

# Student Information

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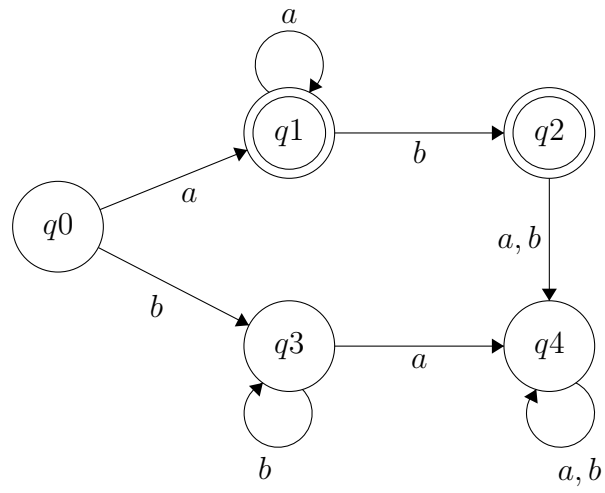
## Answer 1

a.

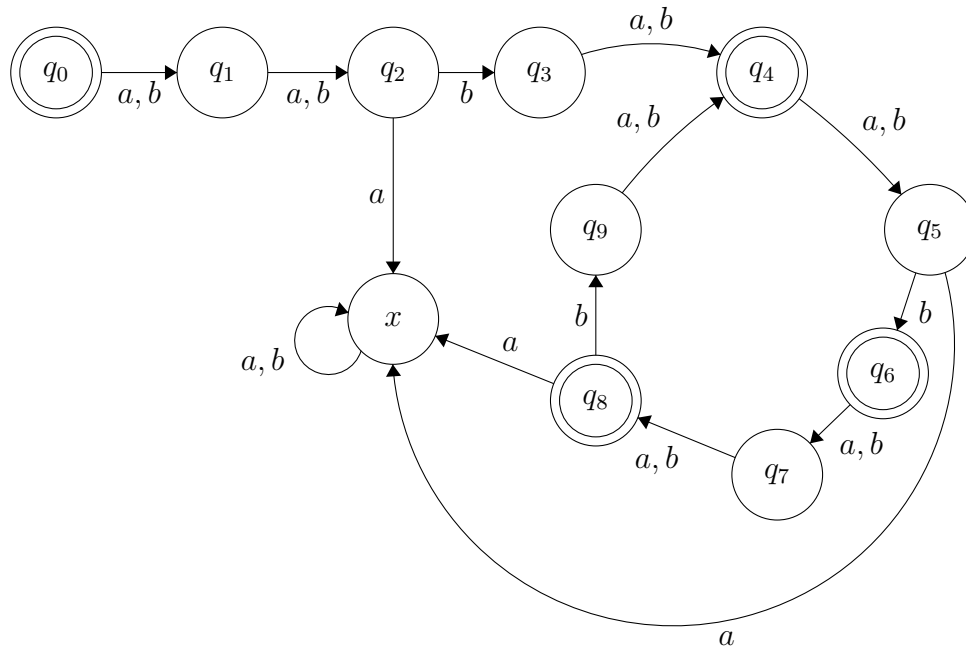
b.

## Answer 2

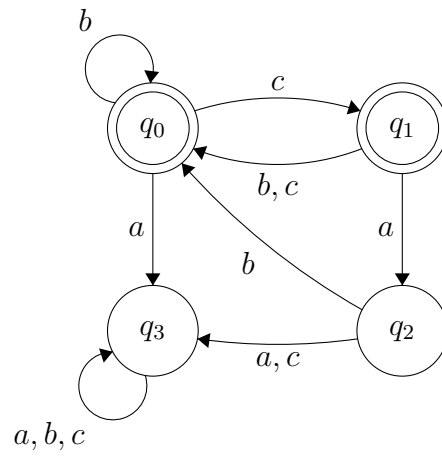
a.



b.



c.



## Answer 3

a.

