CSE 211: Discrete Mathematics **(Due: 04/01/21)**

Homework #3

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Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1: Representing Graphs (10 points)

Represent the graph in Figure ?? with an adjacency matrix. Explain your representation clearly.

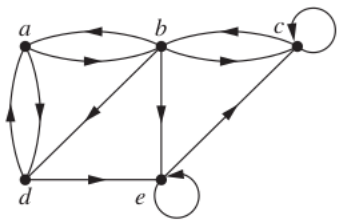


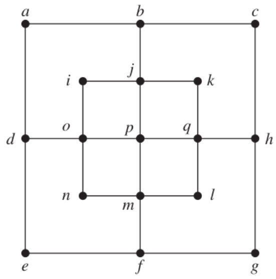
Figure 1: The graph for Problem 1

(Solution)
Given the graph, we examine the nodes in the graph, we give 1 for the value of that region of the node if there is a connection between the two nodes and this direction is in the right direction.For example, there is a connection from node A to node d, and since node a-b is shaped, the value in the matrix Table is 1.But since there is no connection from node A to node e, its value must be 0.

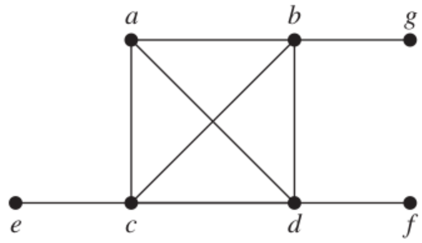
$$\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Problem 2: Hamilton Circuits (10+10+10=30 points)

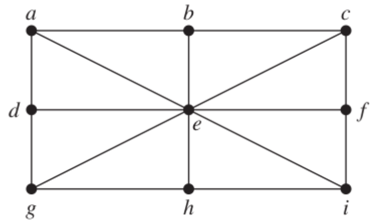
Determine whether there is a Hamilton circuit for each given graph (See Figure 2a, Figure 2b, Figure 2c). If the graph has a Hamilton circuit, show the path with its vertices which gives a Hamilton circuit. If it does not, explain why no Hamilton circuit exists.



(a) The graph G_1



(b) The graph G_2



(c) The graph G_3

Figure 2: The graphs to find Hamilton circuits for Problem 1

(a)
(Solution)
No Hamilton circuit exists.
Reason: Suppose there is a Hamiltonian circuit, then it must contain both edges located at the vertices of two degrees. This implies that it must contain 16 edges that occur at the vertices a, c, g, e,i, k, l, n . Also, at least 2 Edge Events at the Center Vertex will be included in P . So there will be at least 18 edges on the Hamilton Circuit . Since any circuit containing 17 vertices contains exactly 17 edges, here a contradiction arises. So there is such a graph given.

(b)
(Solution)
No Hamilton circuit exists.
Reason: If a graph has a vertex of 1 degree, it cannot have a Hamilton Circuit. Since there are three vertices of degree 1 in the given graph, the Hamiltonian circuit cannot exist.
(c)
(Solution)
Yes Hamilton circuit exists.
The circuit is : a$\rightarrow d \rightarrow g \rightarrow h \rightarrow i \rightarrow e \rightarrow f \rightarrow c \rightarrow b \rightarrow a$

Problem 3: Applications on Graphs (20 points)

Schedule the final exams for Math 101, Math 243, CSE 333, CSE 346, CSE 101, CSE 102, CSE 273, and CSE 211, using the fewest number of different time slots, if there are no students who are taking:

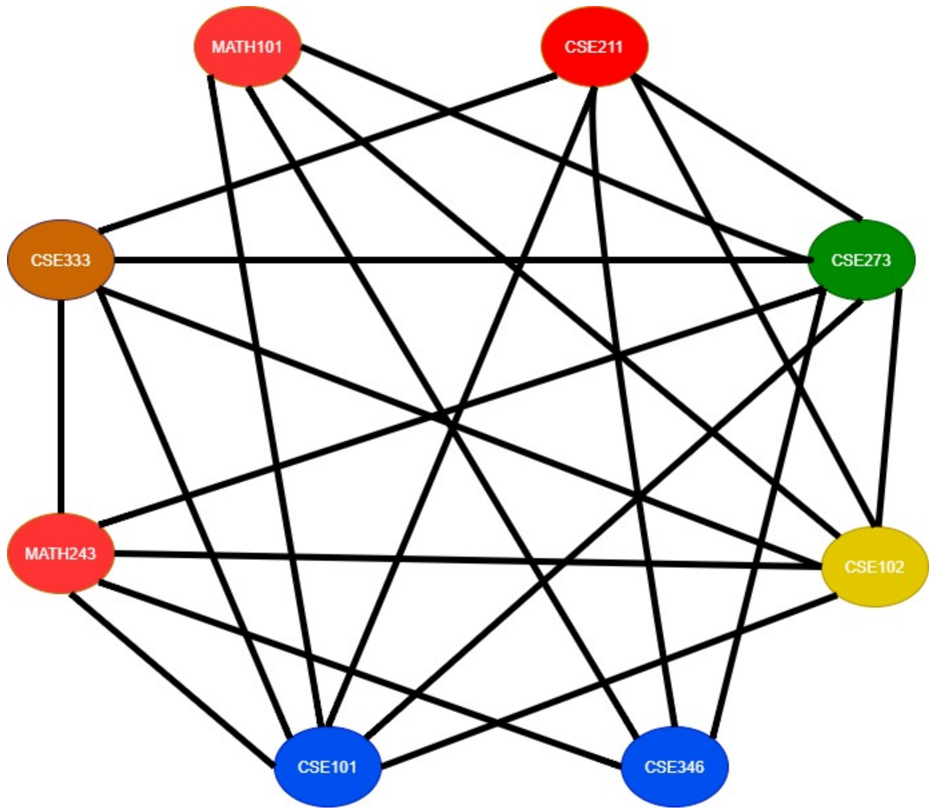
- both Math 101 and CS 211,
- both Math 243 and CS 211,
- both CSE 346 and CSE 101,
- both CSE 346 and CSE 102,
- both Math 101 and Math 243,
- both Math 101 and CSE 333,
- both CSE 333 and CSE 346

but there are students in every other pair of courses together for this semester.
Note: Assume that you have only one classroom.

Hint 1: Solve the problem with respect to your problem session notes.
Hint 2: Check the website

(Solution)

We require at least 5 colors to color the vertices of the graph and thus we require 5 different periods.
Math 101, Math 243 and CSE 211 are all unconnected and thus we can give the same color red to these classes.
CSE 333 is remaining that is connected to class of all previous colors, thus we need to assign a fifth color orange to CSE 333.
CSE 273 is linked to all other classes, so it needs to be given a unique color. We give CSE273 its green color.
CSE101 is not linked to all classes except CSE346, we give these two lessons blue.
CSE 102 is linked to all classes except CSE 346, so CSE 102 has a unique yellow color.
Period 1: MATH101, MATH 243 and CSE 211
Period 2: CSE 333
Period 3: CSE 273
Period 4: CSE 346 and CSE 101
Period 5: CSE 102



Problem 4: Applications for Hasse Diagram of Relations (40 points)

Remember the Problem 3 in Homework 2.

Write an algorithm to draw Hasse diagram of the given relations in "input.txt" which is given for HW2.

Your code should meet the following requirements, standards and accomplish the given tasks.

- Read the relations from the text file "input.txt". You can use your code from HW2 if you implemented to read the file. If you didn't implement it, please check HW2 to learn how to read the relations from the file.
- Determine each relation in "input.txt" whether it is reflexive, symmetric, anti-symmetric and transitive with your algorithm from HW2.
- In order to draw Hasse diagram, each relation must be POSET. Hence, the relation obeys the following rules:
  - Reflexivity
  - Anti-symmetric
  - Transitivity

  If the relation is not a POSET, your algorithm is responsible to CONVERT it to POSET.
  - If the relation is not reflexive, add new pairs to make the relation reflexive.
  - If the relation is symmetric, remove some pairs which make the relation symmetric. For instance, if the relation has (a, b) and (b, a), remove one of them randomly.
  - If the relation is not transitive, add new pairs which would make the relation transitive.
- After the relation becomes POSET, your algorithm should obtain Hasse diagram of the relation and write the diagram with the following format.

  - An example of the output format is given in ”exampleoutput.txt”. The file has the result of the first relation in ”input.txt”.
  - In ”output.txt”, each new Hasse diagram starts with ”n”.
  - The relation is (a, a), (a, b), (a, e), (b, b), (b, e), (c, c), (c, d), (d, d), (e, e)
  - The relation is already a POSET so we don’t need to add or remove any pairs.
  - After ”n”, write the POSET in the next line as it is shown in ”exampleoutput.txt”.
  - Since the relation is POSET, it becomes (a, b), (b, e), (c, d) after removing reflexive and transitive pairs.
  - The following lines give each pair of Hasse diagram.
- You can implement your algorithm in Python, Java, C or C++.
- **Important:** Put comments almost for each line of your code to describe what the line is going to do.
- You should put your source code file (file name is problem1.$\{.c, .java, .py, .cpp\}$) and output.txt into your homework zip file (check Course Policy).