Student Information

Full Name :
Id Number :

Answer 1

Answer 2

Answer 3

Answer 4

Answer for 4.a
There are three cases to consider:
(i)A and B are both finite,
(ii)A is infinite and B is finite,
(iii) A and B are both countably infinite.

case(i): When A and B are both finite, AxB is also finite and therefore, countable.
csae(ii): Because A is countably infinite, its elements can be listed in an infinite sequence a_1, a_2, ... ,a_n, ... and because B is finite, its terms can be listed as b_1,b_2, ... ,b_m for some positive integer m. We can list the elements of AxB as (a_1,b_1),(a_1,b_2), ... ,(a_1,b_m), ... ,(a_n,b_1),(a_n,b_2), ..., (a_n,b_m), ...
case(iii): Because A and B are both countably infinite, its elements can be listed in an infinite sequence. Their elements can be listed as a_1, a_2, ... ,a_n, ... and b_1, b_2, ... ,b_n, ... respectively. And we can list the elements of AxB by alternating and putting into tuples as (a_1,b_1), (a_1,b_2),(a_2,b_1), (a_2,b_2), (a_1,b_3), (a_3,b_1), (a_2,b_3), (a_3,b_2), (a_3,b_3), ...

Answer for 4.b
If A is uncountable and A⊆B, then B is uncountable. Suppose A represents all real numbers between (0,1). A is an uncountable set as proved in the textbook p.173-174. And suppose B represents all real numbers between (0,2). B has two parts: (0,1) and (1,2). As we said that the set of all real numbers between (0,1) is uncountable, B has this uncountable part too. So we say that a set with an uncountable subset is uncountable. So, if A is uncountable and A⊆B, then B is uncountable.

Answer for 4.c
If B is countable and A⊆B, then A is countable. Suppose B represents the positive integers such

that there is a one-to-one correspondence function f(x)=x from Z^+ to B. And suppose A represents the positive even integers such that f(k)=t where t=2k k$\in$ Z^+. B can be listed as b_1=1, b_2=2, b_3=3, ... , b_n=n, ... and A can be listed as a_1=2, a_2=4, a_3=6, ... , a_n=2n, ... So we say that any countable subset of a countable set is countable. So, if B is countable and A$\subseteq$B, then A is countable.

Answer 5

Answer 6