By utilizing

$$E(q_0) = q_0, q_2$$

$$E(q_1) = q_1, q_2, q_3$$

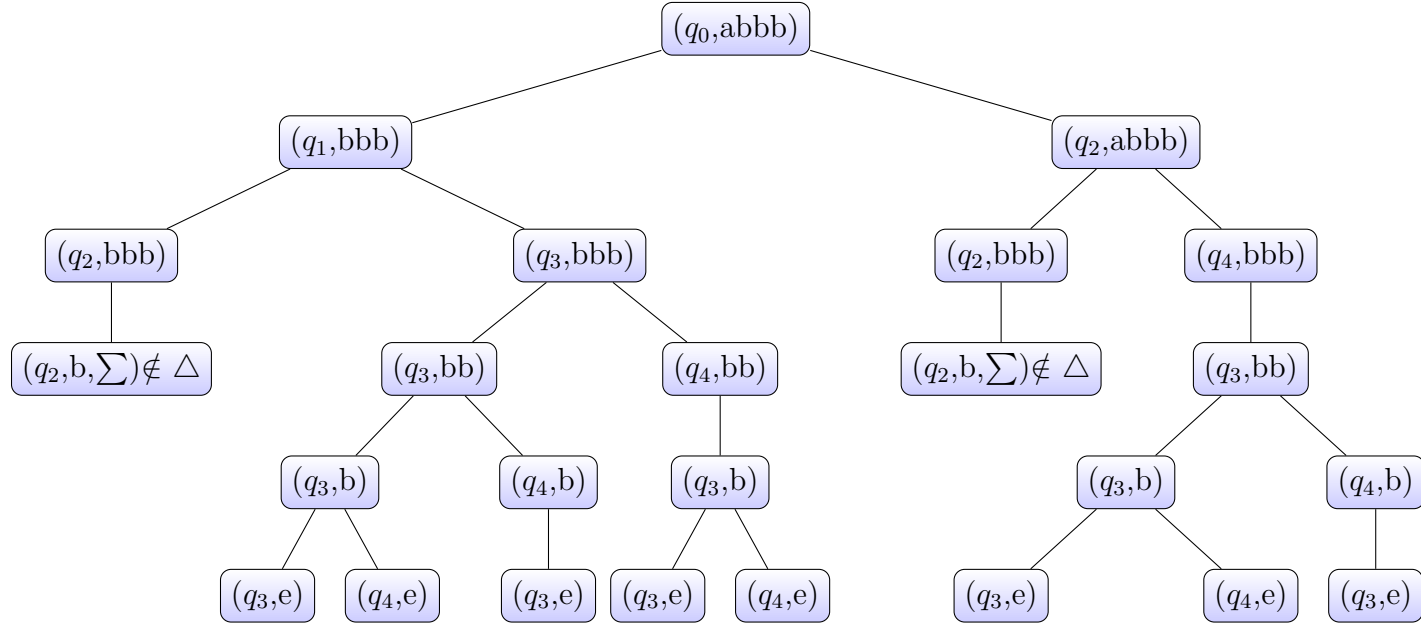
$$E(q_2) = q_2$$

$$E(q_3) = q_3$$

$$E(q_4) = q_4$$

$$E(q_5)=q_1, q_5$$

Here are all possibilities:



So, as seen from above tree, when the machine finishes reading there is no  $q_5$  as last state. Hence,  $w_1$  is not in  $L(N)$

**b.**

$$(q_0, ababa) \vdash_M (q_1, baba)$$

$$(q_1, baba) \vdash_M (q_3, baba)$$

$$(q_3, baba) \vdash_M (q_4, aba)$$

$$(q_4, aba) \vdash_M (q_5, ba)$$

$$(q_5, ba) \vdash_M (q_1, ba)$$

$$(q_1, ba) \vdash_M (q_3, ba)$$

$$(q_3, ba) \vdash_M (q_3, a)$$

$$(q_3, a) \vdash_M (q_5, e)$$

Since  $q_5 \in F$ ,  $w_2$  is in  $L(N)$

## Answer 4

**a.**

Let's call GFA as G.

