CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 2

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1. (a) If we come in reverse of integral, it will be derivative of y(t): y'(t).
What contributes to summation is; x(t), -4y(t). Thus, $\mathbf{y'(t) = x(t) - 4y(t)}$.

(b) We know y(t) = $y_h(t)$+$y_p(t)$
$y_h(t)$ is actually homogeneous differential equation, which is:

$\frac{dy(t)}{dt}$+4y(t)=0, if we rearrange;

$\frac{dy(t)}{y(t)}$ = -4dt

if we integrate both sides,

$\int \frac{dy(t)}{y(t)} = \int -4dt$

Where LHS is ln(y(t)) and RHS is -4t

ln(y(t)) = -4t if we convert base of logaritm function ;

$y_h(t) = Ae^{-4t}$

Since we try to find a particular solution for an exponential input, we look at a similar signal of the same form as the input:
$y_p(t) = Ae^{-t} + Be^{-2t}$
if we insert this into the equation :
$-Ae^{-t} - 2Be^{-2t} + 4Ae^{-t} + 4Be^{-2t} = e^{-t} + e^{-2t}$ (Here, we are looking for the case $t > 0$, becasue of $u(t)$.)

After using equality of polinomials: A=$\frac{1}{3}$ and B=$\frac{1}{2}$

So, $y_p(t) = \frac{e^{-t}}{3} + \frac{e^{-2t}}{2}$

And y(t)= $y_h(t)$+$y_p(t)$ $= Ae^{-4t} + \frac{e^{-t}}{3} + \frac{e^{-2t}}{2}$

To find A, we use condition of initial rest:
y(0)=0.

$0 = A + \frac{1}{3} + \frac{1}{2}$, from here A = $\frac{-5}{6}$

Thus, y(t) = ($\frac{-5e^{-4t}}{6} + \frac{e^{-t}}{3} + \frac{e^{-2t}}{2}$)u(t)

2. (a) $x[n] = \begin{cases} 1, & n = 1 \\ -3, & n = 2 \\ 1, & n = 3 \\ 0, & \text{otherwise} \end{cases} \quad h[n] = \begin{cases} 1, & n = -1 \\ 2, & n = 0 \\ -3, & n = 1 \\ 0, & \text{otherwise} \end{cases}$

Before starting to compute x, we should convert x[n] to x[k] and h[n] to h[n-k].

$$x[k] = \begin{cases} 1, & k = 1 \\ -3, & k = 2 \\ 1, & k = 3 \\ 0, & \text{otherwise} \end{cases} \qquad h[n-k] = \begin{cases} -3, & k = n-1 \\ 2, & k = n \\ 1, & k = n+1 \\ 0, & \text{otherwise} \end{cases}$$

For different values of n, $x[n] * h[n]$ will take different values;

$$x[n] * h[n] = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ -1, & n = 1 \\ -8, & n = 2 \\ 11, & n = 3 \\ -3, & n = 4 \\ 0, & n > 4 \end{cases}$$

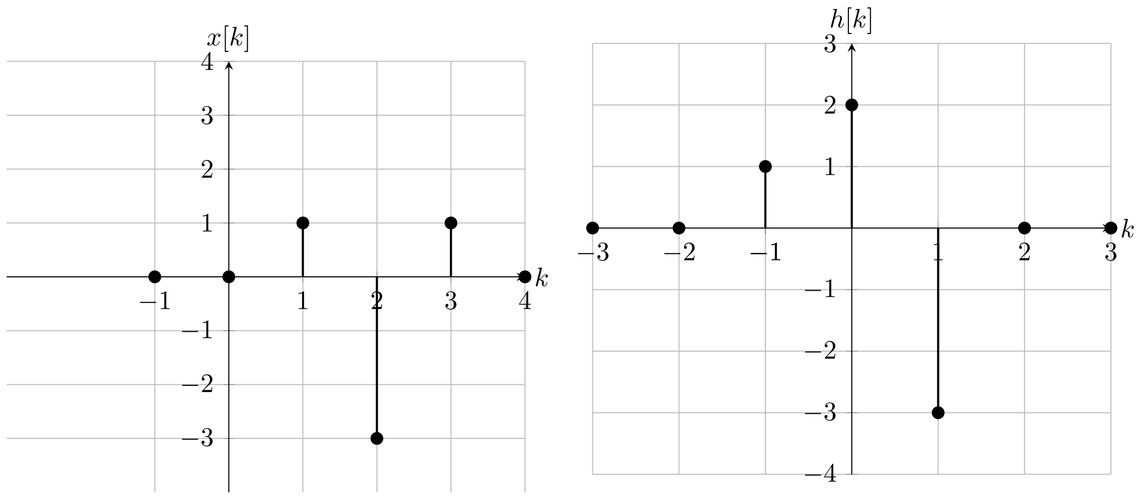

Figure 1: k vs. $x[k]$ and k vs. $h[k]$.

