Hwk 04: ECON 608

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Q1: Simple Options Pricing

1. The 6-month futures price of oil $F_t(t, T) = \$30$ today where $t = 0$ and $T = 6/12$. There are two states of the world: $s \in \{d, u\}$. The spot price of oil $S(T, s)$ in six months will be \$20 if $s = d$ or \$50 if $s = u$. The annual interest rate is $r = 5\%$, known to be constant. What is the price of a European call option on $S(T)$ with a strike price of $K = \$40$?

```
#in 6 months payoffs are
P1<-max(50-40,0)
P2<-max(20-40,0)

delta<-(P1-P2)/(50-20)
#replicating portfolio formula
#delta*strike-PV(lower_spotprice*delta)
p_call<-delta*40-20*delta/1.025

cat(paste("the call price is", p_call, "\n"))
```

```
## the call price is 6.82926829268293
```

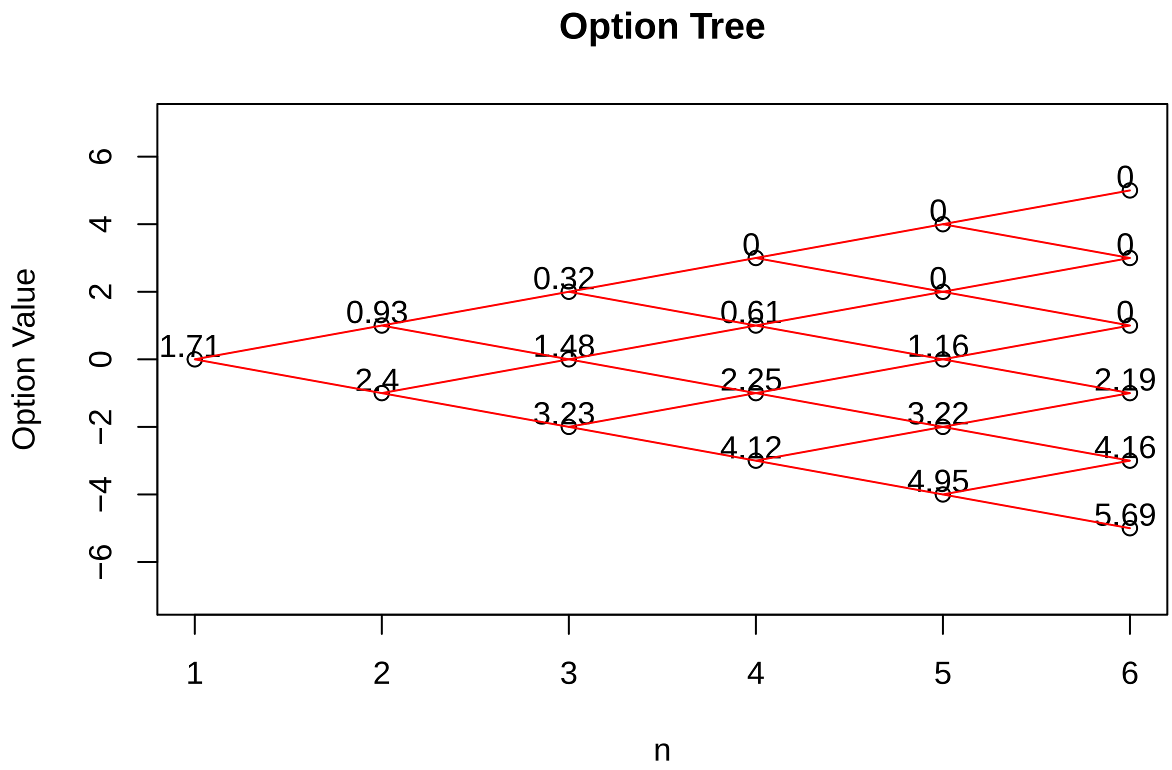
Q2: Binomial Trees

2. Use the `BinomialTreeOption` and `BinomialTreePlot` from the package `fOptions` to price a 6-month European put option on a futures contract. Assume the current price $S = 10$, the strike price is $X = 11$, the annual interest rate is $r = 0.04$, and the annual volatility is $\sigma = 40\%$. Price this option using a binomial option pricing tree with $n = 5$ nodes and plot the tree.
Where does the option price appear in the tree?
Answer: The price appears in the first node.

```
# options functions
library(fOptions)

binomOpt <- BinomialTreeOption(TypeFlag="pe", S=10, X=11, Time=6/12, r=0.04, b=0, sigma=0.4, n=5)

cat(paste("the option price is", round(binomOpt[1,1], 3), "\n"))
```

```
## the option price is 1.706
```

```r
BinomialTreePlot(binomOpt, xlab = "n", ylab = "Option Value", main="Option Tree",  ylim = c(-7, 7))
```