$$G = (K', \Sigma, \Delta', q_4, F')$$

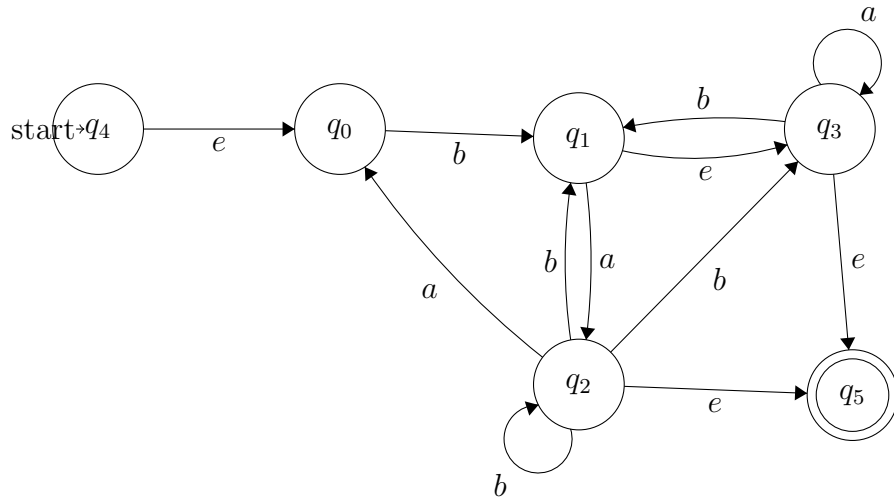
in which  $K' = q_0, q_1, q_2, q_3, q_4, q_5$

$$\Sigma = a, b$$

$$F' = q_5$$

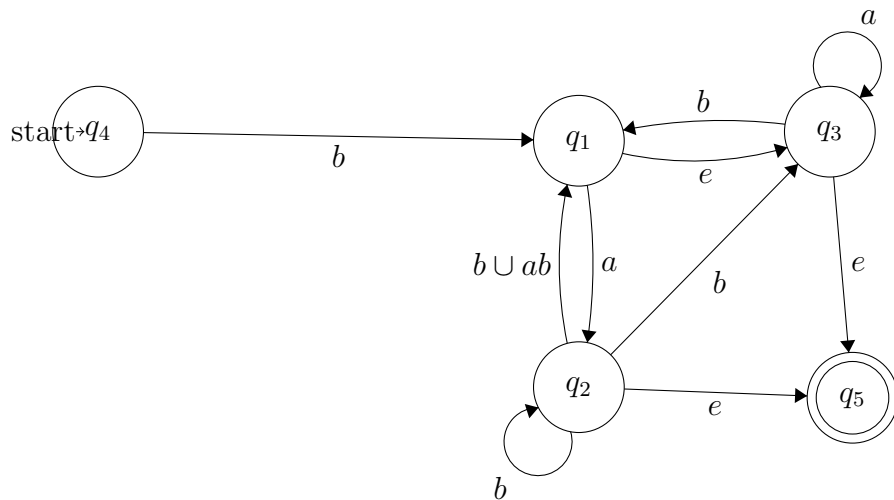
$\Delta' =$   
 $(q_0, b, q_1),$   
 $(q_1, a, q_2), (q_1, e, q_3),$   
 $(q_2, a, q_0), (q_2, b, q_1), (q_2, b, q_2), (q_2, b, q_3), (q_2, e, q_5),$   
 $(q_3, b, q_1), (q_3, a, q_3), (q_3, e, q_5),$   
 $(q_4, e, q_0)$

And here is the diagram:



**b.**

Eliminating state  $q_0$ :

