CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a) i.
   3(x+yj) + 4 = 2j - (x-yj)
   4x + 2yj = 2j - 4
   Hence; x=-1, y=1 and z = -1 + j
   $|z|^2 = x^2 + y^2$=1+1=2

   ii.

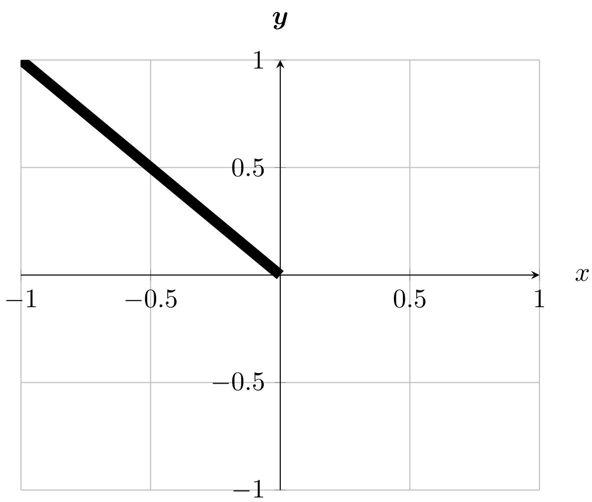

Figure 1: x vs y which shows z = -1 + j

   (b) 64j = $r^3(\cos(3\theta)$ + j.$\sin(3\theta))$
   $\cos(3\theta)$=0 must hold for above equality to hold since real part of left-side is equal to zero.
   $\theta = \{\pi/6, \pi/2,\ 5\pi/6,\ 7\pi/6,\ 9\pi/6,\ 11\pi/6\ \}$

   For $\theta = \pi/6$; 64j = r^3.j.1, r=4
   For $\theta = \pi/2$; 64j = r^3.j.-1, r=-4 which is impossible. So, θ can not be $\pi/2$
   For $\theta = 5\pi/6$; 64j = r^3.j.1, r=4
   For $\theta = 7\pi/6$; 64j = r^3.j.-1, r=-4 which is impossible. So, θ can not be $7\pi/6$
   For $\theta = 9\pi/6$; 64j = r^3.j.1, r=4
   For $\theta = 11\pi/6$; 64j = r^3.j.-1, r=-4 which is impossible. So, θ can not be $11\pi/6$

   So,
   z = 4.$e^{j\pi/6}$ = 4($\cos(\pi/6)$ + j.$\sin(\pi/6)$) or,
   z = 4.$e^{5j\pi/6}$ = 4($\cos(5\pi/6)$ + j.$\sin(5\pi/6)$) or,
   z = 4.$e^{3j\pi/2}$ = 4($\cos(3\pi/2)$ + j.$\sin(3\pi/2)$)

   (c) Let us expand the fraction by (1-j):
   z = $\dfrac{(1-j)^2.(1+\sqrt{3}j)}{1-j^2} = \dfrac{(-2j).(1+\sqrt{3}j)}{2} = \sqrt{3} - j$
   Let r denote the magnitude of z.

So, r = $\sqrt{(-1)^2 + (\sqrt{3})^2} = 2$ is the magnitude.

Let $\theta \in [0, 2\pi)$ be the angle of z.
So, $\tan(\theta) = \frac{-1}{\sqrt{3}}$,
$\theta = tan^{-1}(\frac{-1}{\sqrt{3}}) = \frac{11\pi}{6}$

(d) z = -j(cos(π/2) + j.sin(π/2))
z = -j.cos(π/2) + -j^2.sin(π/2)
z = sin(π/2) - j.cos(π/2) = 1
So, for z = r(cos(θ) + j.sin(θ)),
1 = 1.(cos(0) + j.sin(0)) holds for r=1, θ=0 where $\theta \in [0, 2\pi)$
Hence, z=cos(0) + j.sin(0) in polar form.

2. Here, To derive y(t) we use x(t) doing time scaling by 2 (since t is divided by 2) and time shifting by 2 (x($\frac{t}{2}$+1), "+1" results in time shift but by amount of time scaling).

