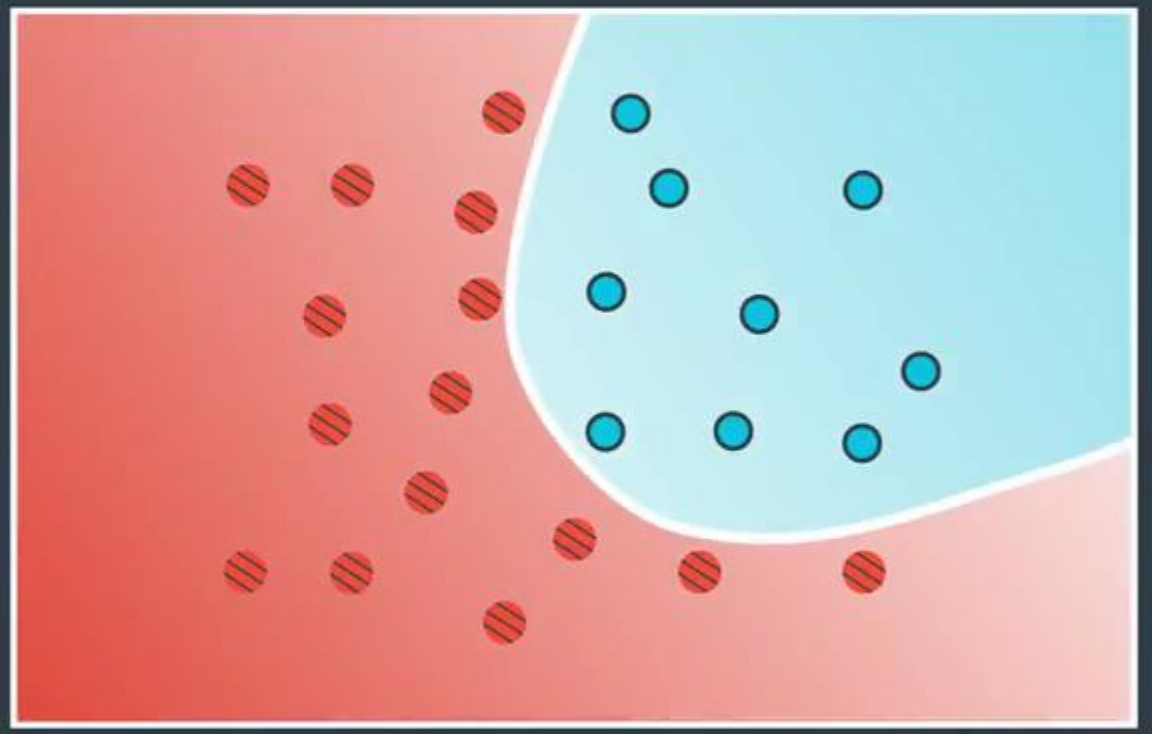
Neural Network

by Hassan Badawy

Acceptance at a University

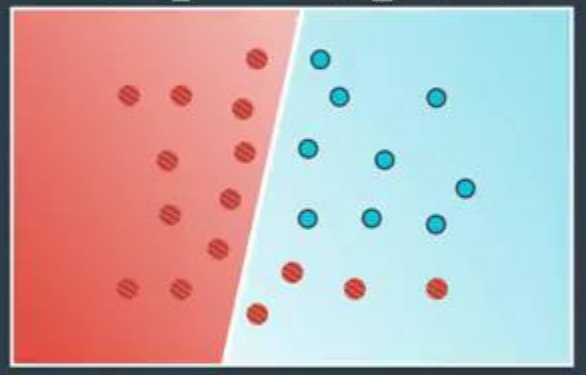
Acceptance at a University

Non-Linear Regions

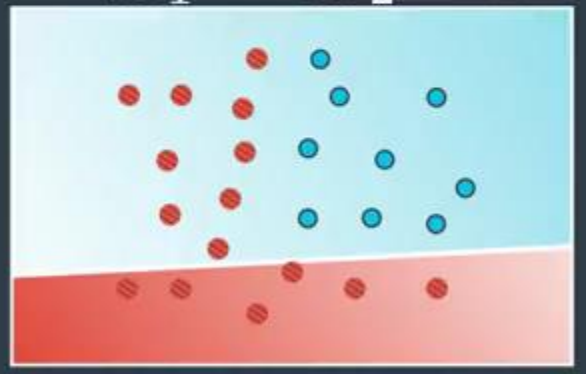


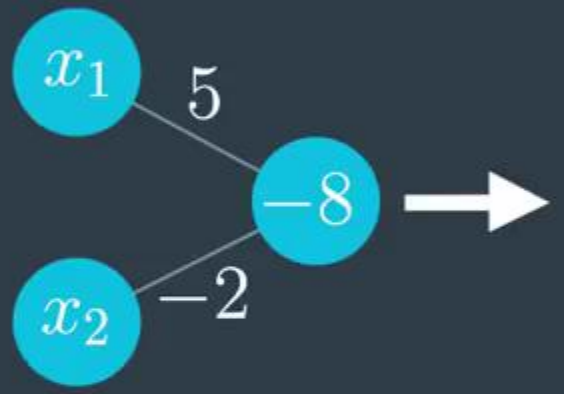
Neural Network

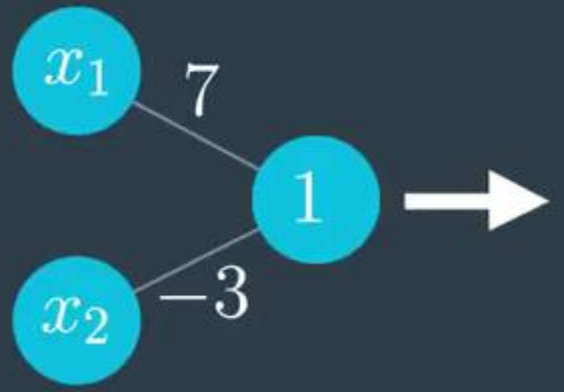
$5x_1 - 2x_2 + 8$



$7x_1 - 3x_2 - 1$







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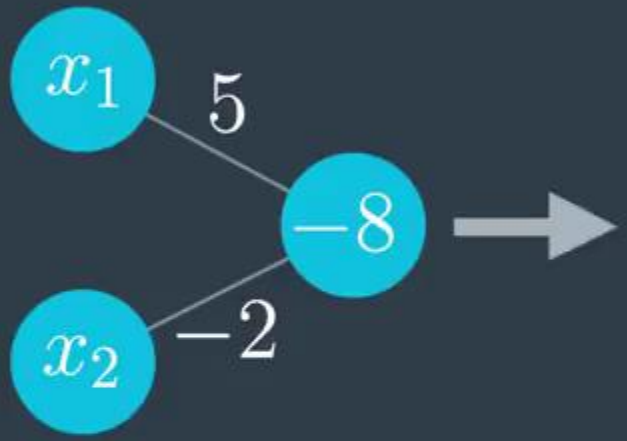

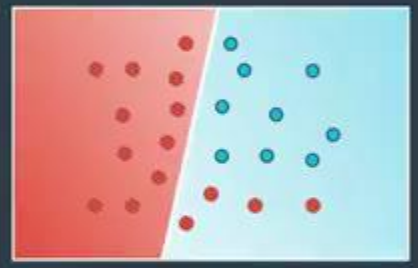

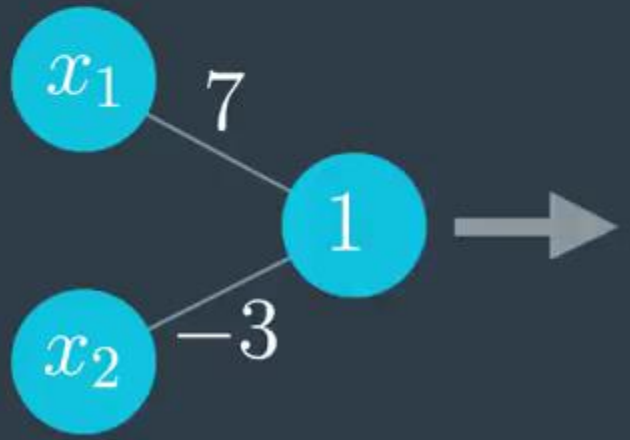

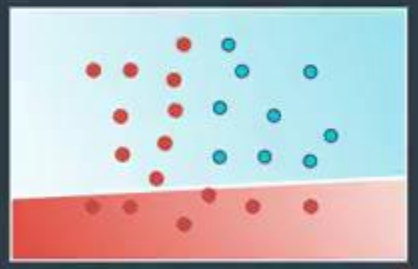

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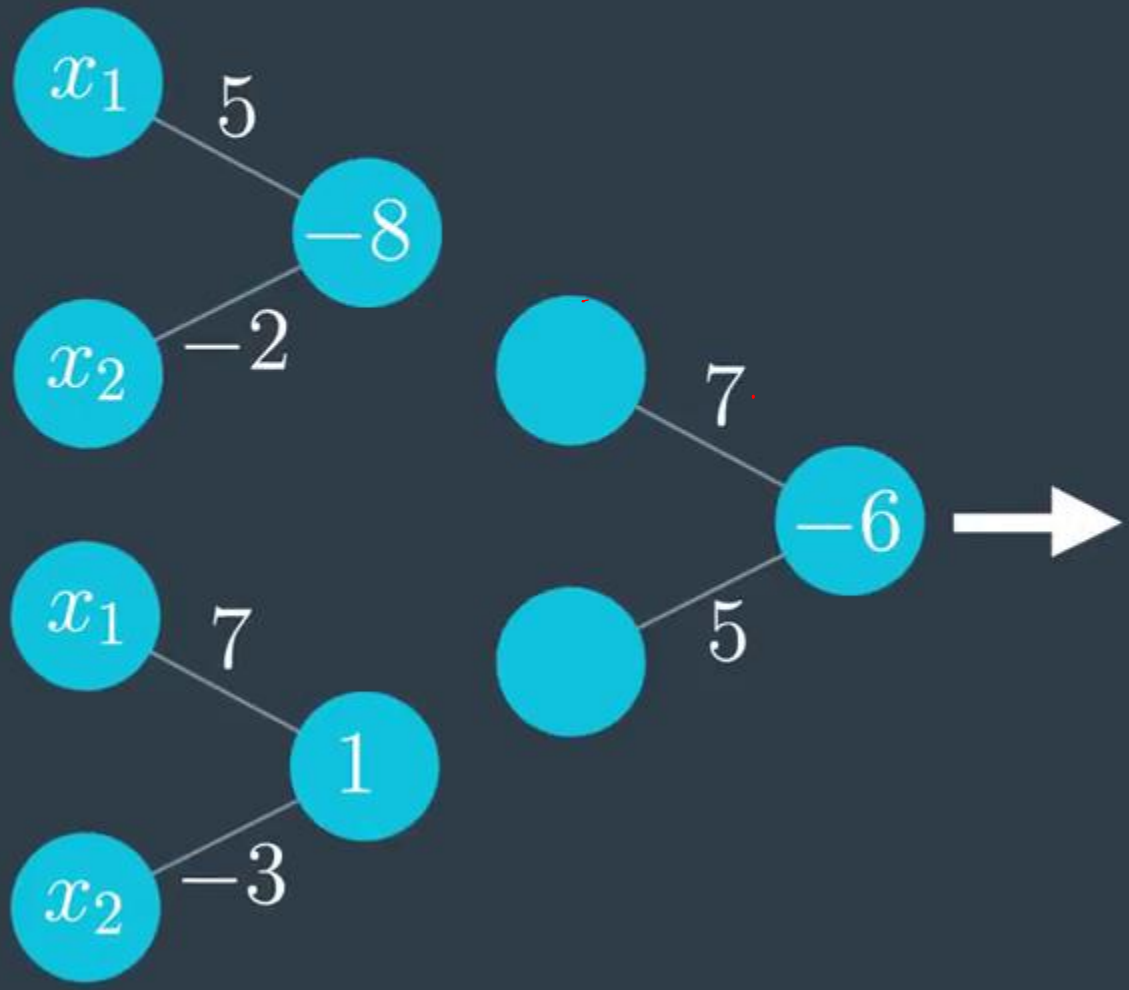

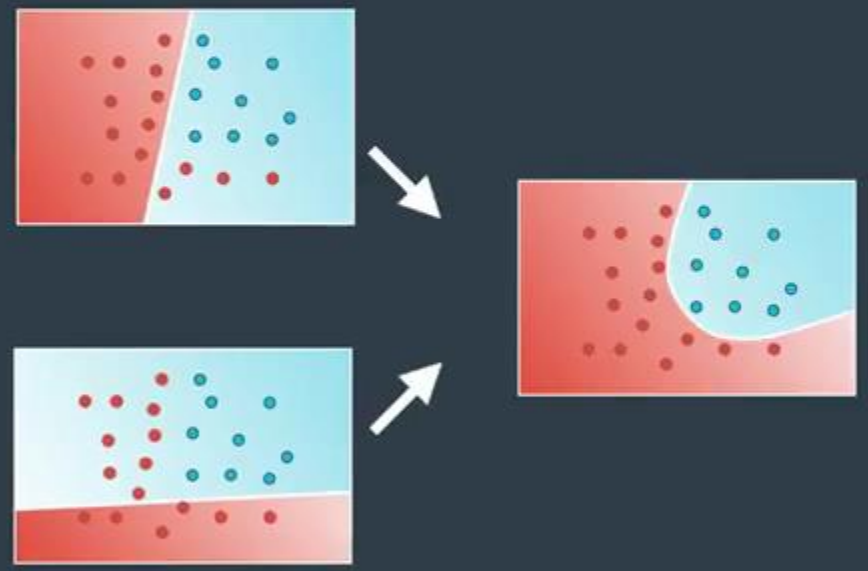

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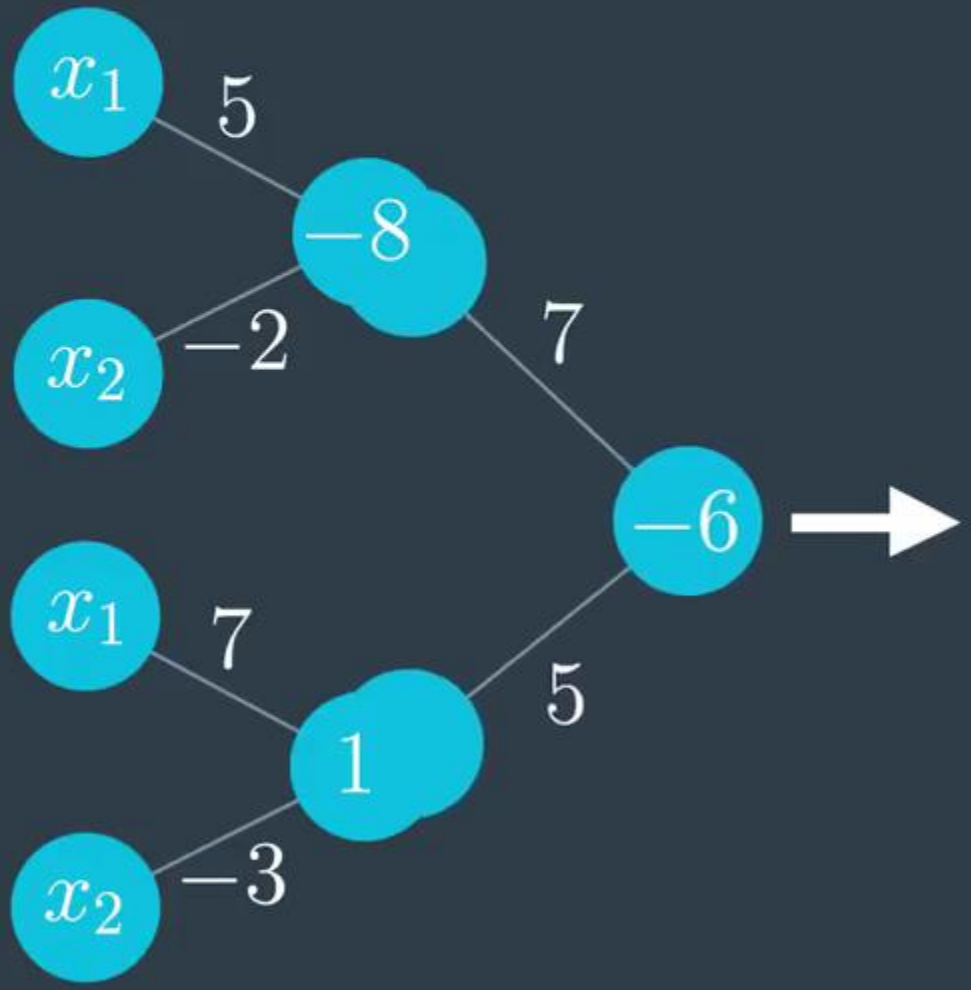

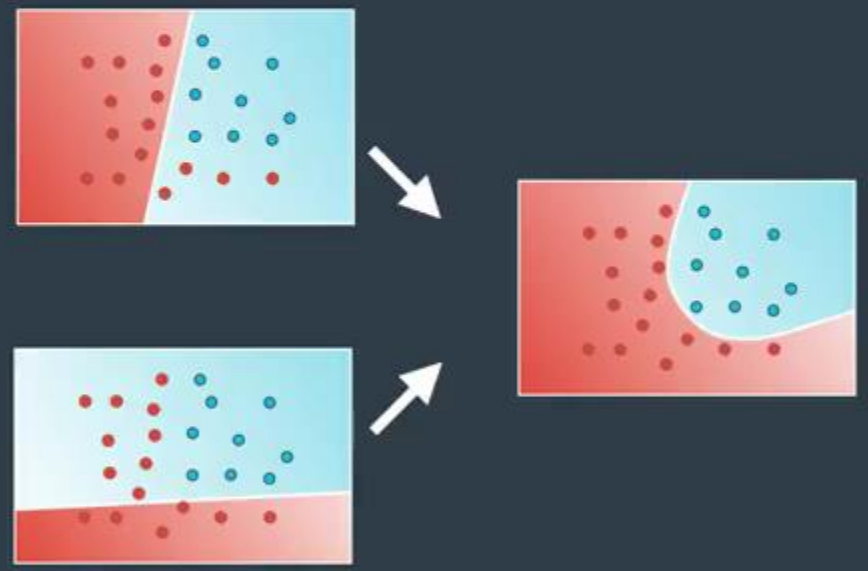

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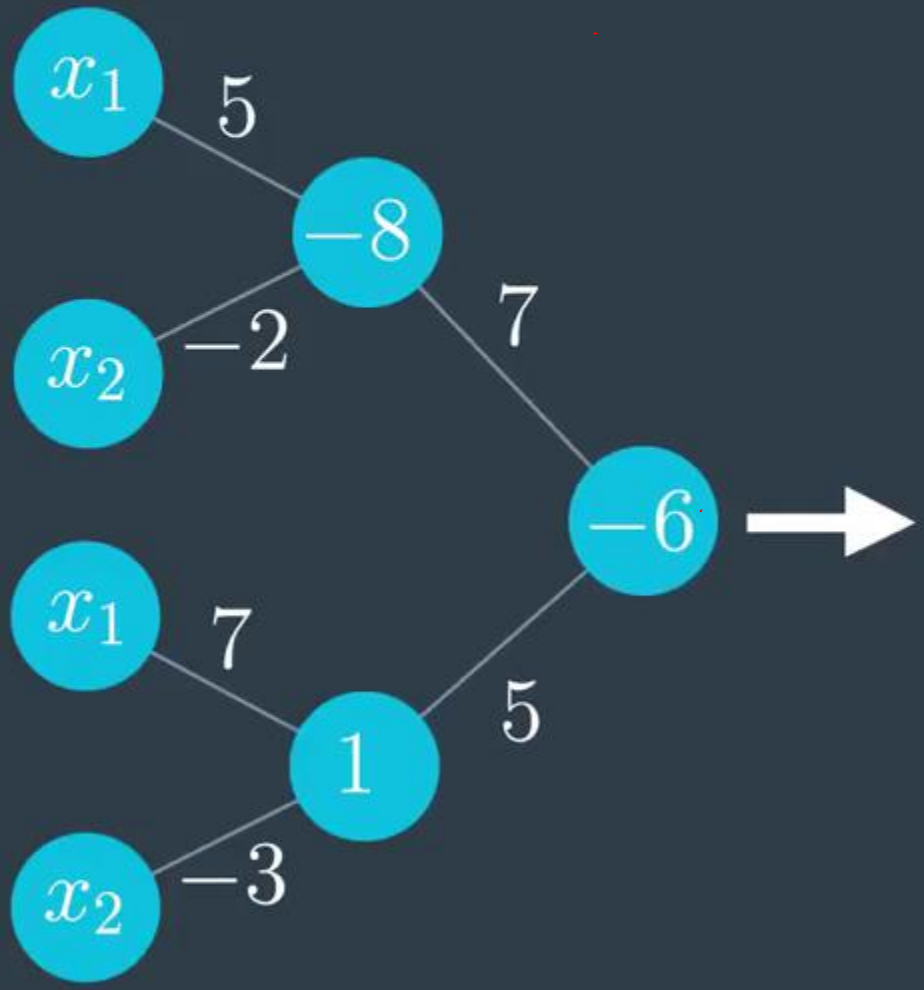

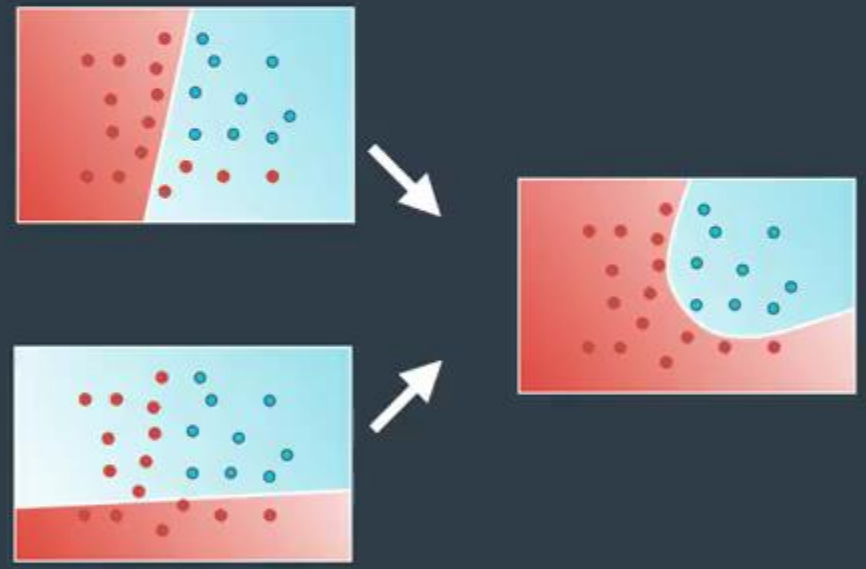

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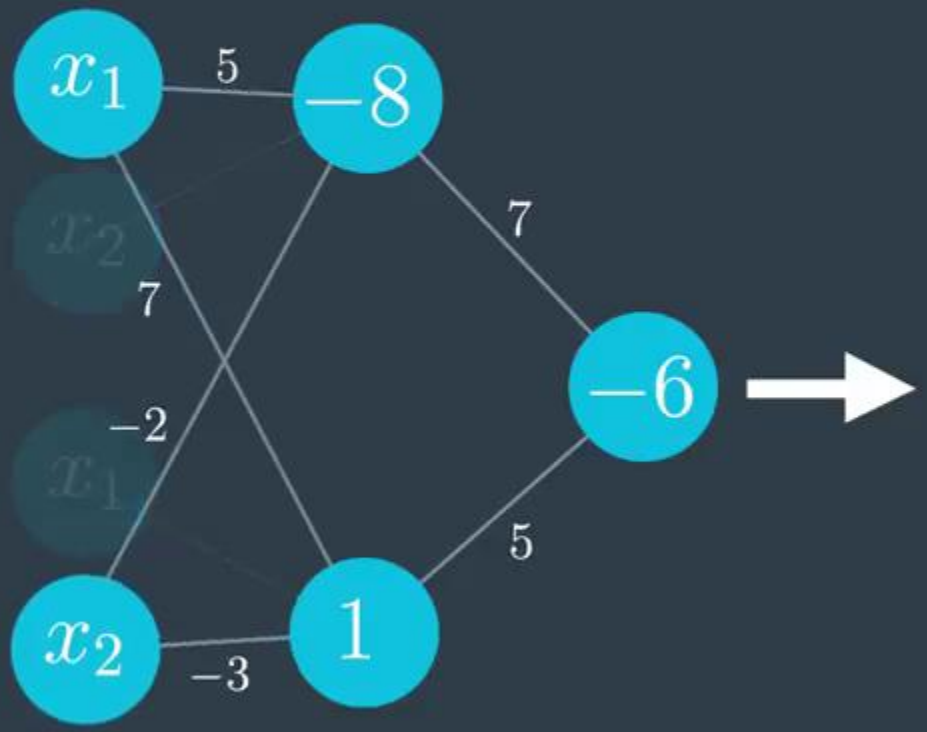

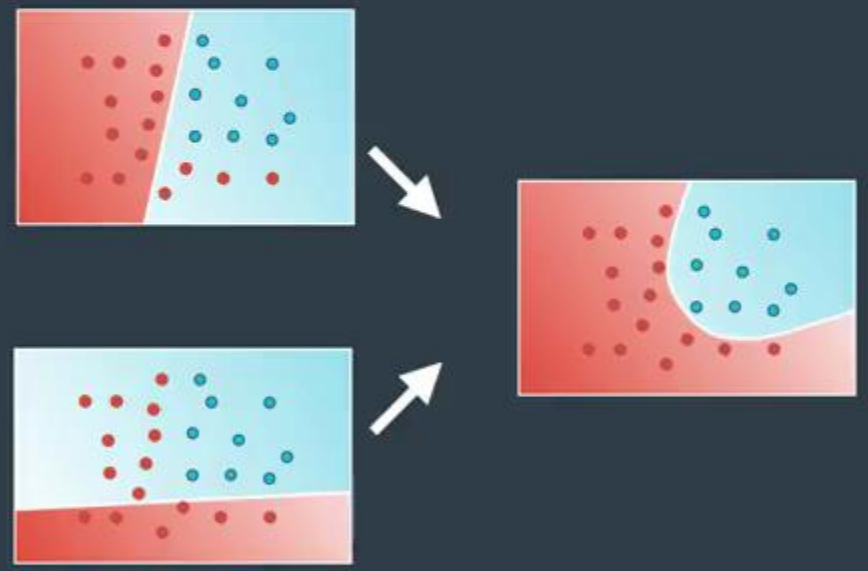

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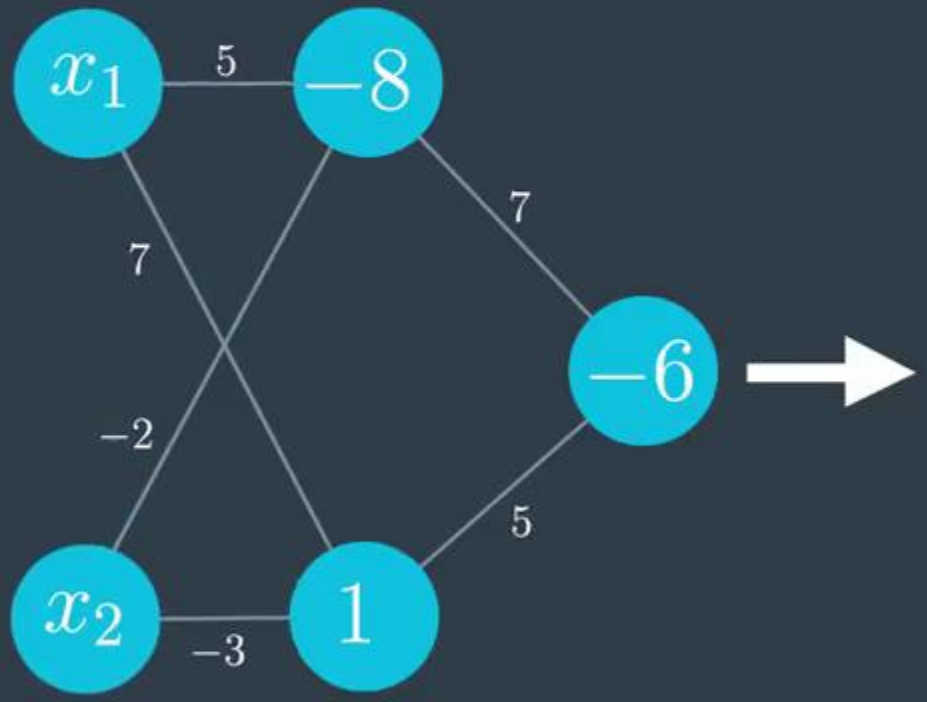

