UNIVERSITY OF BIRMINGHAM

Game Theory
Module Overview

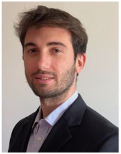
Dr Leonardo Stella

School of Computer Science

2024/2025 - Week 1

Module Team

Dr Jens Christian Claussen
Module Lead



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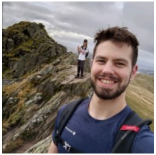

Mr Sam Robbins
Teaching Assistant

Outline

Outline

Module Overview: Module Description

Game theory provides a **mathematical framework** to describe **strategic or economic interactions** among agents, which can act (play) one out of a set of strategies. The core of a game-theoretic analysis are the payoffs (benefits minus costs) [...]. Evolutionary game theory transfers these concepts to biology where reproductive fitness is modelled by benefits and costs, and replicator equations enable analysis of the stability of equilibria as well as numerical investigations of non-equilibrium dynamics.

Game-theoretic concepts are widely transferred and applied to **social, economic or biological problems** to explain opinion formation and decision-making [...], as well as collective behaviour in biology.

In this module, we will study the foundations of economic as well as evolutionary game theory, consider examples of **real-world dilemma situations** (strategic, climate, vaccination) [...].

Module Overview: Learning Outcomes

1. Understand and explain the central concepts of non-cooperative, cooperative and evolutionary game theory including the minimax theorem, dominance, Nash equilibria, replicator equations and evolutionarily stable strategies
2. Understand, explain, and apply strategic and normal form game concepts to analyse real-world conflict situations and aid decision-making
3. Apply game-theoretic concepts in agent-based simulations in unstructured and structured populations, including spreading and decision-making on networks
4. Transfer game-theoretic concepts to model social, economic or biological problems and analyse the models through mathematical reasoning and computer simulations

Outline

Module Organisation: Contact Time

- Lectorials: theory and exercises (synchronous)
  - **Mondays 17:00-19:00 (Building: TLB, Room: TLB2)** – Two-hour lectorials, combining theory, in-class exercises and other group learning activities
- Lectures: theory and exercises (synchronous)
  - **Fridays 12:00-13:00 (Building: Y3, Room: G33)** – One-hour lecture, focussing mostly on theoretical concepts, with some in-class exercises
- Additional Resources and Office Hours (asynchronous/synchronous)
  - Additional materials will be provided on Canvas, including lecture notes and the recordings of all lectures. Also, lecturers and teaching assistants have dedicated office hours (see details on Canvas)

Module Organisation: Teaching Weeks

- The module runs for a total of 12 weeks. Here is a (tentative) plan of the second part of the module:
  - **Week 1** – Module Overview, Introduction

    ⋮

  - **Week 6** – Consolidation

    ⋮

  - **Week 9** – Evolutionary Dynamics
  - **Week 10** – Stochastic Games
  - **Week 11** – Mean-Field Games
  - **Week 12** – Revision

Outline

Evolutionary Tournament

- Let us start from the **Prisoner's Dilemma** game:

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Definition: A **payoff matrix** is a table that shows the possible outcomes of a strategic interaction between players in a game.

Evolutionary Tournament

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Activity. Consider the iterated PD game repeated n times. In pairs or small groups, design a strategy (pseudo-code) to maximise your payoff.

- Your strategy will be played against every other strategy
- Input: the whole history of interactions with each player
- Output: either *C*ooperate or *D*efect

Evolutionary Tournament

- Robert Axelrod designed a similar tournament in the '70s and '80s
- Participants from different backgrounds, e.g., economics, computer science, physics, etc., and different positions, including people from industry and academia, but also students
- All strategies were played against all other strategies, and three known strategies: **ALLC**, **ALLD**, **Random**
- The simplest strategy was the most successful, Tit for Tat:
    - **Generous**, starts with C
    - **Immediately retaliates**, D straight after a D
    - Enforces long-term cooperation

Evolutionary Tournament

	C	D
C	$3{+}\nu$, $3{+}\nu$	0, 5
D	5, 0	1, 1

Activity. Consider the above game repeated n times. In pairs or small groups, design a strategy in Python to maximise your payoff.

- Your strategy will be played against every other strategy
- Input: the whole history of interactions with each player
- Output: either *C*ooperate or *D*efect
- You know how much $\nu \in [-1, 3]$ is at time $t + 1$ if both players selected C at time t. How much does this impact your strategy?

Evolutionary Tournament

Assignment (formative). Develop your strategy as a Python function. Send your code to me via email to participate by 5pm next Mon.

- Your function must take only one argument as input. The first time it is called, the argument is the integer 2. Then, from the second iteration onwards, the argument is going to be an array containing past interactions as 0s or 1s
- The output of your function must be a value, either 0 or 1. The former represents D, the latter represents C
- The game will end after T iterations, which is a random number between 100 and 1000
- Your function will be played against all the functions of all other groups, plus **ALLC**, **ALLD** and **Random**

References

Axelrod, R. and Hamilton, W.D., 1981. The evolution of cooperation. *Science*, 211(4489), pp.1390-1396.

Thank you!