Game Theory

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- Joker's Social Experiment. Analysis.
- Prisoner's Dilemma.
  Which other games exist (group activity)
- Other games ...
- Matrix games
- Zero-Sum Games
- Extensive Form Games
- Real-world conflict example
- Evolutionary Dynamics: Replicator equation (the basic idea)

- Learning Outcomes
- Practical issues: 20% coursework (report-based)
  5(10) days possible welfare extension, waivable
  (the 100% resit exam may include related questions)
- Exercise Sheets (formative): do not count but valuable!

LH/LM Game Theory

- Rational Behaviour, Strategic Games. Nash Equilibrium. Extensive Form Games (aka Decision Trees)
- Payoff Matrices. Two-person games. Zero sum games. Minimax Theorem
- Mixed Strategies. Existence of Nash Equilibrium in symmetric games (Theorem)
- Dominance — dominant strategies
- Evolutionary Game Theory and Replicator Equations (Evolutionary Dynamics I) — evolutionarily stable strategies
- Spatial games and games on networks
- Evolutionary Dynamics in Finite Populations; Update Processes, Agent-based simulations
- Ultimatum Game, Climate Game, Opinion formation
- Evolutionary Dynamics II, Learning in Games, Mean-field games (Weeks 9-11)

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Game Theory - Learning Outcomes

On successful completion of the module, the student will be able to:

(LO1) Understand and explain the central concepts of non-cooperative, cooperative and evolutionary game theory including the minimax theorem, dominance, Nash equilibria, replicator equations and evolutionarily stable strategies

(LO2) Understand, explain, and apply strategic and normal form game concepts to analyse real-world conflict situations and aid decision-making

(LO3) Apply game-theoretic concepts in agent-based simulations in unstructured and structured populations, including spreading and decision-making on networks

(LO4) Transfer game-theoretic concepts to model social, economic or biological problems and analyse the models through mathematical reasoning and computer simulations

(LO5) Demonstrate an awareness of the current literature in this area

LH/LM Game Theory - Prerequisites? (not formal)

- Linear Algebra (matrix-vector mult; transpose; eigenvalues
- Probability (expected values)
- Boolean Logic, logical reasoning, basic proof techniques
- Calculus (product + chain rule of differentication)
  Appreciation that for a differential equation $(dx/dt) = F(x)$ implies that solving $F(x)=0$ provides points x for which $(dx/dt)=0$, also called fixed points.

  (we will also analyze stability, easy technical concept)
- Where we try to deal with real-world situations:
  Respect of each other (diversity of opinions, backgrounds, gender indentities)

  We need sometimes to simplify, m+f (=96%)
  Not everyone is native speaker; showing tolerance & discussing in private may facilitate understanding
- Humor *(Thanks for a former colleague for pointing this out)*
- Curiosity
- Game Theory is a typical "minimal models" approach (Occam's Razor, Einstein: Keep it Simple)

Joker's Social Experiment (movie: "Dark Knight")

- If they both chose C (coperate, stay silent until the deadline) both ships will be blown up, no one survives, hence payoff 0.
- If one chooses D (detonate), but the opponent chooses C, the player would survive (payoff 1)
- Convention: Focal player on the left (rows), Opponent on the top (rows)

opponent

focal player

	Cooperate	Detonate
Cooperate	0 0	0 1
Detonate	1 0	0 0

(0,0) (0,1)
(1,0) (0,0)

- Convention: Payoffs are written as (Focal, Opponent)

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	Cooperate	Detonate
Cooperate	$0, 0$	0,1
Detonate	$1, 0$	$0, 0$

- Convention: Payoffs are written as (Focal, Opponent)

Joker's Social Experiment (movie: "Dark Knight")

- If they both chose C (coperate, stay silent until the deadline) both ships will be blown up with probability 0.9 , hence payoff 0.1

	Cooperate	Detonate
Cooperate	$0.1, 0.1$	$0,1$
Detonate	$1, 0$	$0, 0$

- Alternative model (more complicated) has been discussed (Hummert, Schuster, Hummert 2010)

Activity

Activity: What games (related to game theory) have you heard of? Discuss in a group of 3-8 and write your suggestions on one sheet of paper

The Stag Hunt

- A collection of hunters are hunting for food; either a stag or a hare.
- They individually prefer a share of a stag, but a hare is also acceptable (they would share the stag but whoever catches a hare keeps the hare to themselves).

So, each of the hunters individually decides on one of two courses of action: she may remain focussed on the task of working together to catch a stag, or, she may catch a hare. If all hunters pursue the stag, they will with certainty catch it and share the stag equally. If any hunter diverts her energy to catching a hare, which is much easier, she will keep the hare but the stag will escape and the rest of the group will not eat anything.

	Stag	Hare
Stag	$2, 2$	$0, 1$
Hare	$1, 0$	$1, 1$

This gives one version of this game in a strategic form payoff matrix where there are just two hunters.