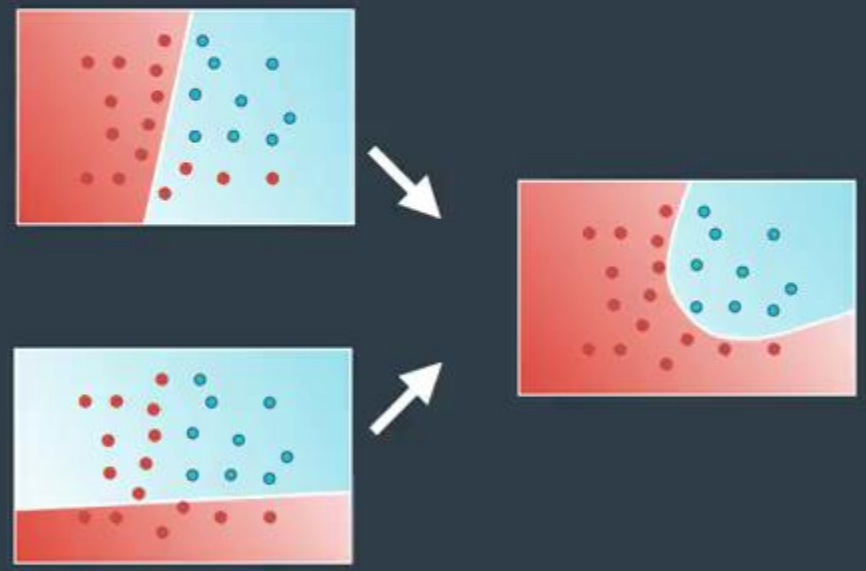

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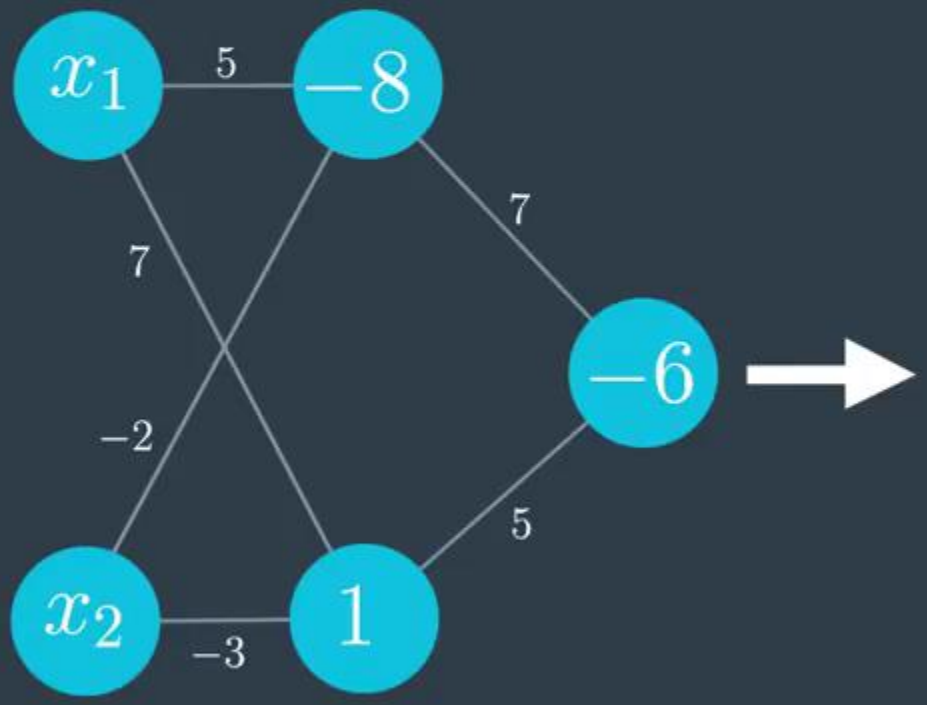

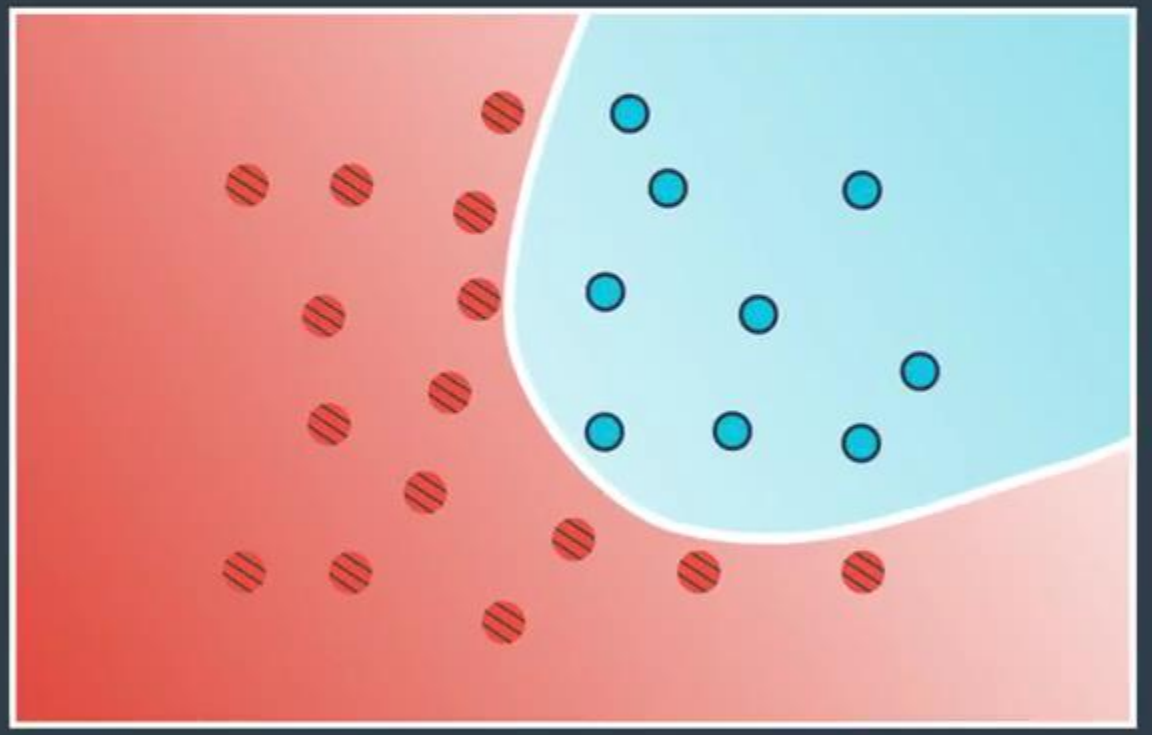

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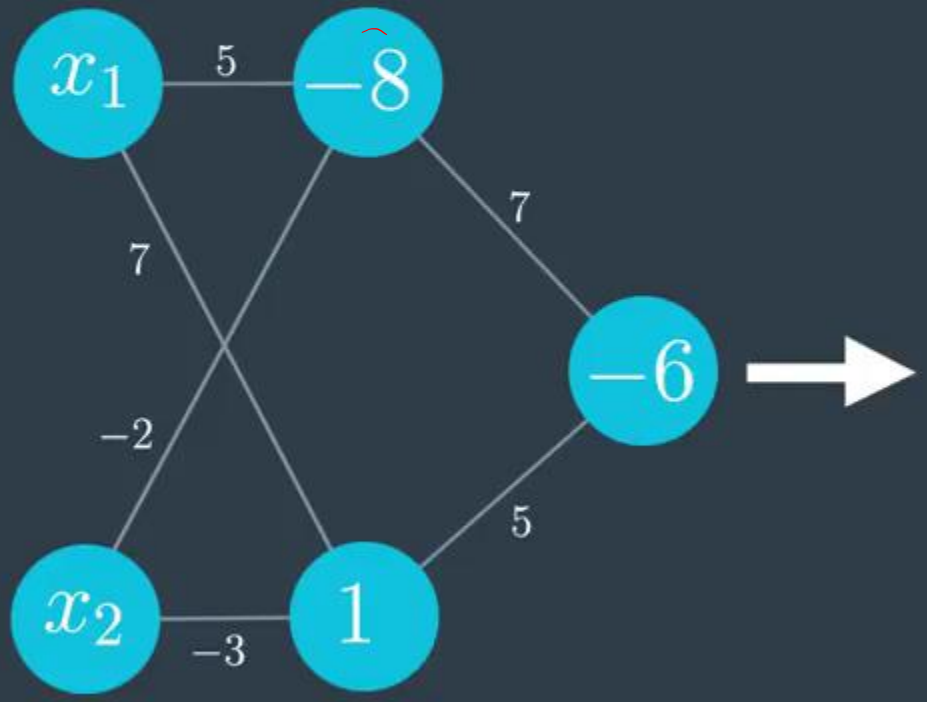
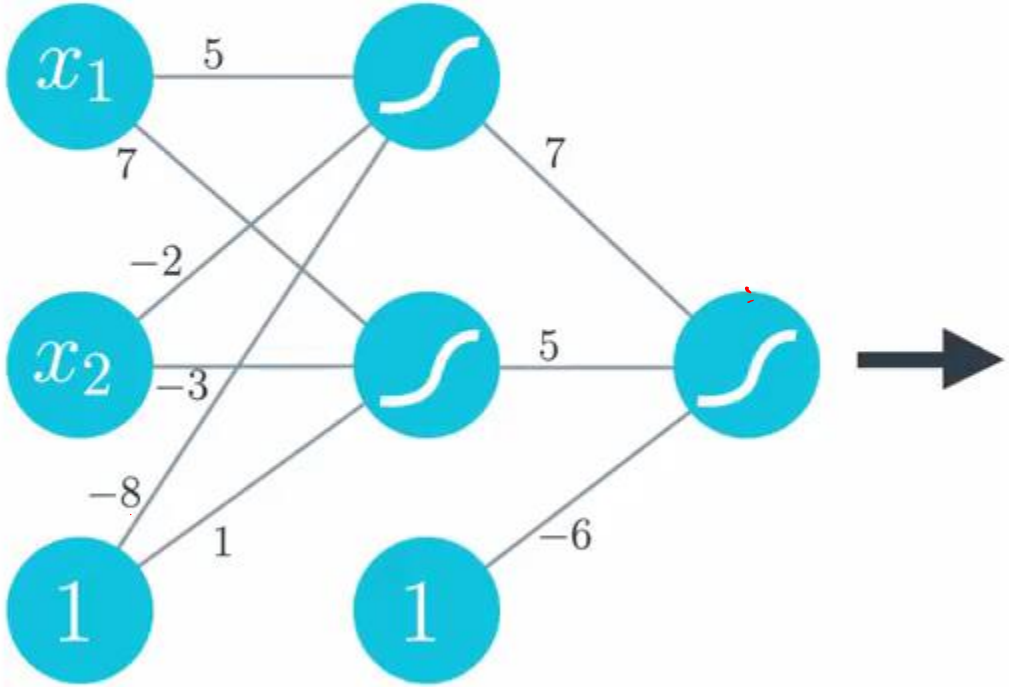
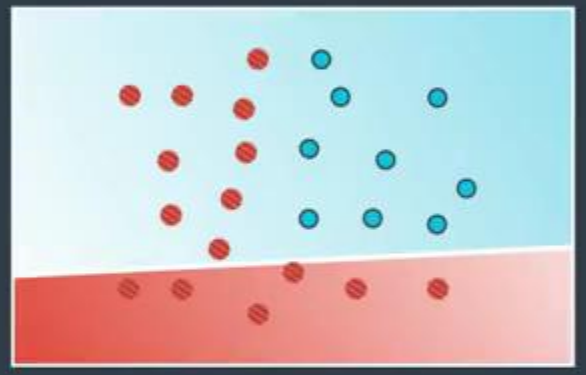
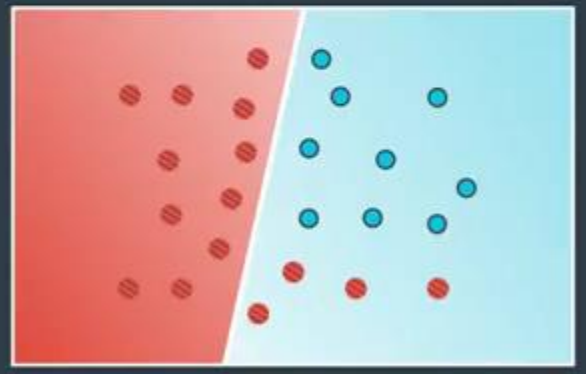
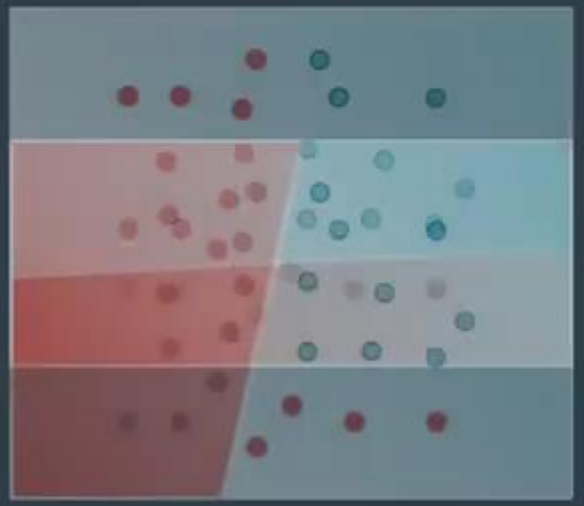




Combining Regions







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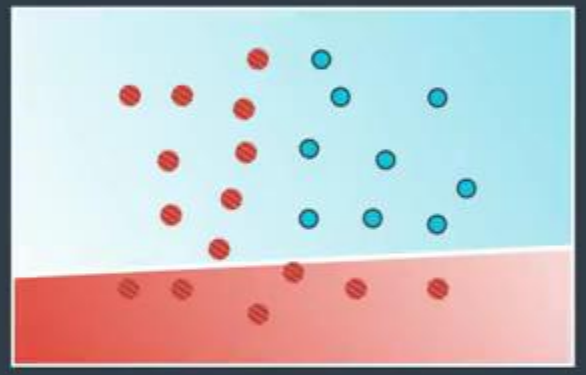

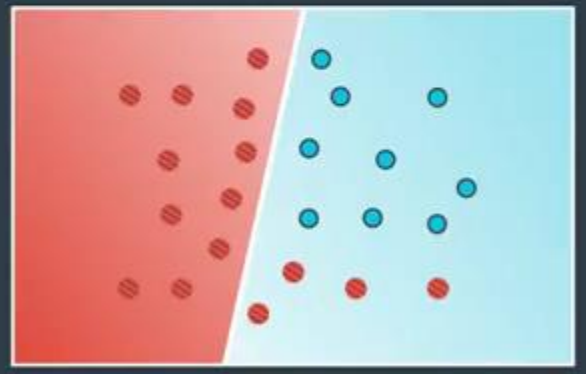

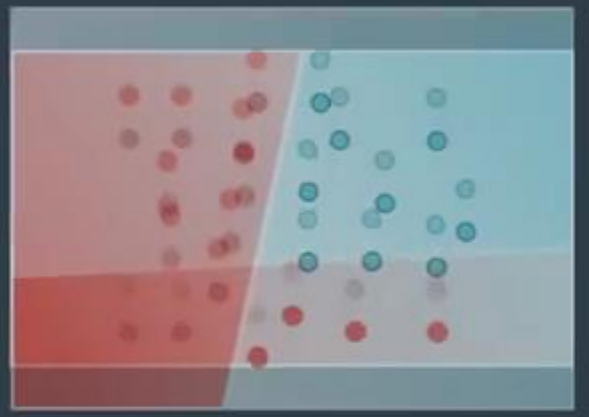

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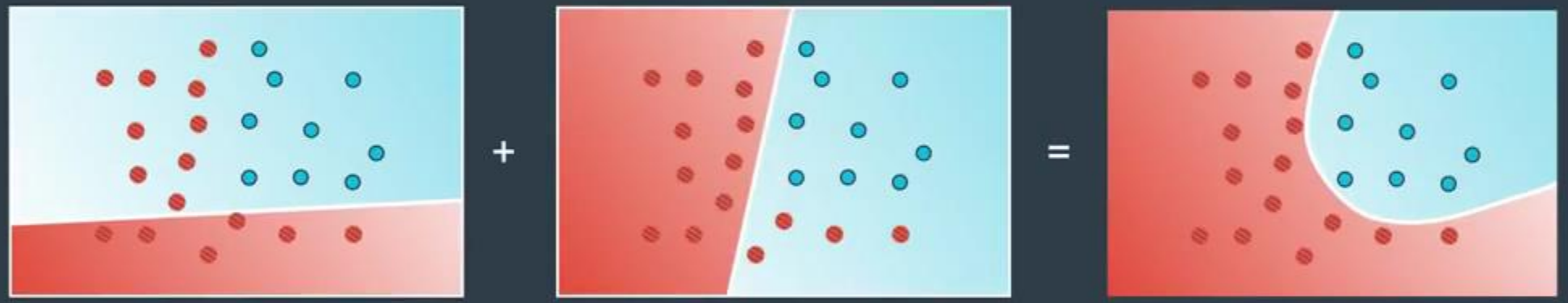
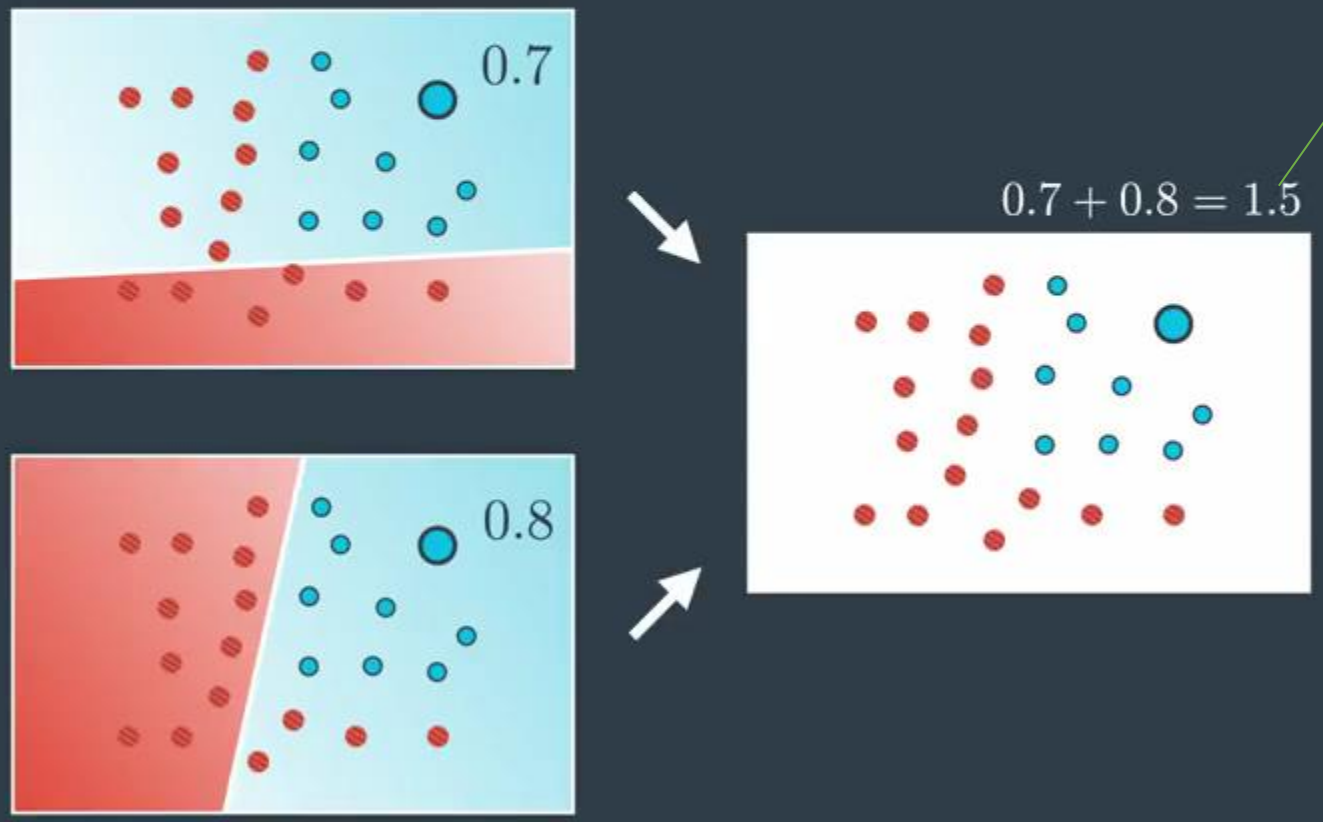
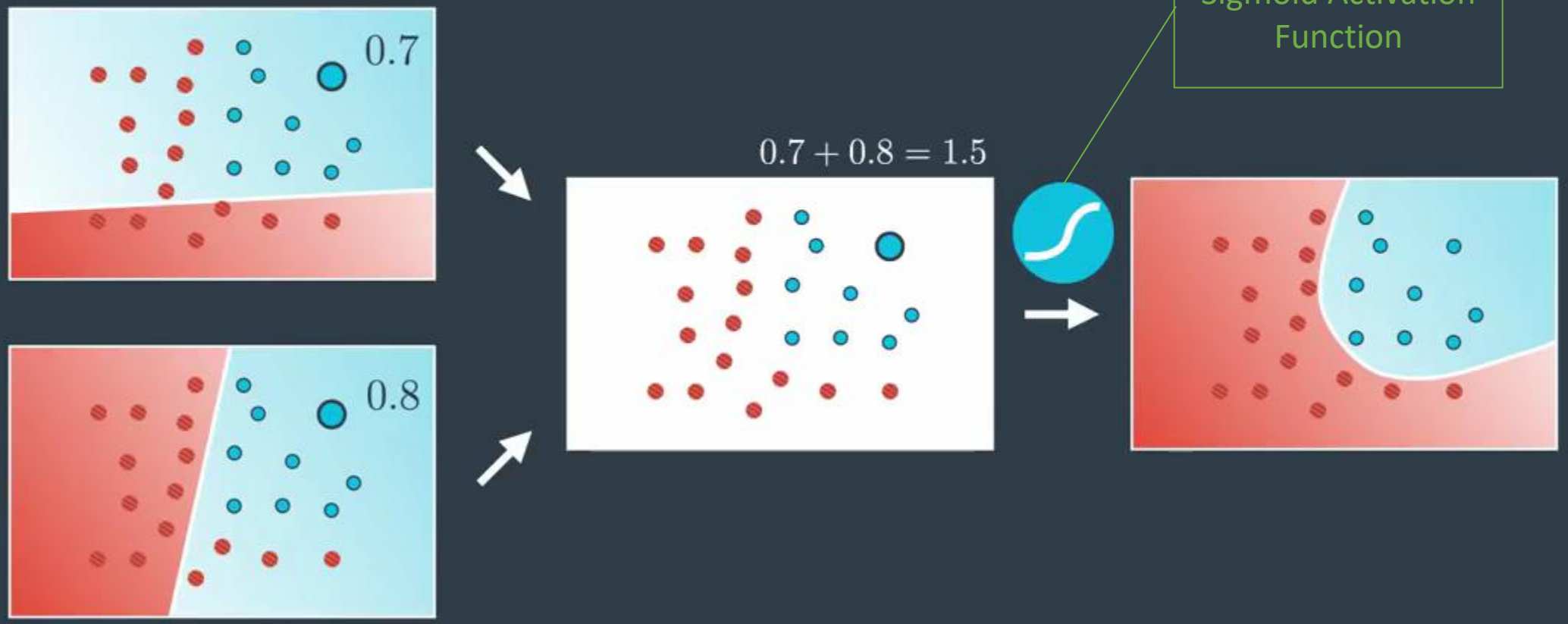


Neural Network



It's bigger than 1
We have to do something

Neural Network



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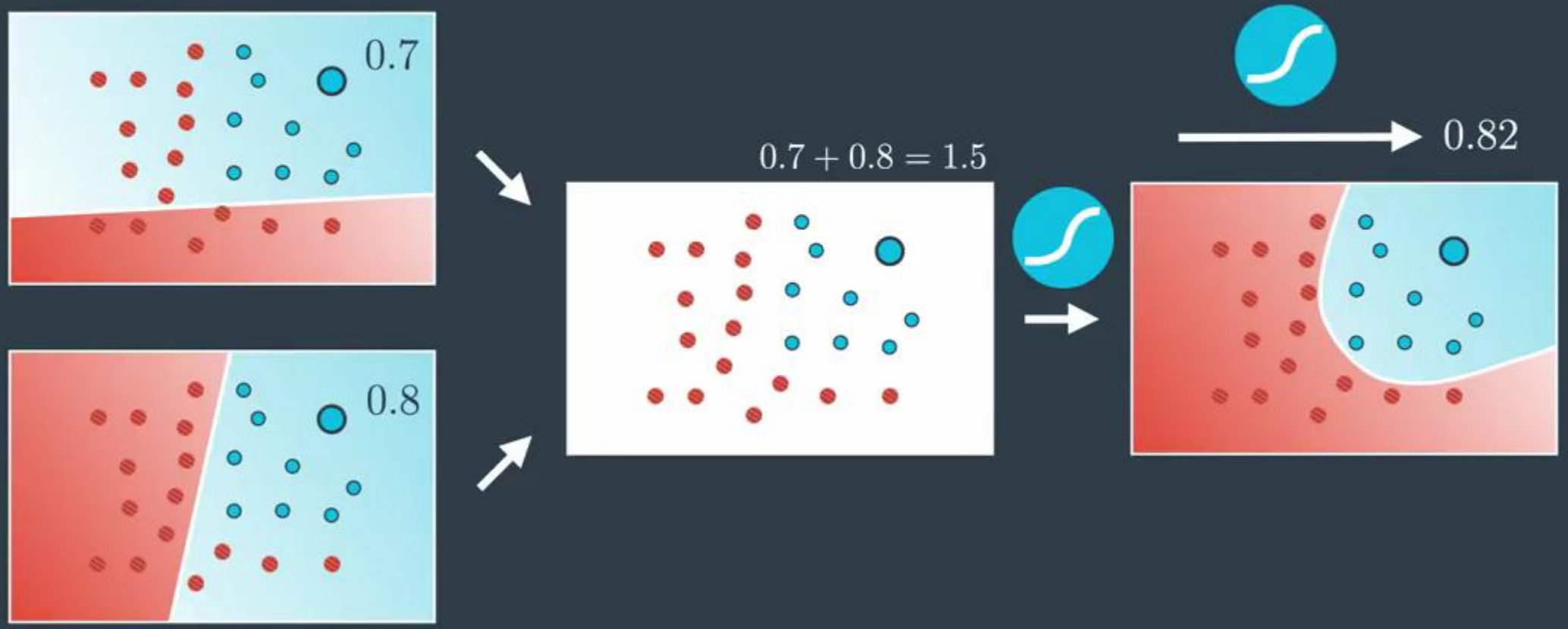
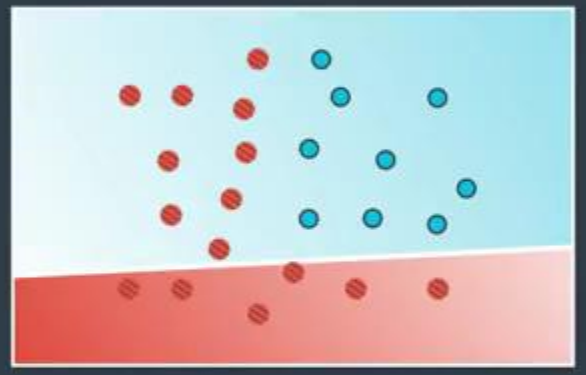
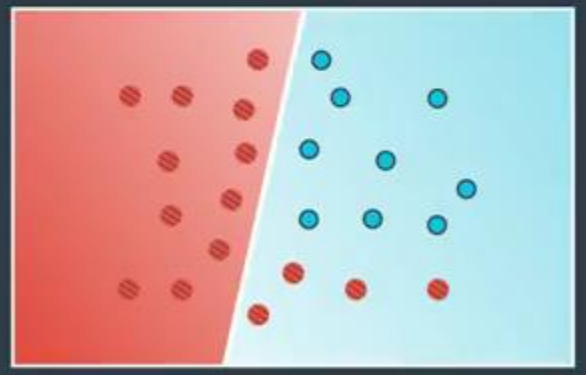


Combining Regions

What Weights can do?