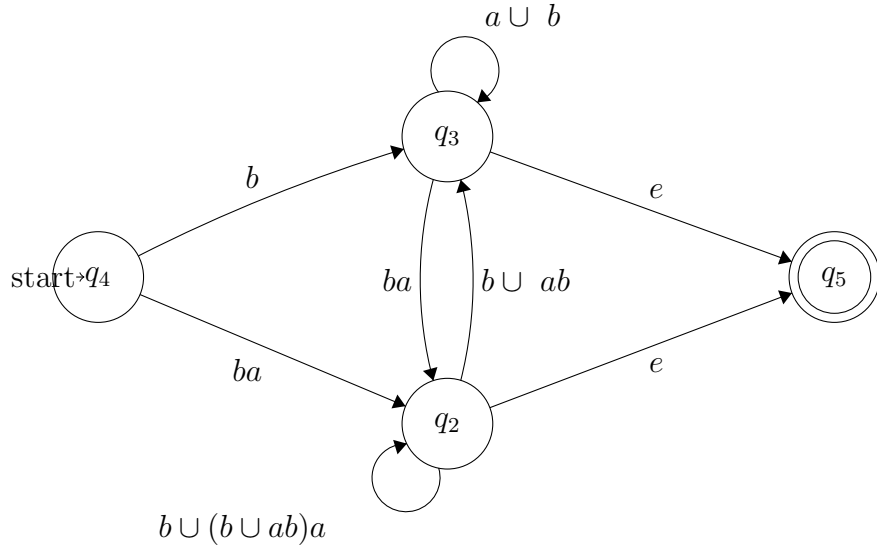


Eliminating state  $q_1$ :

$$R(2,3,1) = R(2,3,0) \cup R(2,1,0)R(1,1,0)^*R(1,3,0) = b \cup (b \cup ab) = b \cup ab$$

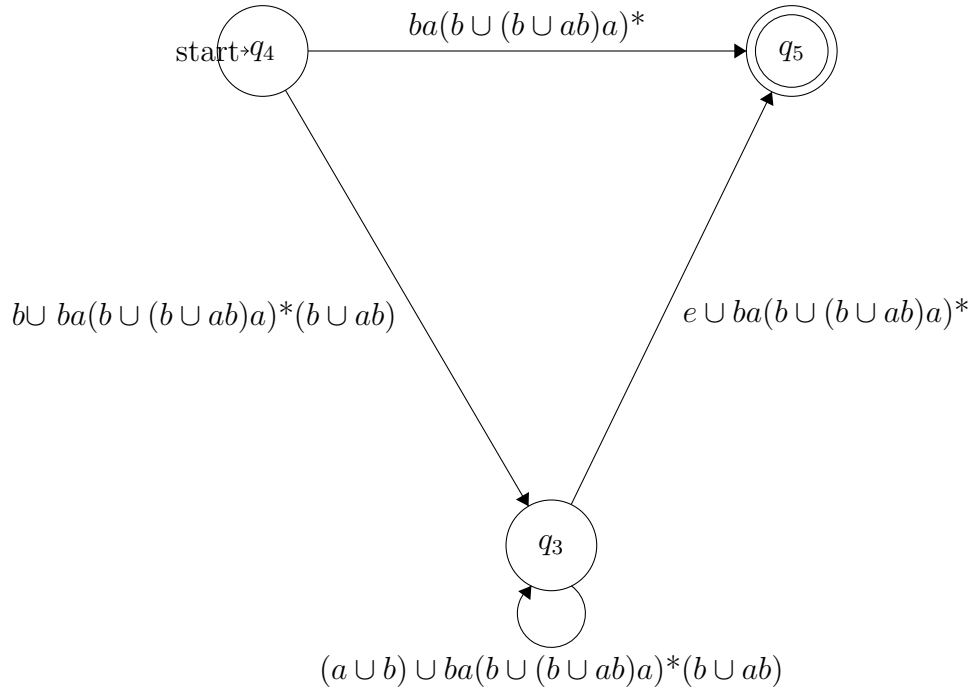
$$R(2,2,1) = R(2,2,0) \cup R(2,1,0)R(1,1,0)^*R(1,2,0) = b^* \cup (b \cup ab)a$$

$$\begin{aligned}
R(3,3,1) &= R(3,3,0) \cup R(3,1,0)R(1,1,0)^*R(1,3,0) = a^* \cup b \\
R(3,2,1) &= R(3,2,0) \cup R(3,1,0)R(1,1,0)^*R(1,2,0) = ba \\
R(4,2,1) &= R(4,2,0) \cup R(4,1,0)R(1,1,0)^*R(1,2,0) = ba \\
R(4,3,1) &= R(4,3,0) \cup R(4,1,0)R(1,1,0)^*R(1,3,0) = b
\end{aligned}$$