The Stag Hunt

	Stag	Hare
Stag	$2, 2$	$0, 1$
Hare	$1, 0$	$1, 1$

This gives one version of this game in a strategic form payoff matrix where there are just two hunters.
There is a benefit (payoff) of zero if a hunter ends up with no food, a benefit of "1" if the hunter catches a hare and a benefit of "2" for a share in a stag. We will see later that the two equilibrium solutions are `[Hare, Hare]` and `[Stag, Stag]`. (ie if one hunter remains focussed on the task of stag-catching, the other hunter prefers to remain focussed, and similarly if one hunter prefers to hunt for hare, then so does the other hunter.)

Stag Hunt

Prisoner's Dilemma (PD)

	C	D
C	$2,2$	$0,3$
D	$3,0$	$1,1$

Stag Hunt: results in PD with -2 punishment for defection

	Stag	Hare
Stag	$2,2$	$0,1$
Hare	$1,0$	$1,1$

Activity

Activity: Pick four random (but pairwise unequal) integer numbers (out of 1, ... 9) and fill them randomly into the 4 entries of a 2×2 matrix.

Generate 3 such matrices (per person)

Analyze the preference relations in each column (does player 1 or player 2 have higher payoff)?

How many cases of (1,1), (2,1) (1,2) and (2,2) did you find?

Collate your results in a group of 3-8. Conclusions?

Prisoner's Dilemma

In the Prisoner's Dilemma:

- Mutual cooperation rewards by 3 remitted years of prison
- If one defects and the opponent cooperates, it is 5 years, but none for the opponent who is considered guilty
- If both defect, they are awarded by 1 remitted year, as they help the police.

We can represent this strategic game as:

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

Battle of the Sexes (the economic BOTS)

In the Prisoner's Dilemma, the issue was whether the prisoners would choose the 'best' option of remaining Quiet. In the Battle of the Sexes (BoS) example, the players agree that it is better to cooperate, but they disagree about the best outcome for each of them.

Two people who really like each other (eg boyfriend/girlfriend) wish to go out together to an event.

- Bob would like to go to a football match.
- Sally likes football, but prefers going to the cinema.
- Bob doesnt mind going to the cinema.
- If they go to separate events by themselves, they are both equally very unhappy as they are alone.

So the ‘quality-of-life’ payoff for Bob would be ‘2’ if he can go to the football with Sally, and ‘1’ if he goes to the cinema with her.

But if Bob goes to the football whilst Sally chooses the cinema he has a payoff of zero.

Similarly, Sally has a payoff of ‘2’ if she can go to the cinema with Bob, a payoff of ‘1’ if she goes to the football with him, but zero if she goes to the cinemas without him (and zero if she goes to the football and he goes to the cinema).

We can represent this strategic game as:

	Football	Cinema
Football	$2, 1$	$0, 0$
Cinema	$0, 0$	$1, 2$

Solution???

We will see later ($\rightarrow$ **Nash equilibria**), that this game has 2 stable choices in which *each player would reduce their payoff by changes their choices.*
In this case both players prefer (*Football*, *Football*) or (*Cinema*, *Cinema*) equally, although their individual preferences differ.

The previous games had at least some element of cooperation between the players. **Matching Pennies** is a game that is purely about conflict.
Two people each have a coin and choose, simultaneously, whether to show the Head or the Tail of their coin:

- If they show the same side then Player 1 gets $\$1$ from Player 2.
- If they are different then Player 1 pays player 2 $\$1$.

So one player can only gain at the expense of the other player.

	Heads	Tails
Heads	$1, -1$	$-1, 1$
Tails	$-1, 1$	$1, -1$

This is an example of a 'zero sum' game.

(but it is played between "match wins" vs "non-match wins" players which makes it an asymmetric conflict (bimatrix game)

Extensive Form Games: Decision trees

Assume the game would be played sequentially

(one player spotting what the other player will do).

Then the game can be represented by a decision tree.

Matching Pennies:

Extensive Form Games: Decision trees

Rock Paper Scissors (yanken)

Extensive Form Games: Entry game (economics)

Chicken

Snowdrift game

Hawk-Dove game

Hawk-Dove game

How many different games (2 players, 2 strategies) c

A Real-World Example (including a 3rd player)

Take-home

- Analyzing a dilemma / conflict (text / narrative)
- Set of preference relations (payoffs as cardinal placeholders)
- Payoffs can be derived from an economic benefit - cost analysis *(to some extent, payoffs could be estimated from experiments)*
- Normal form games (matrix games)
- Extensive form games (decision trees)
- Zero-sum games. Bimatrix games. Cyclic games.

To be continued:

- Two-player games and methods to analyze.
- Central theorems (e.g. minimax theorem, dominance theorem)
- Nash equilibira (static analysis of a strategic situation) vs.
- evolutionarily stable strategy: stable FP of the replicator eq.
- Spatial Games and Games on Networks
- Opinion Formation, Public goods games, Climate Game
- Learning in Games (wk 9-11)