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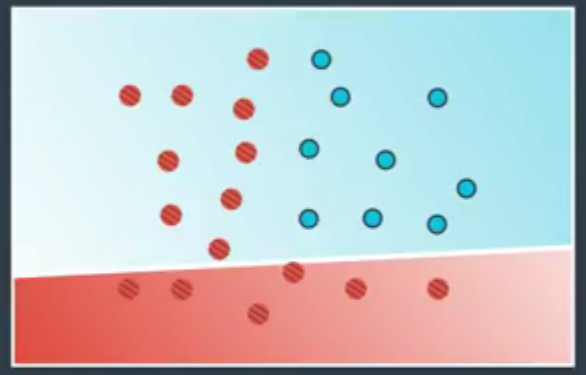

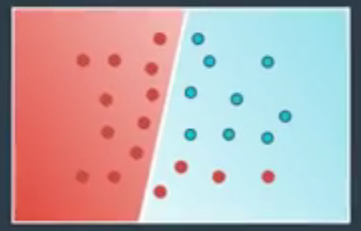

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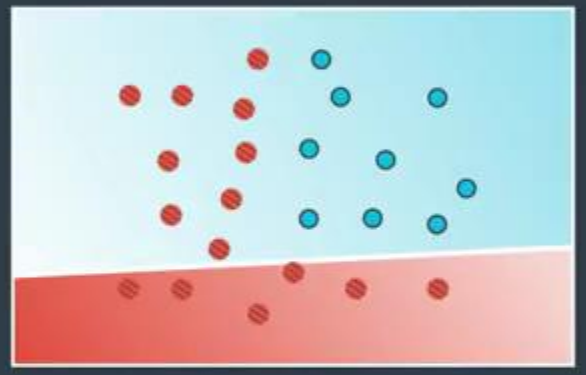

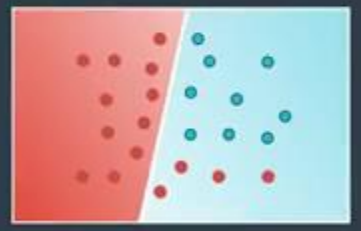

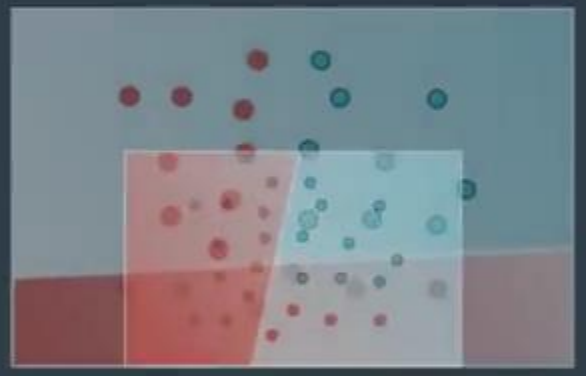

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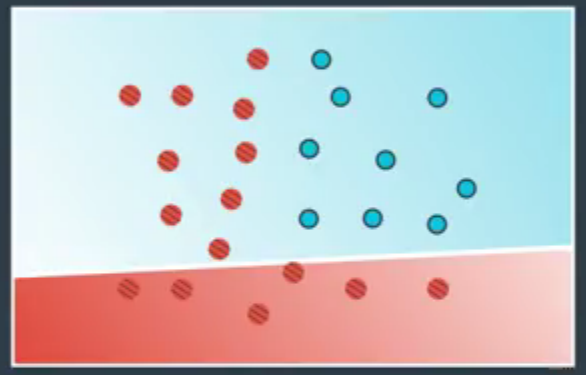

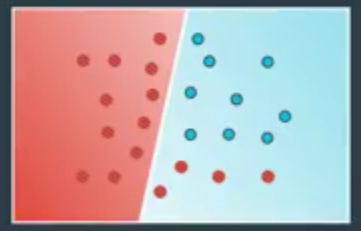

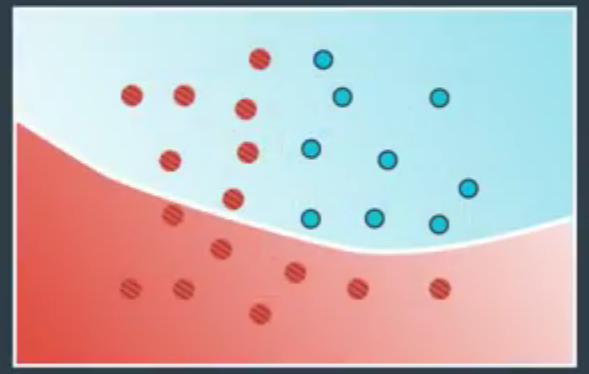

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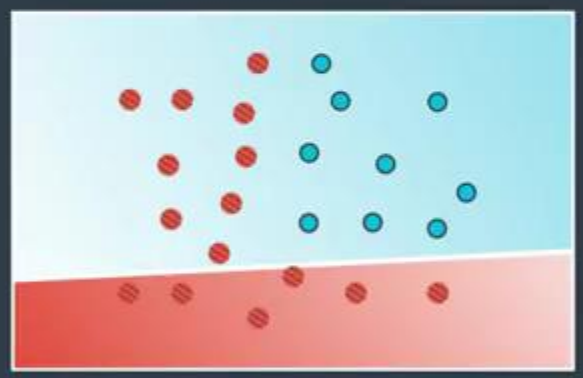

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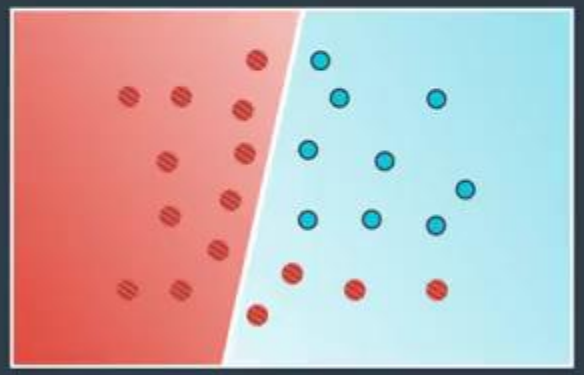

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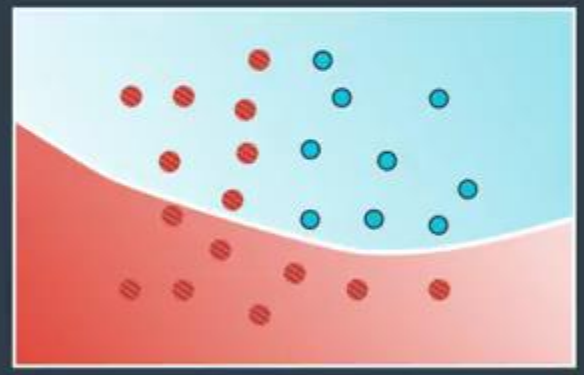

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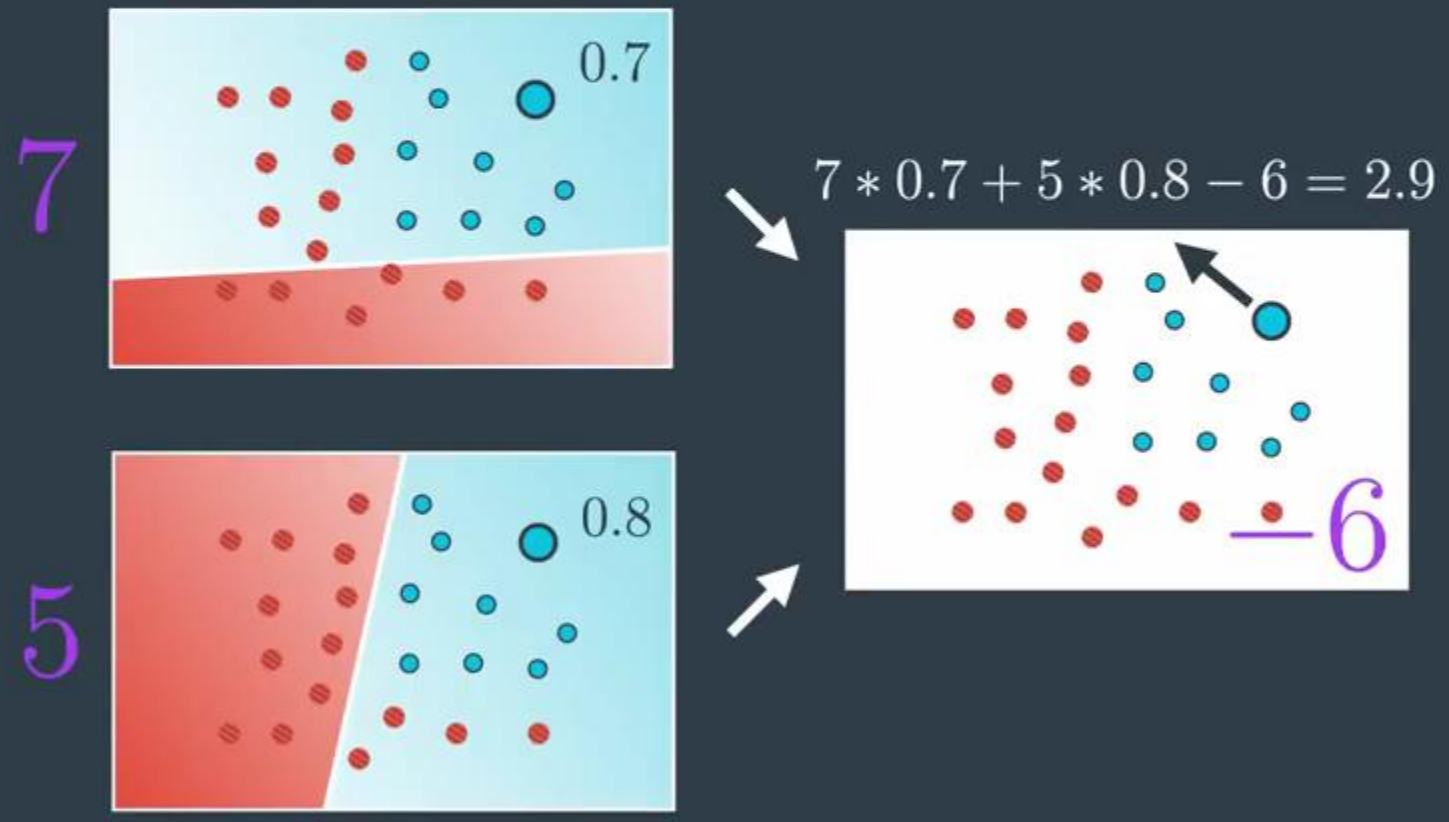

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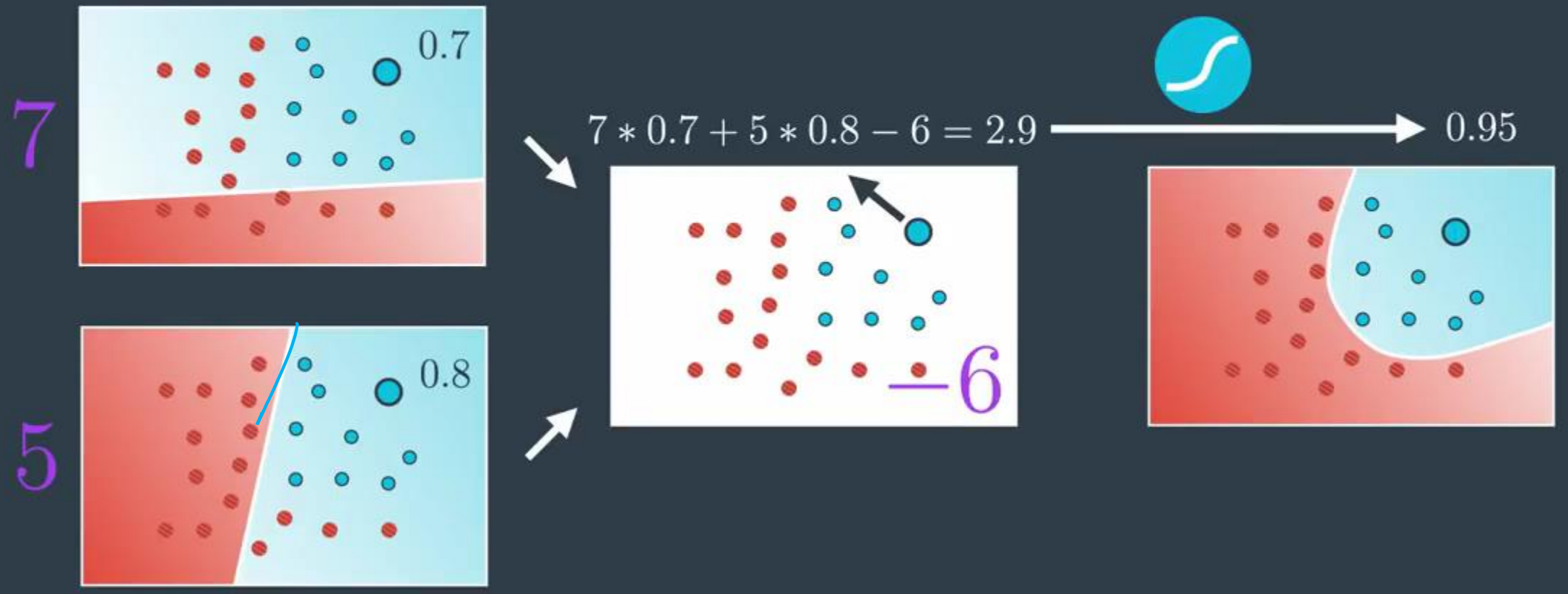
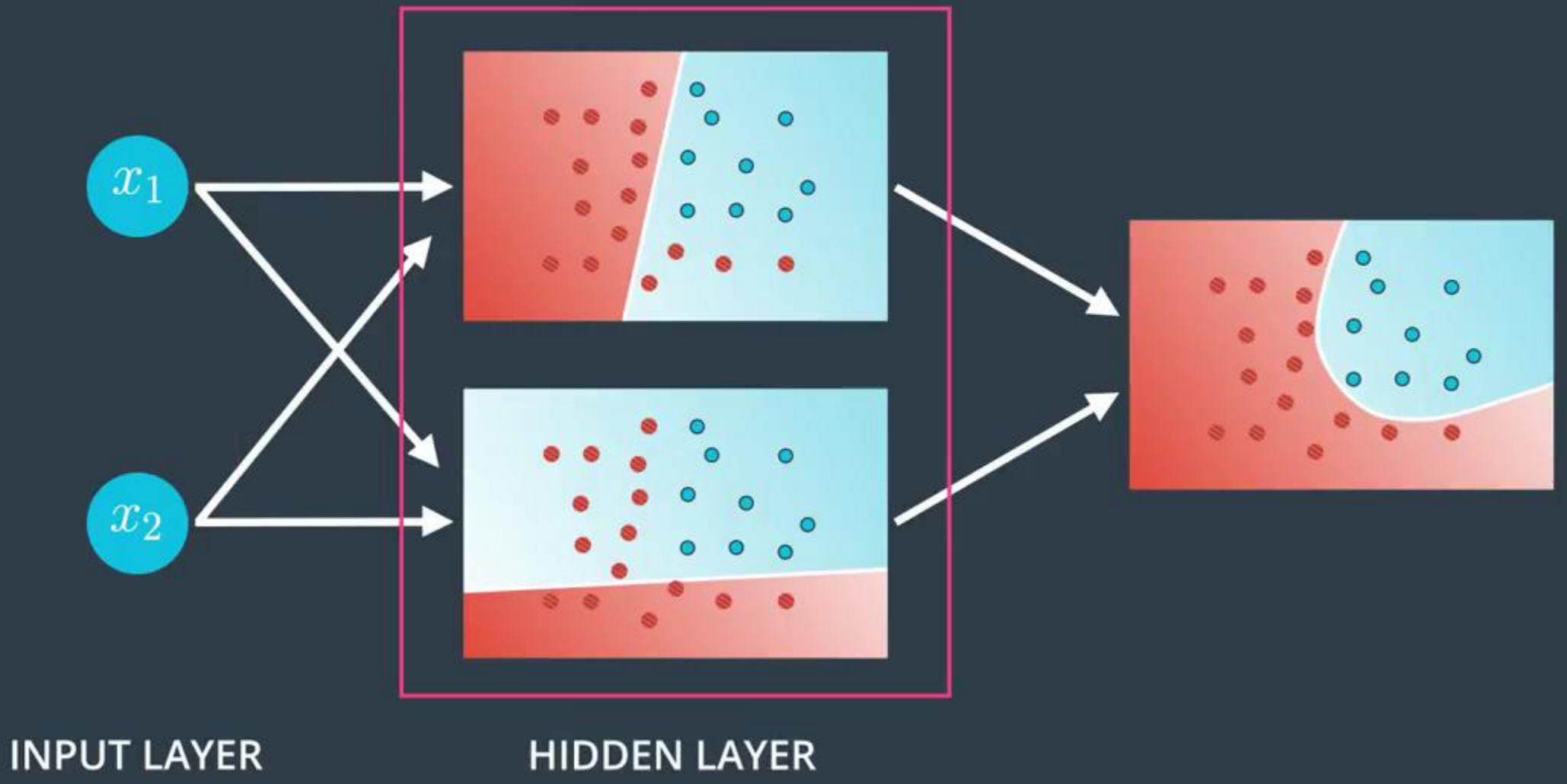


What Layers can do?

Neural Network



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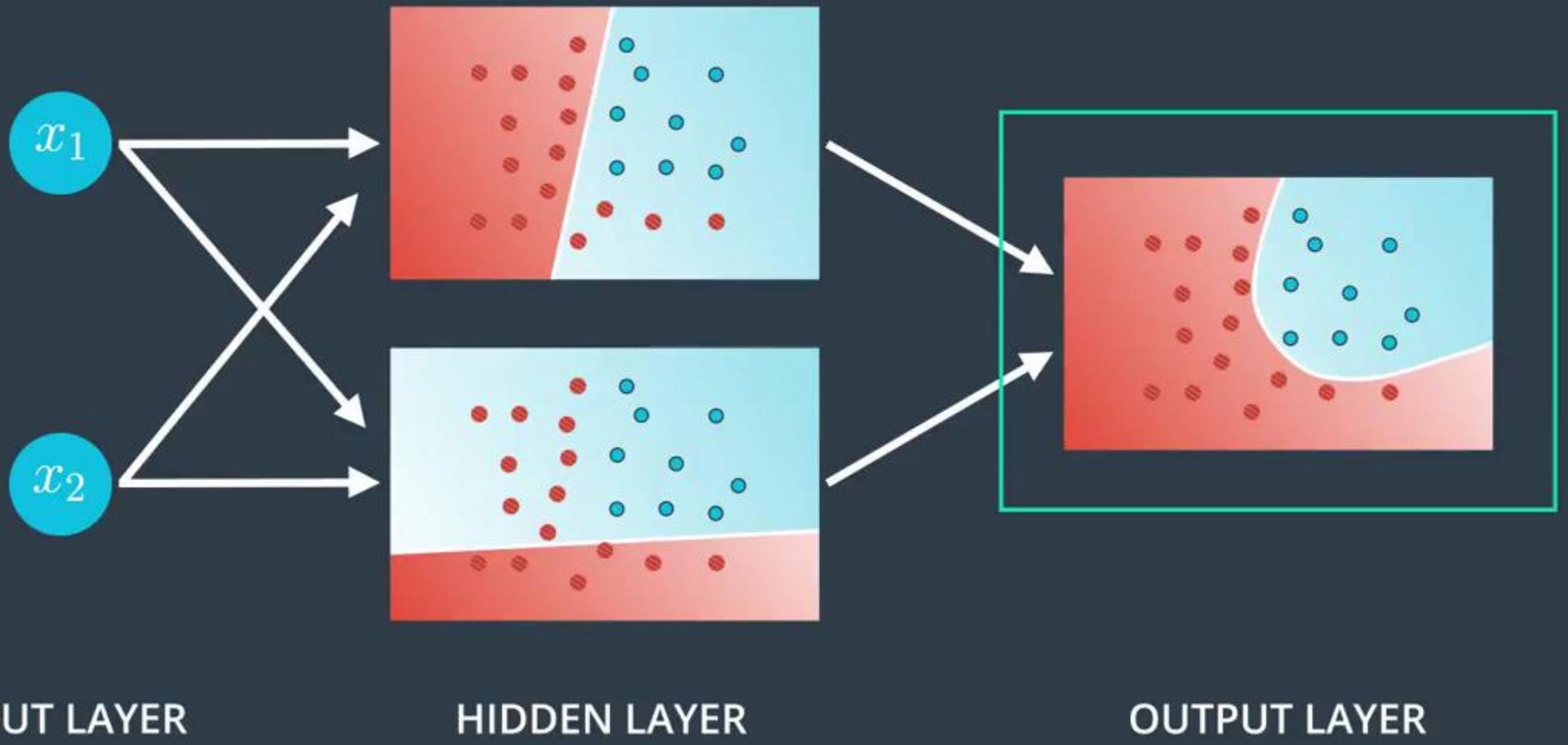

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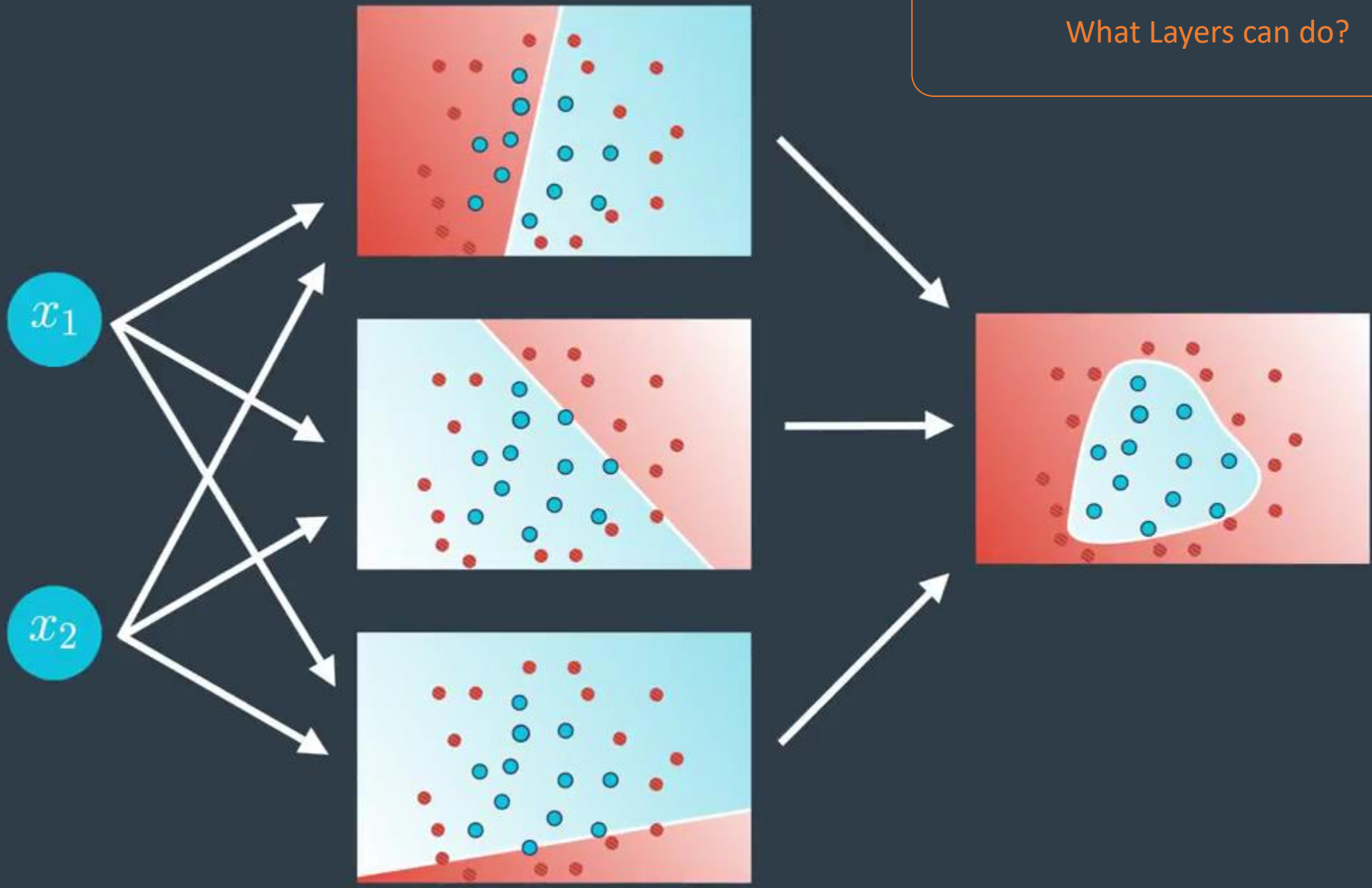

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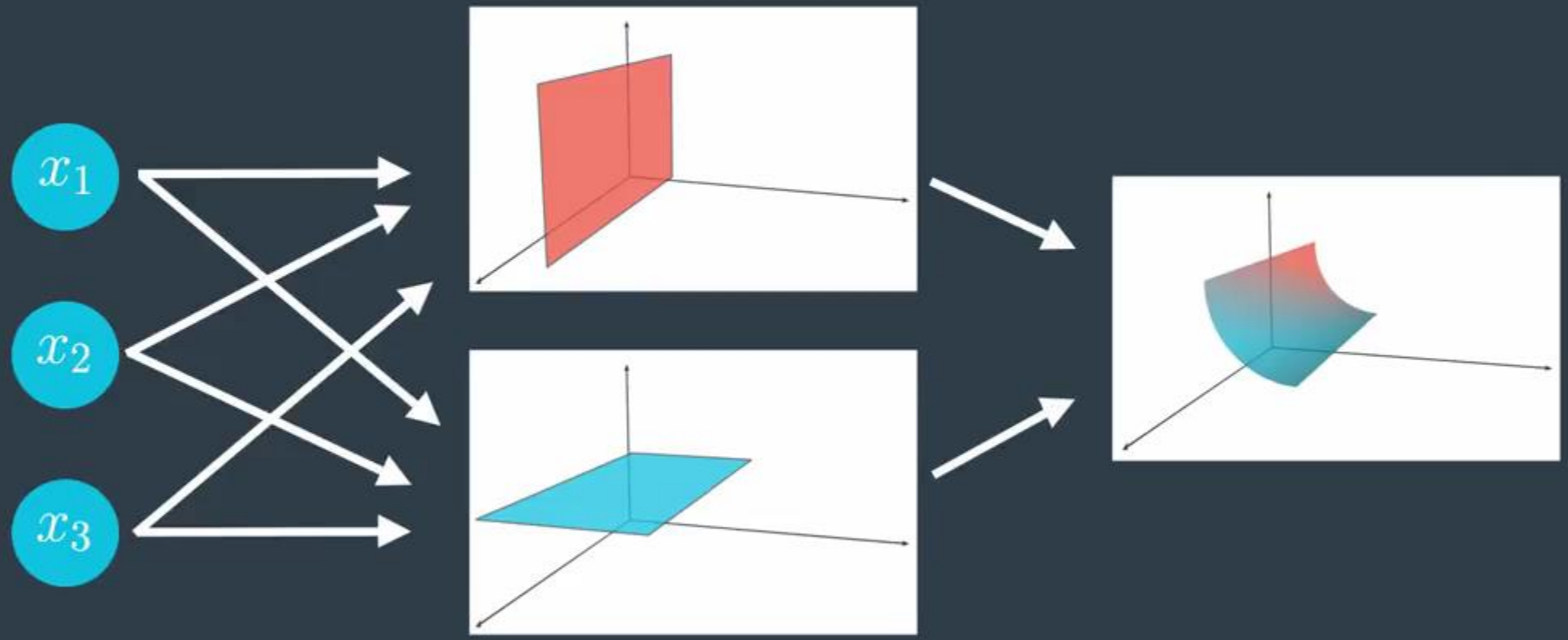
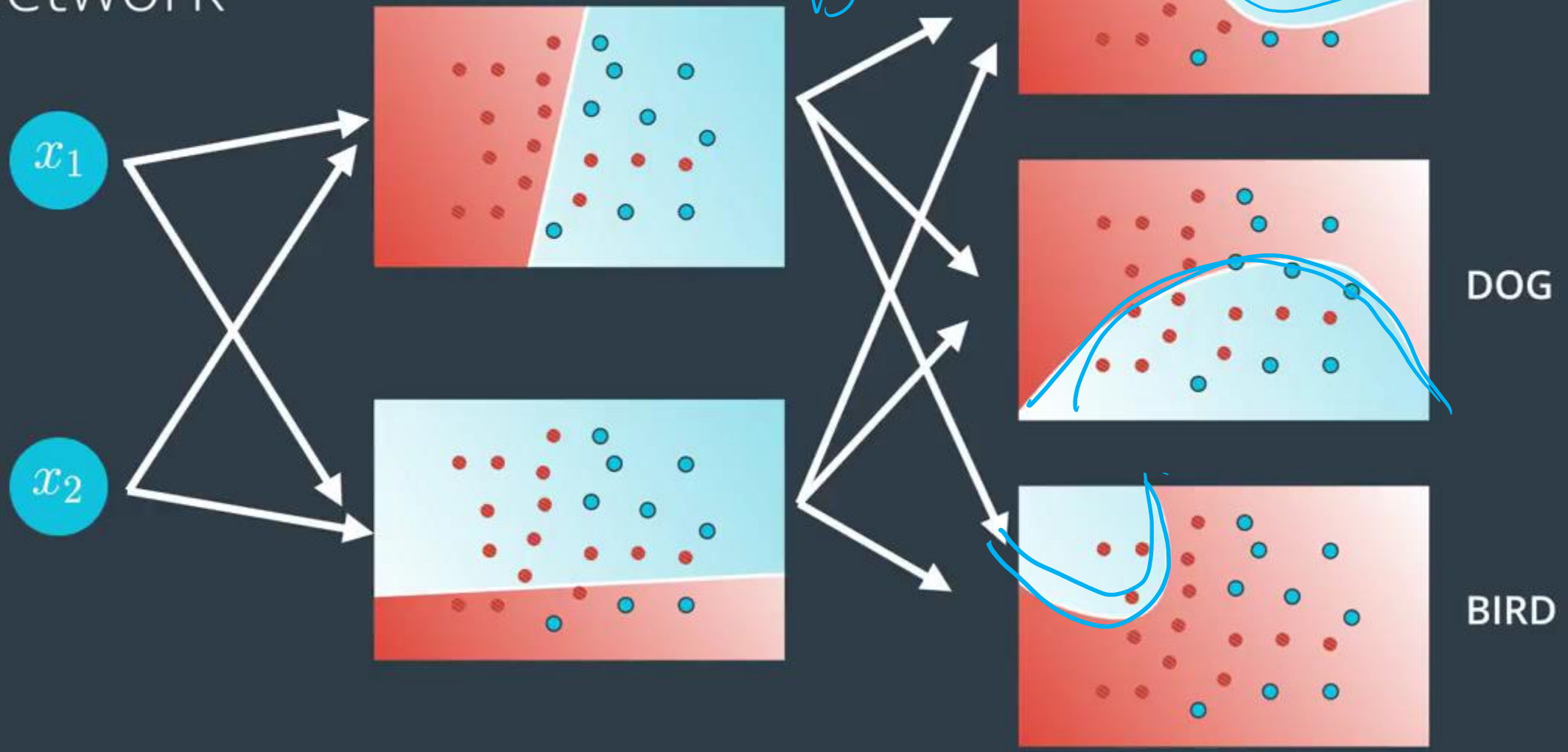