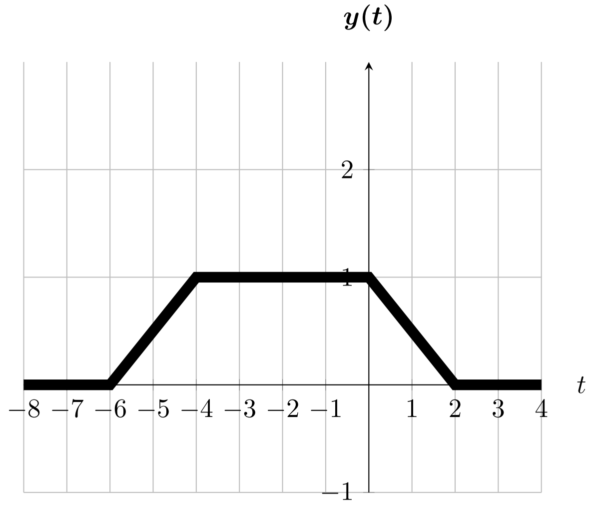

Figure 2: t vs. $y(t)$

3. (a) Below Figure 3 is the answer for this question

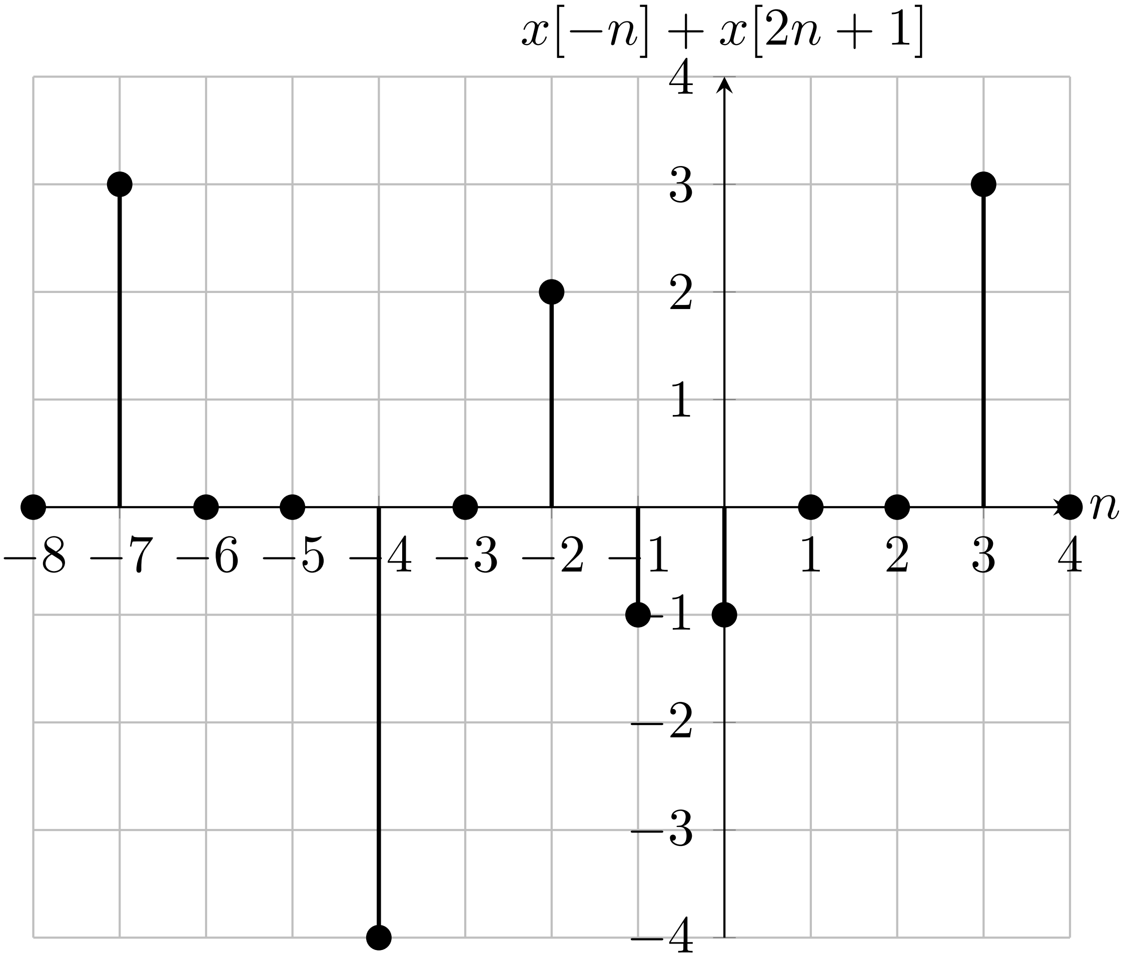

Figure 3: n vs. $x[-n] + x[2n+1]$.