Eliminating state  $q_2$ :

$$\begin{aligned}
R(4,3,2) &= R(4,3,1) \cup R(4,2,1)R(2,2,1)^*R(2,3,1) = b \cup ba(b \cup (b \cup ab)a)^*(b \cup ab) \\
R(4,5,2) &= R(4,5,1) \cup R(4,2,1)R(2,2,1)^*R(2,5,1) = ba(b \cup (b \cup ab)a)^* \\
R(3,3,2) &= R(3,3,1) \cup R(3,2,1)R(2,2,1)^*R(2,3,1) = (a \cup b) \cup ba(b \cup (b \cup ab)a)^*(b \cup ab) \\
R(3,5,2) &= R(3,5,1) \cup R(3,2,1)R(2,2,1)^*R(2,5,1) = e \cup ba(b \cup (b \cup ab)a)^*
\end{aligned}$$



Eliminating state  $q_3$ :

$$ba(b \cup (b \cup ab)a)^* \cup (b \cup ba(b \cup (b \cup ab)a)^*(b \cup ab))((a \cup b) \cup ba(b \cup (b \cup ab)a)^*(b \cup ab))^*(e \cup ba(b \cup (b \cup ab)a)^*)$$



Hence;

$G = ba(b \cup (b \cup ab)a)^* \cup (b \cup ba(b \cup (b \cup ab)a)^*(b \cup ab))((a \cup b) \cup ba(b \cup (b \cup ab)a)^*(b \cup ab))^*(e \cup ba(b \cup (b \cup ab)a)^*)$  is a regular expression form of GFA.

## Answer 5

a.

$$M = (K', \Sigma, \delta, s, F')$$

Firstly, we should find  $s$ :

$$s = E(q_0) = q_0 q_1 q_2$$

Then, we should find  $a$  and  $b$  transitions from  $s$ :

$$\delta(s, a) = E(q_1) \cup E(q_3) = q_1 q_3$$

$$\delta(s, b) = E(q_2) = q_2$$

We should keep going until we don't see any new state.

$$\delta(q_1 q_3, a) = \emptyset$$

$$\delta(q_1 q_3, b) = E(q_1) = q_1$$

$$\delta(q_2, a) = E(q_3) = q_1 q_3$$

$$\delta(q_2, b) = \emptyset$$

$$\delta(q_1, a) = \emptyset$$

$$\delta(q_1, b) = \emptyset$$

At the end, we should also add a trap state  $q_3$  and make the transitions which are empty set to go this state. Also, we should have:

$$\delta(q_3, a) = q_3$$

$$\delta(q_3, b) = q_3$$

$$\text{So } M = (K', \Sigma, \delta, s, F')$$

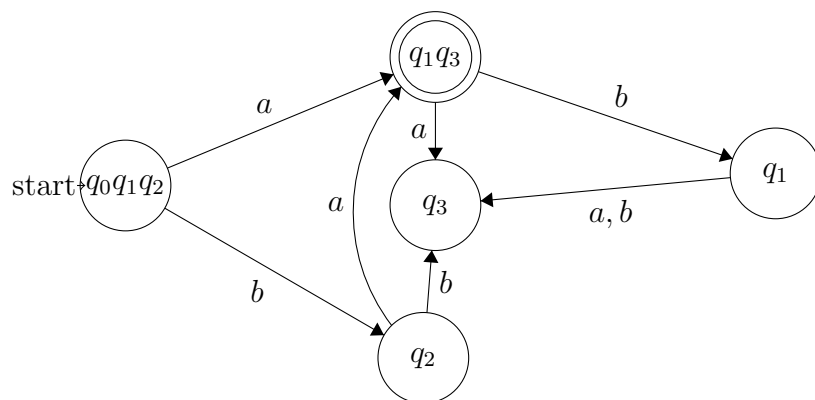
$$K' = q_1, q_2, q_3, q_1 q_3, q_0 q_1 q_2$$