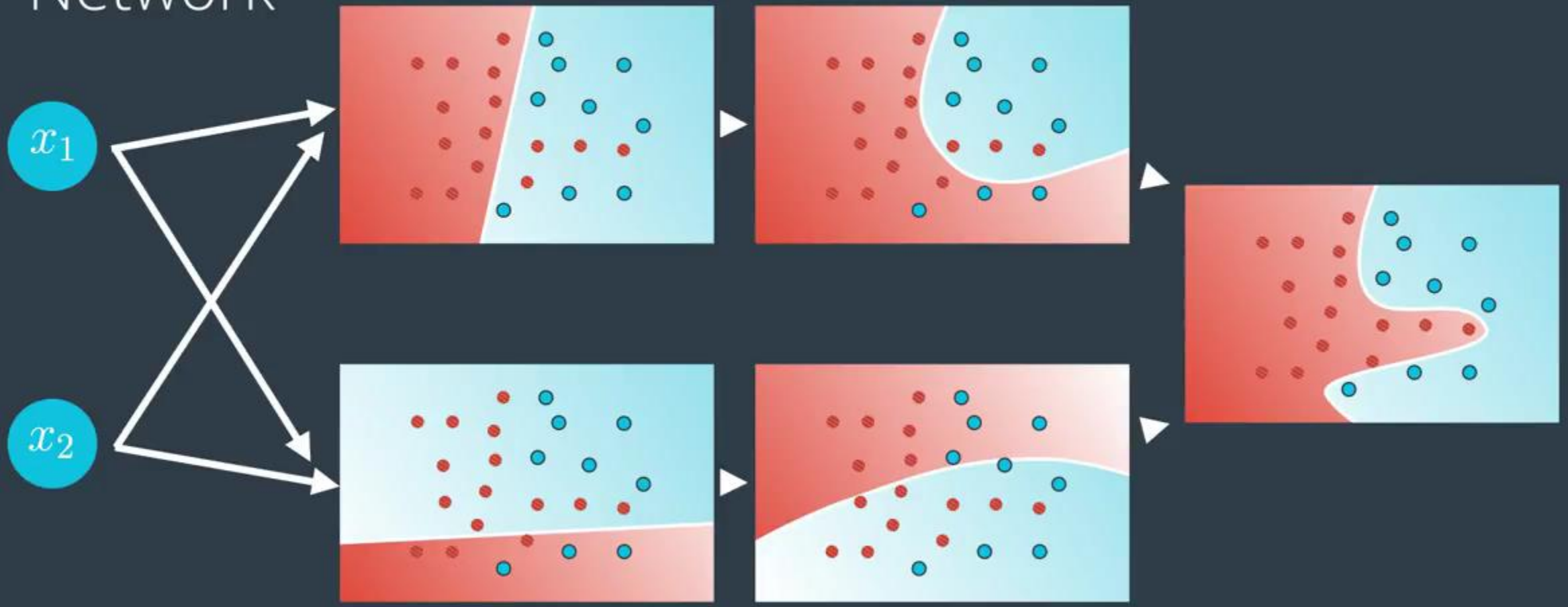
Deep Neural Network

Multi-Class Classification



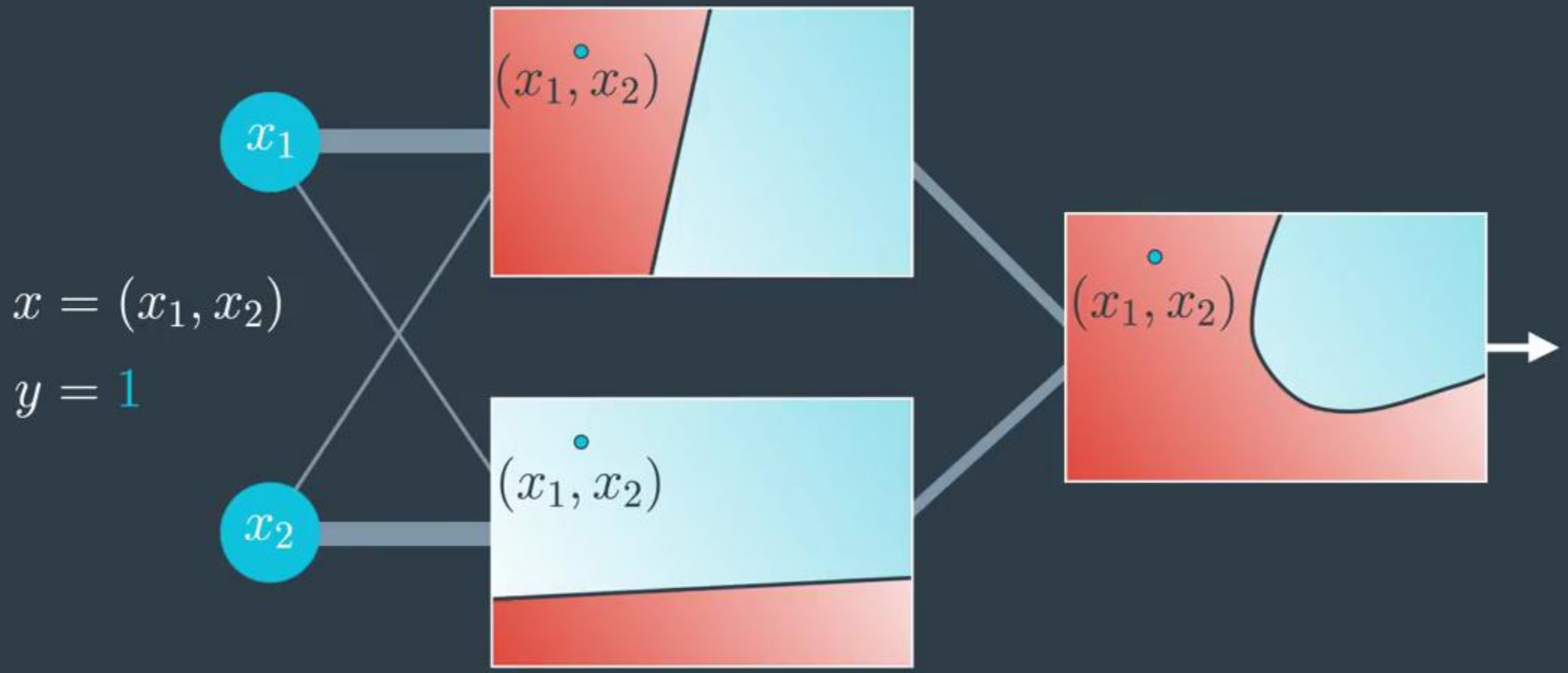
Deep Neural Network

Multi-Layer non-linear Classification

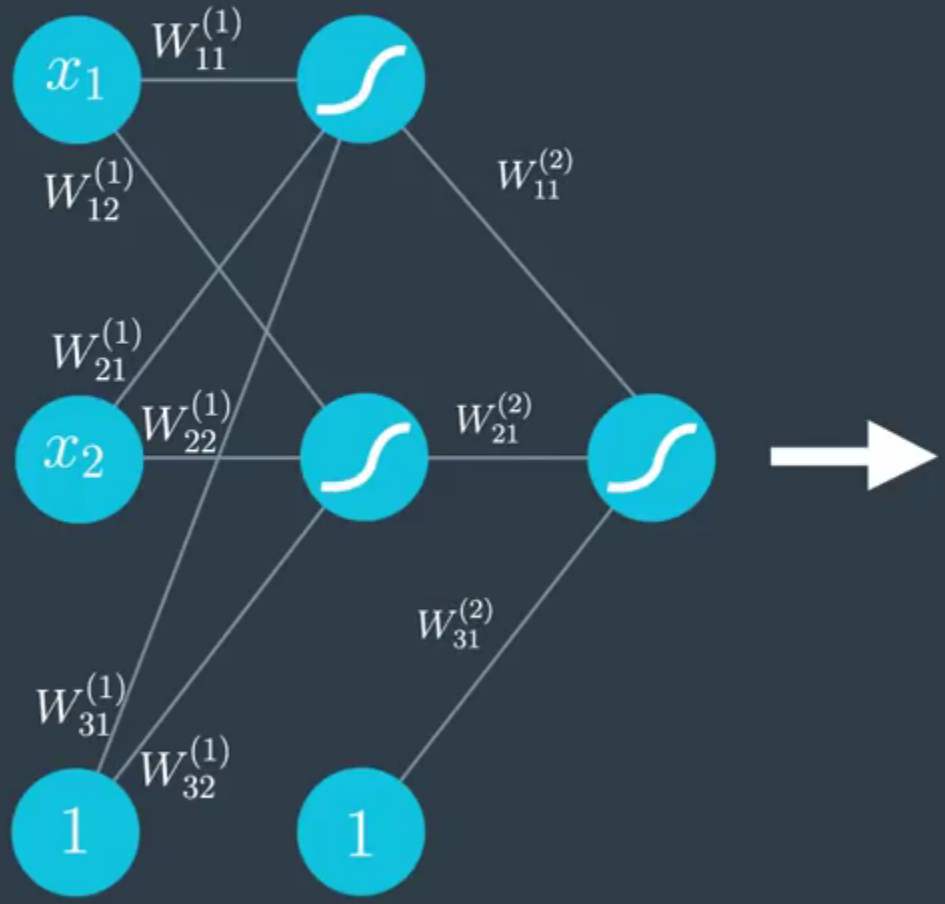


Feed-Forward NN

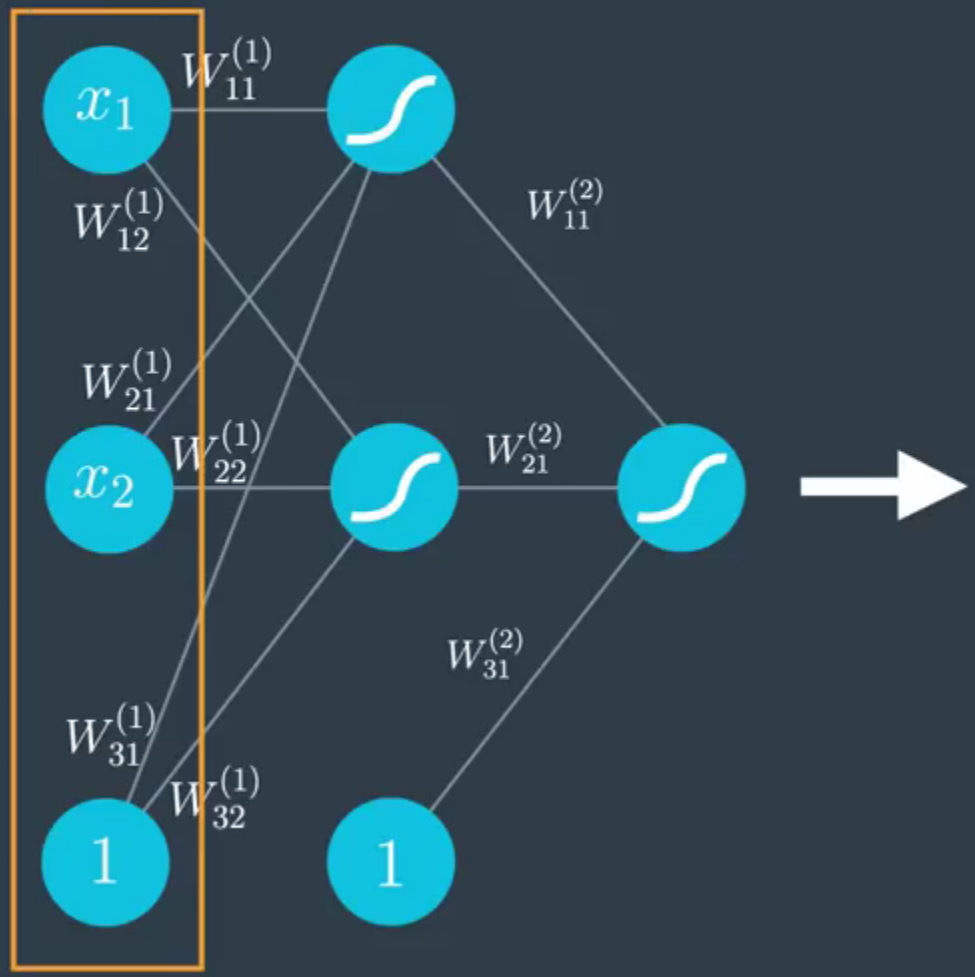
Neural Network



Feedforward

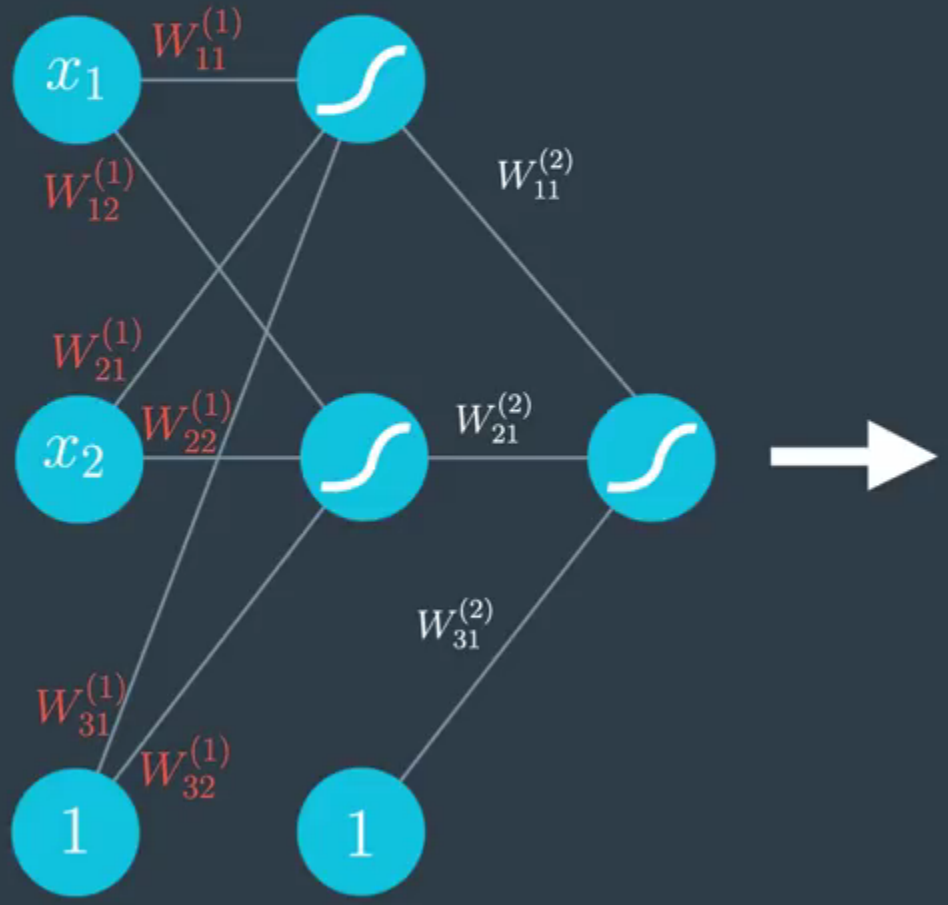


Feedforward



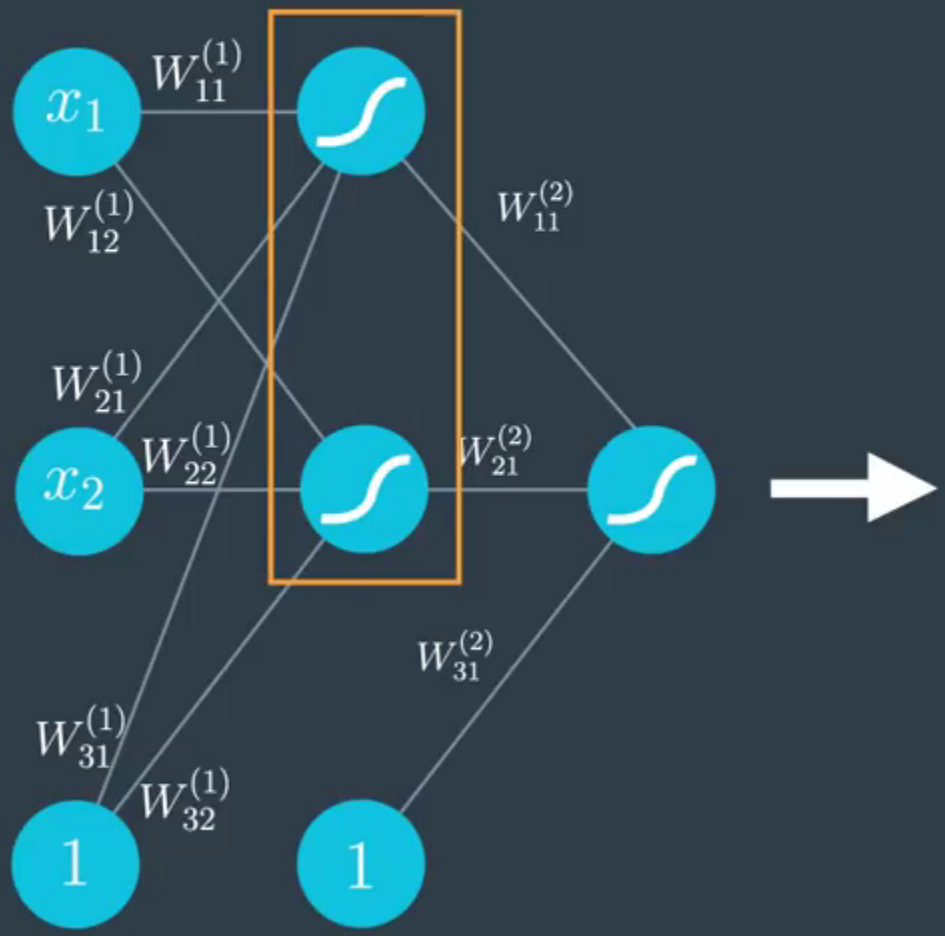
$$\begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

Feedforward



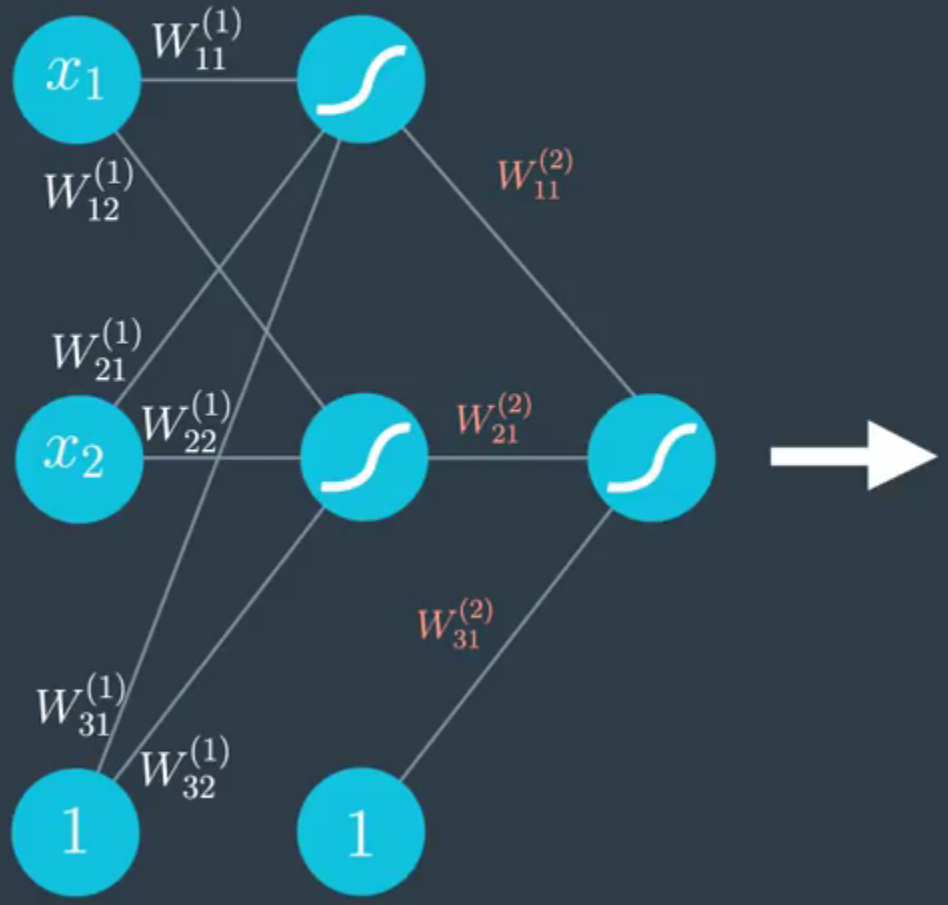
$$\begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

Feedforward



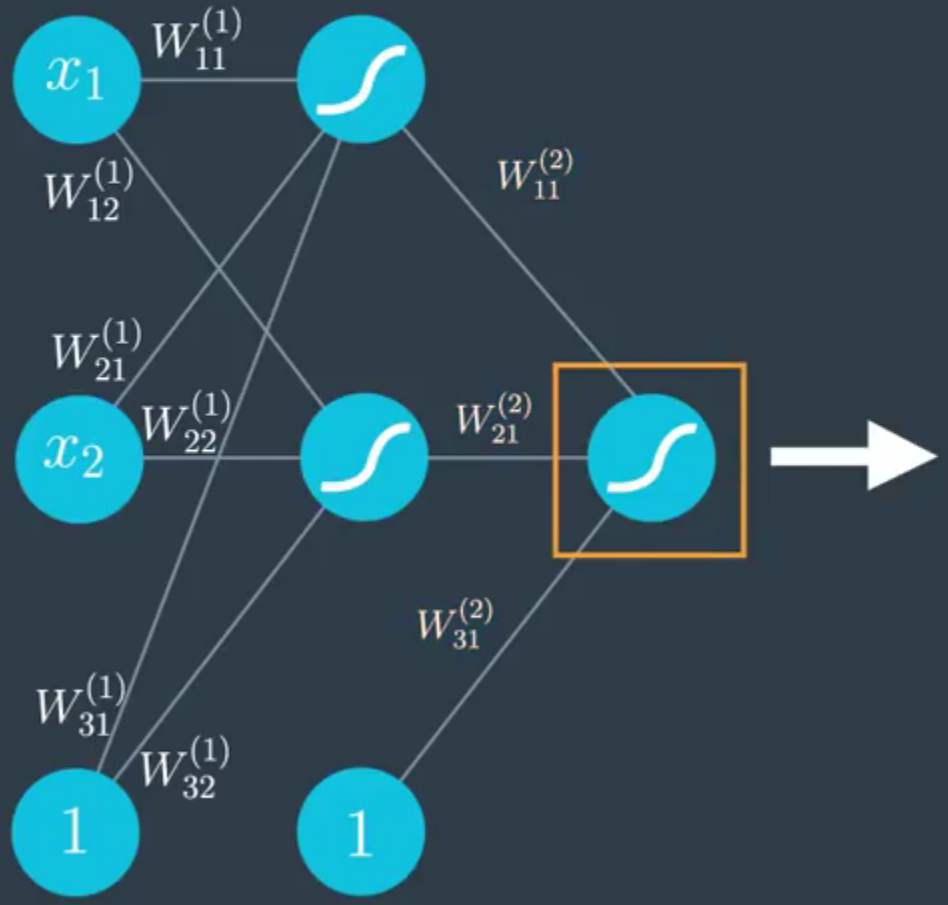
$$\sigma \begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

Feedforward



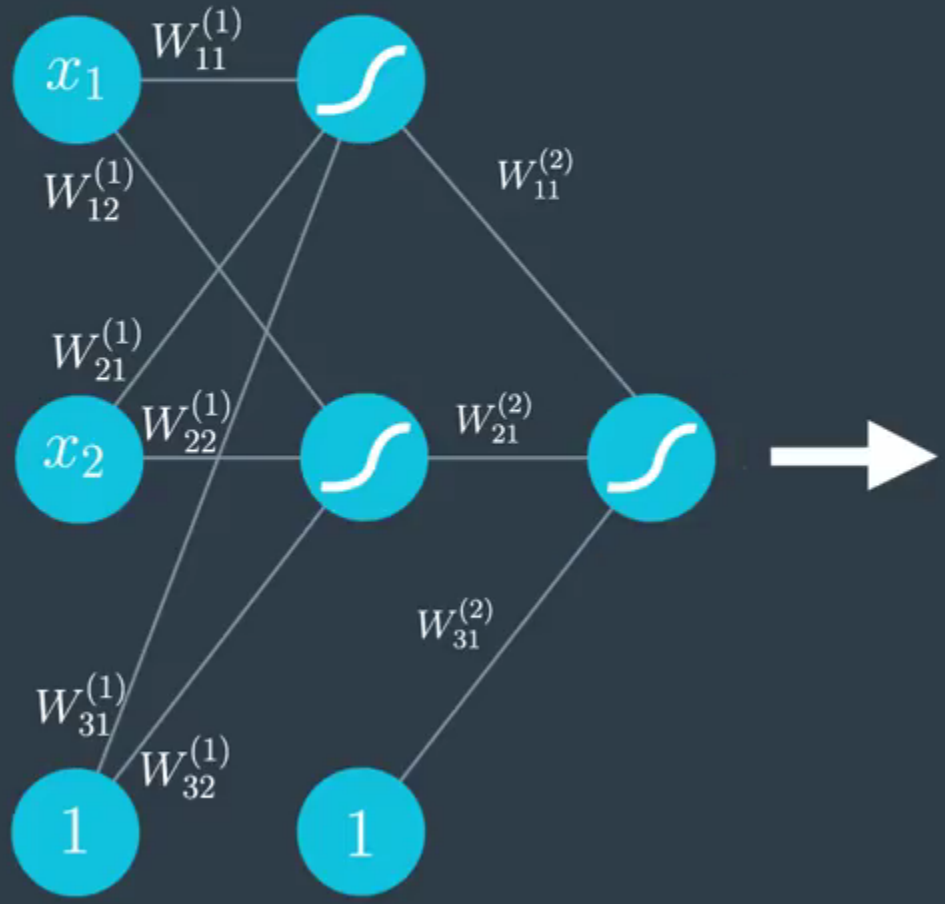
$$\begin{pmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{pmatrix} \sigma \begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

Feedforward



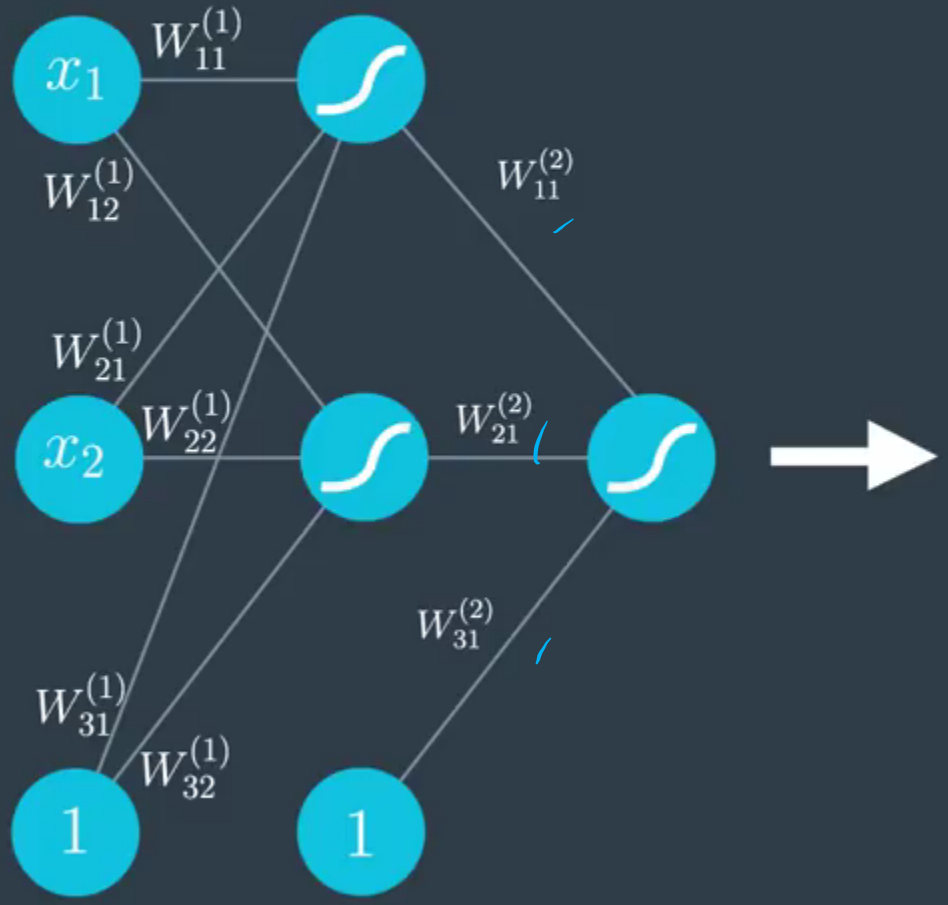
$$\sigma\begin{pmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{pmatrix} \sigma\begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

Feedforward



$$\hat{y} = \sigma \begin{pmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{pmatrix} \sigma \begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