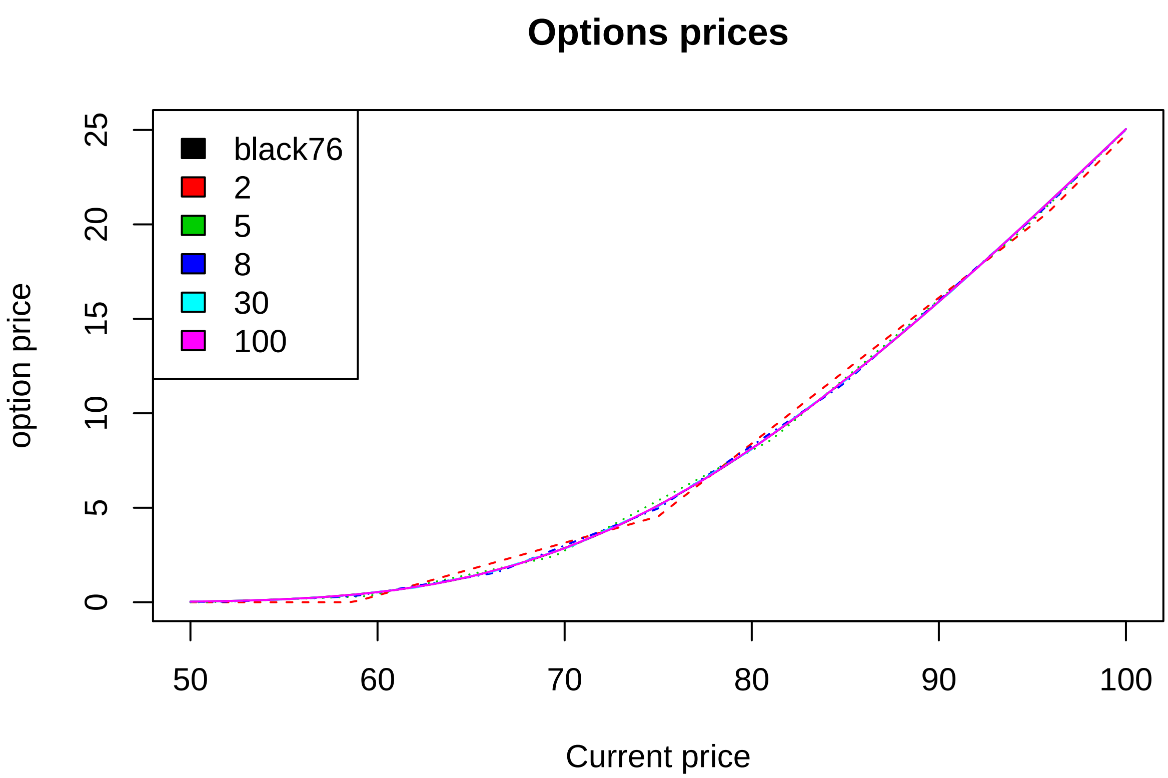


Q3: Binomial Tree & Numerical Error

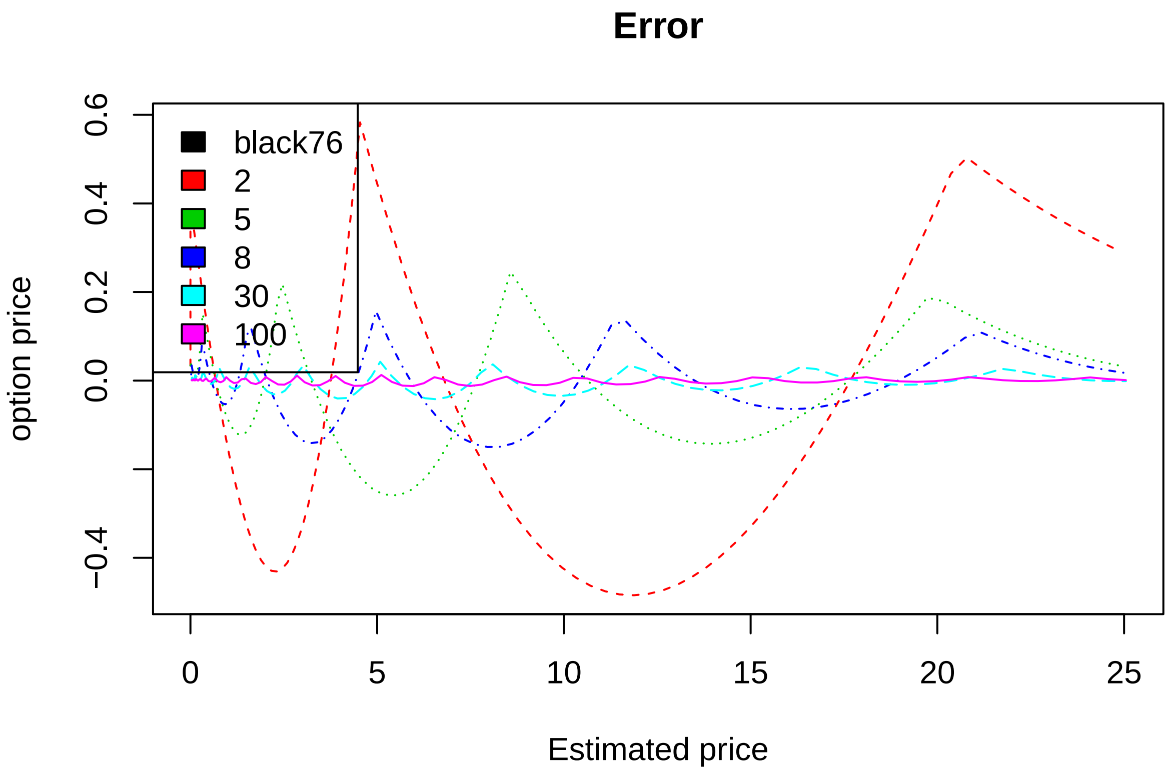
Compute the value of a three-month call option with strike price of $X = \$75$ assuming that $F_0(T) = 50, 50.5, 51, \ldots, 100$. Compute the price of the option using both the Black '76 formula as well as a binomial pricing tree with $M = 2, 5, 8, 30, 100$ grid-points. Assume the annual interest rate is 3%. Assume the annualized volatility is 30%. **Make two plots.** In the first, plot the error of the binomial price and the exact price using Black 1976 where the x-axis is $F_0(T)$. In the second, plot the calculated price of the option on the y-axis using both the binomial prices and exact price. Make sure your plot is color-coded. **What do you notice about the error as M increases?**
Answer: the error decreases so that in the limit it equals the black 76.

```r
   library(fOptions)
M = c(2, 5, 8, 30, 100)
F0 = seq(from=50, to=100, by=0.5)
X = 75
r = 0.03
Time = 4/12
sigma = 0.3

prices <- data.frame(matrix(NA, nrow=length(F0), ncol=length(M) + 1))
errors <- data.frame(matrix(NA, nrow=length(F0), ncol=length(M)))

# name the columns of the data frame
names(prices) <- c("black76", M)
names(errors) <- M
```

```r
# outer loop goes down each row for each diferent F0
for (i in 1:length(F0)){

  # Black '76 price
  prices[i, 1] <- Black76Option(TypeFlag="c", FT=F0[i], X=X, Time=Time, r=r, sigma=sigma)@price

  # inner loop computes binomial trees for different numbers of nodes
  for (j in 1:length(M)) {
    prices[i, 1 + j] <- CRRBinomialTreeOption(TypeFlag="ce", S=F0[i], X=X, Time=Time,
                                              r=r, b=0, sigma=sigma, n=M[j])@price
    errors[i, 1 + j] <- prices[i, 1] - prices[i, 1 + j]
  }
}

# plot prices
matplot(F0, prices, type="l", xlab="Current price", ylab="option price", main="Options prices")
legend(x="topleft", legend=names(prices), fill=1:ncol(prices))
```




```r
# plot errors
matplot(prices, errors, type="l", xlab="Estimated price", ylab="option price", main="Error")
legend(x="topleft", legend=names(prices), fill=1:ncol(errors))
```



