 \quad e = -18$

$f(1) = 2^1 \cdot (d+e) + 1 + 8 + 20 = 2d + 2\overset{-7}{e} + \underset{-36}{29} = 2d - 7 = 5$

$d = 6$

$f(n) = 2^n(6n - 18) + n^2 + 8n + 20$

Problem 3

$a_0 = 1$

$a_1 = 2$

Steps:

1) Find the characteristic equation
2) use algebra to find roots. Call r_1, r_2
3) Set up framework.

$a(n) = c_1 \cdot r_1^n + c_2 r_2^n$

a) $a_n = 2a_{n-1} - 2a_{n-2}$

$a_n = r^2$

$r^2 - 2r + 2 = 0$

$$\frac{2 \pm \sqrt{4i}}{2}$$

- $1 + i$
- $1 - i$

b) ??