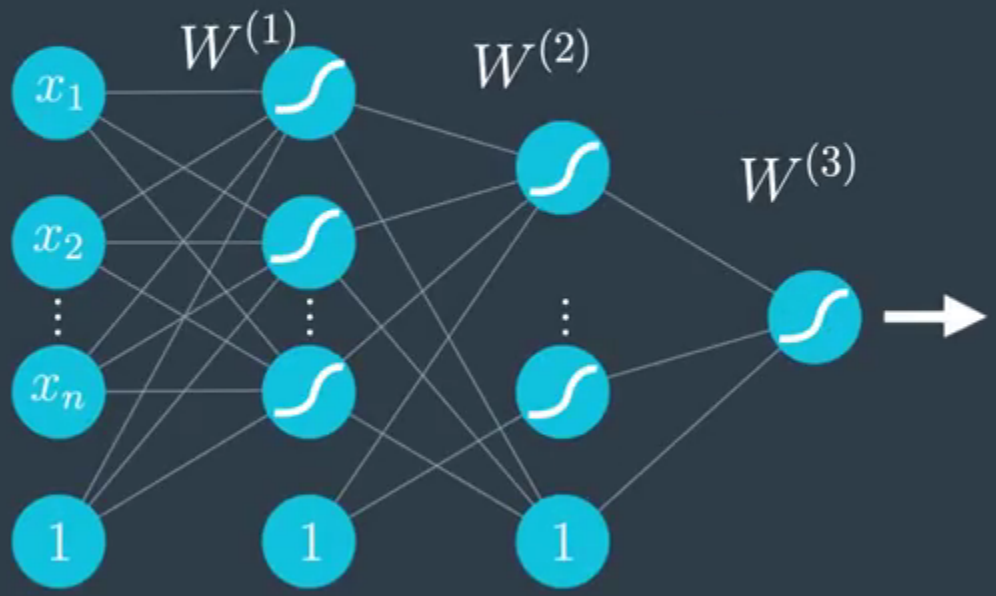
Feedforward



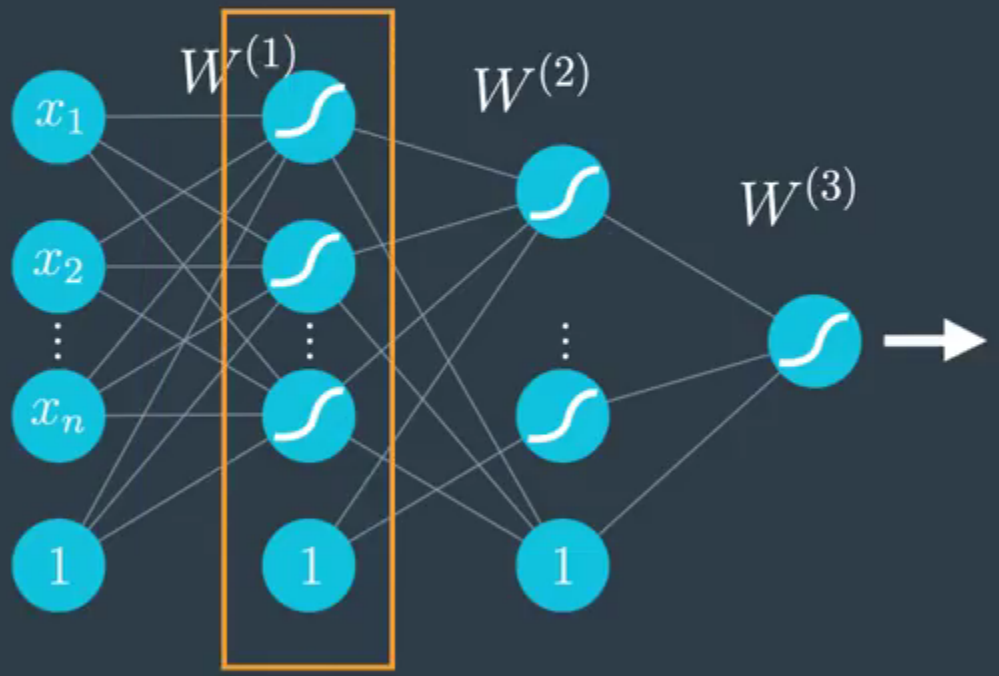
$$\hat{y} = \sigma \begin{pmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{pmatrix} \sigma \begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

$$\hat{y} = \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

Multi-layer Perceptron



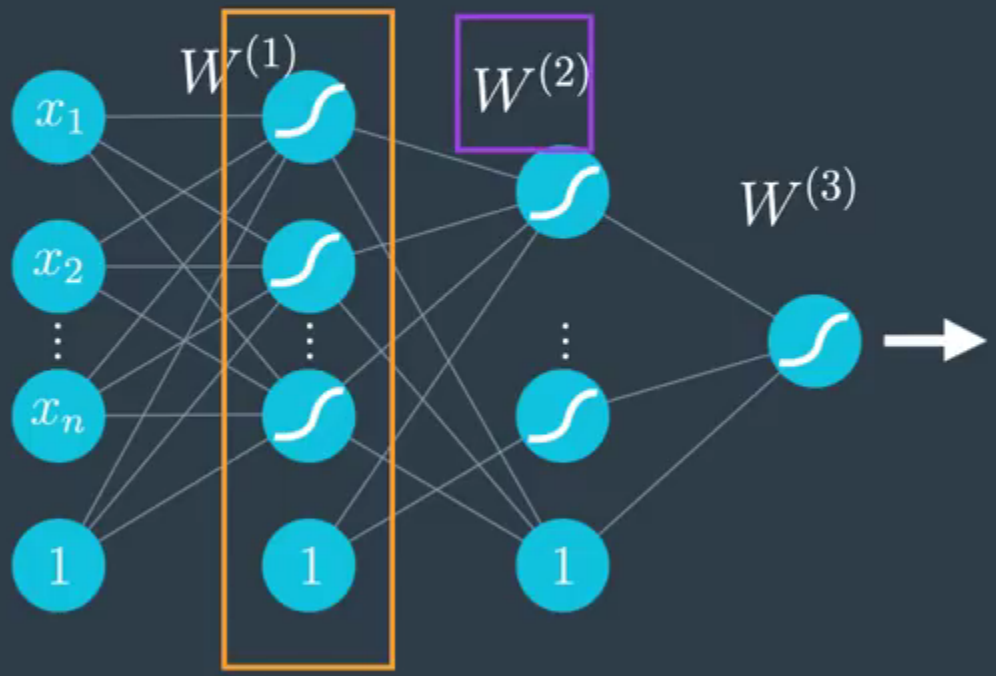
Multi-layer Perceptron



PREDICTION

$$\hat{y} = \sigma \circ W^{(1)}(x)$$

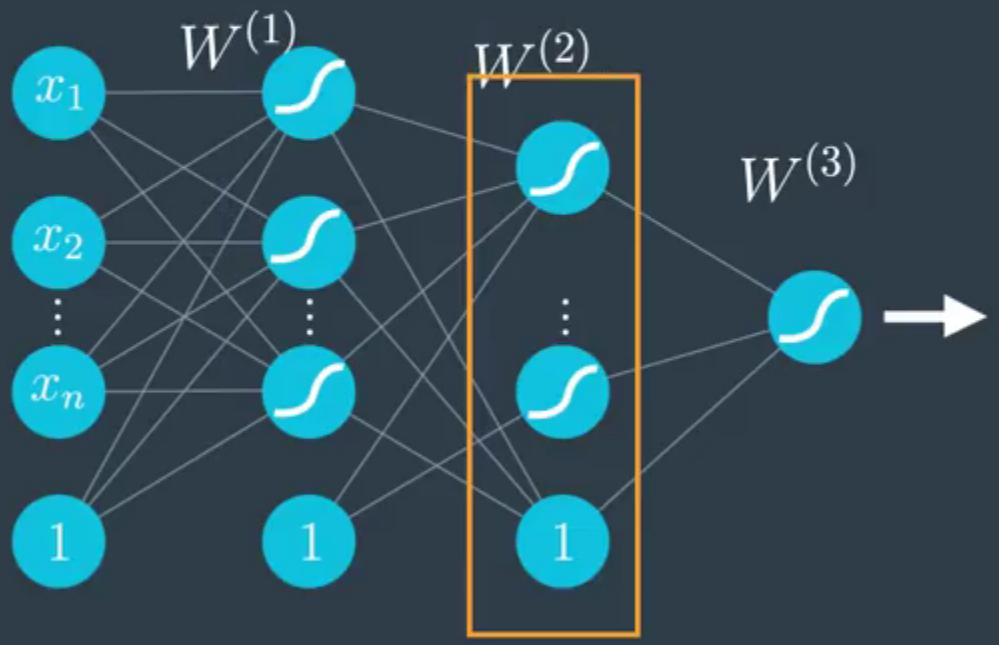
Multi-layer Perceptron



PREDICTION

$$\hat{y} = W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

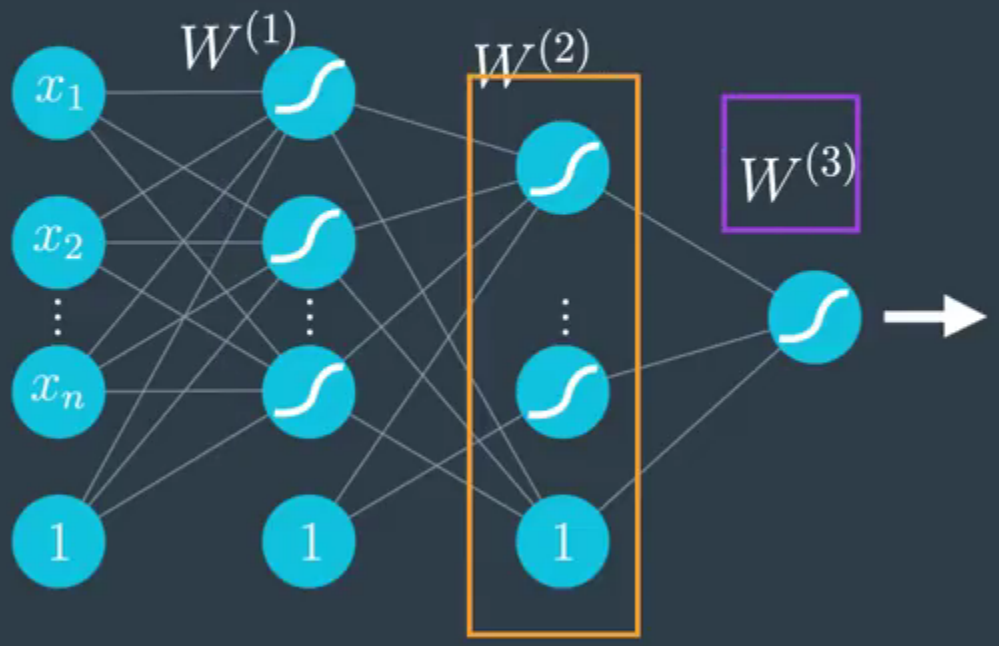
Multi-layer Perceptron



PREDICTION

$$\hat{y} = \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

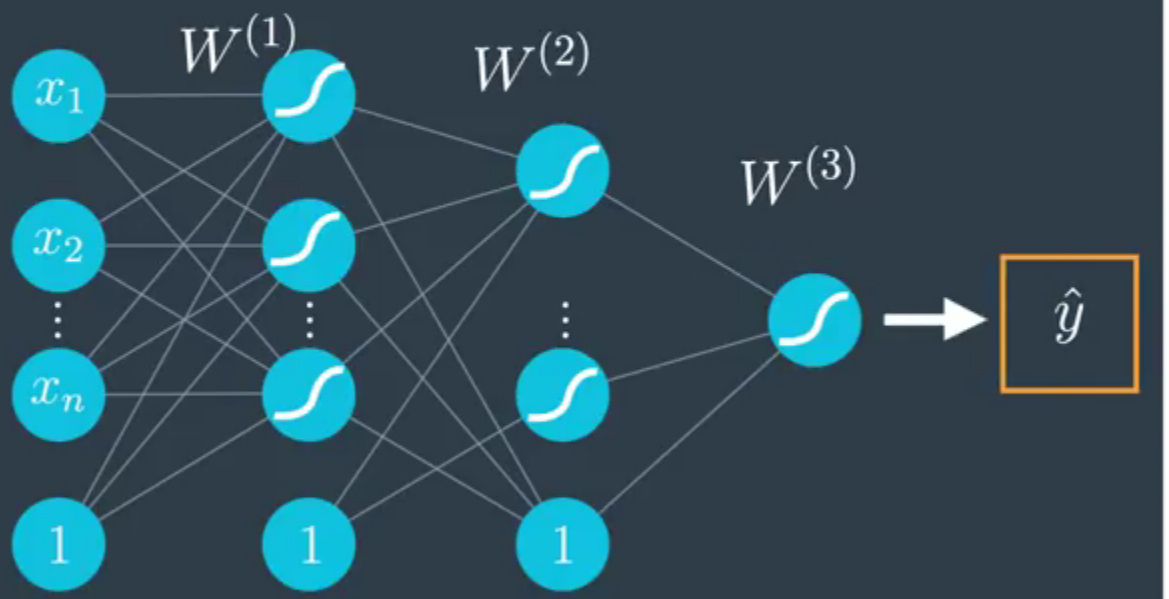
Multi-layer Perceptron



PREDICTION

$$\hat{y} = \sigma \circ W^{(3)} \circ \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

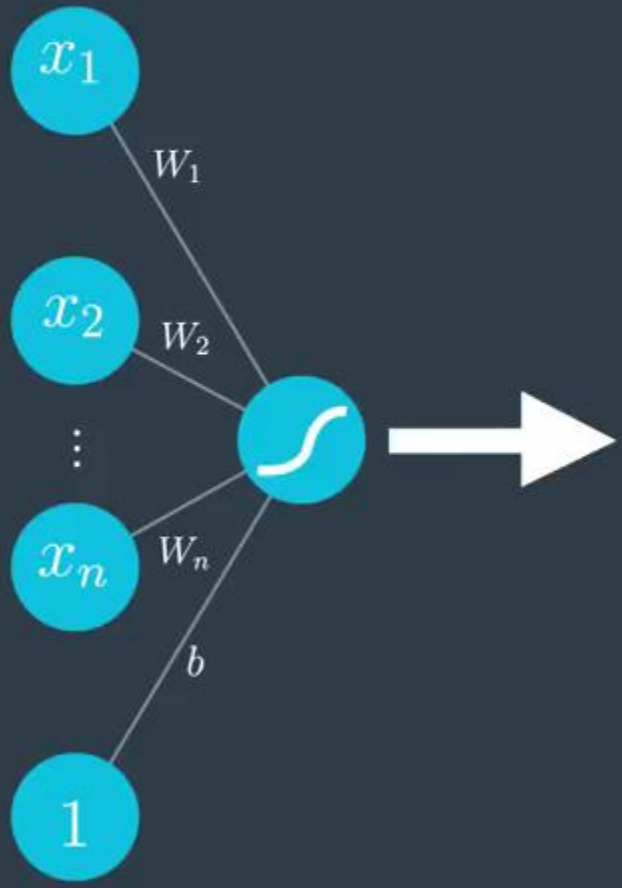
Multi-layer Perceptron



PREDICTION

$$\hat{y} = \sigma \circ W^{(3)} \circ \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

Perceptron

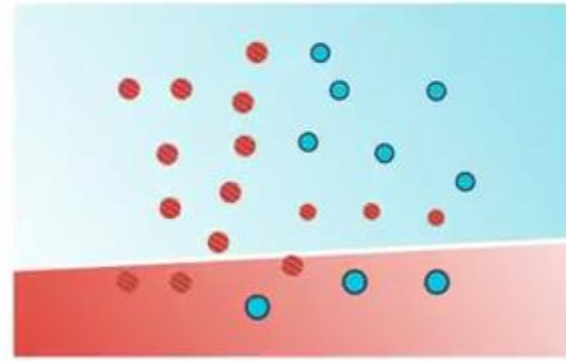


PREDICTION

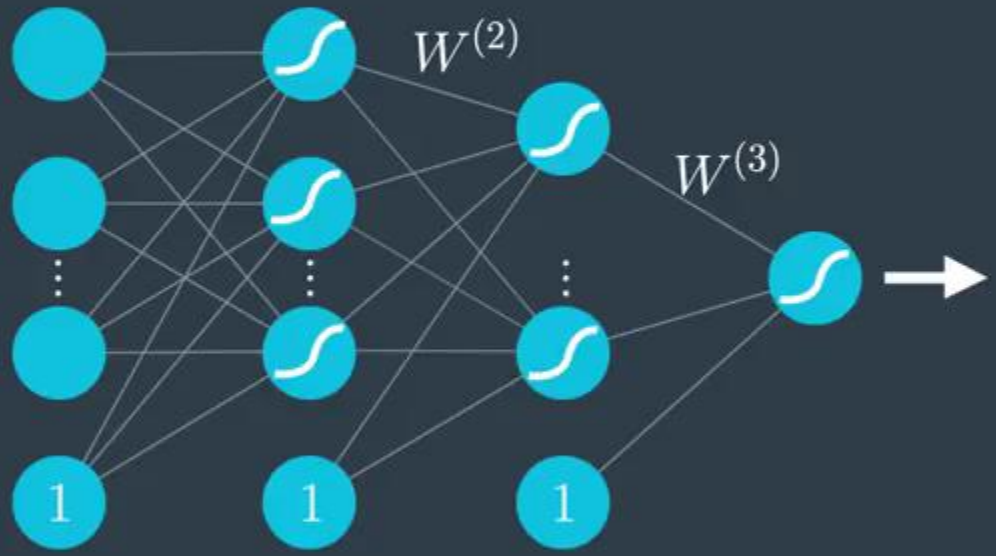
$$\hat{y} = \sigma(Wx + b)$$

ERROR FUNCTION

$$E(W) = -\frac{1}{m}\sum_{i=1}^{m} y_i ln(\hat{y}_i) + (1 - y_i) ln(1 - \hat{y}_i)$$



Multi-layer Perceptron

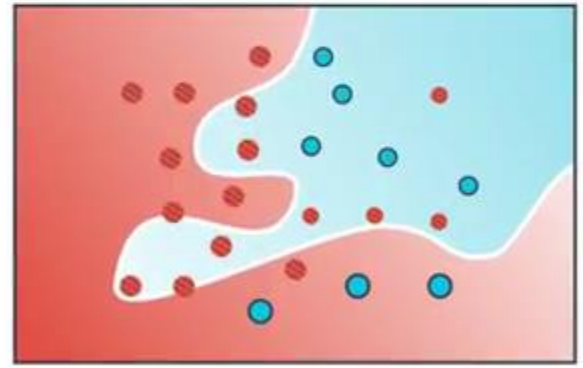


PREDICTION

$$\hat{y} = \sigma \circ W^{(3)} \circ \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

ERROR FUNCTION

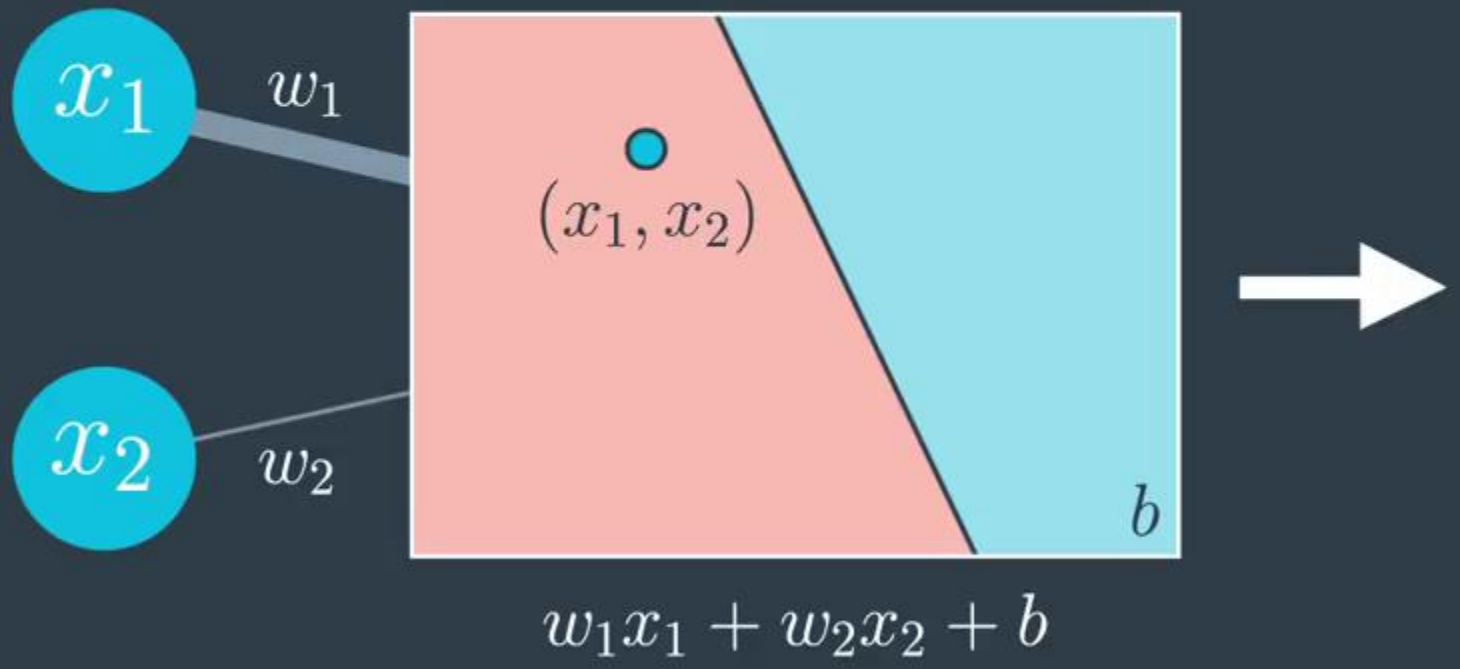
$$E(W) = -\frac{1}{m}\sum_{i=1}^{m} y_i ln(\hat{y}_i) + (1 - y_i) ln(1 - \hat{y}_i)$$



Backpropagation

$x = (x_1, x_2)$

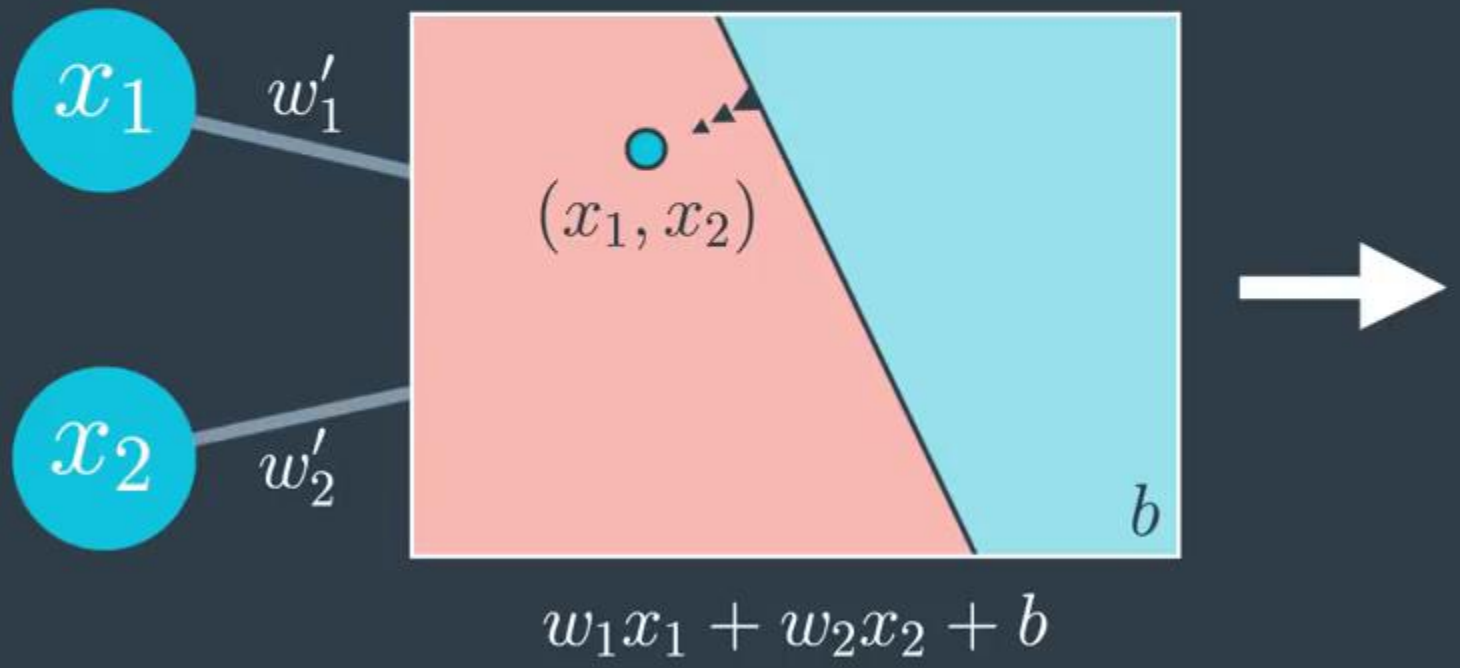
$y = 1$



Backpropagation

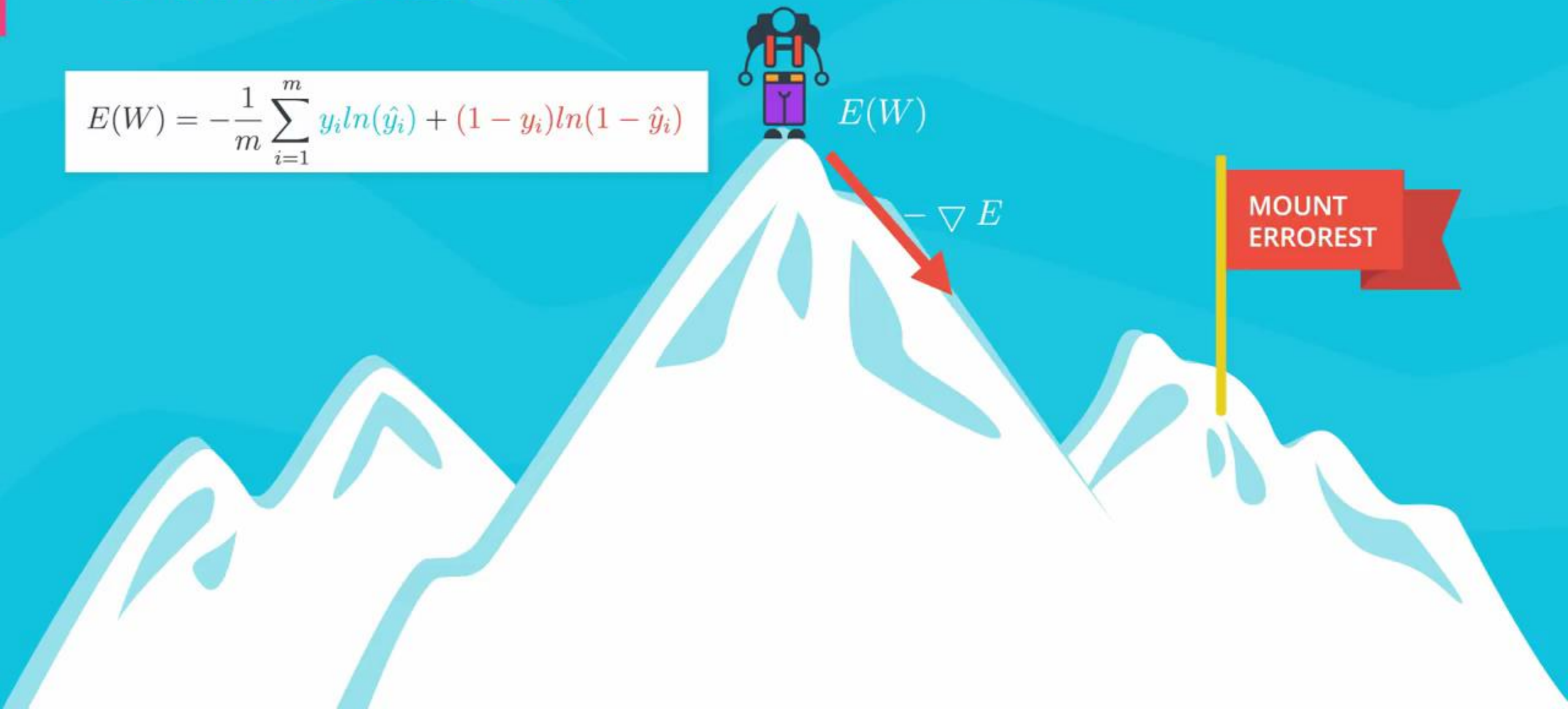
$x = (x_1, x_2)$

$y = 1$



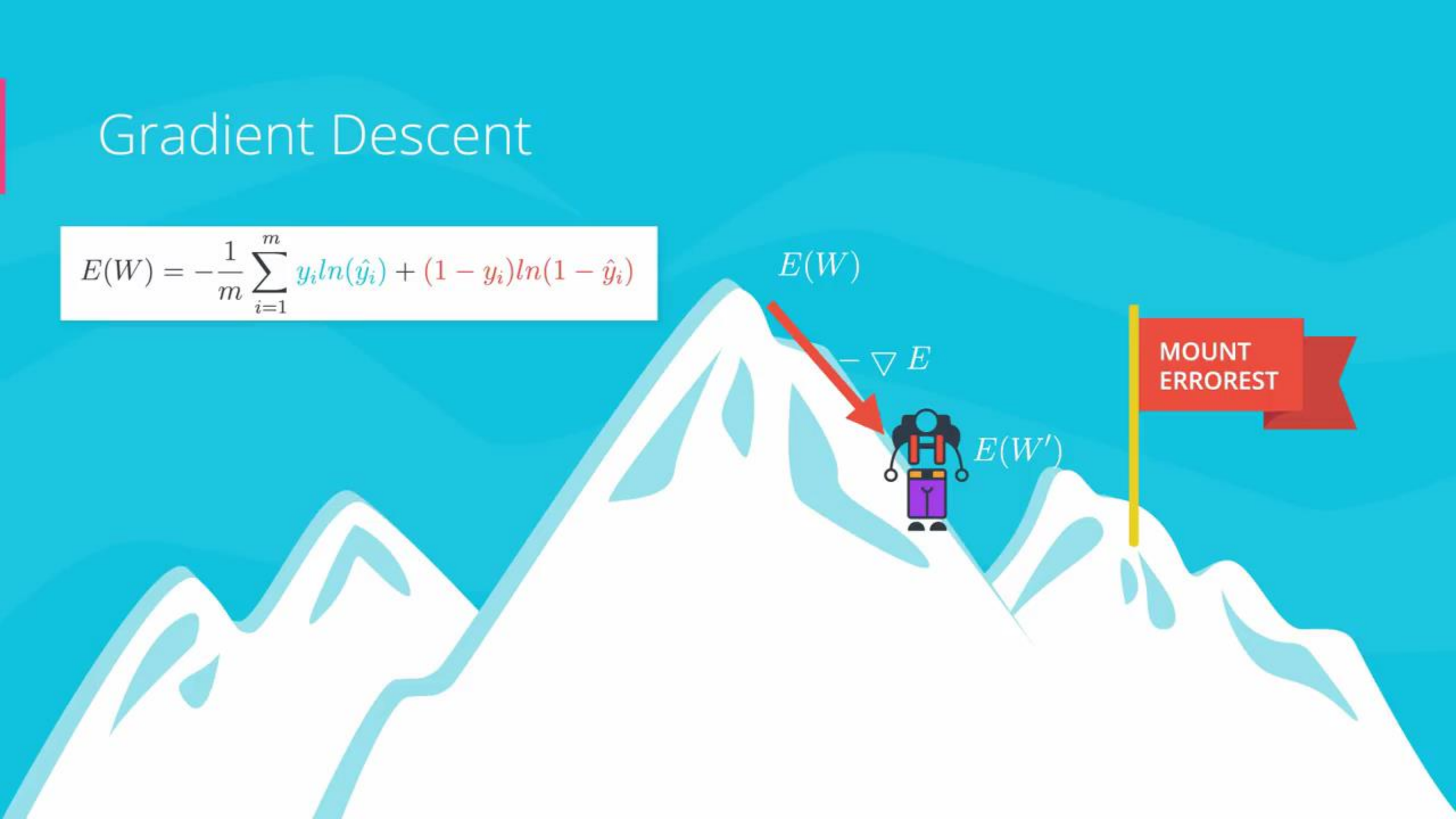
Gradient Descent

$$E(W) = -\frac{1}{m}\sum_{i=1}^{m} y_i ln(\hat{y}_i) + (1-y_i) ln(1-\hat{y}_i)$$