(b) $x[-n] + x[2n+1] = 3\delta[n+7] - 4\delta[n+4] + 2\delta[n+2] - \delta[n+1] - \delta[n] + 3\delta[n-3]$

4. (a) It is **periodic**. if we look at 3cos[$\frac{13\pi}{10}$n] and 5sin[$\frac{7\pi}{3}$n-$\frac{2\pi}{3}$], we will first see that to make $\frac{13\pi}{10}$(n+N_0) multiple of 2π N should be at least 20. On the other hand, for $\frac{7\pi}{3}$(n+N) to be multiple of 2π, N should be at least 6.If we get common least multiple of them, it is 60. Thus, Fundamental period is 60.

(b) **Not periodic** because we can't have an integer N_0 value such that the function will be periodic.That is, there is a value $\frac{2\pi}{3}$ but it can't be an integer because of π.

(c) **Periodic**. 2cos(3πt-$\frac{2\pi}{5}$), here, time shifting doesn't affect the fundamental period. It should be $\frac{2\pi}{3\pi}$. Thus,it has a fundamental period of $\frac{2}{3}$.

(d) **Periodic**. We should look at T_0 value that makes exponential 1, to be periodic. This happens when we add $\frac{2\pi}{5}$ to t.As a result, the value (fundamental period) T_0 for the function is $\frac{2\pi}{5}$.

5. For x[n] to be even: x[n] = x[-n] must hold for each $n \in \mathbf{N}$. However, for n=1; x[1]=-1$\neq$0=x[-1]
Hence given signal is not even.

For x[n] to be odd: x[n] = -x[-n] must hold for each $n \in \mathbf{N}$. However, for n=1; x[1]=-1$\neq$0=-x[-1]
Hence given signal is not odd.

Then, let us find even and odd decompositions of given signal:

$$Ev\{x[n]\} = \frac{1}{2}x[n] + \frac{1}{2}x[-n]$$
$$Ev\{x[n]\}= \frac{3}{2}\delta[n+7] - 2\delta[n+4] + \delta[n+2] + \frac{-1}{2}\delta[n+1] + \frac{-1}{2}\delta[n-1] + \delta[n-2] - 2\delta[n-4] + \frac{3}{2}\delta[n-7]$$

$$Odd\{x[n]\} = \frac{1}{2}x[n] - \frac{1}{2}x[-n]$$
$$Odd\{x[n]\} = \frac{-3}{2}\delta[n+7] + 2\delta[n+4] - \delta[n+2] + \frac{1}{2}\delta[n+1] + \frac{-1}{2}\delta[n-1] + \delta[n-2] - 2\delta[n-4] + \frac{3}{2}\delta[n-7]$$

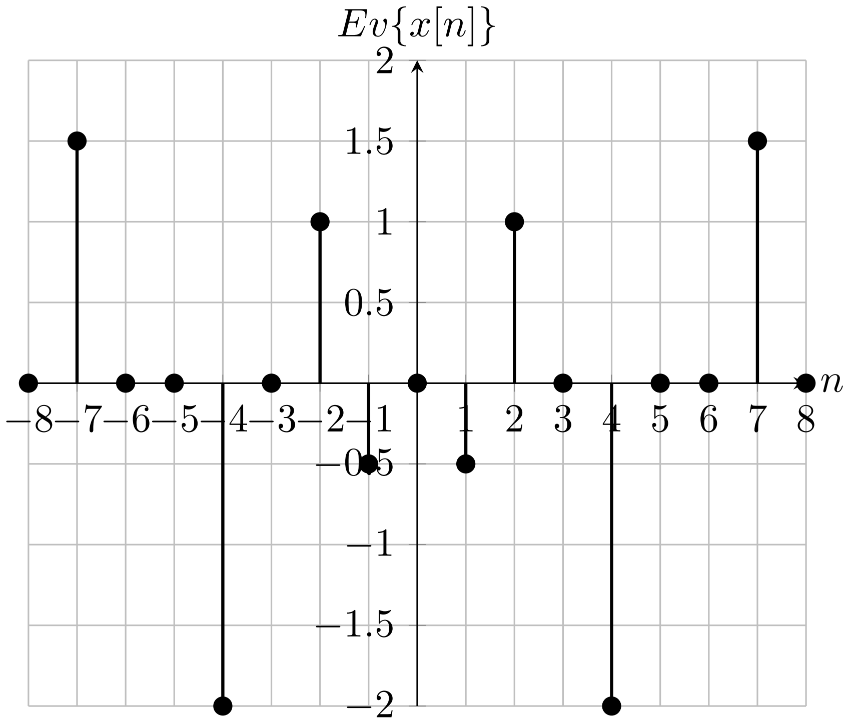

Figure 4: n vs. $Ev\{x[n]\}$ showing even decomposition of x[n].

