B. Construction of a Desired Heading Angle

Following the approach in [20], the desired heading angle comes from a safety filter for the single integrator dynamics:

$$\begin{bmatrix}\dot{x} & \dot{y}\end{bmatrix}^\top = \hat{v}$$

with $\hat{v} \triangleq [\hat{v}_x \;\; \hat{v}_y]^\top \in \mathbb{R}^2$. In order to derive $\theta_s(t,x,y)$, we first introduce a proportional tracking controller for the single integrator dynamics:

$$\hat{v}_p(t,x,y) = K_v \, \mathbf{r_e}(t,\mathbf{x}). \tag{20}$$

A safety filter using the classic CBF-based QP is given by

$$\begin{aligned}\hat{v}_s(t,x,y) = \arg\min_{\hat{v}} \|\hat{v} - \hat{v}_p(t,x,y)\|^2 \\ \text{s.t. } \nabla h_0 \cdot \hat{v} + \alpha h_0(x,y) \geq 0.\end{aligned}$$

This gives an explicit solution:

$$\hat{v}_s(t,x,y) = \hat{v}_p + \max\big(-(\nabla h_0 \cdot \hat{v}_p + \alpha h_0)/\|\nabla h_0\|^2, 0\big)\, \nabla h_0^\top.$$

However, due to the $\max$ function, the above solution may not be differentiable. Instead, a smooth safety filter was proposed in [26] that has an explicit form:

$$\hat{v}_s(t,x,y) \triangleq [\hat{v}_{s,x} \;\; \hat{v}_{s,y}]^\top = \hat{v}_p + \lambda(a,b)\nabla h_0^\top \tag{21}$$

with $a(t,x,y) \triangleq \nabla h_0 \cdot \hat{v}_p + \alpha h_0(x,y)$, $b(x,y) \triangleq \|\nabla h_0\|^2$ and function λ that satisfies certain conditions. One specific choice of the function λ is the half Sontag formula:

$$\lambda(a,b) = \frac{-a + \sqrt{a^2 + q(b)\, b}}{2b},$$

where $q : \mathbb{R} \to \mathbb{R}$, $q(0) = 0$, and $q(b) > 0$ for all $b \neq 0$. We use $q(b) \triangleq \alpha_q b$ with a parameter $\alpha_q > 0$. Then, the safe heading angle is obtained as

$$\theta_s(t,x,y) = \operatorname{atan}(\hat{v}_{s,y}, \hat{v}_{s,x}). \tag{22}$$

C. Differentiation of Modified CBF

In this appendix, we provide the explicit forms of the derivative of CBF h defined in (16). For simplicity, the explicit dependencies on $(t,\mathbf{x})$ are omitted.

Using the definition of the modified CBF (16), we obtain the partial derivatives as

$$\begin{aligned}
\partial_x h &= \partial_x h_0 + \frac{1}{\mu}\sin(\theta - \theta_s)\, \partial_x \theta_s, \\
\partial_y h &= \partial_y h_0 + \frac{1}{\mu}\sin(\theta - \theta_s)\, \partial_y \theta_s, \\
\partial_\theta h &= -\frac{1}{\mu}\sin(\theta - \theta_s), \\
\partial_t h &= \frac{1}{\mu}\sin(\theta - \theta_s)\, \partial_t \theta_s.
\end{aligned}$$

Similarly, the partial derivatives of $\hat{v}_p$ from (20) are:

$$\partial_x \hat{v}_p = K_v \begin{bmatrix}-1 \\ 0\end{bmatrix},\; \partial_y \hat{v}_p = K_v \begin{bmatrix}0 \\ -1\end{bmatrix},\; \partial_t \hat{v}_p = K_v \begin{bmatrix}\dot{x}_d \\ \dot{y}_d\end{bmatrix}.$$

The derivatives of a and b defined in (21) are obtained as

$$\begin{aligned}
\partial_\xi a &= \nabla h_0^\top \partial_\xi \hat{v}_p + (\partial_\xi \nabla h_0)^\top \hat{v}_p + \alpha\, \partial_\xi h_0, \\
\partial_x b &= 2(\partial_x h_0\, \partial_{xx} h_0 + \partial_y h_0\, \partial_{xy} h_0), \\
\partial_y b &= 2(\partial_x h_0\, \partial_{xy} h_0 + \partial_y h_0\, \partial_{yy} h_0),
\end{aligned}$$

where ξ represents t,x,y. Note that, $\partial_t b \equiv 0$. Next, the derivatives of the smooth filtering term λ follow from the application of the chain rule:

$$\begin{aligned}
\lambda_a &= \frac{-1 + a/\sqrt{a^2 + \alpha_q b^2}}{2b}, \\
\lambda_b &= \frac{\alpha_q}{2\sqrt{a^2 + \alpha_q b^2}} + \frac{a - \sqrt{a^2 + \alpha_q b^2}}{2b^2}, \\
\partial_\xi \lambda &= \lambda_a\, \partial_\xi a + \lambda_b\, \partial_\xi b.
\end{aligned}$$

The derivatives of safe velocity $\hat{v}_s$, given in (21), for the single integrator dynamics are:

$$\begin{aligned}
\partial_\xi \hat{v}_s &= \partial_\xi \hat{v}_p + (\partial_\xi \lambda)\, \nabla h_0 + \lambda\, \mathbf{H}_0\, \partial_\xi \mathbf{r}, \\
\partial_x \mathbf{r} &= [1,0]^\top,\; \partial_y \mathbf{r} = [0,1]^\top,\; \partial_t \mathbf{r} = [0,0]^\top,
\end{aligned}$$

where $\mathbf{H}_0$ is the Hessian of h_0, and $\mathbf{r} \triangleq [x,y]^\top$ denotes the planar position vector. The partial derivatives of the heading angle θ_s defined in (22) are:

$$\begin{aligned}
\partial_{\hat{v}_{y,s}} \theta_s &= \frac{\hat{v}_{x,s}}{\hat{v}_{x,s}^2 + \hat{v}_{y,s}^2}, \quad \partial_{\hat{v}_{x,s}} \theta_s = \frac{-\hat{v}_{y,s}}{\hat{v}_{x,s}^2 + \hat{v}_{y,s}^2}, \\
\partial_\xi \theta_s &= \partial_{\hat{v}_{x,s}} \theta_s\, \partial_\xi \hat{v}_{s,x} + \partial_{\hat{v}_{y,s}} \theta_s\, \partial_\xi \hat{v}_{s,y}
\end{aligned}$$

Finally, Lie derivatives required for the R-CBF condition are obtained as

$$\begin{aligned}
L_{g_v} h &= \Big(\partial_x h_0 + a_0 \sin(\theta - \theta_s)\, \partial_x \theta_s\Big)\cos\theta \\
&\quad + \Big(\partial_y h_0 + a_0 \sin(\theta - \theta_s)\, \partial_y \theta_s\Big)\sin\theta, \\
L_{g_\omega} h &= -\, a_0 \sin(\theta - \theta_s),
\end{aligned}$$

where $L_{g_v} h$ and $L_{g_\omega} h$ denote the Lie derivatives along the linear and angular velocity input channels, respectively. Further details can be found on the code repository webpage[‡].