CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 4

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1. (a) Focusing on the four way (three incoming and one outgoing) plus sign, we can deduce the following difference equation for the given system:

   $2x[n] + \frac{-1}{8}y[n-2] + \frac{3}{4}y[n-1] = y[n]$

   (b) Applying Fourier Transform directly, we get the following:

   $H(e^{jw}) = \frac{2}{1-\frac{3}{4}(e^{-jw})+\frac{1}{8}(e^{-jw})^2}$

   (c) Using partial fraction expansion and above frequency response:
   $H(e^{jw}) = \frac{A}{1-\frac{1}{2}(e^{-jw})} + \frac{B}{1-\frac{1}{4}(e^{-jw})}$, we can conclude that equation holds for $A = 4, B = -2$

   Hence,
   $H(e^{jw}) = \frac{4}{1-\frac{1}{2}(e^{-jw})} + \frac{-2}{1-\frac{1}{4}(e^{-jw})}$,

   $h[n] = (4(\frac{1}{2})^n - 2(\frac{1}{4})^n)u[n]$ is the required impulse response of the system.

   (d) Converting x[n] from time domain to frequency domain, we have:
   $X(e^{jw}) = \frac{1}{1-\frac{1}{4}e^{-jw}}$

   $Y(e^{jw}) = X(e^{jw})H(e^{jw})$
   $Y(e^{jw}) = \frac{2}{(1-\frac{1}{2}e^{-jw})(1-\frac{1}{4}e^{-jw})^2}$

   Using partial fraction expansion, we have:
   $Y(e^{jw}) = \frac{A}{1-\frac{1}{2}e^{-jw}} + \frac{B}{1-\frac{1}{4}e^{-jw}} + \frac{C}{(1-\frac{1}{4}e^{-jw})^2}$

   Solving for A, B and C; we can conclude that equation holds for A=8, B=-4, C=-2 values.

   Hence using inverse Fourier Transform:
   $y[n] = (8(\frac{1}{2})^n - 4(\frac{1}{4})^n - 2(n+1)(\frac{1}{4})^n)u[n]$ is the impulse response of the system for given x[n]

2. Being connected in parallel for two discrete time LTI systems means final system's impulse response is going to be a sum of two impulse responses. Hence,
   $h[n] = h_1[n] + h_2[n]$

   From above equality which is in time domain, we can deduce the following in frequency domain using linearity property:
   $H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$

   We are given $H(e^{jw})$ and we can find $H_1(e^{jw})$ using Fourier Transform:
   $H_1(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}}$

   $H(e^{jw}) = \frac{5e^{-jw}-12}{e^{-2jw}-7e^{-jw}+12}$

Using partial fraction expansion, we have:

$H(e^{jw}) = \frac{A}{e^{-jw}-3} + \frac{B}{e^{-jw}-4}$
Solving for A and B, we can conclude that equation holds for A=-3, B=8 values

Hence,
$H(e^{jw}) = \frac{-3}{e^{-jw}-3} + \frac{8}{e^{-jw}-4}$
$H(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}} + \frac{-2}{1-\frac{1}{4}e^{-jw}}$

Since $H_2(e^{jw}) = H(e^{jw}) - H_1(e^{jw})$, we can deduce $H_2(e^{jw})$ as the following:
$H_2(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}} + \frac{-2}{1-\frac{1}{4}e^{-jw}} - \frac{1}{1-\frac{1}{3}e^{-jw}}$
$H_2(e^{jw}) = \frac{-2}{1-\frac{1}{4}e^{-jw}}$

Using inverse Fourier Transform, we can catch $h_2[n]$ as the following:
$h_2[n] = (-2(\frac{1}{4})^n)u[n]$

3. (a) We know $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

we can separate x(t) as $sin\frac{2\pi t}{\pi t}$ and $cos3\pi t$

From equation 4.19 in the textbook;

$X(j\omega)$ for $sin\frac{2\pi t}{\pi t}$ is:

$$X(j\omega) = \begin{cases} 1, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$

$X(j\omega)$ for $cos3\pi t$ (from figure 4.13(b) in textbook) is:

$X(j\omega) = \pi(\delta(w-3\pi) + \delta(w+3\pi))$

Summation of those functions will give us Fourier transform of x(t).

$$X(j\omega) = \begin{cases} \pi(\delta(w-3\pi) + \delta(w+3\pi)), & |\omega| > 2\pi \\ 1, & |\omega| < 2\pi \end{cases}$$

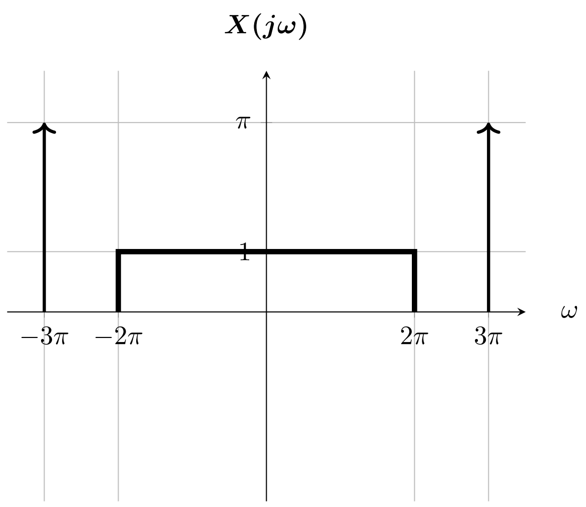

Figure 1: ω vs $X(j\omega)$

(b) $w_s = \frac{2\pi}{T}$ and w_m, Nyquist frequency, is found to be 3π from part (a).