Q4: Computing Futures Options Prices from Real Data

1. Download the crude-oil near month futures price `CHRIS/CME_CL1` from January 1, 1990 through January 31, 2019. Compute the standard deviation of log returns for four different intervals: 1990–2019, 1990–2004, 2004–2009, Jan 2009–June 2014, and July 2014–Jan 2019.

```
# Clear workspace
rm(list=ls())

library(Quandl)
library(xts)

Quandl.api_key("PFrF21zhdga11c2f1BBF")

# this makes up random returns. You'll need to get your own data
type <- "xts"
start <- "1990-01-01"
end <- "2019-01-31"
CL1 <- Quandl("CHRIS/CME_CL1", type=type, start_date=start, end_date=end)

# make returns
CL1$returns <- diff(log(na.omit(CL1$Last)))

# ------- Make a results data-frame to hold F, K, etc ----------

# Periods for to compute Vol
periods <- c( "1990/2019", "1990/2004","2004/2009", "2009-01/2014-06", "2014-07/2019-01")

# for how many samples do we compute the value of a call?
```

```r
nper <- length(periods)

# make data frame to hold our table of results, where row names are the periods
results <- data.frame(matrix(NA, nrow=nper, ncol=6), row.names=periods)

# name the columns of the data frame
names(results) <- c("F0","sigma","K","exactPrice","binomPrice","error")

# Compute SD
for (i in 1:nper ) {

  # pull out a subset of c1$Settle
  x <- CL1[periods[i],]

  # compute sigma and store in results
  # NOTE: We have to scale up to an annual frequency!!
  results[i, "sigma"] <- sd(x[,"returns"], na.rm=TRUE ) * sqrt(252)

  # get the first settlement price and store as F
  results[i, "F0"]   <- x[1,"Last"]
}

print(results)
```

```
##                       F0     sigma  K exactPrice binomPrice error
## 1990/2019          22.89 0.3865364 NA         NA         NA    NA
## 1990/2004          22.89 0.3914292 NA         NA         NA    NA
## 2004/2009          33.78 0.4503732 NA         NA         NA    NA
## 2009-01/2014-06    46.34 0.3314389 NA         NA         NA    NA
## 2014-07/2019-01   105.30 0.3778806 NA         NA         NA    NA
```

2. For the five estimates of volatility of crude oil, compute the value of a one month European **put** option on the first-month crude oil contract. Assume the strike-price is equal to the current price rounded to the nearest dollar (at-the-money). In your computation, use both the binomial tree and the Black 1976 formulas with a time-grid of 30 points. Assume an annual interest rate of $r = 5\%$.

```r
for (i in 1:nper ) {

  F0 <- results[i, "F0"]
  K <- round( F0 )
  sigma <- results[i, "sigma"]
  Time = 1/12
  r = 0.05

  black76 <- Black76Option(TypeFlag="p", FT=F0, X=K, Time=Time, r=r, sigma=sigma)

  binom <- CRRBinomialTreeOption(TypeFlag="pe", S=F0, X=K, Time=Time, r=r, b=0, sigma=sigma, n=30)

  # since black76 & binom are S4 objects, we extract elements with "@" instead of "$"
  results[i, c("K","exactPrice","binomPrice")] <- c(K, black76@price, binom@price)
  results[i, "error"] <- black76@price - binom@price
}
```

3. Make a table of your results. It should list the time period, the standard deviation of returns, the forward and strike prices you assumed, and the binomial and Black 1976 futures options prices, and the

numerical error out to 4 or more decimal places.