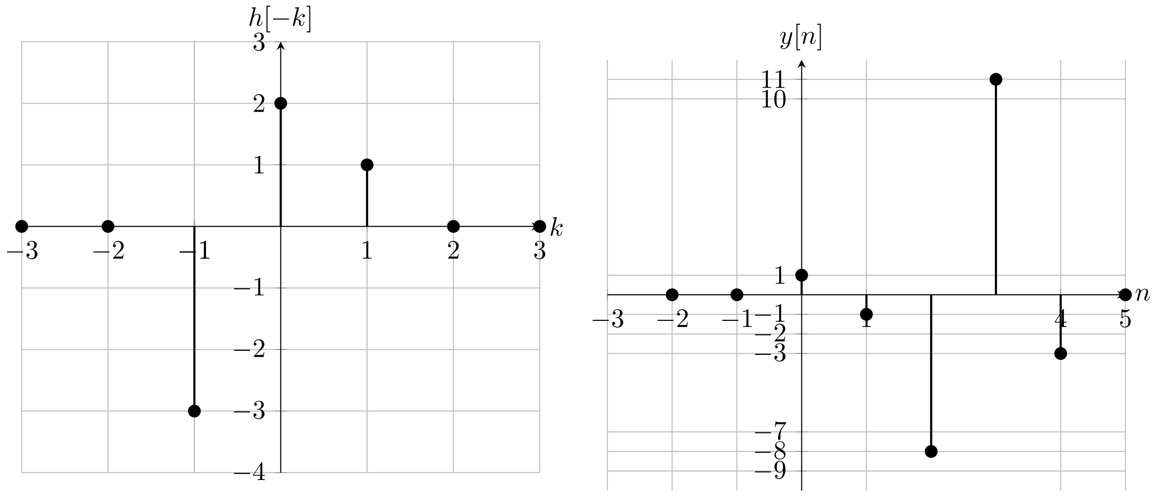

Figure 2: k vs. $h[-k]$ and n vs. $y[n]$.

(b) If we write x'(t) = $\delta(t)$ + $\delta(t-1)$ (because derivative of step function is impulse function) and h(t-τ) in another way;

$$x'(t) = \begin{cases} 0, & t < 0 \\ 1, & t = 0 \\ 1, & t = 1 \\ 0, & \text{otherwise} \end{cases} \qquad h(t-\tau) = \begin{cases} 0, & t < 0 \\ e^{2\tau - 2t} cos(t-\tau), & \text{otherwise} \end{cases}$$

Note that $x(t) * \delta(t)$ = x(t) and $x(t) * \delta(t-1)$= x(t-1) and so on.
Using this note, we can easily calculate convolution;

for $\mathbf{t} < \mathbf{0}$, it is 0
for $\mathbf{t} < \mathbf{0} < \mathbf{1}$, it will just include $\delta(t)$, so it is equal to h(t) $= e^{-2t}cos(t)$
for $\mathbf{t} > \mathbf{1}$, it include both impulse fuctions, as a result it will be h(t)+h(t-1) $= e^{-2t}cos(t) + e^{2-2t}cos(t-1)$. That is,

$$y(t) = \begin{cases} 0, & t < 0 \\ e^{-2t}cos(t), & 0 < t < 1 \\ e^{-2t}cos(t) + e^{2-2t}cos(t-1), & t > 1 \end{cases}$$

3. (a) We just replace τ with t, since it won't change the function.

$$x(\tau) = \begin{cases} 0, & \tau < 0 \\ e^{-\tau}, & \text{otherwise} \end{cases}$$

If we do replace h(t) with h(t-τ), then;

$$h(t-\tau) = \begin{cases} 0, & t < 0 \\ e^{3\tau-3t}, & \text{otherwise} \end{cases}$$

So, there will be two cases;

Case 1: t<0, y(t) = 0 because there will be no overlapping area.