$\delta$  is as above

$$s = q_0 q_1 q_2$$

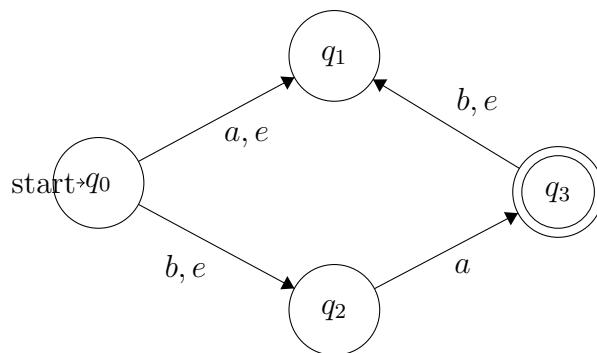
$$F' = q_1 q_3$$

And here is the diagram:

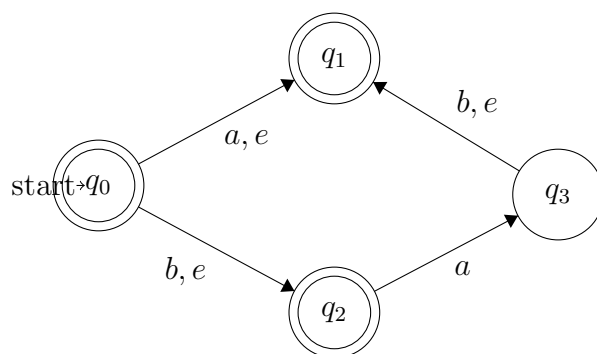


**b.**

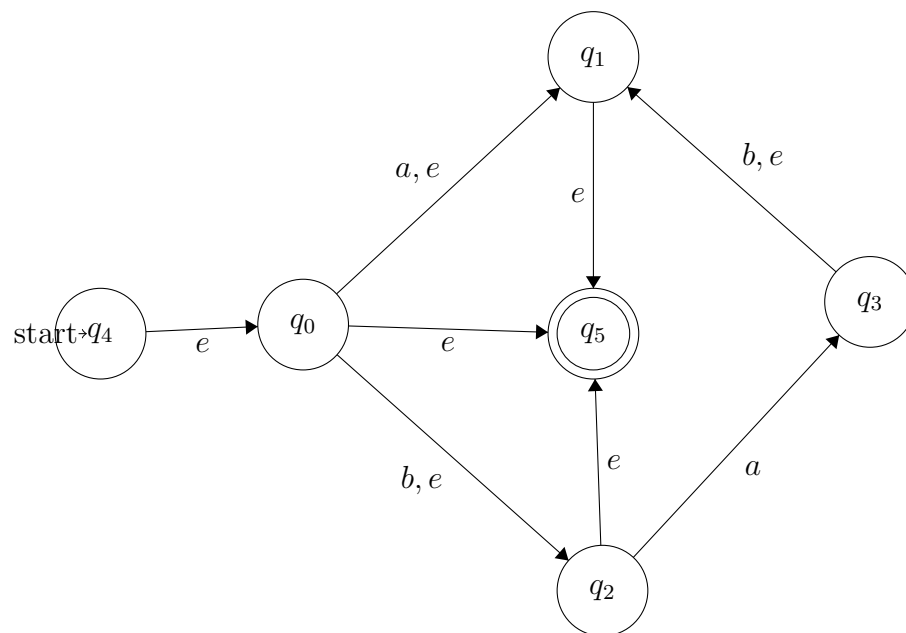
Here is the diagram of  $L$ :