Gradient Descent

$$E(W) = -\frac{1}{m}\sum_{i=1}^{m} y_i ln(\hat{y}_i) + (1 - y_i) ln(1 - \hat{y}_i)$$

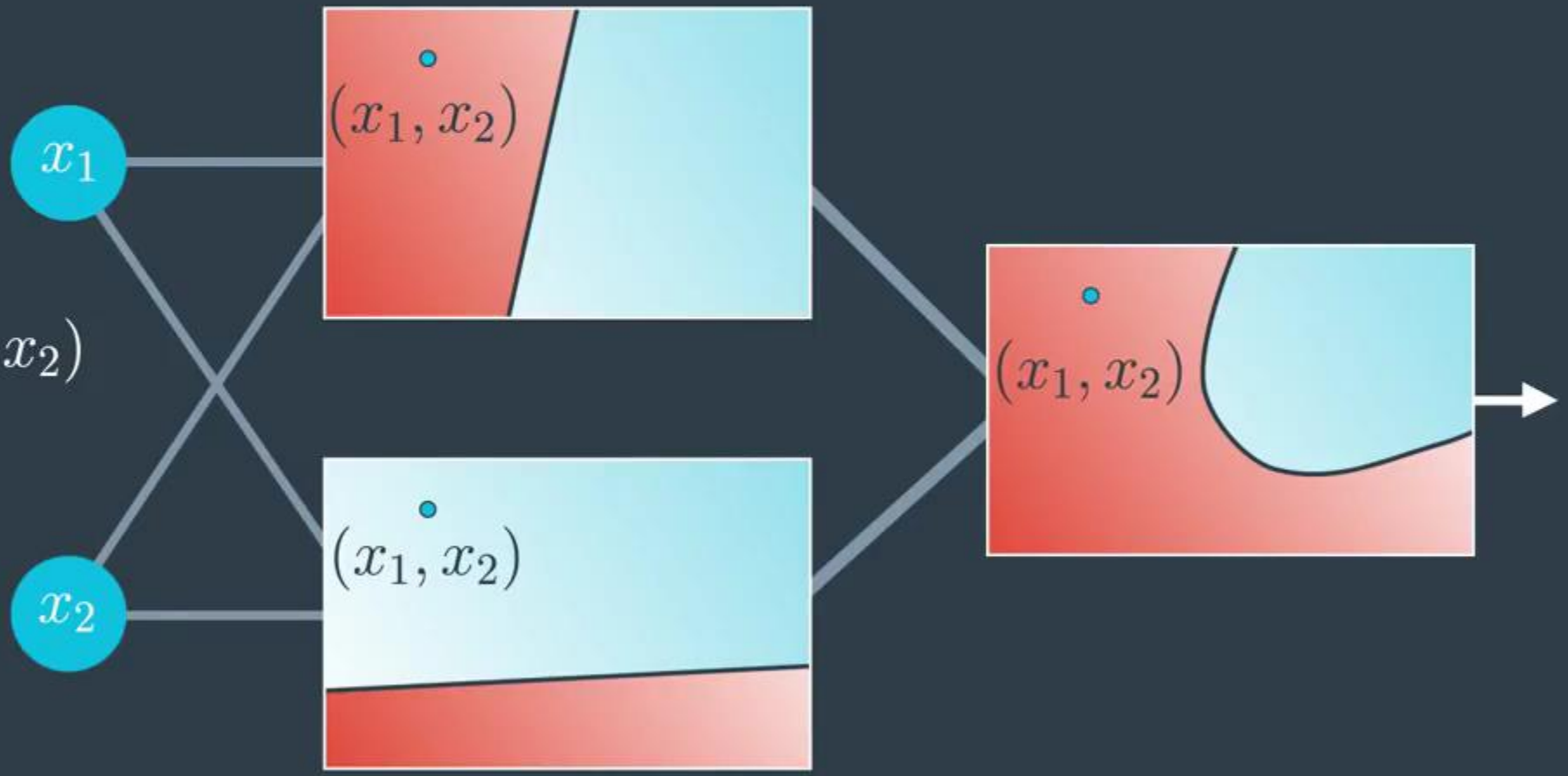


What about Multi-Layer Perceptrons?

FeedForward

$x = (x_1, x_2)$

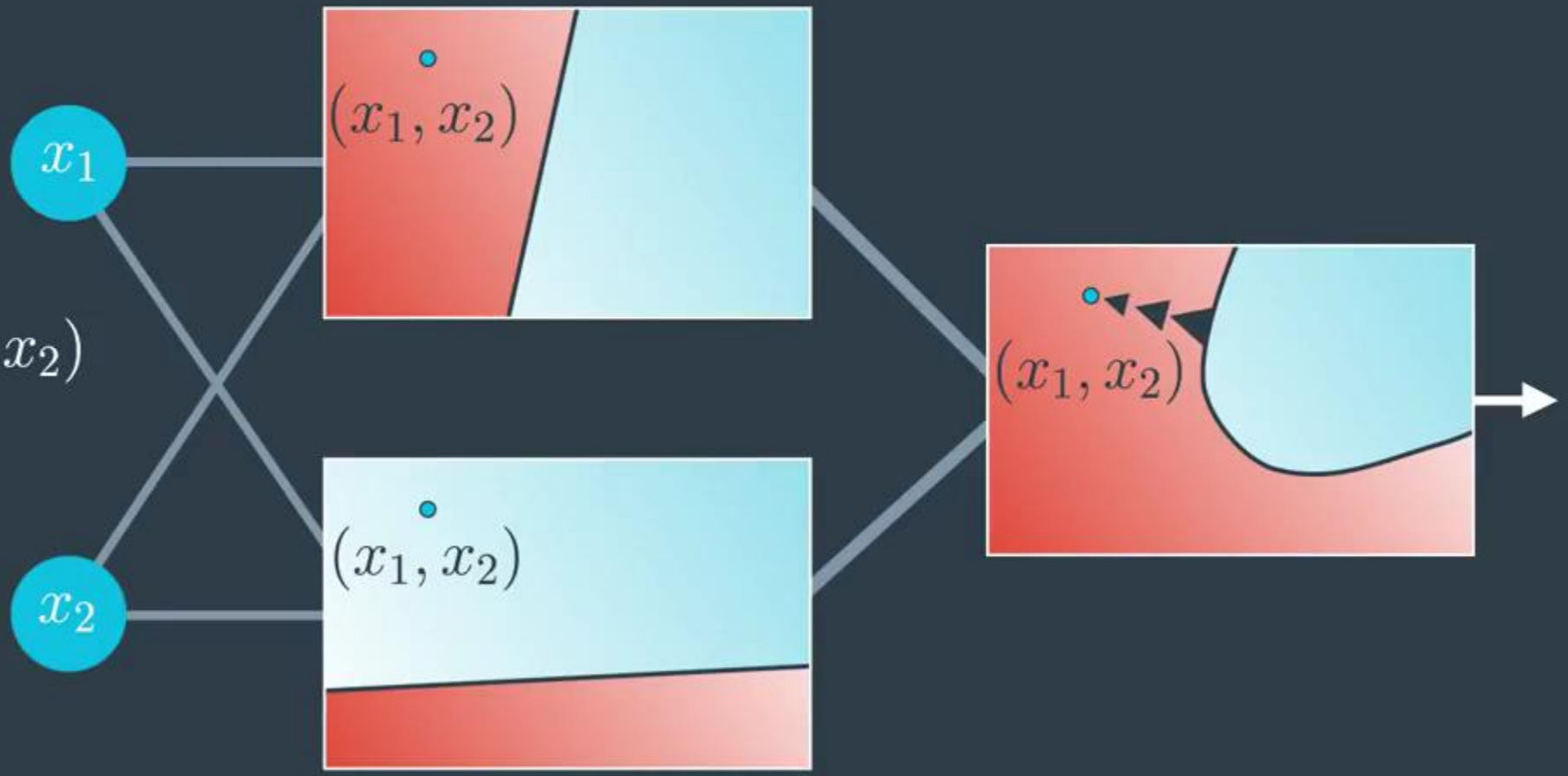
$y = 1$



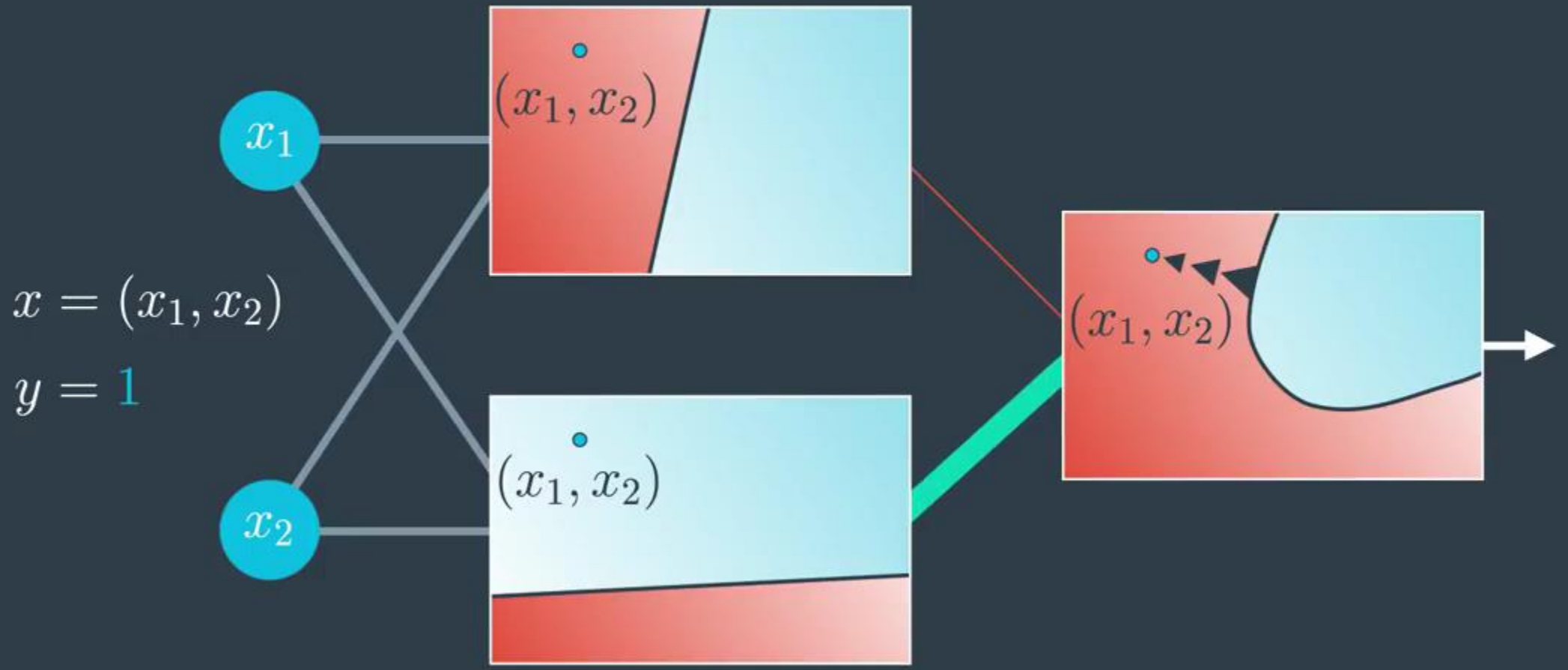
Backpropagation

$x = (x_1, x_2)$

$y = 1$



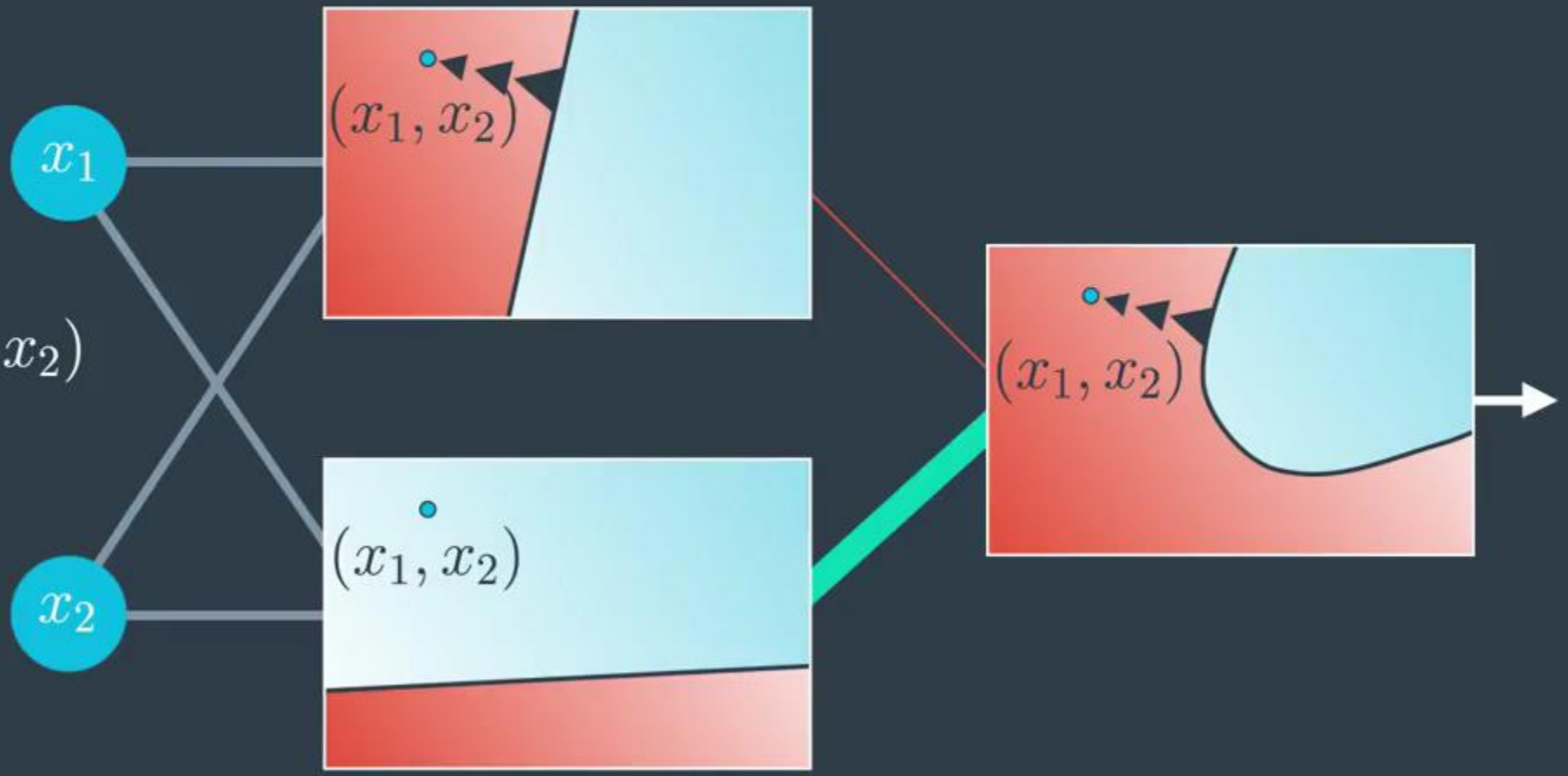
Backpropagation



Backpropagation

$x = (x_1, x_2)$

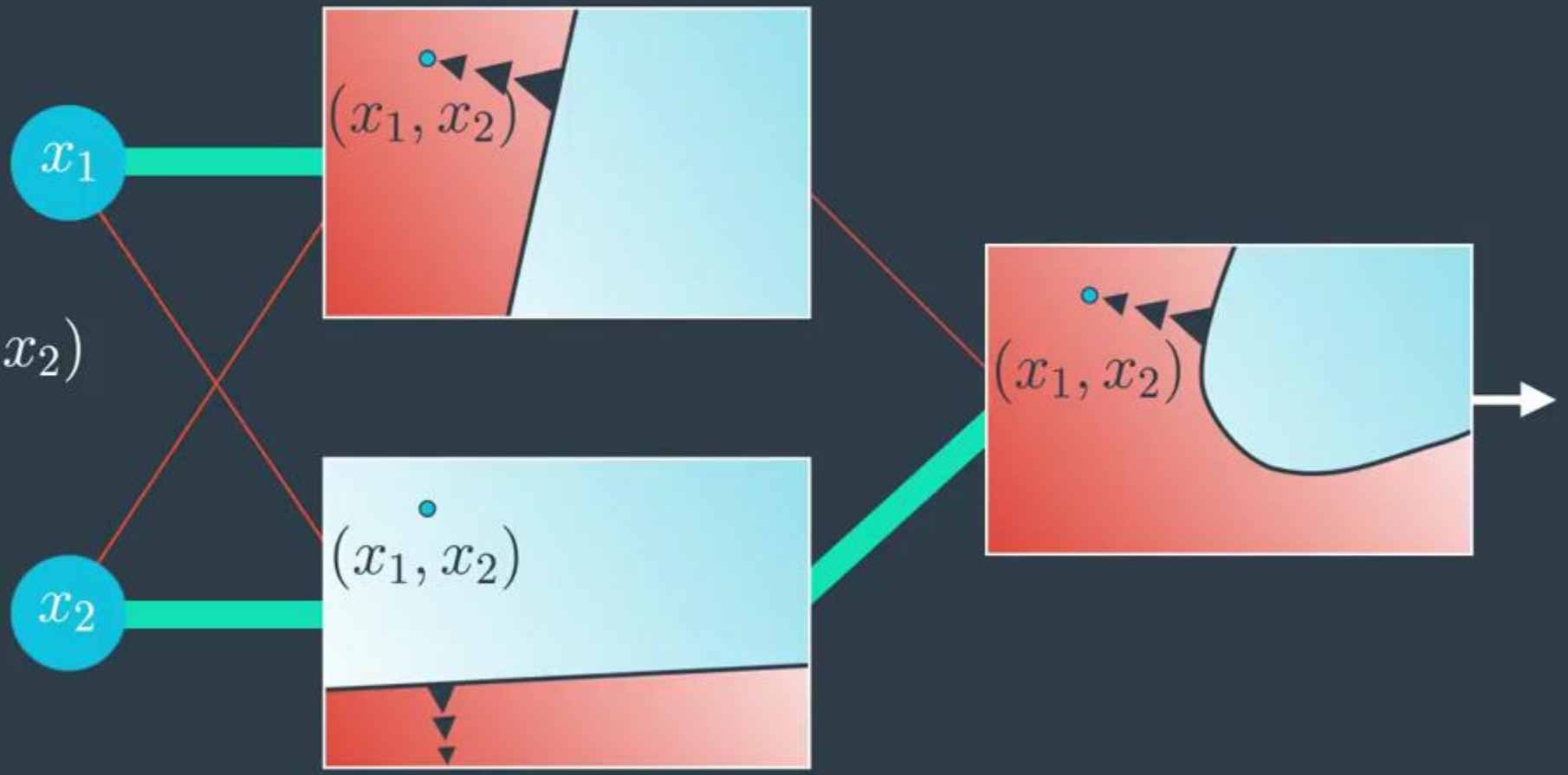
$y = 1$



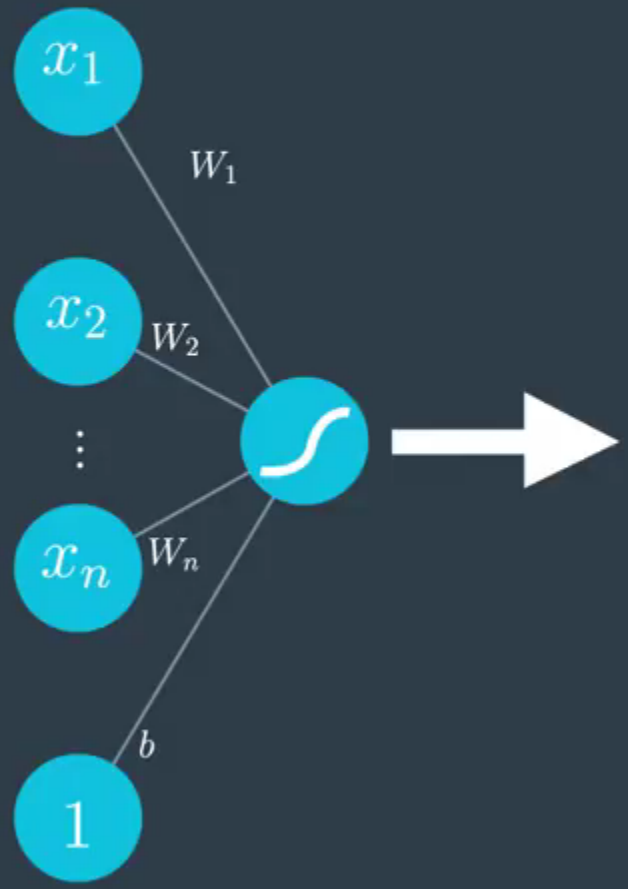
Backpropagation

$x = (x_1, x_2)$

$y = 1$



Perceptron



PREDICTION

$$\hat{y} = \sigma(Wx + b)$$

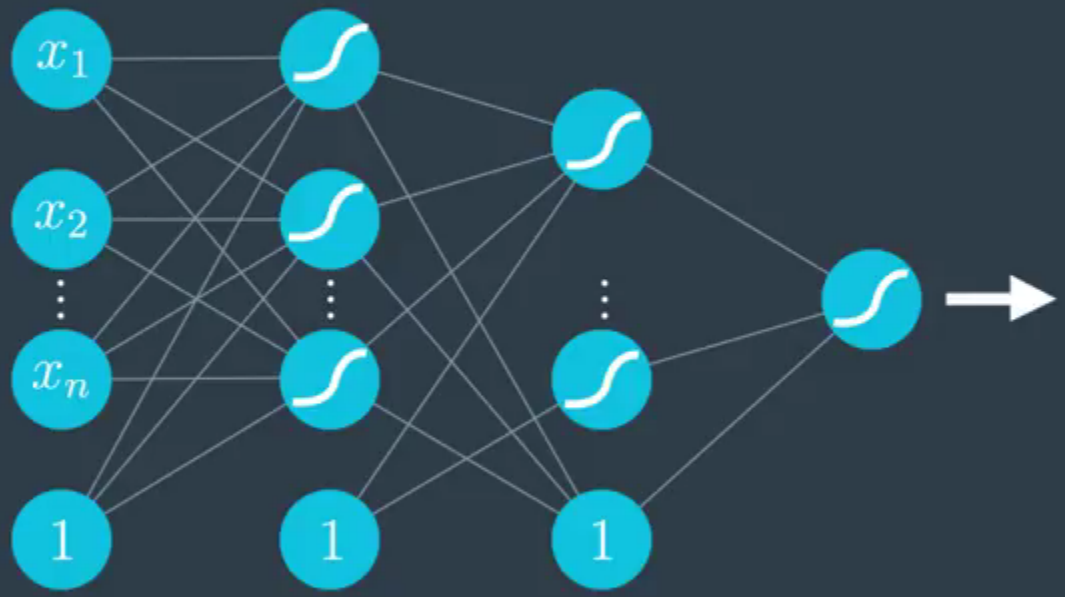
ERROR FUNCTION

$$E(W) = -\frac{1}{m}\sum_{i=1}^{m} y_i ln(\hat{y}_i) + (1 - y_i) ln(1 - \hat{y}_i)$$

GRADIENT OF THE ERROR FUNCTION

$$\nabla E = (\frac{\partial E}{\partial w_1}, ..., \frac{\partial E}{\partial w_n}, \frac{\partial E}{\partial b})$$

Multi-layer Perceptron



PREDICTION

$$\hat{y} = \sigma W^{(3)} \circ \sigma W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

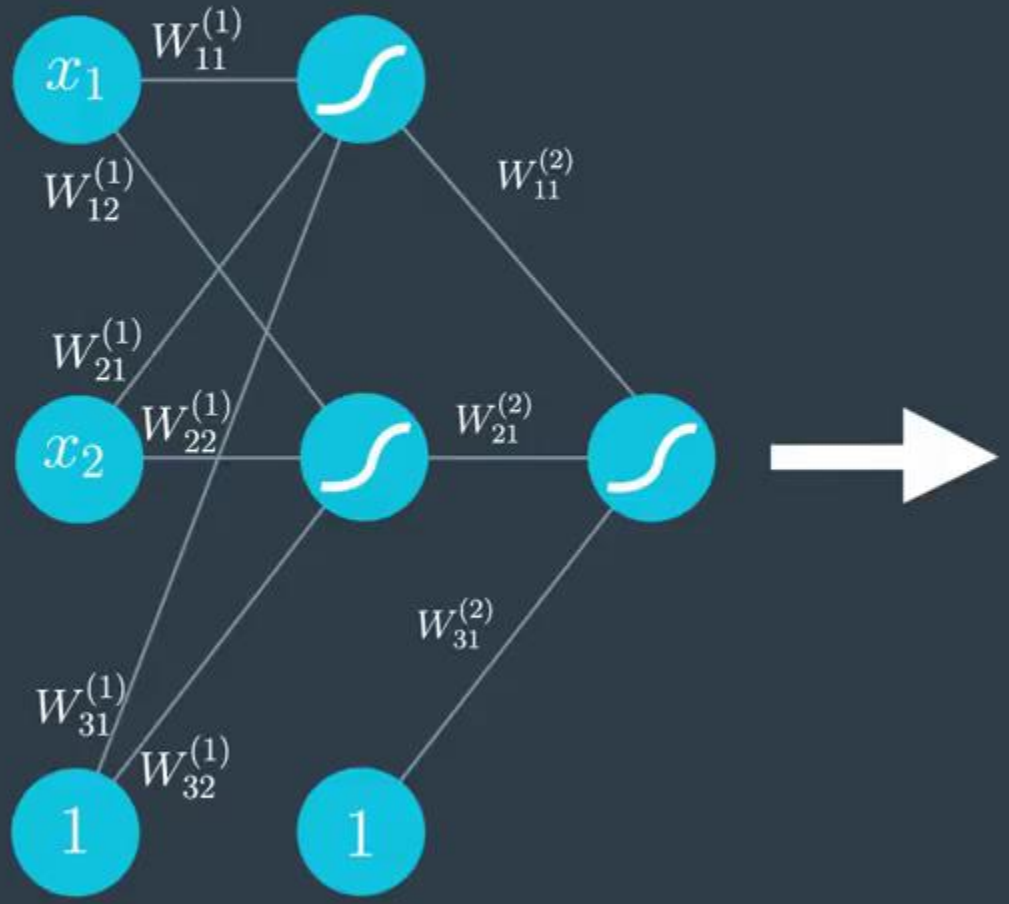
ERROR FUNCTION

$$E(W) = -\frac{1}{m} \sum_{i=1}^{m} y_i ln(\hat{y}_i) + (1 - y_i) ln(1 - \hat{y}_i)$$

GRADIENT OF THE ERROR FUNCTION

$$\bigtriangledown E = (..., \frac{\partial E}{\partial w_j^{(i)}}, ...)$$

Backpropagation



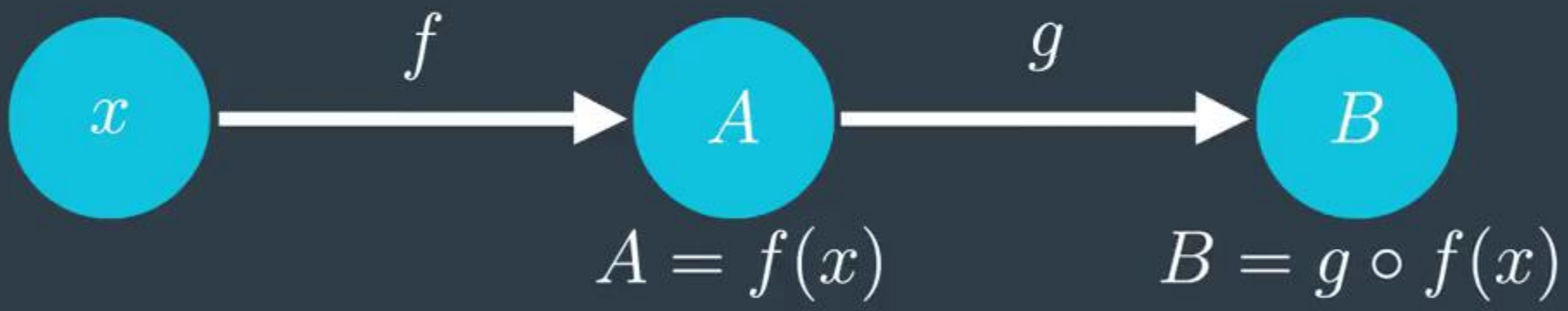
$$\hat{y} = \sigma W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

$$W^{(1)} = \begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \quad W^{(2)} = \begin{pmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{pmatrix}$$