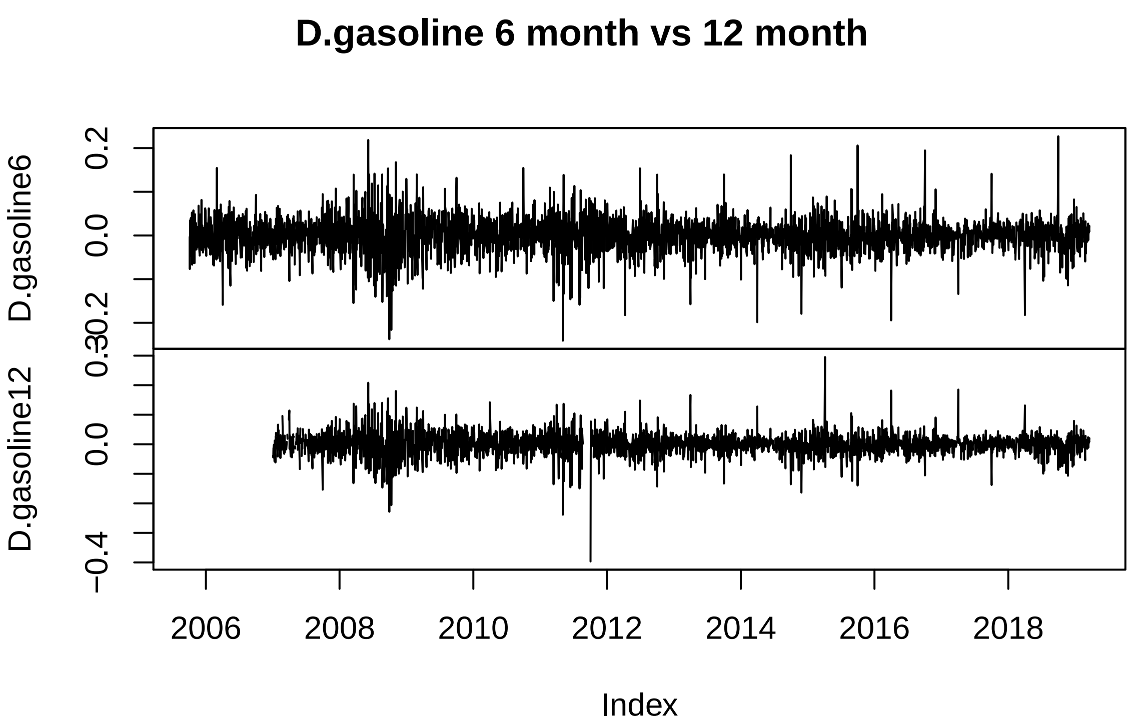
```r
print(results)
```

```
##                        F0     sigma   K exactPrice binomPrice         error
## 1990/2019           22.89 0.3865364  23   1.072341   1.070867  0.0014741664
## 1990/2004           22.89 0.3914292  23   1.085184   1.083615  0.0015694397
## 2004/2009           33.78 0.4503732  34   1.860957   1.860056  0.0009018282
## 2009-01/2014-06     46.34 0.3314389  46   1.590211   1.594895 -0.0046837904
## 2014-07/2019-01    105.30 0.3778806 105   4.406878   4.389066  0.0178123282
```

Q5: Hedge Ratios

Amazon has begun delivering its own packages. This has increased the firm's exposure to gasoline prices. As a risk manager there, you would like to purchase a 12-month hedge on gasoline; however, since the market is not very liquid, you can only use the 6-month futures. Download data on Gulf Coast gasoline for contracts 6 and 12: (`CHRIS/CME_RB6` or `CHRIS/CME_RB12`). **Estimate the optimal hedge ratio using the entire sample available, 2008–2010, and 2010–2017. Do your estimates change?**

```r
gasoline6 <- Quandl("CHRIS/CME_RB6", type="xts", start="2005-10-03")
gasoline12 <- Quandl("CHRIS/CME_RB12", type="xts", start="2007-01-01")

dgasoline6 <- diff(gasoline6$Settle)
dgasoline12 <- diff(gasoline12$Settle)

data <- cbind(dgasoline6, dgasoline12)
names(data) <- c("D.gasoline6", "D.gasoline12")
plot.zoo(data, main="D.gasoline 6 month vs 12 month")
```




```r
for (period in c("2008/2010", "2010/2017")){
  lmresults <- lm(D.gasoline6 ~ D.gasoline12, data=data[period])
  print(coefficients(lmresults))
```

```r
  cat(paste("The optimal hedge ratio over", period, "is", coefficients(lmresults)[2], "\n\n"))
}
```

```
##   (Intercept)  D.gasoline12
## -0.0001997784  0.9184692666
## The optimal hedge ratio over 2008/2010 is 0.918469266622247
##
##  (Intercept) D.gasoline12
## 0.0001764813 0.6705633134
## The optimal hedge ratio over 2010/2017 is 0.670563313368393
```