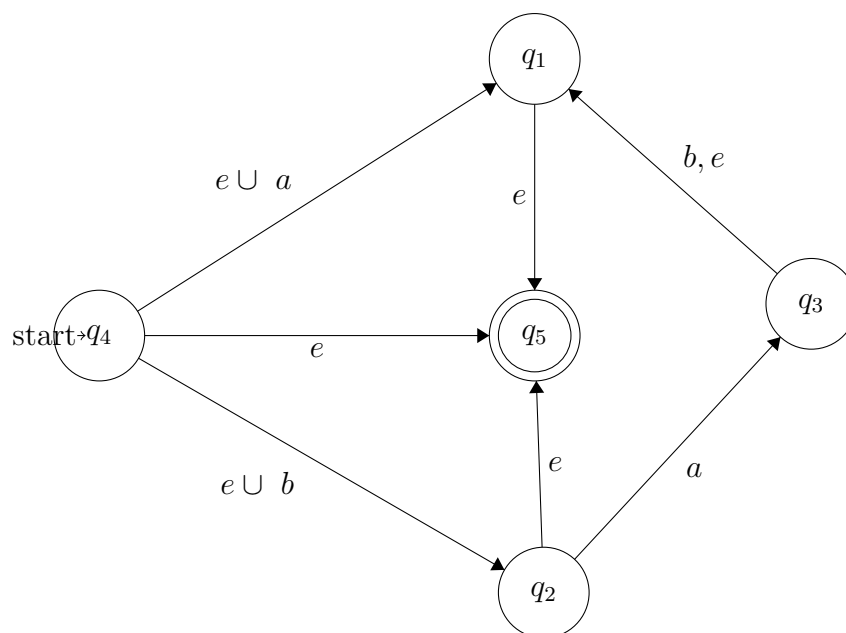
To find  $\bar{L}$  we need to make final states nonfinal, and nonfinal states final. So here is the diagram of  $\bar{L}$ :



To apply state elimination, here is the GFA form of  $\bar{L}$ :

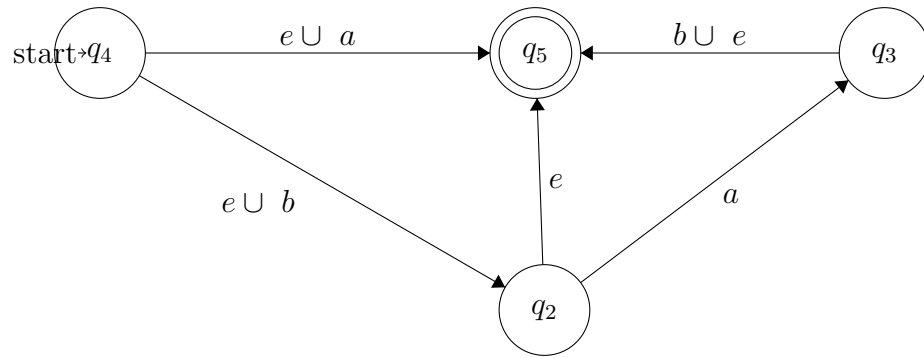


Eliminating  $q_0$ , we have:

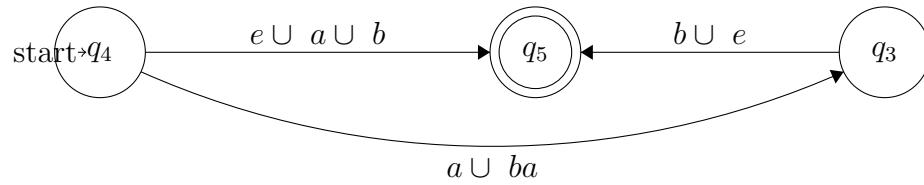


Eliminating  $q_1$ , we have:

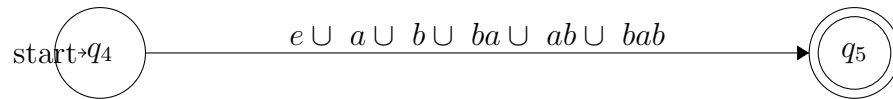




Eliminating  $q_2$ , we have:



Eliminating  $q_3$ , we have:



Hence regular expression for  $\bar{L}$  is:

$$e \cup a \cup b \cup ba \cup ab \cup bab$$

$$\bar{L} = \{w \in a, b^* : (|w| \leq 2 \wedge (aa, bb) \notin \bar{L}) \vee (aba) \in w\}$$

**Answer 6**

**Answer 7**

a.

b.