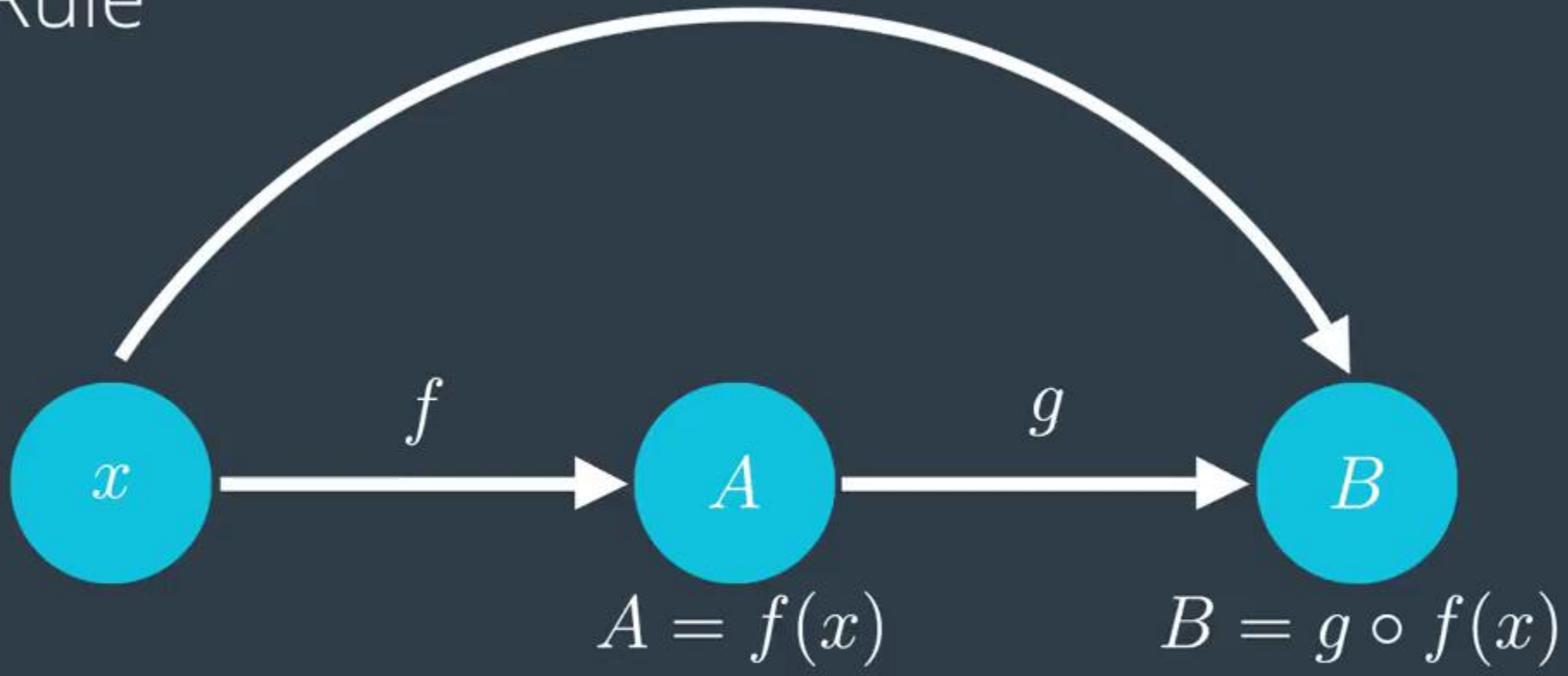
$$\nabla E = \begin{pmatrix} \frac{\partial E}{\partial W_{11}^{(1)}} & \frac{\partial E}{\partial W_{12}^{(1)}} & \frac{\partial E}{\partial W_{11}^{(2)}} \\ \frac{\partial E}{\partial W_{21}^{(1)}} & \frac{\partial E}{\partial W_{22}^{(1)}} & \frac{\partial E}{\partial W_{21}^{(2)}} \\ \frac{\partial E}{\partial W_{31}^{(1)}} & \frac{\partial E}{\partial W_{32}^{(1)}} & \frac{\partial E}{\partial W_{31}^{(2)}} \end{pmatrix}$$

$$W_{ij}^{\prime(k)} \leftarrow W_{ij}^{(k)} - \alpha \frac{\partial E}{\partial W_{ij}^{(k)}}$$

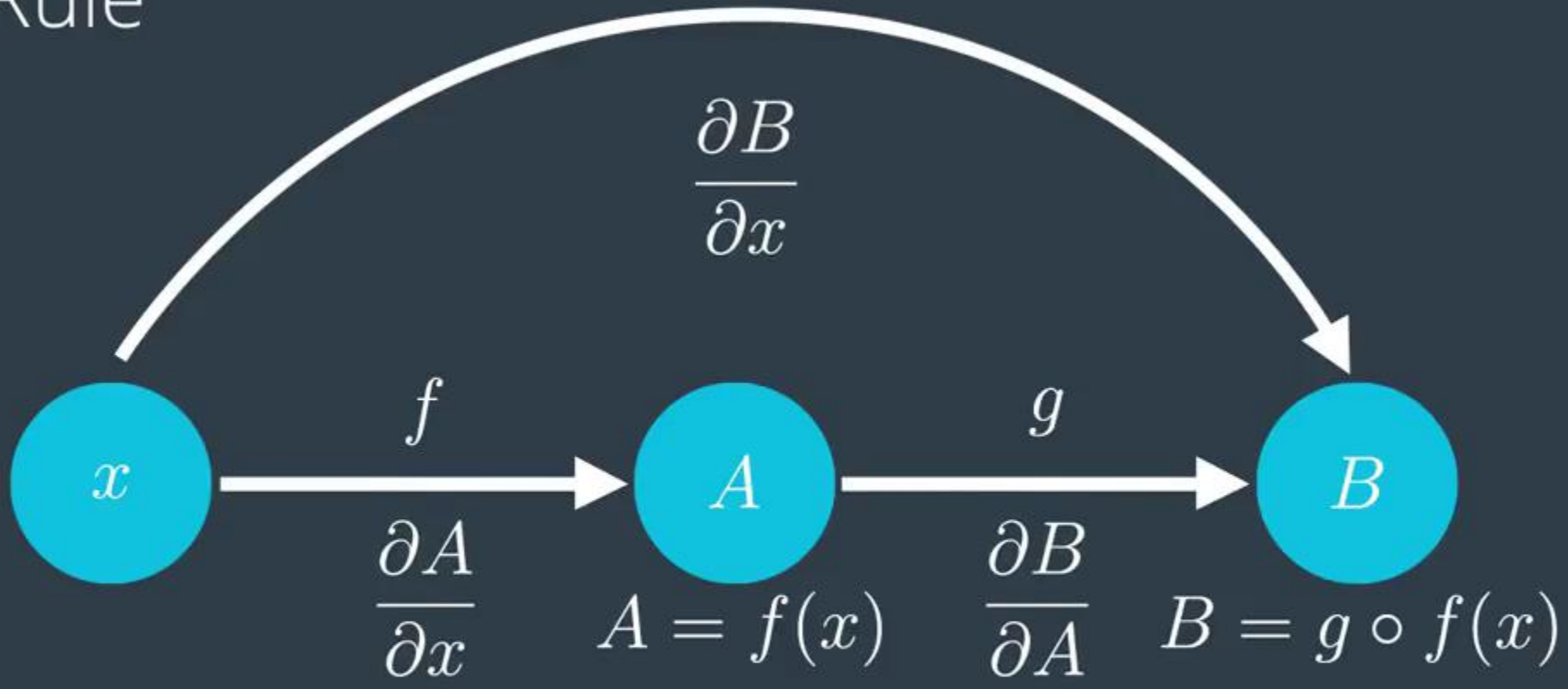
Chain Rule



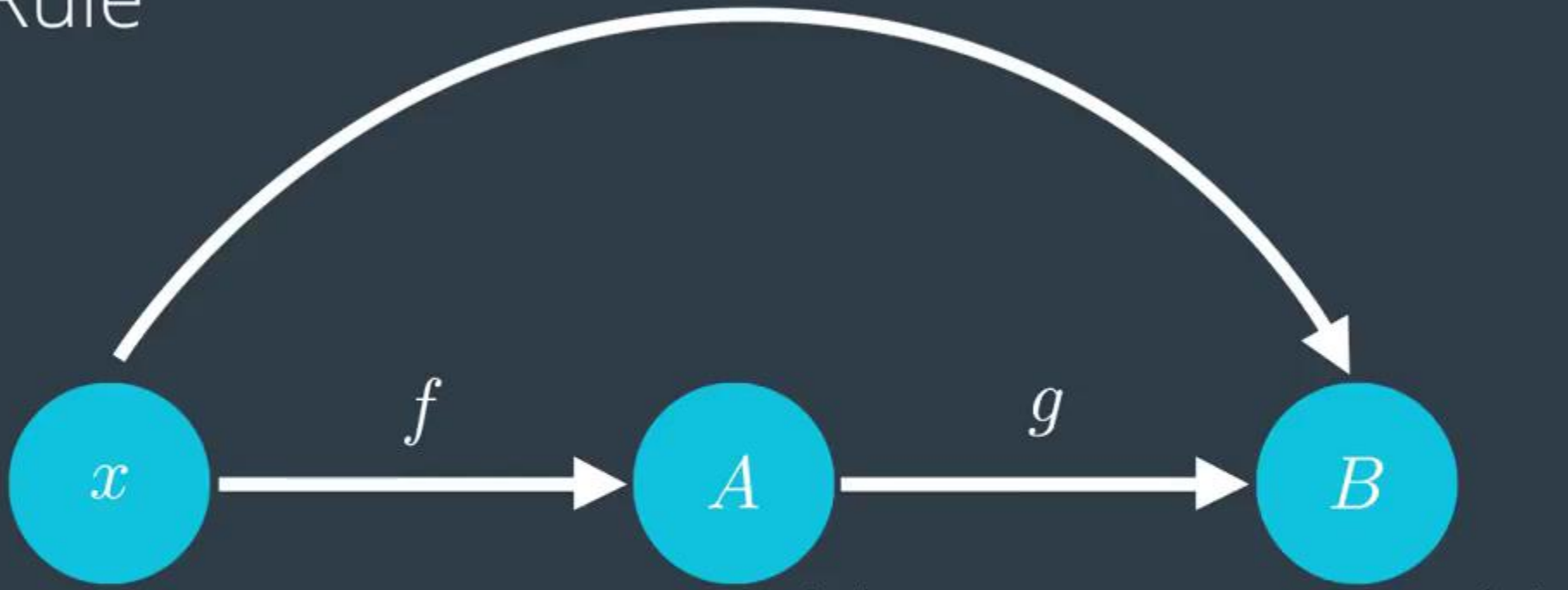
Chain Rule



Chain Rule

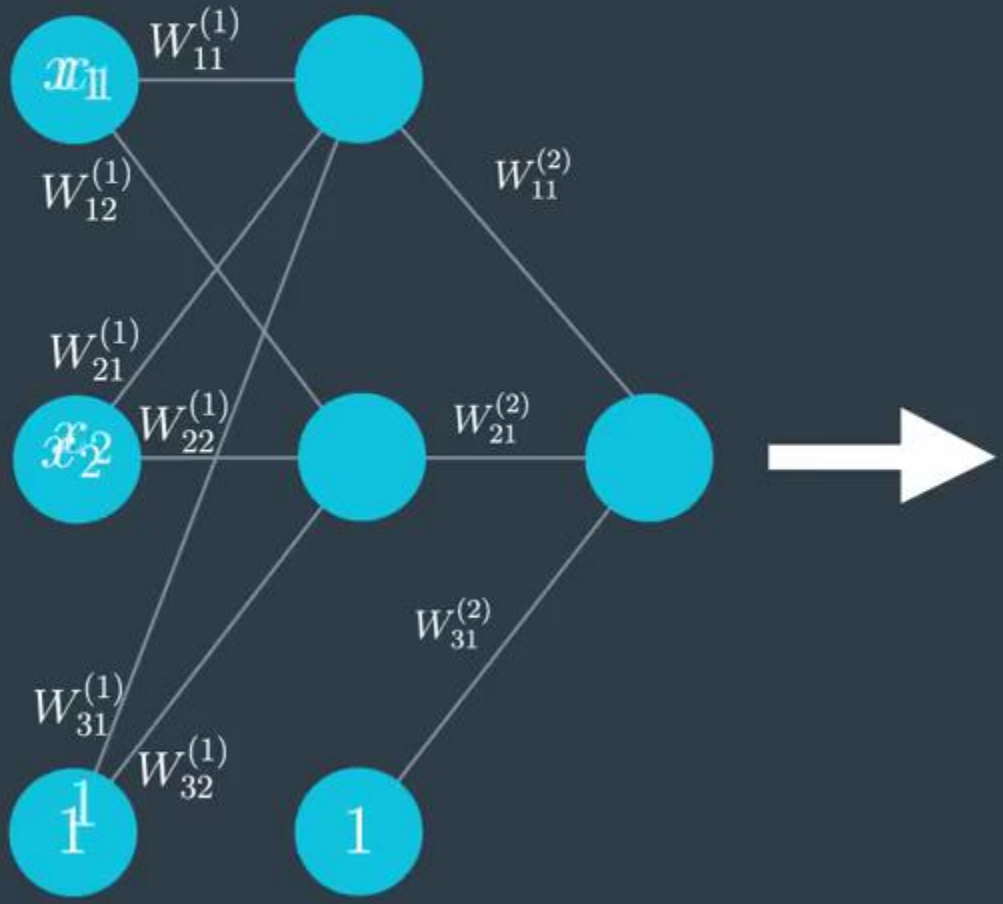

$\frac{\partial B}{\partial x}$
x
f
A
g
B
$\frac{\partial A}{\partial x}$
$A = f(x)$
$\frac{\partial B}{\partial A}$
$B = g \circ f(x)$

Chain Rule



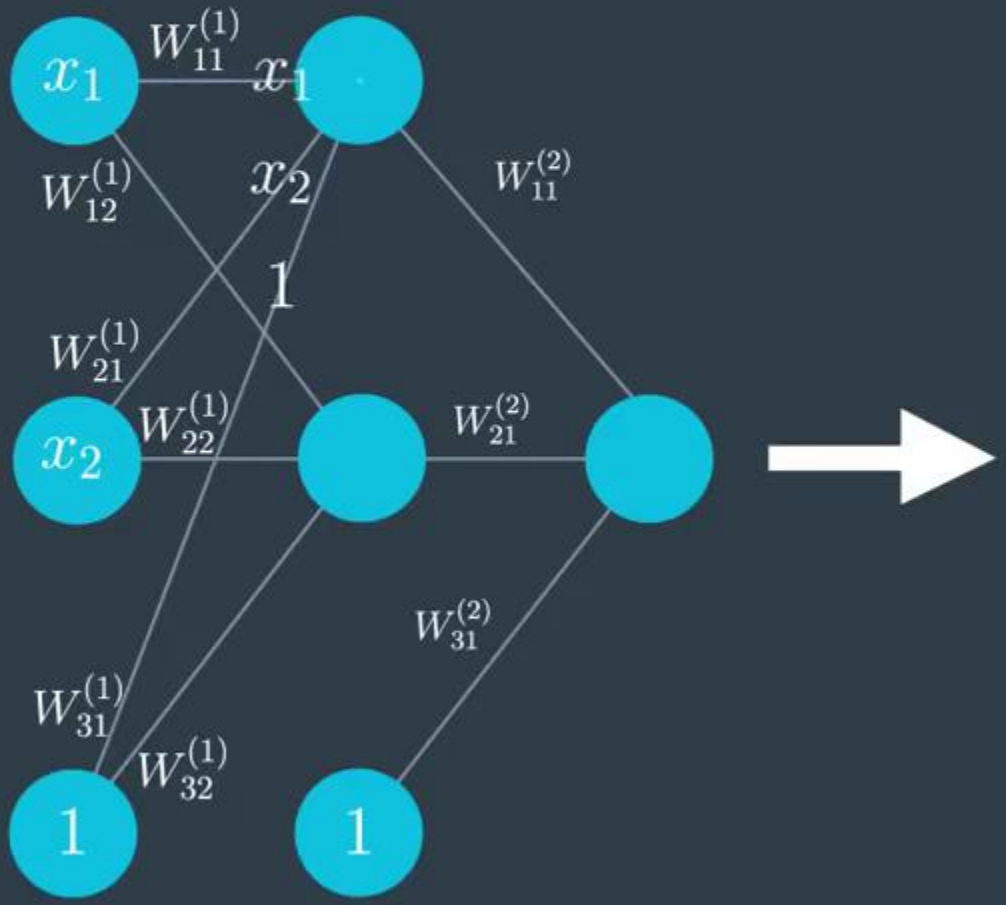
$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial A}\frac{\partial A}{\partial x}$$

Feedforward



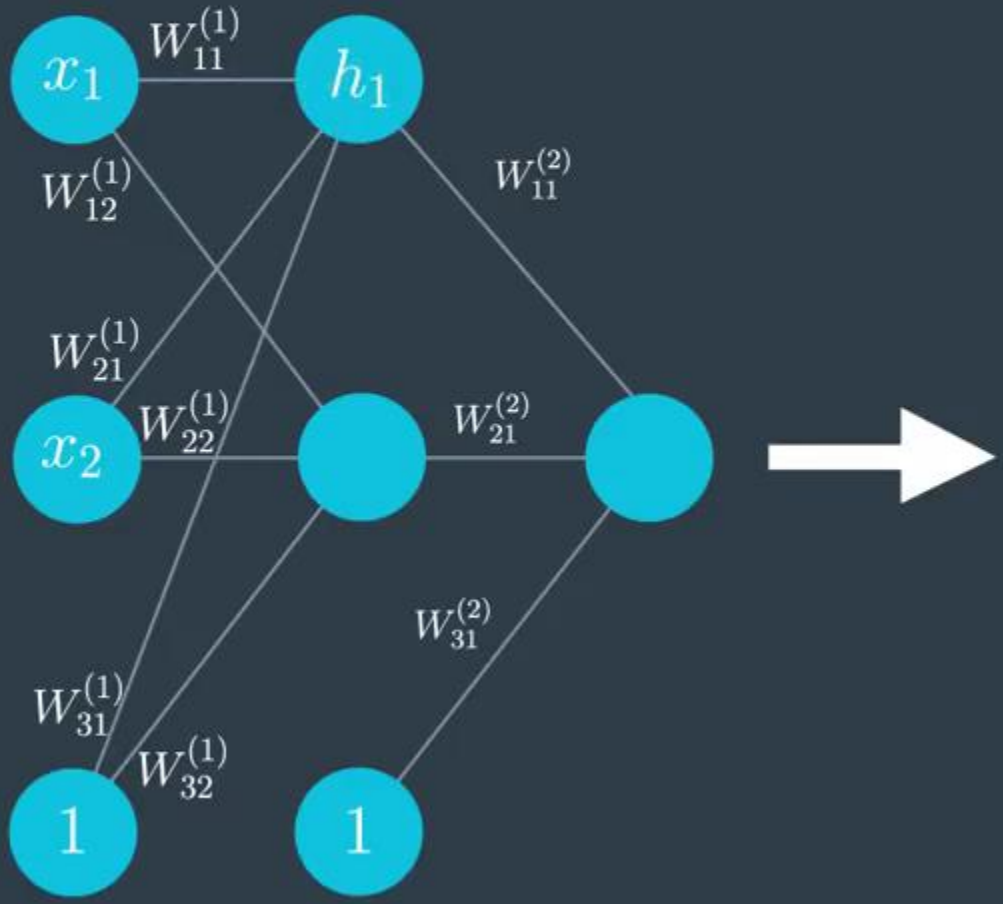
$$h_1 = W_{11}^{(1)} x_1 + W_{21}^{(1)} x_2 + W_{31}^{(1)}$$

Feedforward



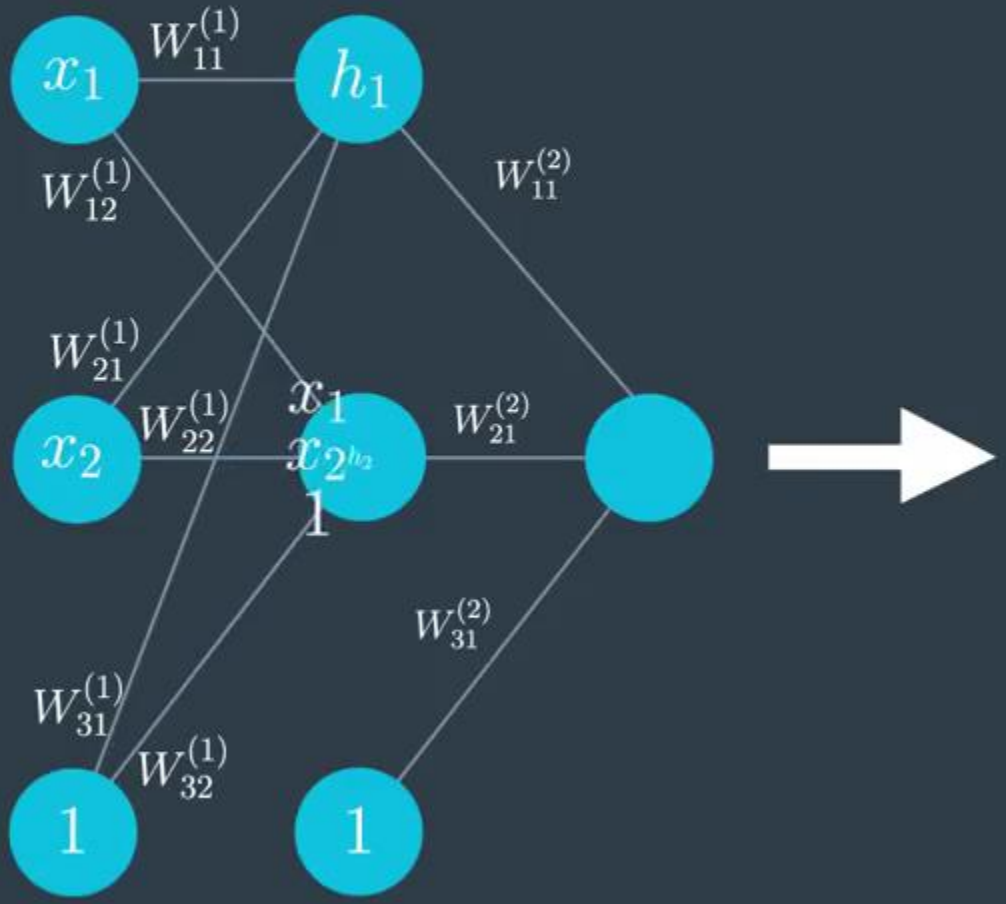
$$h_1 = W_{11}^{(1)} x_1 + W_{21}^{(1)} x_2 + W_{31}^{(1)}$$

Feedforward



$$h_1 = W_{11}^{(1)} x_1 + W_{21}^{(1)} x_2 + W_{31}^{(1)}$$

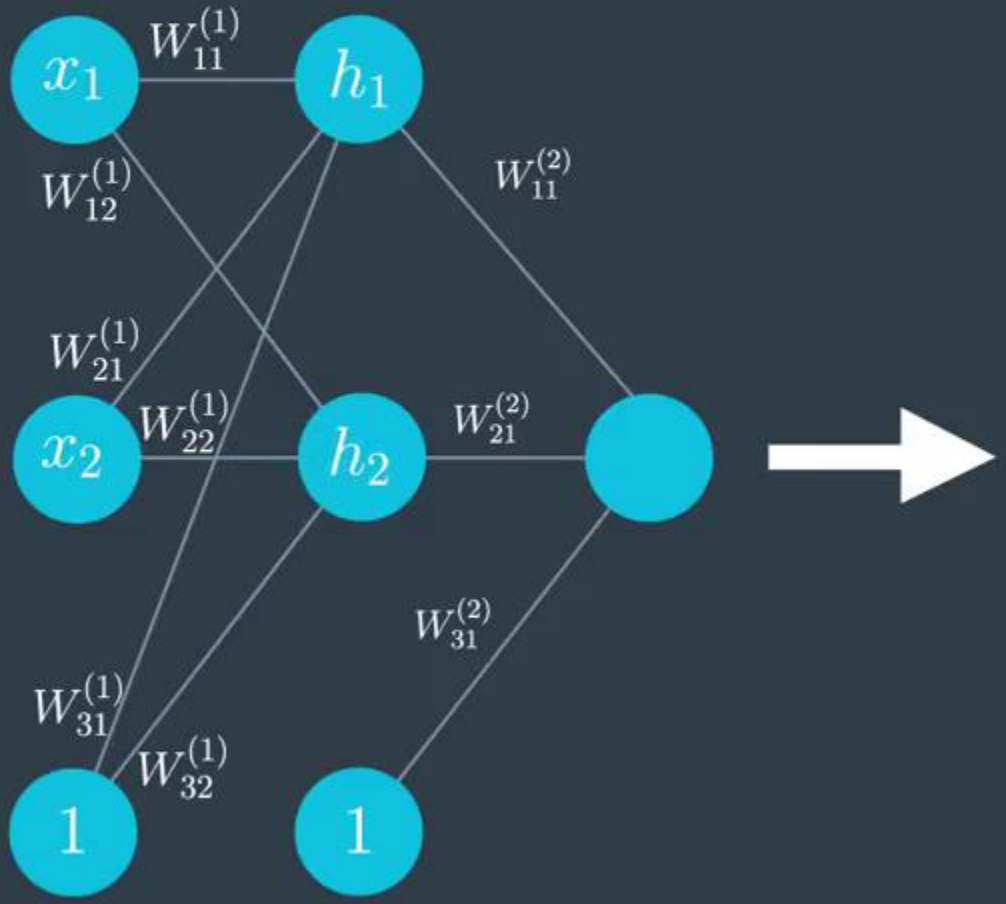
Feedforward



$$h_1 = W_{11}^{(1)} x_1 + W_{21}^{(1)} x_2 + W_{31}^{(1)}$$

$$h_2 = W_{12}^{(1)} x_1 + W_{22}^{(1)} x_2 + W_{32}^{(1)}$$

Feedforward



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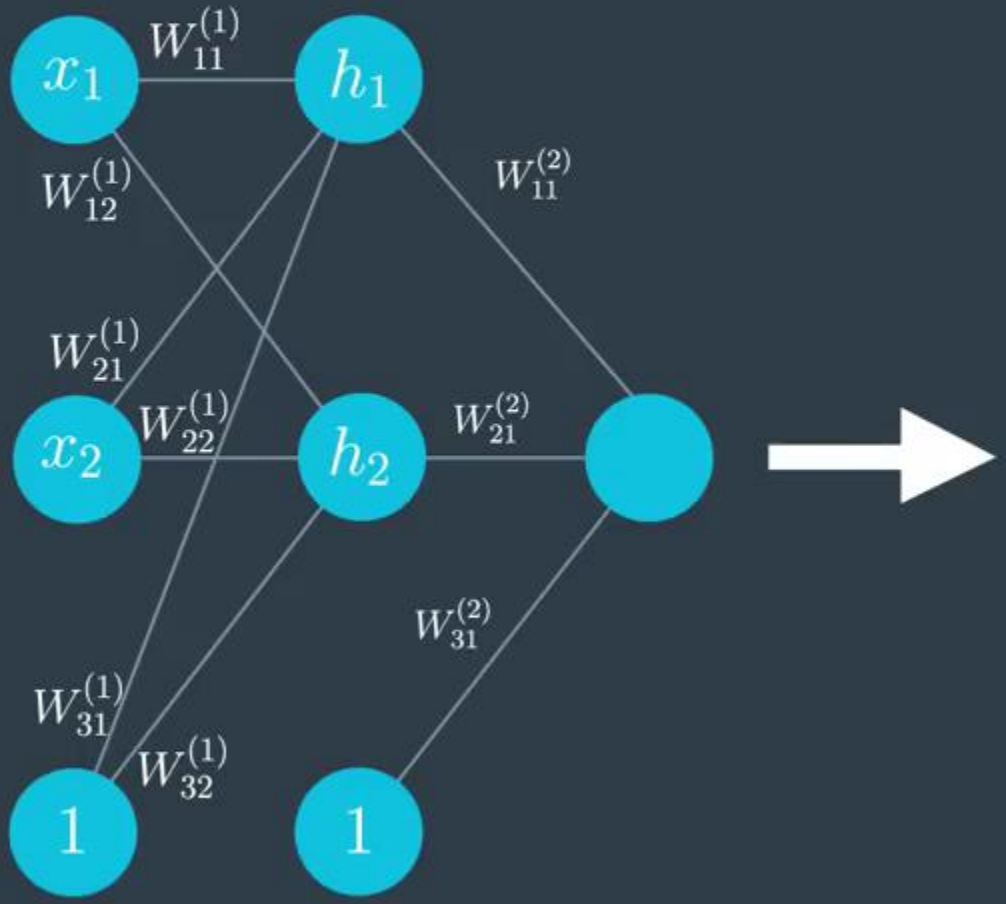
Feedforward