$w_s = 2w_m$, $\frac{2\pi}{T} = 6\pi$, so T= $\frac{1}{3}$. So, it shall be as : T = $\frac{1}{3}$, and $w_s = 6\pi$.

(c) We know the formula,

$$X_p(j\omega) = \frac{1}{T}\sum_{-\infty}^{\infty} X(j(w - kw_s))$$

So if we insert T and $X(jw)$ (from part (a)) into the formula,

$$X_p(j\omega) = \begin{cases} 3\pi, & |\omega| = 3\pi \\ 3, & |\omega| < 2\pi \\ 0, & 2\pi \leq |\omega| < 3\pi \end{cases}$$

and $X_p(j\omega)$ is periodic with 6π and at $3\pi \pm 6\pi k$, successive δ's will overlap, which makes them to be 6π.

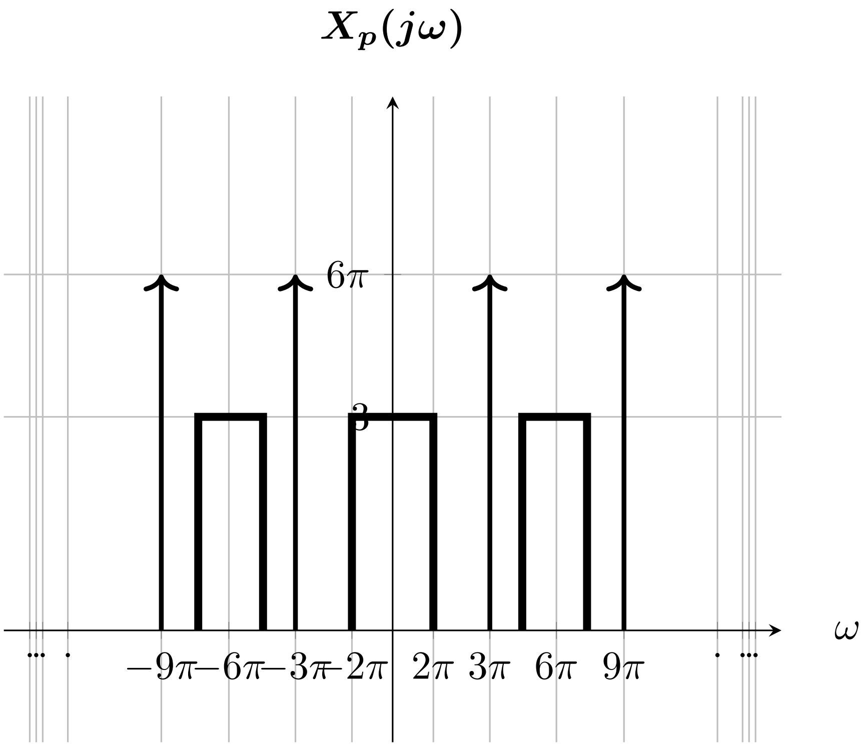

Figure 2: ω vs $X_p(j\omega)$

4. (a) Given $w_s = \pi$, T $= \frac{2\pi}{w_s} = 2$

$X_p(j\omega) = \frac{1}{T}\sum_{-\infty}^{\infty} X(j(w - kw_s))$

$X_d(e^{j\omega}) = X_p(j\frac{\omega}{T})$

Thus, $X_d(e^{j\omega}) = \begin{cases} \frac{2}{\pi}\omega, & |\omega| \leq \frac{\pi}{2} \\ 0, & otherwise \end{cases}$

(b) h[n] $= cos\pi n$, whose fourier transform is (From table 5.2 in textbook) :

$$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega - \pi - 2\pi l) + \delta(\omega + \pi - 2\pi l)\}$$

(c) We found $X_d(e^{j\omega})$ and $H(e^{j\omega})$ from part (a) and part (b).

So, we should find Y$(e^{j\omega}) = \frac{1}{2\pi}(H(e^{j\omega}) * X_d(e^{j\omega}))$

Now, if we look at $(H(e^{j\omega})$, for $\omega_0 = \pi$, let's convolve $H(e^{j\omega})$ with $X_d(e^{j\omega})$. We see that it comes up

$$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega-\pi)+\delta(\omega+\pi)\} * X_d(e^{j\omega})$$

Thus, $Y_d(e^{j\omega})$ is sliding $X_d(e^{j\omega})$ amount of $\pm\pi$. As a result,

$$Y_d(e^{j\omega}) = \begin{cases} \frac{1}{\pi}\omega, & \frac{\pi}{2} \leq |\omega| \leq \frac{3\pi}{2} \\ 0, & otherwise \end{cases}$$

Note that $Y_d(e^{j\omega})$ is periodic over the period 2π