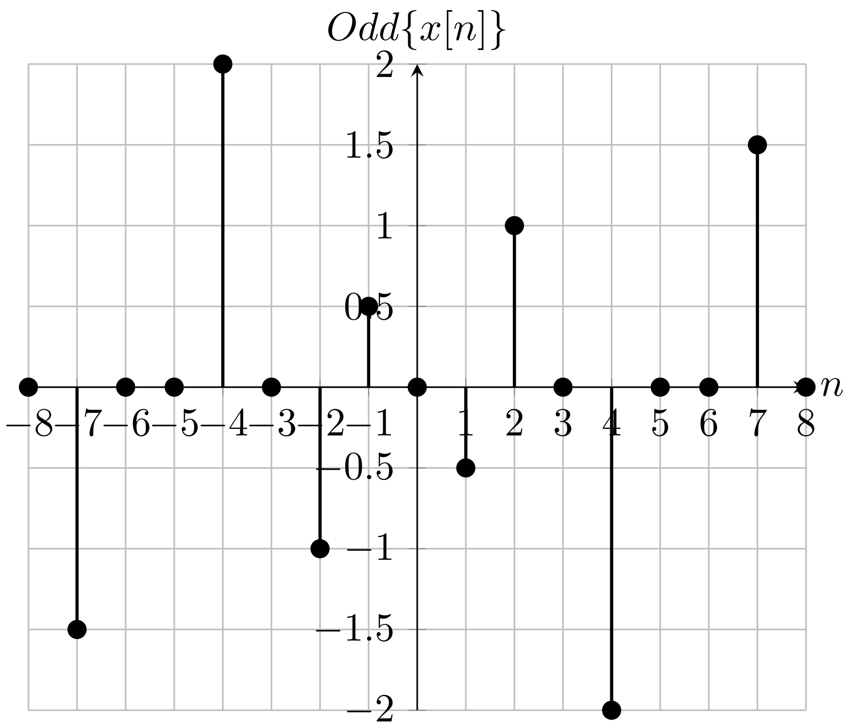

Figure 5: n vs. $Odd\{x[n]\}$ showing odd decomposition of x[n].

6. (a) **memory**: Yes, e.g y(1)=x(-1) , which implies memory.
stability: Yes, it can be bounded for certain bounded inputs.
causality: No, e.g y(4)=x(5), that is output from future input.
linearity: Yes, it holds both scalarity and superposition;
a.x(2t-3) will be equal to a.y(t) (scalarity)
y(t) = a.x_1(2t-3) + b.$x_2(2t-3)$ (superposition)
invertibility: Yes, different outputs for different inputs. (one-to-one)
time − invariance: No, x(2t-3-t_0) will not be equal to y(t-t_0) = x(2(t-t_0)-3).

   (b) **memory**: No, just depends on current value of t.
stability: No, because e.g for x(t)=1, y = t that means not bounded for bounded input.
causality: Yes, it can include up to and current value of t.
linearity: Yes, it has scalarity and superposition principle.

a.tx(t) will be equal to a.y(t) (scalarity)
y(t) = a.tx_1(t) + b.t$x_2(t)$ (superposition)
invertibility: Yes, different outputs for different inputs (one-to-one)
time − invariance: No, tx(t-t_0) $\neq$ y(t-t_0) = (t-t_0)x(t-t_0)

(c) **memory**: Yes, y[1]=x[-1], that is, it needs memory.
stability:Yes, it can be bounded for bounded inputs.
causality: No, y[4]=x[5] meaning that it needs output of a future input.
linearity: Yes, it has scalarity and superposition principle.
a.x[2n-3] will be equal to a.y[n] (scalarity)
y[n] = a.x_1[2n-3] + b.$x_2[2n-3]$ (superposition)
invertibility: Yes, different outputs for different inputs. (one-to-one)
time − invariance: No, x[2n-3-n_0] will not be equal to y[n-n_0] = x[2(n-n_0)-3].

(d) **memory**: Yes, it needs memory because it is sum of previous values of current value.
stability: No, it will be infinite, which cannot be bounded
causality: Yes, independent from outputs of future inputs.
linearity: Yes, it holds superposition and scalarity features.
a$\sum_{k=1}^{\infty} x[n-k]$ will be equal to ay[n] (scalarity)
y[n] = a$\sum_{k=1}^{\infty} x_1[n-k] + b\sum_{k=1}^{\infty} x_2[n-k]$ (superposition)
invertibility: Yes, the inverse function will be w[n]=y[n]-y[n-1]
time − invariance: Yes, $\sum_{k=1}^{\infty} x[(n-n_0)-k] = y[n-n_0]$