Case 2: t≥0, $x(t) * h(t) = \int_0^\infty x(\tau)h(t-\tau)d\tau = \int_0^t e^{2\tau-3t}d\tau = \frac{e^{-3t}}{(}\frac{e^{2t}}{2} - \frac{1}{2}) = \frac{e^{-t}-e^{-3t}}{2}$. Thus, the result will be:

$$y(t) = x(\tau) * h(t-\tau) = \begin{cases} 0, & t < 0, \\ \frac{e^{-t}-e^{-3t}}{2}, & \text{otherwise} \end{cases}$$

(b) Given $x(t) = \begin{cases} 1, & \text{if } 1 < t < 2 \\ 0, & \text{otherwise} \end{cases}$

$$h(t) = \begin{cases} 0, & \text{if } t < 0 \\ e^t, & \text{otherwise} \end{cases}$$

Hence, it is easier to compute $x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$ rather than $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ where these two equations yield the same result.
So, there are three cases that one needs to examine:

Case 1: $-\infty < t < 1$

y(t)=0 for this case since there is no overlapping region between $x(t-\tau)$ and $h(\tau)$

Case 2: $1 \leq t < 2$

$y(t) = \int_0^{t-1} x(t-\tau)h(\tau)d\tau = \int_0^{t-1} e^\tau d\tau$
$y(t) = e^{t-1} - 1$

Case 3: $2 \leq t < \infty$

$y(t) = \int_{t-2}^{t-1} x(t-\tau)h(\tau)d\tau = \int_{t-2}^{t-1} e^\tau d\tau$
$y(t) = e^{t-1} - e^{t-2}$

Hence, the answer is:

$$y(t) = \begin{cases} 0, & -\infty < t < 1 \\ e^{t-1} - 1, & 1 \leq t < 2 \\ e^{t-1} - e^{t-2}, & 2 \leq t < \infty \end{cases}$$

4. (a) Characteristic equation of given equation is:
$s^2 - 15s + 26 = (s-13)(s-2) = 0$ where roots are s={2,13}
So, $y[n] = A.13^n + B.2^n$
Using y[0]=10 initial condition, $A + B = 10$
Using y[1]=42 initial condition, $13A + 2B = 42$
Solving above two equations, $A = 2, B = 8$
Hence, the answer is:
$y[n] = 2.13^n + 8.2^n$

(b) Characteristic equation of given equation is:
$s^2 - 3s + 1 = (s - \frac{3}{2})^2 - \frac{5}{4} = 0$
So, the roots are $s = \{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\}$

And, $y[n] = A.(\frac{3+\sqrt{5}}{2})^n + B.(\frac{3-\sqrt{5}}{2})^n$ Using y[0]=1 initial condition, $A + B = 1$
Using y[1]=2 initial condition, $\frac{3}{2}.(A + B) + \frac{\sqrt{5}}{2}.(A - B) = 2$
Solving above two equations, $A = \frac{\sqrt{5}+1}{2\sqrt{5}}, B = \frac{\sqrt{5}-1}{2\sqrt{5}}$
Hence the answer is:
$y[n] = \frac{\sqrt{5}+1}{2\sqrt{5}}.(\frac{3+\sqrt{5}}{2})^n + \frac{\sqrt{5}-1}{2\sqrt{5}}.(\frac{3-\sqrt{5}}{2})^n$

5. (a) Characteristic equation for the homogeneous version of the system is:
$s^2 + 6s + 8 = (s + 4)(s + 2) = 0$, so the roots are s={-2,-4}
Homogeneous solution is $y_h(t) = A.e^{-4t} + B.e^{-2t}$

Using initial conditions of $y_h(0) = 0, y_h'(0) = 1$, below two equations comes up:

$A + B = 0,$
$-4A - 2B = 1$
Solving above two equations, $A = \frac{-1}{2}, B = \frac{1}{2}$

Hence, the impulse response for homogeneous equation is $\frac{-1}{2}.e^{-4t} + \frac{1}{2}.e^{-2t}$ for $t > 0$

Since the system is linear, finding the impulse response for full equation means putting the impulse response for homogeneous equation in full equation.

As a result the impulse response for given system is,
$h(t) = 2.(\frac{-1}{2}.e^{-4t} + \frac{1}{2}.e^{-2t})$ for $t > 0$
$h(t) = (-e^{-4t} + e^{-2t}).u(t)$

(b) i. **Causal**, since $h(t) = 0$ along the interval $-\infty < t < 0$ which makes the system not to depend on future.

ii. **Not memoryless**, since for an LTI system to be memoryless, $h(t) = 0$ for $t \neq 0$ must hold. However, in this case for $t = 1$, $h(1) = 0,117 \neq 1$.

iii. **Stable**, since $\int_{-\infty}^{\infty} |h(t)|dt < \int_{-\infty}^{\infty} |e^{-2t}.u(t)|dt = \int_{0}^{\infty} e^{-2t}dt \leq \frac{1}{2}$ is bounded by a finite value.

iv. **Not invertible**, since the impulse response of two distinct inputs give the same output. An example can be $t = \{0.059787, 1.09151\}$ which both result in $h(t) = 0.1$