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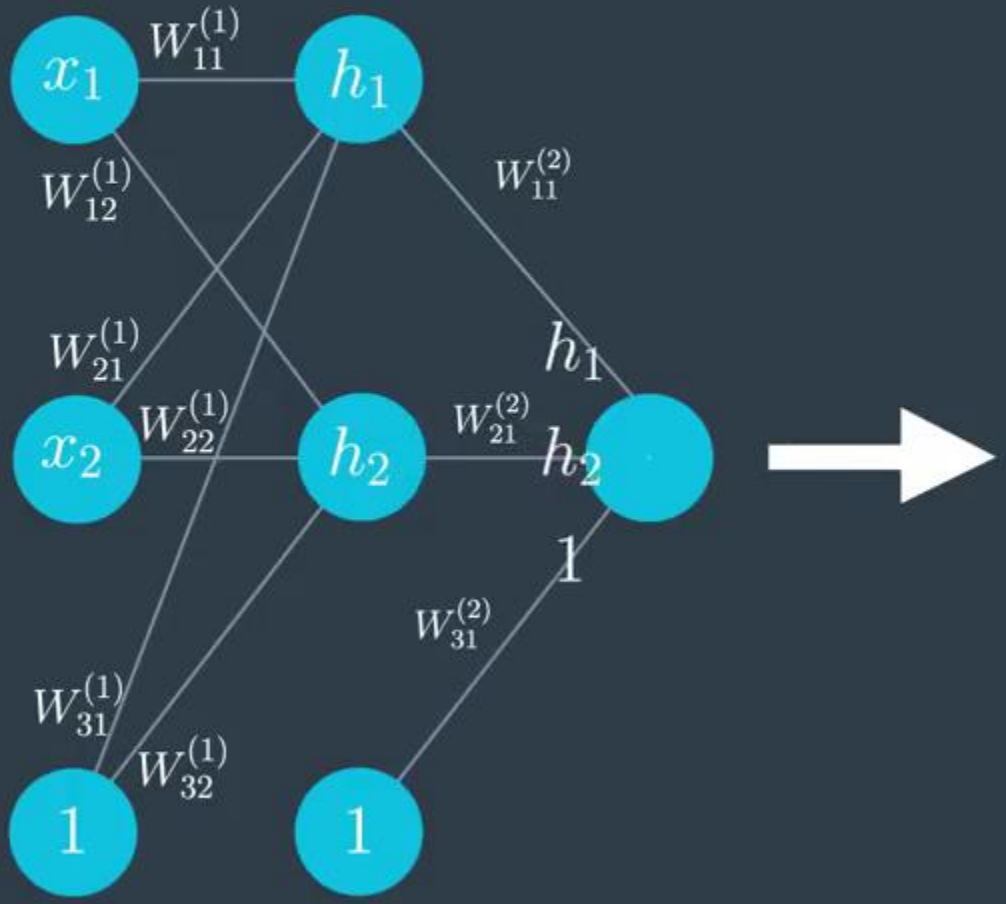
$$h_2 = W_{12}^{(1)} x_1 + W_{22}^{(1)} x_2 + W_{32}^{(1)}$$

$$h = W_{11}^{(2)} \sigma(h_1) + W_{21}^{(2)} \sigma(h_2) + W_{31}^{(2)}$$

Feedforward



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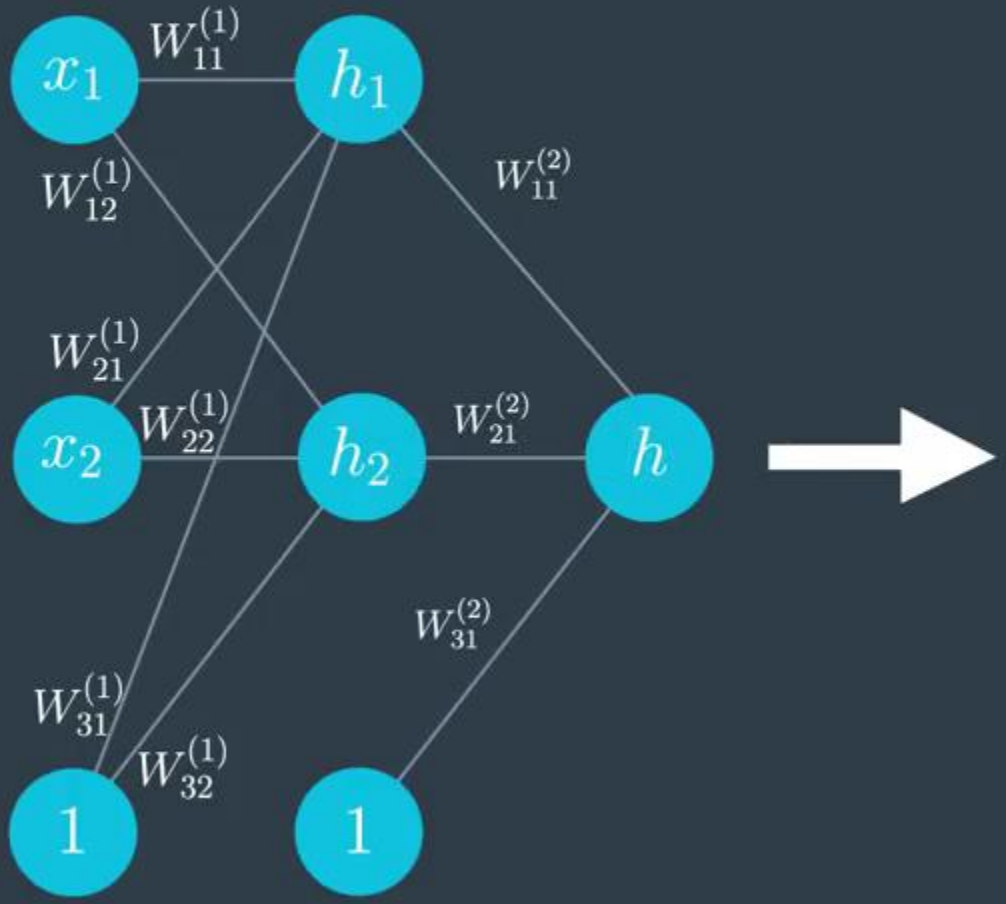
Feedforward



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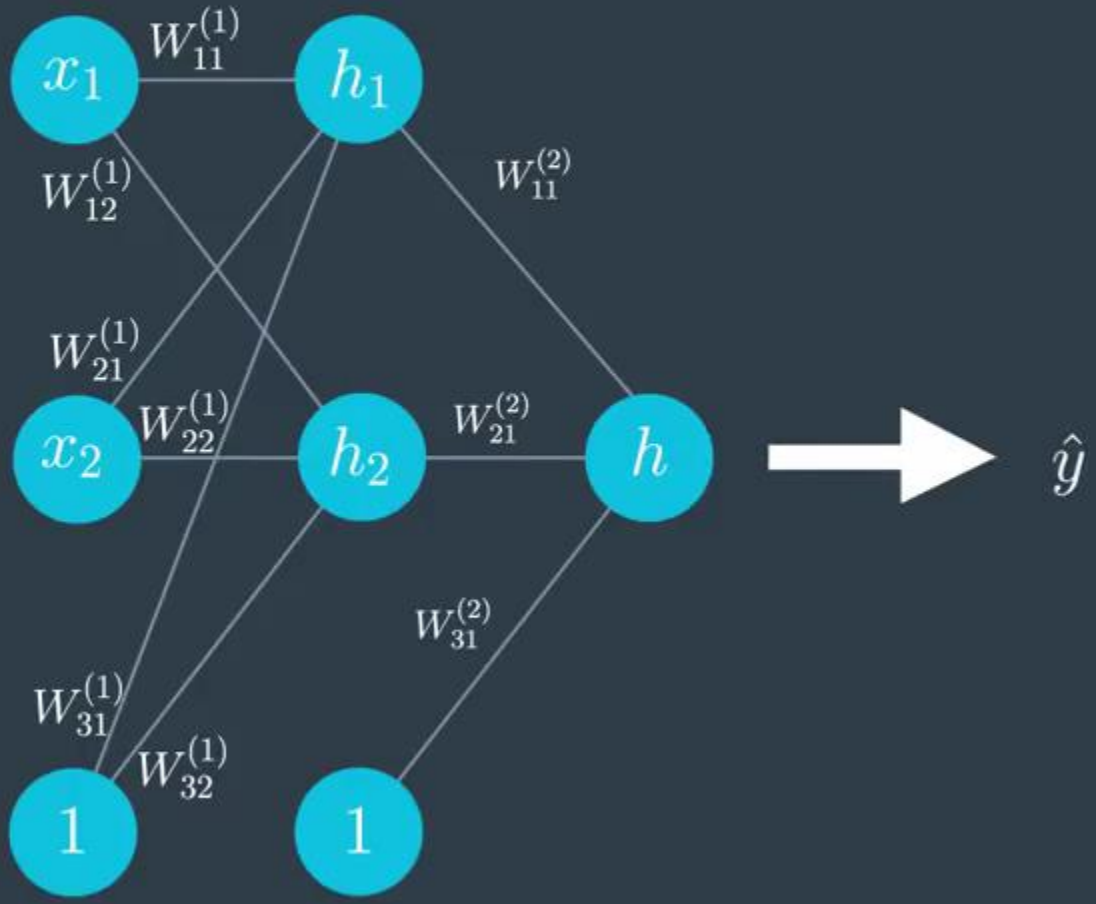
Feedforward



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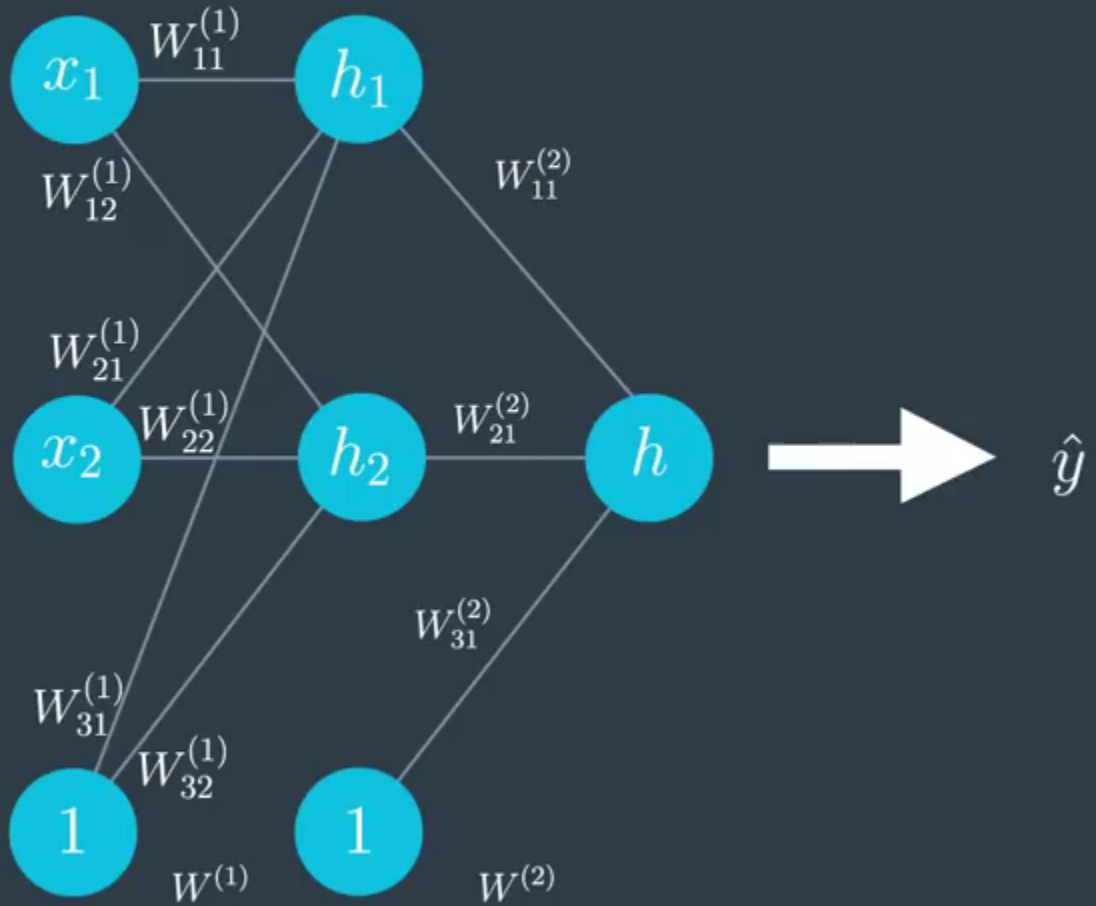
$$h = W_{11}^{(2)} \sigma(h_1) + W_{21}^{(2)} \sigma(h_2) + W_{31}^{(2)}$$

$$\hat{y} = \sigma(h)$$

Feedforward



$$h_1 = W_{11}^{(1)} x_1 + W_{21}^{(1)} x_2 + W_{31}^{(1)}$$

$$h_2 = W_{12}^{(1)} x_1 + W_{22}^{(1)} x_2 + W_{32}^{(1)}$$

$$h = W_{11}^{(2)} \sigma(h_1) + W_{21}^{(2)} \sigma(h_2) + W_{31}^{(2)}$$

$$\hat{y} = \sigma(h)$$

$$\hat{y} = \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

Sigmoid Derivative

$$
\begin{aligned}
\sigma'(x) &= \frac{\partial}{\partial x}\frac{1}{1+e^{-x}} \\
&= \frac{e^{-x}}{(1+e^{-x})^2} \\
&= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} \\
&= \sigma(x)(1-\sigma(x)) \\
&= \sigma(h)(1-\sigma(h))
\end{aligned}
$$

$$\frac{dh}{dz} = h(1-h)$$

Log loss derivative

$$L(h, y) = -[\, y \log(h) + (1-y) \log(1-h) \,]$$

$$\frac{d(C \log(x))}{dx} = \frac{C}{x}$$

$$\therefore L(h, y) = -\frac{y}{h} - \frac{1-y}{1-h}$$

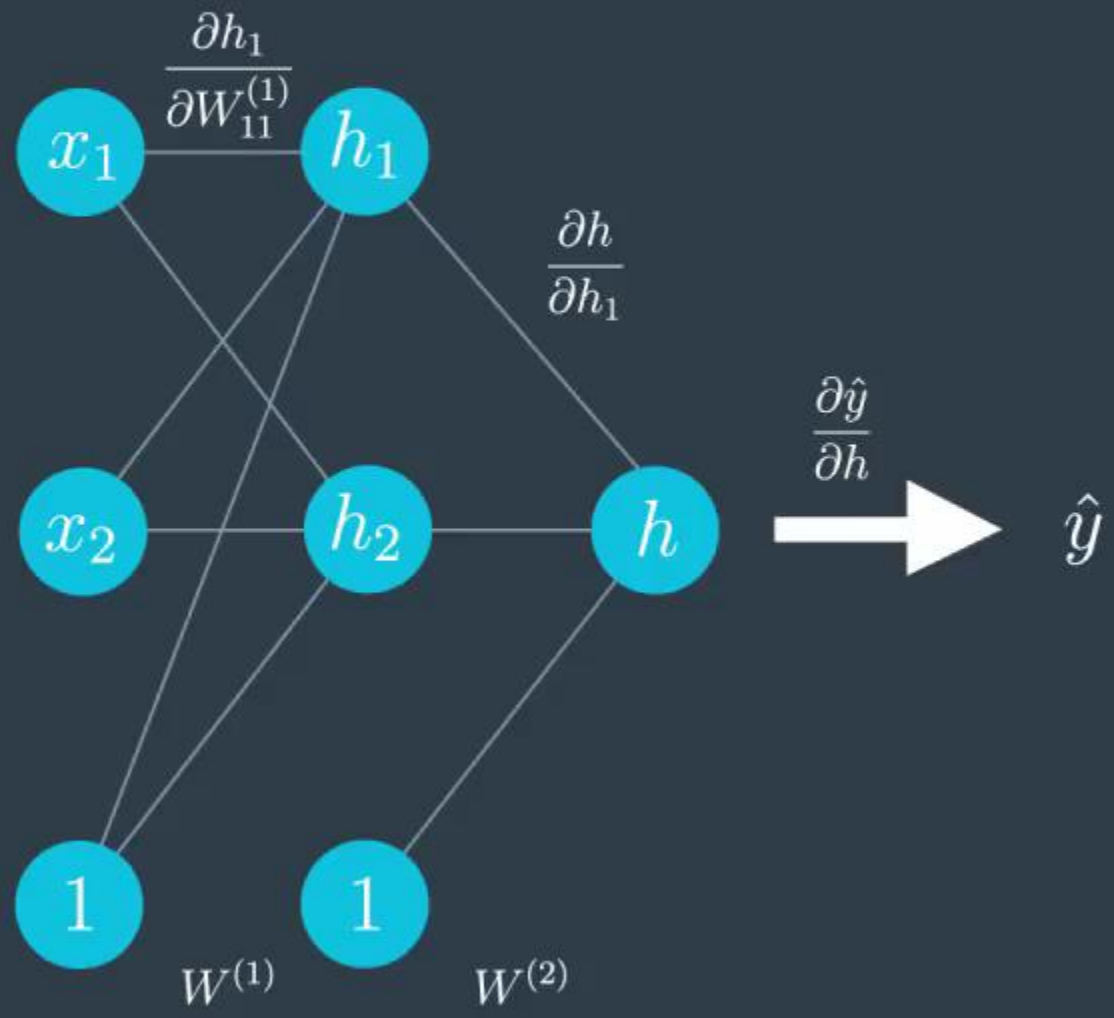
By using Chain Rule

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial z}$$

$$\frac{\partial L}{\partial z} = \left(-\frac{y}{h} - \frac{1-y}{1-h}\right)\left(h(1-h)\right)$$

$$= -y(1-h) - (1-y)h$$

BACKPROPAGATION

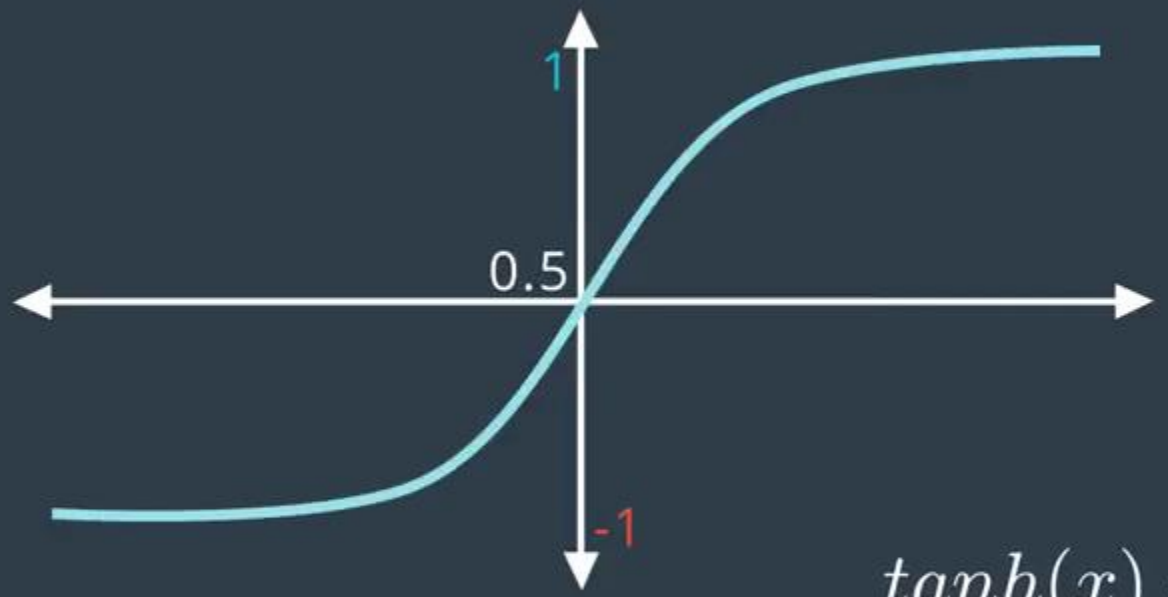


$$\frac{\partial E}{\partial W_{11}^{(1)}} = \frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h} \frac{\partial h}{\partial h_1} \frac{\partial h_1}{\partial W_{11}^{(1)}}$$

TINY

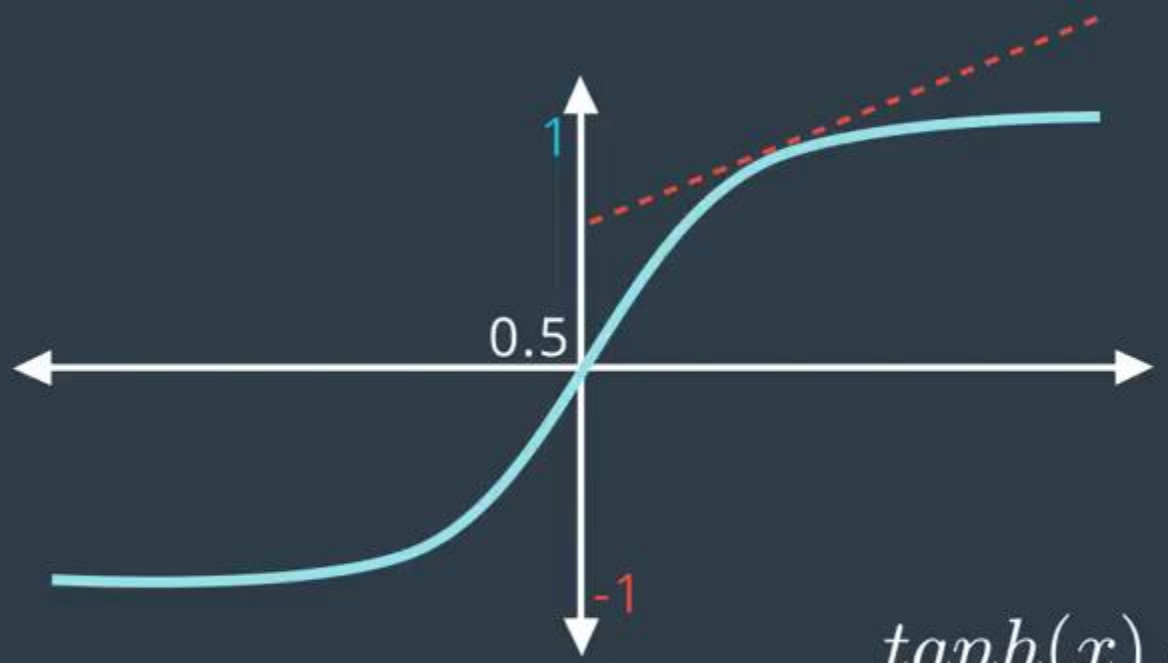
SMALL

HYPERBOLIC TANGENT FUNCTION



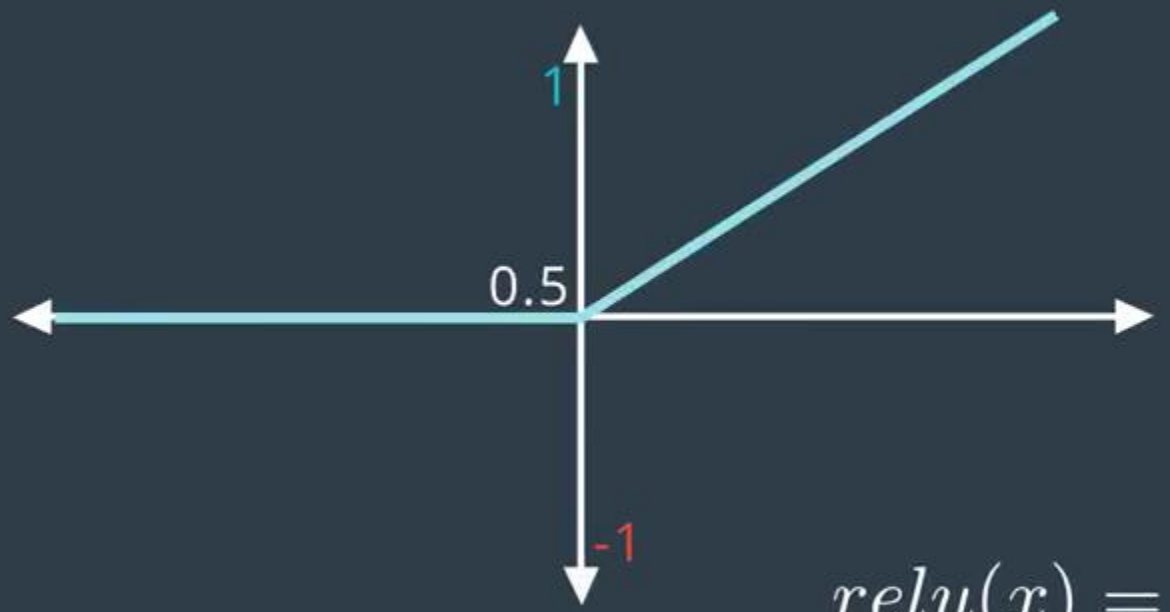
$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

HYPERBOLIC TANGENT FUNCTION



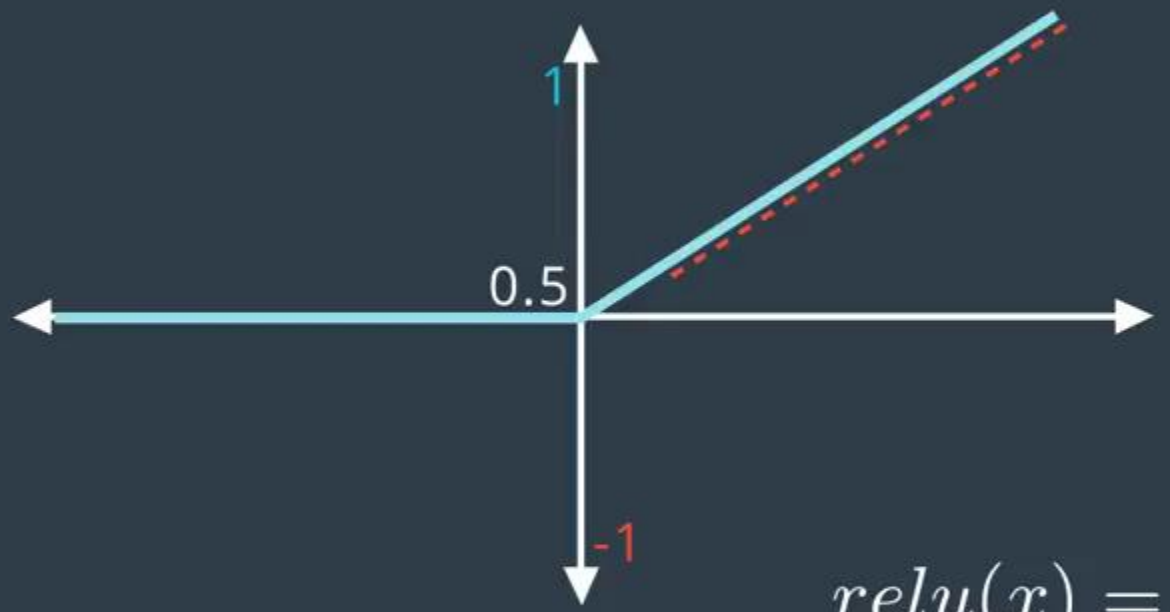
$$tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

RECTIFIED LINEAR UNIT (ReLU)



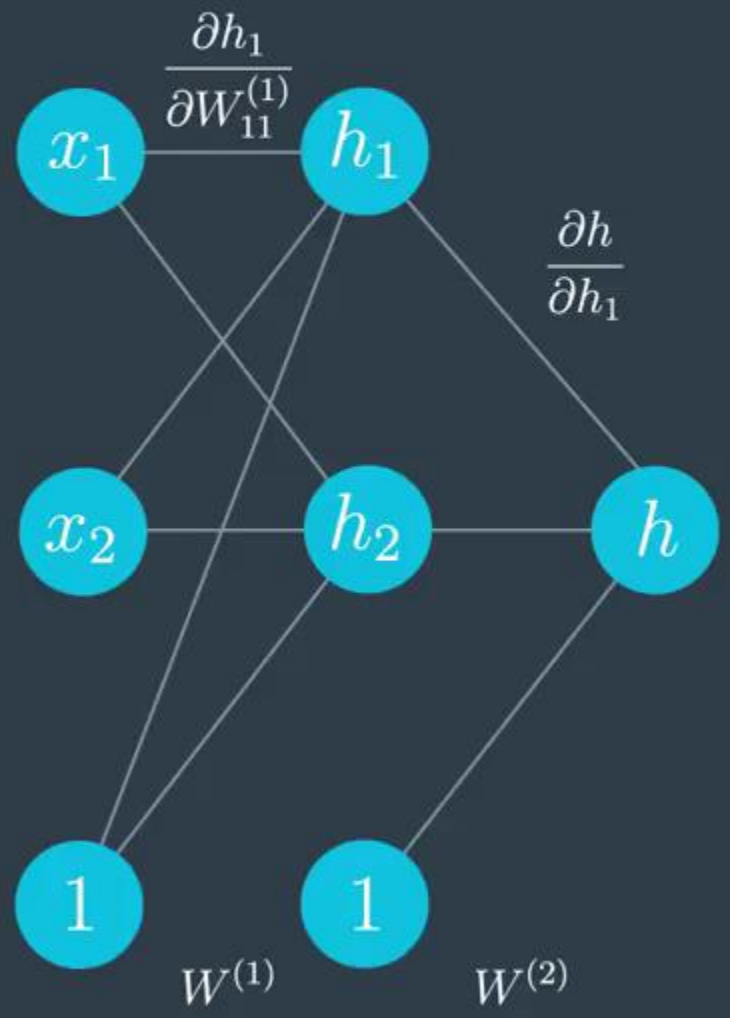
$$relu(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

RECTIFIED LINEAR UNIT (ReLU)



$$relu(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

BACKPROPAGATION



$$\boxed{\frac{\partial E}{\partial W_{11}^{(1)}}} = \boxed{\frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h} \frac{\partial h}{\partial h_1} \frac{\partial h_1}{\partial W_{11}^{(1)}}}$$

OK!

OK